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Determination of the neutrino flavor ratio at the astrophysical source

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We discuss the reconstruction of neutrino flavor ratios at astrophysical sources through the future neutrino-telescope measurements. Taking the ranges of neutrino mixing parameters θ_{ij} as those given by the current global fit, we demonstrate by a statistical method that the accuracies in the measurements of energy-independent ratios $R \equiv \phi(\nu_\mu)/(\phi(\nu_e) + \phi(\nu_\tau))$ and $S \equiv \phi(\nu_e)/\phi(\nu_\tau)$ among integrated neutrino flux should both be better than 10% in order to distinguish between the pion source and the muon-damped source at the 3σ level. The 10% accuracy needed for measuring R and S requires an improved understanding of the background atmospheric neutrino flux to a better than 10% level in the future. We discuss the applicability of our analysis to practical situations that the diffuse astrophysical neutrino flux arises from different types of sources and each point source has a neutrino flavor ratio varying with energies. We also discuss the effect of the leptonic CP phase on the flavor-ratio reconstruction.

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I. INTRODUCTION

The operation of the IceCube detector [1] and the research and development effort of KM3Net [2] are important progresses toward a km^3-sized detection capability in the neutrino astronomy [3]. Furthermore, the radio and air-shower detectors, such as ANITA [4] and the Pierre Auger detector [5], respectively, are also taking the data. These detectors are sensitive to neutrinos with energies higher than those probed by IceCube and KM3Net. Finally, the radio extension of the IceCube detector, the IceRay [6], is also under consideration. It is expected to detect a score of cosmogenic neutrinos [7] per year. Motivated by the development of neutrino telescopes, numerous efforts were devoted to studying neutrino mixing parameters with astrophysical neutrinos as the beam source [8–24]. Because of the large neutrino propagation distance, the neutrino oscillation probabilities only depend on the mixing angles θ_{ij} and the CP phase δ [25,26], which make the astrophysical beam source favorable for extracting the above parameters, provided there are a sufficient number of events.

Most of the astrophysical neutrinos are believed to be produced by the decay of the charged pion through the following chain: $\pi^+ \to \mu^+ + \nu_\mu \to e^+ + \nu_\mu + \nu_e + \bar{\nu}_\mu$ or $\pi^- \to \mu^- + \bar{\nu}_\mu \to e^- + \bar{\nu}_\mu + \bar{\nu}_e + \nu_\mu$. This leads to the neutrino flux ratio $\phi_0(\nu_e):\phi_0(\nu_\mu):\phi_0(\nu_\tau) = 1:2:0$ at the astrophysical source where $\phi_0(\nu_\alpha)$ is the sum of ν_α and $\bar{\nu}_\alpha$ flux. Such a flux ratio results from an implicit assumption that the muon decays into neutrinos before it loses a significant fraction of its energy. However, in some source the muon quickly loses its energy by interacting with strong magnetic fields or with matter [27–29]. Such a muon eventually decays into neutrinos with energies much lower than that of $\nu_\mu(\bar{\nu}_\mu)$ from $\pi^+(\pi^-)$ decays. Consequently this type of source has a neutrino flavor ratio $\phi_0(\nu_e):\phi_0(\nu_\mu):\phi_0(\nu_\tau) = 0:1:0$, which is referred to as the muon-damped source. The third type of source emits neutrons resulting from the photodisassociation of nuclei. As neutrons propagate to the Earth, $\bar{\nu}_e$ are produced from neutron β decays [30], leading to a neutrino flavor ratio $\phi_0(\nu_e):\phi_0(\nu_\mu):\phi_0(\nu_\tau) = 1:0:0$. Finally, neutrinos might be produced deep inside optically thick sources so that the flavor ratio at the source surface is significantly different from the flavor ratio at the production point due to the oscillations [31]. Hence, unlike the previous three cases, the ν_τ fraction can be significant at the surface of such sources. In the class of sources studied by Mena *et al.* [31], which are referred to as the astrophysical hidden sources, the neutrino flux ratio for $E_\nu > 10^4$ GeV approaches to $1/3:a:b$ with both a and b oscillating with the neutrino energy under the constraint $a + b = 2/3$.

