HYDROLOGICAL PROCESSES
Hydrol. Process. **23**, 3382–3390 (2009)
Published online 16 September 2009 in Wiley InterScience
(www.interscience.wiley.com) DOI: 10.1002/hyp.7449

Radial groundwater flow to a finite diameter well in a leaky confined aquifer with a finite-thickness skin

Shaw-Yang Yang[1] and Hund-Der Yeh[2]*

[1] *Department of Civil Engineering, Vanung University, Chungli, Taiwan*
[2] *Institute of Environmental Engineering, National Chiao Tung University, Hsinchu, Taiwan*

Abstract:

A mathematical model that describes the drawdown due to constant pumpage from a finite radius well in a two-zone leaky confined aquifer system is presented. The aquifer system is overlain by an aquitard and underlain by an impermeable formation. A skin zone of constant thickness exists around the wellbore. A general solution to a two-zone leaky confined aquifer system in Laplace domain is developed and inverted numerically to the time-domain solution using the modified Crump (1976) algorithm. The results show that the drawdown distribution is significantly influenced by the properties and thickness of the skin zone and aquitard. The sensitivity analyses of parameters of the aquifer and aquitard are performed to illustrate their effects on drawdowns in a two-zone leaky confined aquifer system. For the negative-skin case, the drawdown is very sensitive to the relative change in the formation transmissivity. For the positive-skin case, the drawdown is also sensitive to the relative changes in the skin thickness, and both the skin and formation transmissivities over the entire pumping period and the well radius and formation storage coefficient at early pumping time. Copyright © 2009 John Wiley & Sons, Ltd.

KEY WORDS groundwater; skin effect; Laplace transforms; leaky confined aquifer; sensitivity analysis

Received 27 November 2007; Accepted 17 July 2009

INTRODUCTION

Hantush and Jacob (1955) presented the non-steady drawdown solution in a leaky confined aquifer due to constant well discharge. They assumed that the rate of leakage from an aquitard is proportional to the drawdown at any point and the storage of the aquitard is negligible. In addition, the early-time and late-time approximate solutions were also developed by Hantush and Jacob (1955). Hantush (1960) dealt with a flow system in which the effect of storage in the semi-pervious layers was taken into consideration. The solutions to the boundary-value problems were obtained using the Hankel and Laplace transforms. The investigations of Hantush and Jacob (1955) and Hantush (1960) did not consider the effect of well radius and hence might not accurately describe the early-time drawdown response. Cheng and Morohunfola (1993) presented an analytical drawdown solution for the problem of radially convergent flow towards a well pumping at a constant rate in a multi-layered leaky aquifer system. They modelled the aquifer system based on the methodology of Neuman and Witherspoon (1969) and Herrera (1970), and utilized a numerical inversion algorithm to evaluate the drawdowns in associated layers.

Sekhar *et al*. (1994) presented a procedure for the determination of flow parameters in an anisotropic aquifer in which the direction of principal axes is unknown. They used a modified parameter perturbation technique to determine the sensitivity coefficients. Shan *et al*. (1995) studied the problem of saturated water flow to a single pumping test well in an aquifer-fault-aquifer system. They developed analytical solutions and presented methods in determining the fault transmissivity from pumping test data. Zlotnik (2004) introduced a new concept of maximum stream depletion rate (MSDR), defined as a maximum fraction of pumping rate contributed by the stream depletion. The MSDR was determined from the aquifer hydro-stratigraphic conditions, geometry of recharge and discharge zones, and locations of pumping wells. Yeh and Huang (2005) employed the extended Kalman filter to determine aquifer parameters in leaky aquifer systems with and without considering storage effect in the aquitard. Copty *et al*. (2006) assessed the effect of leakage on equivalent transmissivity for a steady-state radial flow in heterogeneous leaky aquifers. Zhan and Bian (2006) provided the analytical and semi-analytical solutions for use in calculating leakage rate and volume in a leaky confined aquifer bounded by a relatively thin aquitard. Yeh *et al*. (2007) developed a novel approach based on global optimization methods such as simulated annealing or a genetic algorithm to determine the best-fit aquifer parameters for leaky aquifer systems. Hunt and Scott (2007) obtained an approximate solution for the aquifer–aquitard–aquifer problem from numerical inversions of exact analytical solutions for Laplace transforms and reduced it to a well-known aquifer–aquitard problem. Li (2007a) presented a new analytical solution to investigate the aquifer horizontal movement driven by hydraulic forces. His solution described the aquifer

* Correspondence to: Hund-Der Yeh, Institute of Environmental Engineering, National Chiao Tung University, Hsinchu, Taiwan.
E-mail: hdyeh@mail.nctu.edu.tw

radial transient movement caused by well discharge and recharge in a leaky confined aquifer. In addition, Li (2007b) also developed new analytical solutions in the velocity and cumulative displacement fields describing transient radial movement of an unconfined leaky aquifer. He indicated that the large leakage is important in slowing radial movement and reducing aquifer deformation. Li and Neuman (2007) interpreted the pumping test in the Oxnard basin by coupling the Neuman solution (Neuman, 1968) with a numerical inversion algorithm in a five-layer system. Trinchero *et al*. (2008) developed a double inflection point (DIP) method for the interpretation of pumping tests in the leaky aquifers. Their DIP method does not involve any curve fitting, requiring the estimation of the position of three points on the time-drawdown curve instead.

A skin is usually developed near the wellbore due to an extensive well development or the intrusion of drilling mud into the adjacent formation during well construction. A positive wellbore skin (also called positive skin or low-conductivity skin) has a lower permeability than that of the original formation. In contrast, a disturbed formation with a higher permeability near the bore well is referred to as a negative bore well skin (also called negative skin or high-conductivity skin). Novakowski (1989) mentioned that the thickness of the skin zone might range from a few millimetres to several metres and thus should be considered in the pumping-test data analysis. The effect of wellbore skin on the results of pumping tests had been investigated by Barker and Herbert (1982) without considering the well radius effect and by Novakowski (1989), Novakowski (1990), and Yeh *et al*. (2003) accounting for the well radius effect. However, these papers did not consider the leaky condition in a two-zone confined aquifer system. Moench (1985) developed the conceptual models combining the Hantush theory of the storage in the aquitard (Hantush, 1960) with the Papadopulos and Cooper theory of a large-diameter well (Papadopulos and Cooper, 1967). The solution for the dimensionless drawdown in the comprising aquifer and aquitard due to pumpage was provided in Laplace domain and inverted numerically. However, his model treated the skin effect as a factor of head loss and therefore, the skin thickness was neglected.

The drawdown solution, which accounts for the effects of the skin zone, finite radius well, and storage in the aquitard, has not been developed before for the case of a leaky confined aquifer. The purpose of this paper is to present a mathematical model that describes the drawdown due to constant pumpage from a finite radiuswell in a two-zone leaky confined aquifer accounting for the effects of storage of the aquitard and the finite thickness skin. The drawdown solutions in the skin and formation zones are developed in Laplace domain and evaluated to the time-domain solutions by a numerical inversion algorithm. Hypothetical leaky aquifer systems are used to illustrate the effects of the skin zone and leakage on drawdown distribution in a two-zone leaky confined aquifer system. In addition, the sensitivity analysis is performed to investigate the aquifer drawdown in response to the relative changes of parameters of the aquifer and aquitard.