As mentioned before, almost all previous studies treat astrophysical neutrinos as the beam source for extracting neutrino mixing parameters [32]. To have a better determination of certain neutrino mixing parameters, for instance the atmospheric mixing angle θ_{23} or the CP phase δ, a combined analysis on the terrestrially measured flavor ratios of astrophysical neutrinos coming from different sources, such as the pion source and the muon-damped source, has been considered [20,23]. A natural question to ask is then how well one can distinguish these neutrino sources. The answer to this question depends on our knowledge of neutrino mixing parameters and the achievable accuracies in measuring the neutrino flavor ratios on the Earth such as $R \equiv \phi(\nu_\mu)/(\phi(\nu_e) + \phi(\nu_\tau))$ and $S \equiv \phi(\nu_e)/\phi(\nu_\tau)$. In this article, we shall provide an answer to this question with a statistical analysis.

The possibility of measuring the neutrino flavor fraction by IceCube has been discussed in Ref. [33]. It is through the measurement of the muon track to shower ratio. It was demonstrated that the ν_e fraction can be extracted from the above ratio by assuming flavor independence of the neutrino spectrum and the $\nu_\mu - \nu_\tau$ symmetry, i.e., $\phi(\nu_\mu) = \phi(\nu_\tau)$. Taking a pion source with $E^2\phi(\nu_\mu) = 10^{-7}\ \mathrm{GeV\,cm^{-2}\,s^{-1}}$ [34] and thresholds for muon and shower energies taken to be 100 GeV and 1 TeV, respectively, the ν_e fraction can be determined to an accuracy of 25% at IceCube for 1 yr of data taking, or equivalently to an accuracy of 8% for a decade of data taking. However, the tau neutrino events are too rare to provide additional information on the neutrino flavor composition. The analysis in Ref. [33] as summarized above provides a feasibility of measuring R in good precision at IceCube and detectors with comparable capacities. In fact one may repeat the analysis in [33] and extract R and its associated uncertainty directly. The uncertainty in R is expected to be comparable to that of the ν_e fraction. We note that the precisions on measuring R and S should depend on neutrino energies. However, for simplicity in discussions, we shall take R and S as ratios of integrated neutrino flux with appropriate energy thresholds for suppressing the atmospheric neutrino background. These ratios and the corresponding precisions, $\Delta R/R$ and $\Delta S/S$, are therefore energy independent. In our analysis, we do not assume $\nu_\mu - \nu_\tau$ symmetry for the neutrino flux measured on the Earth. We shall argue that, besides measuring R, it is essential to measure S in order to reconstruct the neutrino flavor ratio at the astrophysical source. This implies that a neutrino telescope beyond the capability of IceCube is needed to study the neutrino flavor astronomy.

It is important to understand the atmospheric neutrino background which affects the precisions on measuring R and S. The flux spectrum of conventional atmospheric neutrinos which arise from pion and kaon decays is well understood [35,36]. The measurement on such a spectrum [37] has reached to the energy of 10^5 GeV. The prompt atmospheric neutrino flux arising from charm decays still contains large uncertainties [38–40] and it has not yet been measured experimentally. The prompt atmospheric neutrino flux takes over the conventional one around 10^5 GeV for ν_e and 10^6 GeV for ν_μ [41]. The flavor ratio of the conventional atmospheric neutrino beyond TeV energies is approximately $\phi_c^{\mathrm{atm}}(\nu_e){:}\phi_c^{\mathrm{atm}}(\nu_\mu){:}\phi_c^{\mathrm{atm}}(\nu_\tau) =$ 1:20:0; while the flavor ratio of prompt atmospheric neutrino flux is approximately $\phi_p^{\mathrm{atm}}(\nu_e){:}\phi_p^{\mathrm{atm}}(\nu_\mu){:}\phi_p^{\mathrm{atm}}(\nu_\tau) =$ 1:1:0.1. Such flavor ratios differ significantly from those of astrophysical neutrinos which arrive on Earth with $\phi(\nu_\mu) \approx \phi(\nu_\tau)$. To suppress atmospheric neutrino background in the search of astrophysical neutrinos, energy distributions of astrophysical neutrino events and cuts on photomultiplier tube hits are imposed [42].

The paper is organized as follows. In Sec. II, we discuss properties of the probability matrix that links the initial neutrino flavor ratio to the ratio measured on the Earth. In Sec. III, we begin with a brief review on the current understanding of neutrino mixing angles. We then present the reconstructed neutrino flavor ratio at the source from the simulated data, which is generated by the chosen true values of the neutrino flavor ratio at the source and best-fit values of neutrino mixing parameters. The statistical analysis is performed with different measurement accuracies in R and S, as well as different ranges of neutrino mixing parameters. The implications of our results are discussed in Sec. IV.

II. NEUTRINO MIXING PARAMETERS AND OSCILLATIONS OF ASTROPHYSICAL NEUTRINOS

The neutrino flux at the astrophysical source and that detected on the Earth are related by

$$\begin{pmatrix} \phi(\nu_e) \\ \phi(\nu_\mu) \\ \phi(\nu_\tau) \end{pmatrix} = \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{\mu e} & P_{\mu\mu} & P_{\mu\tau} \\ P_{\tau e} & P_{\tau\mu} & P_{\tau\tau} \end{pmatrix} \begin{pmatrix} \phi_0(\nu_e) \\ \phi_0(\nu_\mu) \\ \phi_0(\nu_\tau) \end{pmatrix} \equiv P \begin{pmatrix} \phi_0(\nu_e) \\ \phi_0(\nu_\mu) \\ \phi_0(\nu_\tau) \end{pmatrix}, \quad (1)$$

where $\phi(\nu_\alpha)$ is the neutrino flux measured on the Earth while $\phi_0(\nu_\alpha)$ is the neutrino flux at the astrophysical source, and the matrix element $P_{\alpha\beta}$ is the probability of the oscillation $\nu_\beta \to \nu_\alpha$. The exact analytic expressions for $P_{\alpha\beta}$ are given in Eq. (A1). It is seen that $P_{e\mu} = P_{e\tau}$ and $P_{\mu\mu} = P_{\mu\tau} = P_{\tau\tau}$ in the limit $\Delta = 0 = D$, i.e., $\theta_{23} = \pi/4$ and $\theta_{13} = 0$. In this case, the probability matrix P is singular with a vanishing determinant. In general, the determinant of this matrix remains suppressed since both Δ and D are expected to be small. For $\Delta = 0 = D$, the eigenvectors of P are given by

$$V^a = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad V^b = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \quad V^c = \frac{1}{\sqrt{6}}\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \quad (2)$$

with the corresponding eigenvalues

$$\lambda_a = 1, \qquad \lambda_b = 0, \qquad \lambda_c = \tfrac{1}{4}(4 - 3\omega), \quad (3)$$

where $\omega = \sin^2 2\theta_{12}$. Therefore, those initial flavor ratios that differ from one another by a multiple of V^b shall oscillate into the same flavor ratio on the Earth. To illustrate this explicitly, we write the initial flux Φ_0 at the astrophysical source as

$$\Phi_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \frac{\sqrt{2}}{2}(\phi_0(\nu_\mu) - \phi_0(\nu_\tau))V^b + \frac{\sqrt{6}}{2}(\phi_0(\nu_\mu) + \phi_0(\nu_\tau))V^c, \quad (4)$$

where we have imposed the normalization condition $\phi_0(\nu_e) + \phi_0(\nu_\mu) + \phi_0(\nu_\tau) = 1$. This normalization convention will be adopted throughout this paper. The first term on the right-hand side of Eq. (4) can be expressed as $(\sqrt{3}V^a - \sqrt{6}V^c)/3$. Hence the neutrino flux measured by the terrestrial neutrino telescope is

$$\begin{aligned}\Phi &= P\phi_0 \\ &= \frac{\sqrt{3}}{3}V^a - \frac{\sqrt{6}}{3}\left(1 - \frac{3}{4}\omega\right)V^c + \frac{\sqrt{6}\lambda_c}{2}(\phi_0(\nu_\mu) \\ &\quad + \phi_0(\nu_\tau))V^c. \end{aligned} \tag{5}$$