THEORY

A schematic cross-section of an idealized leaky confined aquifer system is depicted in Figure 1. The aquifer of constant thickness is overlain by an aquitard and underlain by an impermeable formation. A skin zone of finite thickness is assumed to exist around the wellbore. The pumping well penetrates the entire thickness of the aquifer and the pumping rate is maintained constant. In this study, the assumptions made for the conceptual model are:

1. The upper aquifer is highly permeable with an infinite amount of water supply such that it maintains a constant head at any time.
2. The formation zone is homogeneous, isotropic, of a constant thickness, and infinite in radial extent.
3. The skin zone is also homogeneous, isotropic, and of a constant thickness around the wellbore.
4. The flow direction is vertical in the aquitard and horizontal in the confined aquifer.

Mathematical model

Based on the above assumptions, the governing equation describing the drawdown distribution, $s(r, t)$, for the skin and formation zones are, respectively,

$$\frac{\partial^2 s_1(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial s_1(r,t)}{\partial r} + \frac{q'}{T_1} = \frac{S_1}{T_1}\frac{\partial s_1(r,t)}{\partial t}, \; r_w \le r \le r_1 \tag{1}$$

and

$$\frac{\partial^2 s_2(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial s_2(r,t)}{\partial r} + \frac{q'}{T_2} = \frac{S_2}{T_2}\frac{\partial s_2(r,t)}{\partial t}, \; r_1 \le r < \infty \tag{2}$$

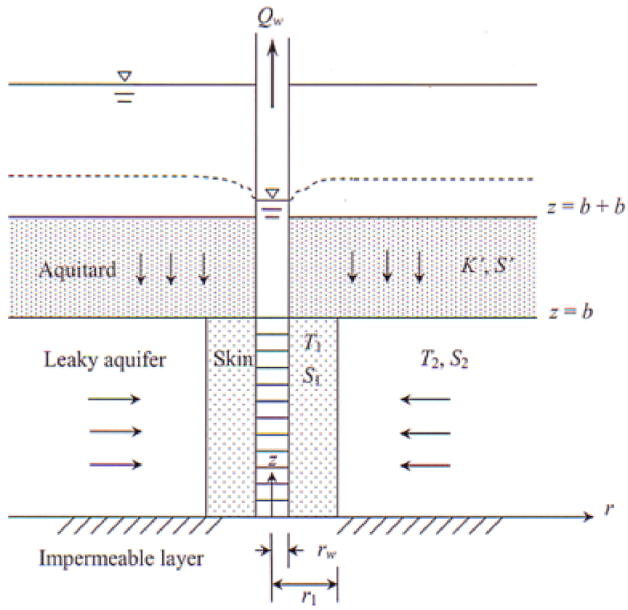

Figure 1. The schematic cross-sectional diagram of an idealized two-zone leaky confined aquifer system

where subscripts 1 and 2 respectively denote the skin and formation zones; q' is the leakage of the aquitard; T is the transmissivity; S is the storage coefficient; r is the radial distance from the centerline of pumping well; r_w is the radius of pumping well; r_1 is the radial distance from the centerline of the well to the outer skin envelope; and t is the time from the start of the pumping test.

The drawdown of the aquifer is initially zero within the skin and formation zones. Thus, the initial conditions for Equations (1) and (2) are written as

$$s_1(r, 0) = s_2(r, 0) = 0 \tag{3}$$

As r approaches infinity, the drawdown in the formation zone tends to be zero. Therefore, the outer boundary condition at an infinite distance is specified as

$$s_2(\infty, t) = 0 \tag{4}$$

Applying the Darcy law, the boundary condition for a constant flow rate across the screen is assumed to be uniform and expressed as

$$Q_w = -2\pi T_1 r \frac{\partial s_1(r, t)}{\partial r}, \quad r = r_w \tag{5}$$

where Q_w is a constant pumping rate.

The continuities of the drawdown and the flux between the skin and formation zones, respectively, require that

$$s_1(r_1, t) = s_2(r_1, t) \tag{6}$$

and

$$T_1 \frac{\partial s_1(r_1, t)}{\partial r} = T_2 \frac{\partial s_2(r_1, t)}{\partial r} \tag{7}$$

Aquitard flow

Considering the effect of the aquitard storage, the governing equation describing drawdown distribution within the aquitard is

$$b'K' \frac{\partial^2 s'(z, t)}{\partial z^2} = S' \frac{\partial s'(z, t)}{\partial t} \tag{8}$$

where $s'(z, t)$ is the drawdown of the aquitard; z is the vertical distance from the lower impermeable layer; b' is the thickness of the aquitard; and K' and S' are the hydraulic conductivity and storage coefficient of the aquitard, respectively.

The drawdown in the aquitard is initially assumed zero and expressed as

$$s'(z, 0) = 0 \tag{9}$$

The boundary condition at the interface between the aquitard and the lower aquifer is

$$s'(z, t) = s_1(r, t) = s_2(r, t), \quad z = b \tag{10}$$

In addition, the boundary condition on the top of the aquitard is

$$s'(z, t) = 0, \quad z = b + b' \tag{11}$$

Applying Laplace transforms to Equations (8), (10) and (11) leads to

$$\frac{d^2\overline{s}'(z, p)}{dz^2} = \alpha'^2 \overline{s}'(z, p), \quad \alpha'^2 = \frac{pS'}{b'K'} \tag{12}$$

$$\overline{s}'(z, p) = \overline{s}_1(r, p) = \overline{s}_2(r, p) = \overline{s}(r, p), \quad for\ z = b \tag{13}$$

and

$$\overline{s}'(z, p) = 0, \quad for\ z = b + b' \tag{14}$$

The general solution to Equation (12) is

$$\overline{s}'(z, p) = C_1 \sinh(\alpha' z) + C_2 \cosh(\alpha' z) \tag{15}$$

where C_1 and C_2 are undetermined constants.

Substituting Equation(15) into Equations (13) and (14), one obtains

$$C_1 = -\frac{\cosh(\alpha'(b + b'))}{\sinh(\alpha' b')} \overline{s}_i(r, p) \tag{16}$$

and

$$C_2 = \frac{\sinh(\alpha'(b + b'))}{\sinh(\alpha' b')} \overline{s}_i(r, p) \tag{17}$$

where $\overline{s}_i(r, p)$ is the drawdown within the skin for $i = 1$ and within the formation for $i = 2$. The solution for drawdown within the aquitard can be obtained by substituting Equations (16) and (17) into Equation (15) after some manipulations as

$$\overline{s}'(z, p) = \frac{\sinh(\alpha'(b + b' - z))}{\sinh(\alpha' b')} \overline{s}_i(r, p) \tag{18}$$

Based on the mass conservation, the leakage of the aquitard is

$$\overline{q}' = K' \left. \frac{d\overline{s}'(z, p)}{dz} \right|_{z=b} \tag{19}$$