It is seen that the vector V^b, with a coefficient proportional to $\phi_0(\nu_\mu) - \phi_0(\nu_\tau)$, does not appear in the terrestrially measured flux Φ. Hence the terrestrial measurement can not constrain $\phi_0(\nu_\mu) - \phi_0(\nu_\tau)$ in this case.

The above degeneracy is lifted by either a nonvanishing θ_{13} ($D \neq 0$) or a deviation of θ_{23} from $\pi/4$ ($\Delta \neq 0$). To simplify our discussions, let us take $D = 0$ and $\Delta \neq 0$. One can show that the flux combination $(1 + 4\omega\Delta/(4 - 3\omega))\phi_0(\nu_\mu) - (1 - 2\omega\Delta/(4 - 3\omega))\phi_0(\nu_\tau)$ remains poorly constrained due to the suppression of det$P$. To demonstrate this, we observe that

$$P = \frac{1}{8}\begin{pmatrix} 8 - 4\omega & 2(1+\Delta)\omega & 2(1-\Delta)\omega \\ 2(1+\Delta)\omega & (4-\omega)(1+\Delta^2) - 2\Delta\omega & (4-\omega)(1-\Delta^2) \\ 2(1-\Delta)\omega & (4-\omega)(1-\Delta^2) & (4-\omega)(1+\Delta^2) + 2\Delta\omega \end{pmatrix} \tag{6}$$

for $D = 0$ and $\Delta \neq 0$. The eigenvalues of P expanded to the second order in Δ are given by

$$\begin{aligned} \lambda'_a &= 1, \qquad \lambda'_b = \left(\frac{4 - 4\omega}{4 - 3\omega}\right)\Delta^2, \\ \lambda'_c &= \frac{1}{4}(4 - 3\omega) + \frac{3\omega^2\Delta^2}{4(4 - 3\omega)}, \end{aligned} \tag{7}$$

and the corresponding eigenvectors to the same order in Δ are

$$V'^a = N^a\begin{pmatrix}1\\1\\1\end{pmatrix}, \qquad V'^b = N^b\begin{pmatrix}2r\Delta(1 + r\Delta)\\ -1 - 2r\Delta(1 + r\Delta)\\ 1\end{pmatrix},$$
$$V'^c = N^c\begin{pmatrix}-2 + 6r\Delta\\ 1 - 6r\Delta(1 - 3r\Delta)\\ 1\end{pmatrix}, \tag{8}$$

with $r = \omega/(4 - 3\omega)$ and $N^{a,b,c}$ the appropriate normalization factors. It is interesting to note that the corrections to the eigenvectors of P begin at $\mathcal{O}(\Delta)$ while the corrections to the corresponding eigenvalues begin at $\mathcal{O}(\Delta^2)$. With the above eigenvectors, we write the source neutrino flux as

$$\begin{aligned}\Phi_0 &= N^aV'^a - [(1 + 4r\Delta)\phi_0(\nu_\mu) - (1 - 2r\Delta)\phi_0(\nu_\tau) \\ &\quad - 2r\Delta]N^bV'^b + 3\Big[(1 - 4r\Delta)\phi_0(\nu_\mu) \\ &\quad + (1 - 2r\Delta)\phi_0(\nu_\tau) - \tfrac{2}{3}(1 - 3r\Delta)\Big]N^cV'^c. \end{aligned} \tag{9}$$

It is easy to show that the measured flux $P\Phi_0$ depends on V'^b through the combination $-B\lambda'_bN^bV'^b$ with

$$B = [(1 + 4r\Delta)\phi_0(\nu_\mu) - (1 - 2r\Delta)\phi_0(\nu_\tau) - 2r\Delta]. \tag{10}$$

Clearly the flux combination $(1 + 4r\Delta)\phi_0(\nu_\mu) - (1 - 2r\Delta)\phi_0(\nu_\tau)$ is poorly constrained due to the smallness of λ'_b, of the order Δ^2.

III. STATISTICAL ANALYSIS

To reconstruct the neutrino flavor ratio at the source with a statistical analysis, we employ the following best-fit values and 1σ ranges of neutrino mixing angles [43]

$$\begin{aligned} \sin^2\theta_{12} = 0.32^{+0.02}_{-0.02}, \qquad \sin^2\theta_{23} = 0.45^{+0.09}_{-0.06}, \\ \sin^2\theta_{13} < 0.019, \end{aligned} \tag{11}$$

for the major part of our analysis. In the above parameter set, the best-fit value of θ_{23} is smaller than $\pi/4$. There exist proposals to probe $\sin^2\theta_{23}$ by future atmospheric neutrino experiments [44,45] and long baseline neutrino experiments [46]. We therefore include in our analysis the hypothetical scenario that $(\sin^2\theta_{23})_{\text{best fit}} = 0.55$ with an error identical to the one associated with $(\sin^2\theta_{23})_{\text{best fit}} = 0.45$. Finally, we also consider a θ_{13} range suggested by Ref. [47] where

$$\sin^2\theta_{13} = 0.016 \pm 0.010(1\sigma) \tag{12}$$

by a global analysis.

In this work, we investigate uncertainties in the reconstruction of neutrino flavor ratios at the source for the pion source and the muon-damped source. Different choices of neutrino mixing parameters in our analysis are listed in Table I. Employing these mixing parameters, the true values of neutrino flavor ratios on the Earth and the corresponding values for R and S are presented in Table II. The true values of the neutrino flavor ratios on the Earth are denoted by Φ_π and Φ_μ for the pion source and the muon-

TABLE I. Parameter sets chosen for our analysis.

Parameter set	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δ
1	$0.32^{+0.02}_{-0.02}$	$0.45^{+0.09}_{-0.06}$	<0.019	0
2	$0.32^{+0.02}_{-0.02}$	$0.55^{+0.09}_{-0.06}$	<0.019	0
3a	$0.32^{+0.02}_{-0.02}$	$0.45^{+0.09}_{-0.06}$	0.016 ± 0.010	0
3b	$0.32^{+0.02}_{-0.02}$	$0.45^{+0.09}_{-0.06}$	0.016 ± 0.010	$\pi/2$
3c	$0.32^{+0.02}_{-0.02}$	$0.45^{+0.09}_{-0.06}$	0.016 ± 0.010	π

damped source, respectively. They are calculated with Eq. (1) where P is evaluated with neutrino mixing parameters at their best-fit values. The flux ratios R_π and S_π are obtained from Φ_π while R_μ and S_μ are obtained from Φ_μ.

Given a precision on measuring R, $\Delta R_i/R_i$, we estimate $\Delta S_i/S_i$ with two approaches. The first approach assumes that both ΔR_i and ΔS_i are dominated by the statistical errors. In this case, one has

$$\left(\frac{\Delta S_i}{S_i}\right) = \frac{1+S_i}{\sqrt{S_i}}\sqrt{\frac{R_i}{1+R_i}}\left(\frac{\Delta R_i}{R_i}\right), \tag{13}$$