Substituting Equation (18) into Equation (19), the leakage to the confined aquifer is

$$\overline{q}' = -K'\alpha' \coth(\alpha' b') \overline{s}_i(r, p) \tag{20}$$

Drawdown of leaky aquifer

The solutions to Equations (1) and (2) with respect to boundary conditions Equations (4)–(7) can be found using Laplace transform method. The detailed derivation for the Laplace-domain solutions is given in Appendix A and the drawdown solutions within the skin and formation zones are respectively

$$\overline{s}_1(r, p) = \left(\frac{Q_w}{2\pi r_w T_1} \right) \left(\frac{-\phi_1 I_0(\alpha_1 r) + \phi_2 K_0(\alpha_1 r)}{\alpha_1 p[\phi_1 I_1(\alpha_1 r_w) + \phi_2 K_1(\alpha_1 r_w)]} \right) \tag{21}$$

and

$$\overline{s}_2(r, p) = \left(\frac{Q_w}{2\pi r_w r_1} \right) \left(\frac{K_0(\alpha_2 r)}{\alpha_1 p[\phi_1 I_1(\alpha_1 r_w) + \phi_2 K_1(\alpha_1 r_w)]} \right) \tag{22}$$

with

$$\phi_1 = T_2\alpha_2 K_0(\alpha_1 r_1)K_1(\alpha_2 r_1) - T_1\alpha_1 K_1(\alpha_1 r_1)K_0(\alpha_2 r_1) \quad (23)$$

and

$$\phi_2 = T_2\alpha_2 I_0(\alpha_1 r_1)K_1(\alpha_2 r_1) + T_1\alpha_1 I_1(\alpha_1 r_1)K_0(\alpha_2 r_1) \quad (24)$$

where p is the Laplace variable; $\bar{s}(r, p)$ is the transformed drawdown; $\alpha_1^2 = (S_1/T_1)p + (K'\alpha'/T_1)\coth(\alpha' b')$; $\alpha_2^2 = (S_2/T_2)p + (K'\alpha'/T_2)\coth(\alpha' b')$; $I_0(.)$ and $I_1(.)$ are the modified Bessel functions of the first kind of order zero and one, respectively; and $K_0(.)$ and $K_1(.)$ are the modified Bessel functions of the second kind of order zero and one, respectively.

Dimensionless solutions

Define dimensionless variables as following:

$$t_D = \frac{T_2 t}{S_2 r_w^2},\ r_D = \frac{r}{r_w},\ r_{1D} = \frac{r_1}{r_w},$$
$$b'_D = \frac{b'}{r_w},\ T_D = \frac{T_1}{T_2}$$
$$T'_D = \frac{K'b'}{T_2},\ S_D = \frac{S_1}{S_2},\ S'_D = \frac{S'}{S_2},$$
$$\bar{s}_{1D} = \frac{2\pi T_2}{Q_w}\bar{s}_1,\ \bar{s}_{2D} = \frac{2\pi T_2}{Q_w}\bar{s}_2 \quad (25)$$

Using the above-defined dimensionless variables, Equations (21) and (22) can be expressed in dimensionless forms as

$$\bar{s}_{1D}(r_D, p) = \left(\frac{1}{T_D\alpha_{1D}}\right) \left(\frac{-\phi_{1D}I_0(\alpha_{1D}r_D) + \phi_{2D}K_0(\alpha_{1D}r_D)}{p[\phi_{1D}I_1(\alpha_{1D}) + \phi_{2D}K_1(\alpha_{1D})]}\right) \quad (26)$$

and

$$\bar{s}_{2D}(r_D, p) = \left(\frac{1}{\alpha_{1D}r_{1D}}\right) \left(\frac{K_0(\alpha_{2D}r_D)}{p[\phi_{1D}I_1(\alpha_{1D}) + \phi_{2D}K_1(\alpha_{1D})]}\right) \quad (27)$$

with

$$\phi_{1D} = \alpha_{2D}K_0(\alpha_{1D}r_{1D})K_1(\alpha_{2D}r_{1D}) - T_D\alpha_{1D}K_1(\alpha_{1D}r_{1D})K_0(\alpha_{2D}r_{1D}) \quad (28)$$

and

$$\phi_{2D} = \alpha_{2D}I_0(\alpha_{1D}r_{1D})K_1(\alpha_{2D}r_{1D}) + T_D\alpha_{1D}I_1(\alpha_{1D}r_{1D})K_0(\alpha_{2D}r_{1D}) \quad (29)$$

where $\alpha_D'^2 = (S'_D/T'_D)\mathrm{p}$, $\alpha_{1D}^2 = (S_D/T_D)p + (T'_D\alpha'_D/T_D b'_D)\coth(\alpha'_D b'_D)$, and $\alpha_{2D}^2 = p + (T'_D\alpha'_D/b'_D)\coth(\alpha'_D b'_D)$.

SIMPLIFIED SOLUTIONS

Solution without considering skin effect

The aquifer formation is a single-layer system if the skin zone is absent. Under this condition, the aquifer properties $T_1 = T_2 = T$ and $S_1 = S_2 = S$ and the variables $\phi_1 = 0$, $\phi_2 = T/r_1$, and $\alpha_1 = \alpha_2 = \alpha$. Then both Equations (21) and (22) reduce to

$$\bar{s}(r, p) = \left(\frac{Q_w}{2\pi r_w T}\right)\left(\frac{K_0(\alpha r)}{\alpha p K_1(\alpha r_w)}\right),$$
$$\alpha = \sqrt{(S/T)p + (K'\alpha'/T)\coth(\alpha' b')} \quad (30)$$

If the well radius is negligible (i.e. $r_w \to 0$), the modified Bessel function $K_1(\alpha r_w)$ approaches $1/(\alpha r_w)$ and Equation (30) reduces to

$$\bar{s}(r, p) = \left(\frac{Q_w}{2\pi T}\right)\left(\frac{K_0(\alpha r)}{p}\right) \quad (31)$$

which is the drawdown solution in Laplace domain given by Hantush and Jacob (1955) for a leaky confined aquifer under the assumption of an infinitesimal radius well.

Solution without considering leakage effect

If the aquitard is impervious (i.e. $K' = 0$), then one can write $\alpha_1^2 = (S_1/T_1)p$ and $\alpha_2^2 = (S_2/T_2)p$ in Equations (21) and (22), which, consequently, lead these two solutions to those presented in Yeh *et al.* (2003) for the two-zone nonleaky aquifer case. After neglecting the well radius ($r_w \to 0$), the solutions of Yeh *et al.* (2003) reduce respectively to

$$\bar{s}_1(r, p) = \left(\frac{Q_w}{2\pi T_1}\right)\left(\frac{-\phi_1 I_0(\alpha_1 r) + \phi_2 K_0(\alpha_1 r)}{p\phi_2}\right) \quad (32)$$

and

$$\bar{s}_2(r, p) = \left(\frac{Q_w}{2\pi T_1}\right)\left(\frac{T_1 K_0(\alpha_2 r)}{r_1 p\phi_2}\right) \quad (33)$$

Note that Equations (32) and (33) were given in different forms by Barker and Herbert (1982).