with $i = \pi, \mu$ [20]. Using the values of R_i and S_i from Table II, we obtain $\Delta S_\pi/S_\pi = (1.1 - 1.2)(\Delta R_\pi/R_\pi)$ and $\Delta S_\mu/S_\mu = (1.1 - 1.4)(\Delta R_\mu/R_\mu)$. The second approach takes into account the specific complications for identifying tau neutrinos. Since tau lepton decays before it loses a significant fraction of its energy, the tau neutrino is identified by the so-called double-bang or lollipop events [25,33,48]. In IceCube or other detectors with a comparable size, double-bang events are observable only in a narrow energy range between 2 and 20 PeV [25,48] while the probability for observing a lollipop event, though increasing with the neutrino energy, is still less than 10^{-3} for $E_\nu = 1$ EeV [33]. In view of these, we do not correlate $\Delta S_i/S_i$ with $\Delta R_i/R_i$ in the second approach. Rather we fix $\Delta S_i/S_i$ while we vary $\Delta R_i/R_i$ for achieving the goal of distinguishing astrophysical neutrino sources. The results of both approaches will be presented. Before presenting the details of our analysis, we point out that the decays $\tau \to \nu_\tau \mu \bar{\nu}_\mu$ and $\tau \to \nu_\tau \mu \bar{\nu}_\mu$, each with a 18% branching ratio, produce extra muon events or secondary ν_e and ν_μ [49,50]. Cares are needed to separate these events from those of primary ν_e and ν_μ or muons produced by the charged current interaction.

The fitting to the neutrino flavor ratios at the source is facilitated through

$$\chi_i^2 = \left(\frac{R_{i,\rm th} - R_{i,\rm exp}}{\sigma_{R_{i,\rm exp}}}\right)^2 + \left(\frac{S_{i,\rm th} - S_{i,\rm exp}}{\sigma_{S_{i,\rm exp}}}\right)^2 + \sum_{jk=12,23,13}\left(\frac{s_{jk}^2 - (s_{jk})^2_{\rm best\ fit}}{\sigma_{s^2_{jk}}}\right)^2 \tag{14}$$

with $i = \pi, \mu$, $\sigma_{R_{i,\rm exp}} = (\Delta R_i/R_i)R_{i,\rm exp}$, $\sigma_{S_{i,\rm exp}} = (\Delta S_i/S_i)S_{i,\rm exp}$, $s^2_{jk} \equiv \sin^2\theta_{jk}$, and $\sigma_{s^2_{jk}}$ the 1σ range for s^2_{jk}. Here $R_{i,\rm th}$ and $S_{i,\rm th}$ are theoretically predicted values for R_i and S_i, respectively, while $R_{i,\rm exp}$ and $S_{i,\rm exp}$ are experimentally measured values. The values for $R_{i,\rm exp}$ and $S_{i,\rm exp}$ are listed in Table II, which are generated from input true values of neutrino flavor ratios at the source and input true values of neutrino mixing parameters. In $R_{i,\rm th}$ and $S_{i,\rm th}$, the variables s^2_{jk} can vary between 0 and 1 while $\cos\delta$ can vary between -1 and 1. We note that similar χ^2 functions have been used for fitting the CP violation phase and the mixing angle θ_{23}, respectively [20,23], assuming the source flavor ratio is known. In our analysis, we scan all possible neutrino flavor ratios at the source that give rise to a specific χ^2_i value. Since we have taken $R_{i,\rm exp}$ and $S_{i,\rm exp}$ as those generated by input true values of initial neutrino flavor ratios and neutrino mixing parameters, we have $(\chi^2_i)_{\rm min} = 0$ occurring at these input true values of parameters. Hence the boundaries for 1σ and 3σ ranges of initial neutrino flavor ratios are given by $\Delta\chi^2_i = 2.3$ and $\Delta\chi^2_i = 11.8$, respectively, where $\Delta\chi^2_i \equiv \chi^2_i - (\chi^2_i)_{\rm min} = \chi^2_i$ in our analysis.

A. The reconstruction of initial neutrino flavor ratio by measuring R alone

It is instructive to see how well one can determine the initial neutrino flavor ratio by measuring R alone. We perform such an analysis by neglecting the second term on the right-hand side of Eq. (14). The 1σ and 3σ ranges for the reconstructed flavor ratios at the source are shown in Fig. 1. For an input muon-damped source, it is seen that, with $\Delta R_\mu/R_\mu = 10\%$, the reconstructed 3σ range of the neutrino flavor ratio almost covers the entire physical region. For an input pion source with $\Delta R_\pi/R_\pi = 10\%$, all possible initial neutrino flavor ratios are allowed at the 3σ level. Clearly it is desirable to measure both R and S.