As the aquitard is impervious and the skin zone is absent, both Equations (21) and (22) reduce to

$$\bar{s}(r, p) = \left(\frac{Q_w}{2\pi r_w T}\right)\left(\frac{K_0(\alpha r)}{\alpha p K_1(\alpha r_w)}\right), \quad \alpha = \sqrt{(S/T)p} \quad (34)$$

which is the drawdown equation in Laplace domain for a single-layer confined aquifer.

NUMERICAL INVERSION OF THE SOLUTIONS

The Laplace-domain solutions to Equations (21) and (22) for drawdowns consist of the products of the Bessel functions. These Bessel functions can be approximated by the formulas given in Watson (1958) and Abramowitz and Stegun (1964). The application of the Shanks method (Shanks, 1955; Wynn, 1956) will be computational efficient to numerically evaluate the Bessel functions. Similar approximations of the Bessel functions can be found in the works of Yeh and Yang (2006) and Yang and Yeh (2007).

The analytical inversion of the Laplace-domain solutions Equations (21) and (22) may not be possible. Therefore, the method of numerical Laplace inversion such as the Crump (1976) algorithm is employed. The routine INLAP of IMSL (2003), developed on the basis of the Crump (1976) algorithm, can be used to evaluate the time-domain solutions to Equations (21) and (22) with accuracy to the fourth decimal place in comparison to the analytical inversion. This routine had also been successfully applied to the groundwater problems mentioned in the study by Yang and Yeh (2005) or Yang *et al.* (2006).

RESULTS AND DISCUSSION

Effect of leakage

Several examples with hypothetical data are used to illustrate the effects of the skin and leakage in a two-zone leaky confined aquifer system. The parameters of the formation zone are $T_2 = 10^{-3}$ m^2 s^{-1}, $S_2 = 10^{-3}$, and the thickness of the confined aquifer $b = 30$ m. The storage coefficient of the aquitard is $S' = 10^{-3}$. The well radius r_w is 0·05 m and the pumpage is maintained constant at $Q = 10^{-3}$ m^3 s^{-1}.

Figure 2 illustrates the effects of the conductivity and thickness of the aquitard on drawdowns in a pumping well for the positive-skin case. Figure 2a displays the time-drawdown curves for $T_1 = 10^{-4}$ m^2 s^{-1}, $r_1 =$ 0·50 m, $b' = 5$ m, and $K' = 0$ (no leakage), 10^{-8}, 10^{-7} or 10^{-6} m s^{-1} when the time ranges from 10^{-1} to 10^6 s. Figure 2b displays the time-drawdown curves for $T_1 =$ 10^{-4} m^2 s^{-1}, $r_1 = 0{\cdot}50$ m, $K' = 10^{-7}$ m s^{-1}, and $b' =$ 5, 10 or 20 m for the same time. The time-drawdown curves shown in Figure 2a are the same at early pumping time, and the drawdown increases with the decrease of K' at a later pumping time (say, $t \geq 2 \times 10^3$ s). The drawdown in a leaky aquifer is less than that in a non-leaky aquifer after $t \geq 2 \times 10^3$ s. Furthermore, the aquifer without having the skin and leakage produces the least drawdown among these five drawdown curves for the positive-skin case. The drawdown tends to be stabilized when the pumping time is larger than 10^5 s (1·157 day). Figure 2b also shows that the drawdowns are the same at early pumping time and increase with b' at the later pumping time. These results indicate that a smaller K' and/or a larger b' results in a larger drawdown for the positive-skin case. In addition, the influence of K' and b' is slightly more profound on drawdown at later time.

Figure 3 displays the distance-drawdown curves in a leaky confined aquifer with $r = 0{\cdot}05$ m, $b' = 5$ m, $T_1 =$ 10^{-4} m^2 s^{-1}, and $r_1 = 0{\cdot}50$ m for $K' = 0$ (no leakage), 10^{-8}, 10^{-7} or 10^{-6} m s^{-1}. The figure shows that the drawdown decreases with time and radial distance. In addition, the drawdown decreases rapidly within the skin zone and slowly within the formation zone. The drawdowns in the skin and formation zones near the interface behave differently. The drawdown is larger within the skin zone and smaller within the formation zone when compared with that of the aquifer without skin and leakage. The figure also indicates that a larger K' results in a smaller drawdown. Figure 4 also displays the distance-drawdown curves in a leaky confined aquifer for $K' = 0$ (no leakage) or 10^{-7} m s^{-1}, $r = 0{\cdot}05$ m, $T_1 = 10^{-4}$ m^2 s^{-1}, and $r_1 = 0{\cdot}50$ m when $b' = 5$, 10, or 20 m. For the positive-skin case, the drawdown decreases rapidly within the skin zone and slowly within the formation zone. In addition, a larger b' leads to a larger drawdown, especially near the wellbore area.

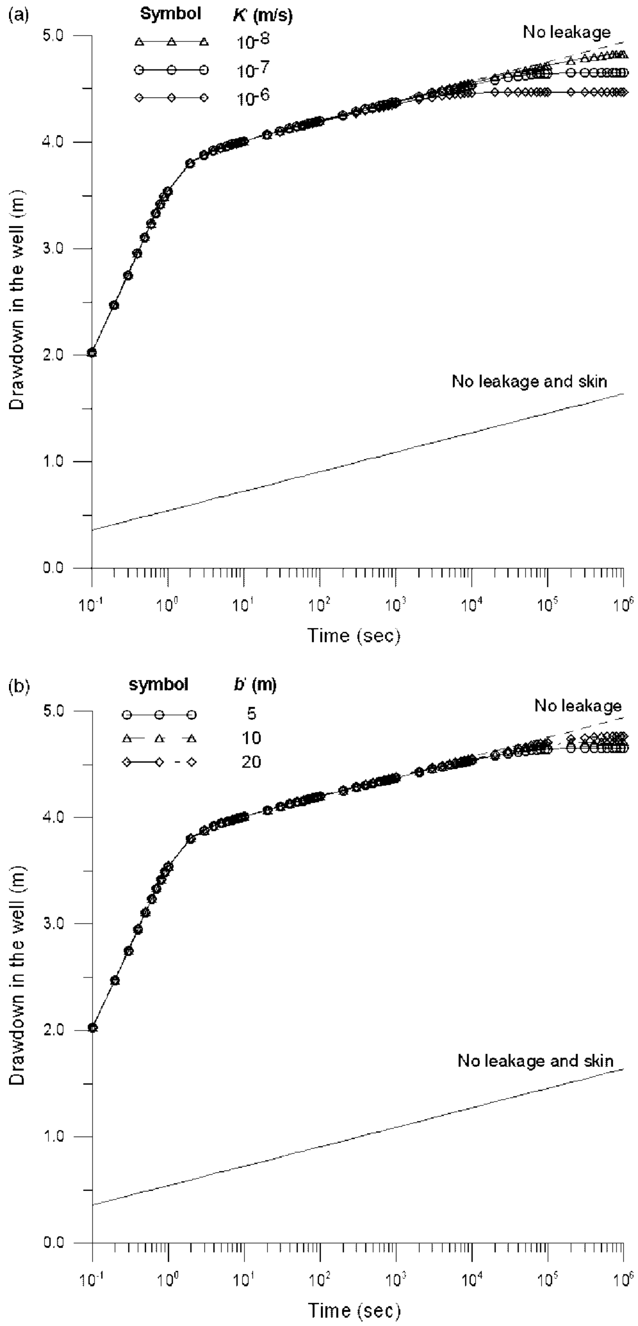

Figure 2. The time-drawdown curves with $r = 0{\cdot}05$ m, $T_1 = 10^{-4}$ m^2 s^{-1}, and $r_1 = 0{\cdot}50$ m for (a) $b' = 5$ m and various $K' = 0$ (no leakage), 10^{-8}, 10^{-7}, or 10^{-6} m s^{-1} and (b) $K' = 10^{-7}$ m s^{-1} and various $b' = 5$, 10, or 20 m

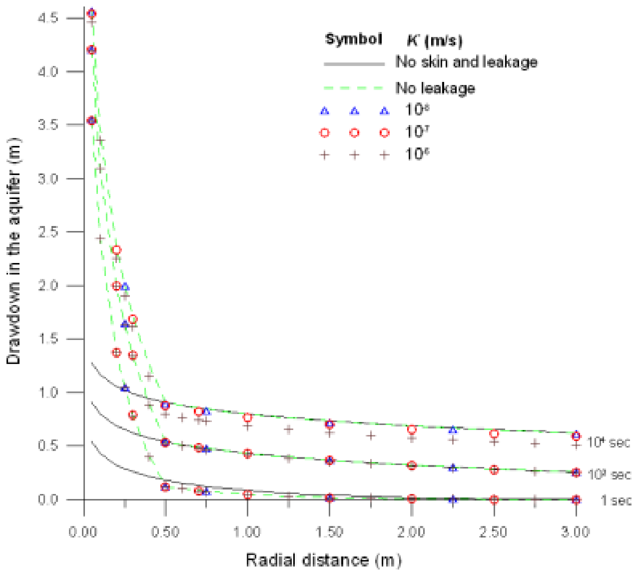

Figure 3. The distance-drawdown curves for $r = 0{\cdot}05$ m, $T_1 = 10^{-4}$ m^2 s^{-1}, $r_1 = 0{\cdot}50$ m, $b' = 5$ m and various $K' = 0$ (no leakage), 10^{-8}, 10^{-7}, or 10^{-6} m s^{-1}

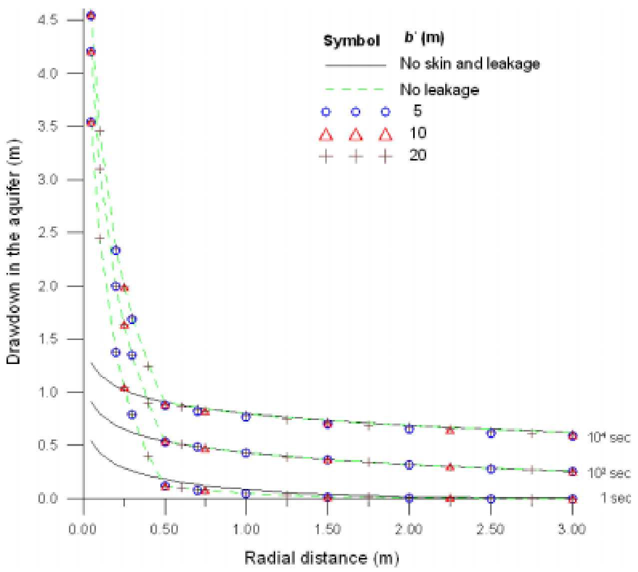

Figure 4. The distance-drawdown curves for $r = 0{\cdot}05$ m, $T_1 = 10^{-4}$ m^2 s^{-1}, $r_1 = 0{\cdot}50$ m, $K' = 10^{-7}$ m s^{-1} and various $b' = 5$, 10, or 20 m

Effect of wellbore skin

Figure 5 shows the time-drawdown curves for $r_w = 0{\cdot}05$ m in the aquifer with the skin zone of $T_1 = 10^{-4}$, 10^{-3} (no skin) or 10^{-2} m^2 s^{-1} and $r_1 = 0{\cdot}25$ or $0{\cdot}50$ m and the aquitard of $K' = 0$ (no leakage) or 10^{-7} m s^{-1} and $b' = 5$ m when the time ranges from 10^{-1} to 10^6 s. Note that the case of $K' = 0$ corresponds to the solution of a non-leaky confined aquifer system and the case of $K' = 0$ and $T_1 = T_2 = 10^{-3}$ m^2 s^{-1} corresponds to the solution for a single-layer non-leaky confined aquifer system. In this figure, the time-drawdown curves for the aquifer with a positive skin when $T_1 = 10^{-4}$ m^2 s^{-1}, without skin when $T_1 = 10^{-3}$ m^2 s^{-1}, and with a negative skin when $T_1 = 10^{-2}$ m^2 s^{-1}. The figure shows that the aquifer with a

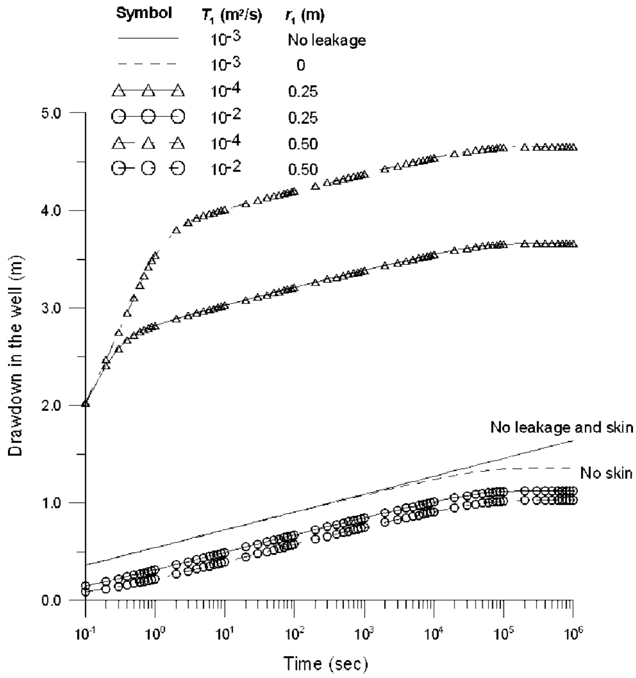

Figure 5. The drawdown-time curves within a pumping well ($r_w = 0{\cdot}05$ m) for $K' = 0$ (no leakage) and 10^{-7} m s^{-1}, $b' = 5$ m, $r_1 = 0{\cdot}25$ and 0·50 m, and $T_1 = 10^{-4}$ in the positive-skin case, 10^{-3} in the no skin case or 10^{-2} m^2 s^{-1} in the negative-skin case

positive skin produces the largest drawdown, the no skin is the second, and the negative skin yields the smallest drawdown. In addition, the difference in drawdown between the aquifer with a positive skin and a single-layer aquifer is larger than that between the aquifer with a negative skin and a single-layer aquifer. It also shows that a larger r_1 has a larger drawdown for the positive-skin case and a smaller drawdown for the negative-skin case. This indicates that the effect of a positive skin on drawdown is larger than that of a negative skin. The results demonstrate that the drawdown is significantly influenced by a positive skin than by a negative skin. In addition, a thicker skin zone has a more profound effect on drawdown. The time-drawdown curves also show that the drawdown of a non-leaky curve is proportional to natural logarithm of time.