TABLE II. True values of neutrino flavor ratios on the Earth.

Parameter set	$\Phi_\mu = P\Phi_{0,\mu}$	R_μ	S_μ	$\Phi_\pi = P\Phi_{0,\pi}$	R_π	S_π
1	(024, 0.37, 0.39)	0.62	0.60	(0.35, 0.33, 0.32)	0.49	1.08
2	(0.19, 0.42, 0.39)	0.71	0.51	(0.32, 0.34, 0.34)	0.52	0.94
3a	(0.27, 0.35, 0.38)	0.55	0.71	(0.36, 0.33, 0.31)	0.48	1.15
3b	(0.25, 0.37, 0.38)	0.59	0.64	(0.35, 0.33, 0.32)	0.49	1.07
3c	(0.23, 0.40, 0.37)	0.67	0.60	(0.33, 0.34, 0.33)	0.52	1.02

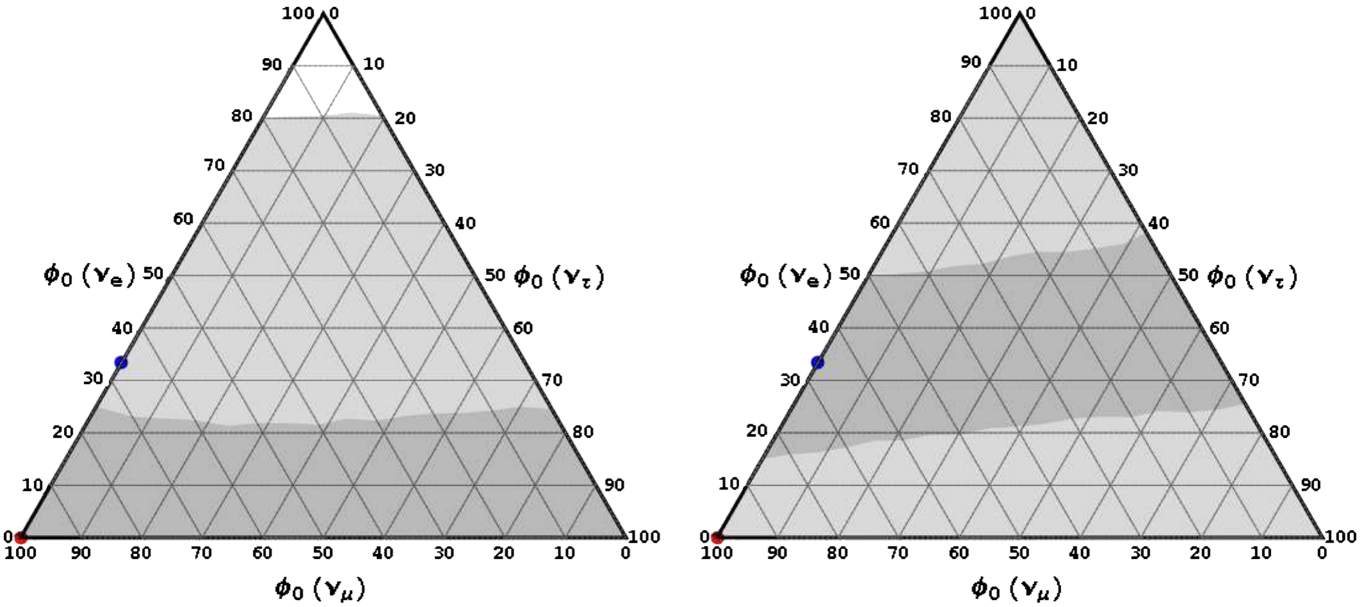

FIG. 1 (color online). The reconstructed ranges for the neutrino flavor ratios at the source with $\Delta R_i/R_i = 10\%$. The left and right panels are results with the muon-damped source and the pion source as the input true source, respectively. The numbers on each side of the triangle denote the flux percentage of a specific flavor of neutrino. The point situated at one of the triangle vertices marks the muon-damped source $\Phi_{0,\mu} = (0, 1, 0)$ while the other point marks the pion source $\Phi_{0,\pi} = (1/3, 2/3, 0)$. The gray and light gray areas, respectively, denote the 1σ and 3σ ranges for the reconstructed neutrino flavor ratios at the source. We choose parameter set 1 in Table I for this analysis.