Sensitivity analysis

Sensitivity analysis is a technique to assess the effects of uncertainty in input parameters on the model result. This method is helpful in assessing how a model responds to the change in certain parameters. The sensitivity of a dependent variable in response to the change in a parameter is defined as (Liou and Yeh, 1997)

$$X_{i,j} = \frac{\partial V_i}{\partial P_j} \tag{35}$$

where $X_{i,j}$ is the sensitivity coefficient of the jth parameter (P_j) at the ith time and V_i is the dependent variable of the model, e.g. the drawdown distribution. Huang and Yeh (2007) provided a normalized sensitivity to assess the effect of relative changes in parameters on dependent variable. The normalized sensitivity of a dependent

DOI: 10.1002/hyp

variable to the relative change in a given parameter is defined as

$$X'_{i,j} = P_j \frac{\partial V_i}{\partial P_j} \quad (36)$$

where $X'_{i,j}$ is the normalized sensitivity of the jth parameter (P_j) at the ith time. The partial derivative in Equation (36) may be laborious to evaluate. A finite-difference formula may be used to approximate the differentiation (Yeh, 1987). That is

$$\frac{\partial V_i}{\partial P_j} = \frac{V_i(P_j + \Delta P_j) - V_i(P_j)}{\Delta P_j} \quad (37)$$

where ΔP_j is a small increment chosen as $10^{-2} \times P_j$.

The sensitivity analyses of parameters of a two-zone leaky confined aquifer system are performed using the hypothetic data. The aquifer has $b = 30$ m, $T_2 = 10^{-3}$ m^2 s^{-1}, and $S_2 = 10^{-3}$ for the formation zone while the aquitard has $b' = 5$ m, $K' = 10^{-7}$ m s^{-1}, and $S' = 10^{-3}$. The well radius r_w is 0·05 m and the pumping rate is maintained constant at $Q = 10^{-3}$ m^3 s^{-1}. For a skin zone with the storage $S_1 = 10^{-3}$, two cases are considered; that is, $T_1 = 10^{-2}$ m^2 s^{-1} for the negative-skin case and $T_1 = 10^{-4}$ m^2 s^{-1} for the positive-skin case. Figure 6a and b plots the time-drawdown curves and the normalized sensitivities of the parameters K', S', b', T_1, S_1, r_1, T_2, S_2, and r_w. Figure 6a shows that a relative change in b' produces a minor positive effect on drawdown and the other parameters produce negative effects on drawdown for the negative-skin case with $T_1 = 10^{-2}$ m^2 s^{-1}. The normalized sensitivity of drawdown with respect to b' starts with a slight increase after 10^3 s and reaches a constant value of 0·08 m after 10^5 s. In contrast, the normalized sensitivity of drawdown with respect to K' produces a similar pattern but has a negative effect. A relative change in r_1 has a negative effect on drawdown and the sensitivity curve shows a slight increase with time at early time and stabilized at a value of $-0{\cdot}14$ m after $t \geq 3$ s. The normalized sensitivity of drawdown with respect to T_2 decreases with time and approaches a constant value of $-0{\cdot}90$ m when $t \geq 10^5$ s. Furthermore, the normalized sensitivities of drawdown with respect to K', b', r_1, and T_2 maintain constant values of $-0{\cdot}08$, 0·08, $-0{\cdot}14$, and $-0{\cdot}90$ m, respectively, at a later time (say, $t \geq 10^5$ s) because the leaky confined aquifer system reaches a steady-state condition. The figure also shows that the drawdown in response to the change of T_2 produces the largest normalized sensitivity in the magnitude, the parameter r_1 is the second, and the other parameters such as S', T_1, S_1, and S_2, r_w are relatively less in the analyses. Those results indicate that the drawdowns are very sensitive to the change in T_2, slightly less sensitive to the change in r_1, and the parameters b' and K' give minor influences on drawdown at a later time in the negative-skin case. Figure 6b plots the time-drawdown curves and the normalized sensitivities for the positive-skin case with $T_1 = 10^{-4}$ m^2 s^{-1}. The figure shows that the normalized sensitivities of drawdown with respect to

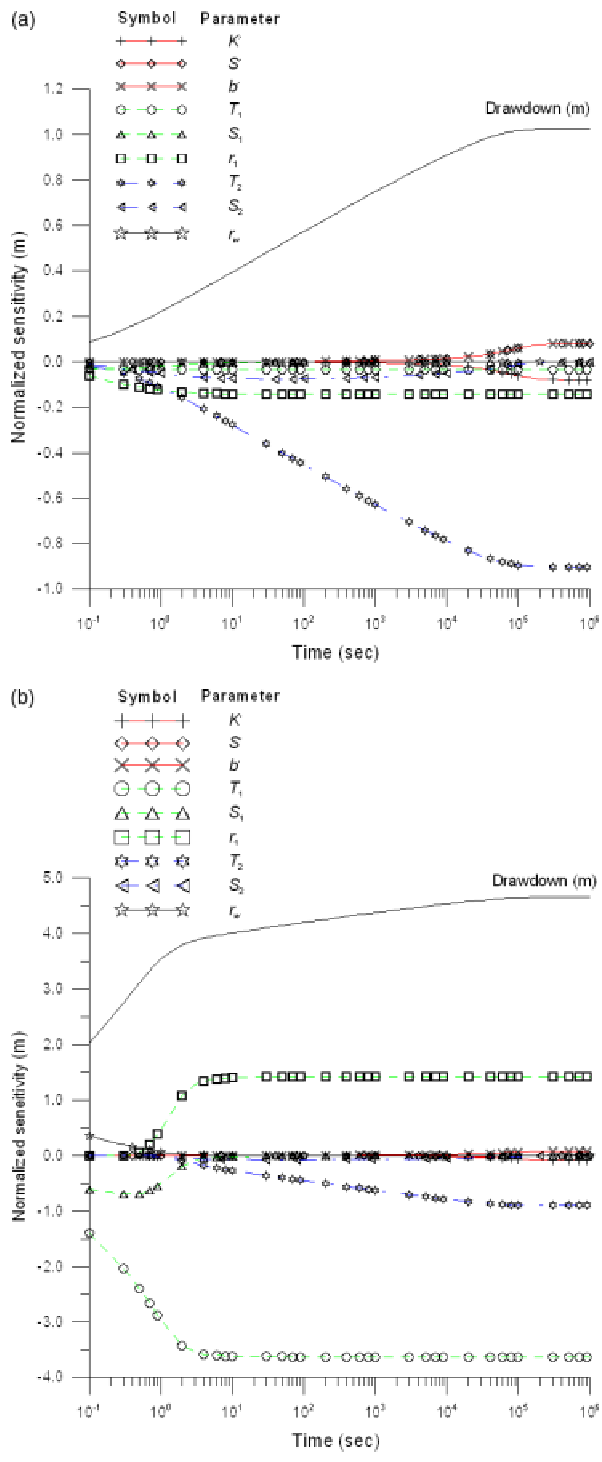

Figure 6. Plots of the time-drawdown curve and the normalized sensitivities of the parameters K', S', b', T_1, S_1, r_1, T_2, S_2, and r_w versus time for (a) $T_1 = 10^{-2}$ m^2 s^{-1} in the negative-skin case and (b) $T_1 = 10^{-4}$ m^2 s^{-1} in the positive-skin case

b', r_1, and r_w are positive and those with respect to other parameters are negative. The normalized sensitivity to the relative change in r_1 starts to increase after about 0·5 s and is stabilized at a value of 1·4 m after about 10 s. The normalized sensitivity of drawdown in response to the change in T_1 decreases with time and keeps a constant value of $-3{\cdot}6$ m after about 5 s. The normalized sensitivity to the relative change in T_2 becomes larger

DOI: 10.1002/hyp

after 1 s and approaches a constant value (−0·9 m) when $t \geq 10^5$ s. The normalized sensitivity of drawdown with respect to r_w is relatively small (say 0·36) at $t = 0{\cdot}1$ s and then decreases with time and approaches zero when $t \geq 600$ s. This figure also shows that the normalized sensitivity to the relative change in S_2 produces a negative effect, gradually decreases in magnitude with time from 1 s, and approaches zero at about 5 s. It indicates that the effect of the formation storage on drawdown occurs only at early time. Furthermore, the relative changes in b', K', S', and S_1 yield very small effects on drawdown. The influences of relative changes in parameters K' and b' on drawdown become profound at a later time. In short, the results of sensitivity analyses indicate that the effects of relative changes in T_1, T_2, and r_1 on drawdown are very significant and the parameters r_w and S_2 are fairly sensitivity only at an early time in the positive-skin case. In addition, the effect of relative change in r_1 on drawdown in the positive-skin case is much larger than those in the negative-skin case.

CONCLUSION

A mathematical model describing the drawdown distribution for a radial flow to a fully penetrating well of finite radius in a two-zone leaky confined aquifer is developed, accounting for the effect of the finite-thickness skin zone. The general solution to a two-zone leaky confined aquifer system in Laplace domain is developed and the time-domain solution is obtained by the modified Crump (1976) algorithm. The conclusions can be drawn as follows:

1. An aquitard with larger hydraulic conductivity and smaller thickness leads to a smaller drawdown in the positive-skin case. Both hydraulic conductivity and thickness of the aquitard influence the drawdown after a large pumping time in a two-zone leaky confined aquifer system.
2. The effect of relative change in formation transmissivity T_2 to the drawdown is significant for the negative-skin case. On the other hand, the effects of relative changes in the transmissivities of the skin and formation zones (i.e. T_1 and T_2) and the radial distance from the centerline of the well to the outer skin envelope (r_1) on the drawdown are significant for the positive-skin case. In addition, the drawdown in response to the relative change in the well radius (r_w) and the formation storage coefficient (S_2) is sensitive only at early time.
3. This solution predicts the drawdown distribution in a leaky confined aquifer system with the skin zone around the wellbore. It is useful in preliminary design for a constant-flux pumping system in a two-zone leaky confined aquifer.

ACKNOWLEDGEMENTS

Research leading to this paper has been partially supported by the grants from Taiwan National Science Council under the contract number NSC 96—2221—E—238—019 and NSC 96—2221—E—009—087-MY3. The authors would also like to thank two anonymous reviewers for their valuable and constructive comments.

APPENDIX A. DERIVATION OF THE LAPLACE-DOMAIN SOLUTIONS TO EQUATIONS (21) AND (22)

Applying Laplace transforms to Equations (1) and (2) with Equation (20) yields the following subsidiary equations, respectively,

$$\frac{d^2\bar{s}_1(r,p)}{dr^2} + \frac{1}{r}\frac{d\bar{s}_1(r,p)}{dr} = \alpha_1^2\bar{s}_1(r,p), \quad r_w \leq r \leq r_1 \tag{A1}$$

and

$$\frac{d^2\bar{s}_2(r,p)}{dr^2} + \frac{1}{r}\frac{d\bar{s}_2(r,p)}{dr} = \alpha_2^2\bar{s}_2(r,p), \quad r_1 \leq r < \infty \tag{A2}$$

where p is the Laplace transform variable corresponding to the time variable t; $\bar{s}(r,p)$ is the transformed drawdown; $\alpha_1^2 = (S_1/T_1)p + (K'\alpha'/T_1)\coth(\alpha' b')$; $\alpha_2^2 = (S_2/T_2)p + (K'\alpha'/T_2)\coth(\alpha' b')$; and $\alpha'^2 = pS'/(b'K')$.

The boundary conditions of Equations (4) and (5) in Laplace domain are

$$\bar{s}_2(\infty, p) = 0 \tag{A3}$$

and

$$\frac{Q_w}{p} = -2\pi T_1 r_w \frac{d\bar{s}_1(r_w,p)}{dr} \tag{A4}$$

Moreover, the continuity conditions of drawdown and flux between the skin and formation zones after applying Laplace transform yield

$$\bar{s}_1(r_1,p) = \bar{s}_2(r_1,p) \tag{A5}$$

and

$$T_1\frac{d\bar{s}_1(r_1,p)}{dr} = T_2\frac{d\bar{s}_2(r_1,p)}{dr} \tag{A6}$$

The general solutions to Equations (A1) and (A2) are

$$\bar{s}_1(r,p) = D_1 I_0(\alpha_1 r) + D_2 K_0(\alpha_1 r) \tag{A7}$$

and

$$\bar{s}_2(r,p) = D_3 I_0(\alpha_2 r) + D_4 K_0(\alpha_2 r) \tag{A8}$$

where D_1, D_2, D_3, and D_4 are undetermined constants.

Substituting Equations(A7) and (A8) into Equations(A3)–(A6), the undetermined constants can then be determined as

$$D_1 = \left(\frac{Q_w}{2\pi r_w T_1 \alpha_1 p}\right)\left(\frac{-\phi_1}{\phi_1 I_1(\alpha_1 r_w) + \phi_2 K_1(\alpha_1 r_w)}\right) \tag{A9}$$

$$D_2 = \left(\frac{Q_w}{2\pi r_w T_1 \alpha_1 p}\right)\left(\frac{\phi_2}{\phi_1 I_1(\alpha_1 r_w) + \phi_2 K_1(\alpha_1 r_w)}\right) \tag{A10}$$

$$D_3 = 0 \tag{A11}$$

and

$$D_4 = \left(\frac{Q_w}{2\pi r_w T_1 \alpha_1 p}\right) \left(\frac{-\phi_1 I_0(\alpha_1 r_1) + \phi_2 K_0(\alpha_1 r_1)}{(\phi_1 I_1(\alpha_1 r_w) + \phi_2 K_1(\alpha_1 r_w))K_0(\alpha_2 r_1)}\right) \quad \text{(A12)}$$

with

$$\phi_1 = T_2\alpha_2 K_0(\alpha_1 r_1)K_1(\alpha_2 r_1) - T_1\alpha_1 K_1(\alpha_1 r_1)K_0(\alpha_2 r_1) \quad \text{(A13)}$$

and

$$\phi_2 = T_2\alpha_2 I_0(\alpha_1 r_1)K_1(\alpha_2 r_1) + T_1\alpha_1 I_1(\alpha_1 r_1)K_0(\alpha_2 r_1) \quad \text{(A14)}$$

Consequently, the solutions of drawdowns within the skin and formation zones can then be respectively obtained by substituting the constants in Equations (A9) and (A10) into Equation (A7) and the constants in Equations (A11) and (A12) into Equation (A8). The final results are given as Equations (21) and (22) in the text.

REFERENCES

Abramowitz M, Stegun IA. 1964. *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables, National Bureau of Standards*. Dover: Washington, DC.

Barker JA, Herbert R. 1982. Pumping tests in patchy aquifers. *Ground Water* **20**(2): 150–155.

Cheng AH-D, Morohunfola OK. 1993. Multilayered leaky aquifer systems: 1. Pumping well solutions. *Water Resources Research* **29**(8): 2787–2800.

Copty NK, Sarioglu MS, Findikakis AN. 2006. Equivalent transmissivity of heterogeneous leaky aquifers for steady state radial flow. *Water Resources Research* **42**: W04416. DOI:10.1029/2005WR004673.

Crump KS. 1976. Numerical inversion of Laplace transforms using a Fourier series approximation. *Journal of the Association for Computing Machinery* **23**(1): 89–96.

Hantush MS. 1960. Modification of the theory of leaky aquifers. *Journal of Geophysical Research* **65**(11): 3713–3725.

Hantush MS, Jacob CE. 1955. Non-steady radial flow in an infinite leaky aquifer. *Transactions AGU* **36**(1): 95–100.

Herrera I. 1970. Theory of multiple leaky aquifers. *Water Resources Research* **6**: 185–193.

Huang YC, Yeh HD. 2007. The use of sensitivity analysis in on-line aquifer parameter estimation. *Journal Hydrology* **335**(3–4): 406–418.

Hunt B, Scott D. 2007. Flow to a well in a two-aquifer system. *Journal of Hydrologic Engineering* **12**(2): 146–155.

IMSL. 2003. *IMSL Fortran Library User's Guide Math/Library Volume 2 of 2*, version 5·0. Visual Numerics: Houston.

Li J. 2007a. Transient radial movement of a confined leaky aquifer due to variable well flow rates. *Journal of Hydrology* **333**: 542–553.

Li J. 2007b. Analysis of radial movement of an unconfined leaky aquifer due to well pumping and Injection. *Hydrogeology Journal* **15**(4): 1063–1076.

Li Y, Neuman SP. 2007. Flow to a well in a five-layer system with application to the Oxnard basin. *Ground Water* **45**(6): 672–682.

Liou TS, Yeh HD. 1997. Conditional expectation for evaluation of risk groundwater flow and transport: one-dimensional analysis. *Journal of Hydrology* **199**: 378–402.

Moench AF. 1985. Transient flow to a large-diameter well in an aquifer with storative semiconfining layers. *Water Resources Research* **21**(8): 1121–1131.

Neuman SP. 1968. *Transient flow of ground water to wells in multiple-aquifer systems*. Ph. D. dissertation, Department of Civil Engineering, University of California: Berkeley, California.

Neuman SP, Witherspoon PA. 1969. Theory of flow in a confined two-aquifer system. *Water Resources Research* **5**: 803–816.

Novakowski KS. 1989. A composite analytical model for analysis of pumping tests affected by wellbore storage and finite thickness skin. *Water Resources Research* **25**(9): 1937–1946.

Novakowski KS. 1990. Analysis of aquifer tests conducts in fractured rock: a review of the physical background and the design of a computer program for generating type curves. *Ground Water* **28**(1): 99–107.

Papadopulos IS, Cooper HH Jr. 1967. Drawdown in a well of large diameter. *Water Resources Research* **3**(1): 241–244.

Sekhar M, Mohan Kumar MS, Sridharan K. 1994. Parameter estimation in an anisotropic leaky aquifer system. *Journal Hydrology* **163**: 373–391.

Shan C, Javandel I, Witherspoon PA. 1995. Characterization of leaky faults: study of water flow in aquifer-fault-aquifer systems. *Water Resources Research* **31**(12): 2897–2904.

Shanks D. 1955. Non-linear transformations of divergent and slowly convergent sequences. *Journal of Mathematical Physics* **34**: 1–42.

Trinchero P, Sanchez-Vila X, Copty N, Findikakis A. 2008. A new method for the interpretation of pumping tests in leaky aquifers. *Ground Water* **46**(1): 133–143.

Watson GN. 1958. *A Treatise on the Theory of Bessel Functions* (2nd edn). Cambridge University Press: Cambridge.

Wynn P. 1956. On a device for computing the $e_m(S_n)$ transformation. *Mathematical Tables and other AIDS to Computation* **10**: 91–96.

Yang SY, Yeh HD. 2005. Laplace-domain solutions for radial two-zone flow equations under the conditions of constant-head and partially penetrating well. *Journal of Hydraulic Engineering, ASCE* **131**(3): 213–214. DOI:10.1061/(ASCE)0733-9429(2005)131:3(209): 209–216.

Yang SY, Yeh HD. 2007. On the solutions of modeling slug test performed in a two-zone confined aquifer. *Hydrogeology Journal* **15**(2): 301–302. DOI: 10.1007/s10040-006-0100-x: 297.

Yang SY, Yeh HD, Chiu PY. 2006. A closed-form solution for constant-flux pumping test under the effect of well partial penetration. *Water Resources Research* **42**(5): W05502. DOI:10.1029/2004WR003889.

Yeh HD. 1987. Theis' solution by nonlinear least-squares and finite-difference Newton's method. *Ground Water* **25**: 710–715.

Yeh HD, Huang YC. 2005. Parameter estimation for leaky aquifers using the extended Kalman filter and considering model and data measurement uncertainties. *Journal of Hydrology* **302**(1–4): 28–45.

Yeh HD, Lin YC, Huang YC. 2007. Parameter identification for leaky aquifers using global optimization methods. *Hydrological Processes* **21**: 862–872. DOI:10.1002/hyp.6274.

Yeh HD, Yang SY. 2006. A novel analytical solution for a slug test conducted in a well with a finite-thickness skin. *Advances in Water Resources* **29**(10): 1479–1489. DOI:10.1016/j.advwatres.2005.11.002.

Yeh HD, Yang SY, Peng HY. 2003. A new closed-form solution for radial two-layer drawdown equation under constant-flux pumping in a finite-radius well. *Advances in Water Resources* **26**(5): 747–757.

Zhan H, Bian A. 2006. A method of calculating pumping induced leakage. *Journal of Hydrology* **328**: 659–667.

Zlotnik VA. 2004. A concept of maximum stream depletion rate for leaky aquifers in alluvial valleys. *Water Resources Research* **40**: W06507. DOI:10.1029/2003WR002932.