B. The flavor reconstruction with measurements on both R and S

In this subsection, we perform a statistical analysis with respect to simultaneous measurements of R and S. The accuracy for the measurement on R is $\Delta R_i/R_i = 10\%$ with $i = \pi, \mu$. Here we adopt the first approach for estimating $\Delta S_i/S_i$ while we present the second approach in the next subsection. With the first approach, we have $\Delta S_\pi/S_\pi = (11 - 12)\%$ and $\Delta S_\mu/S_\mu = (11 - 14)\%$ depending on the parameter set chosen for calculations.

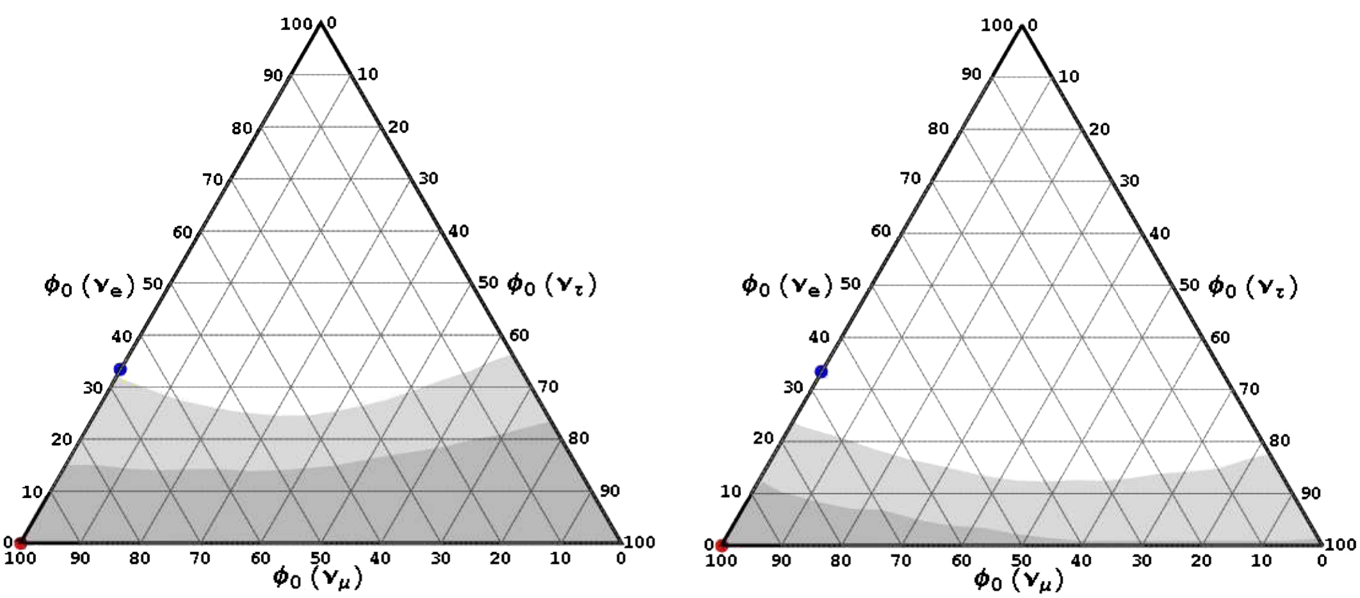

FIG. 2 (color online). The reconstructed ranges for the neutrino flavor ratios for an input muon-damped source with $\Delta R_\mu/R_\mu = 10\%$ and $\Delta S_\mu/S_\mu$ related to the former by the Poisson statistics. Gray and light gray areas in the left (right) panel denote the reconstructed 1σ and 3σ ranges with parameter set 1 (2).

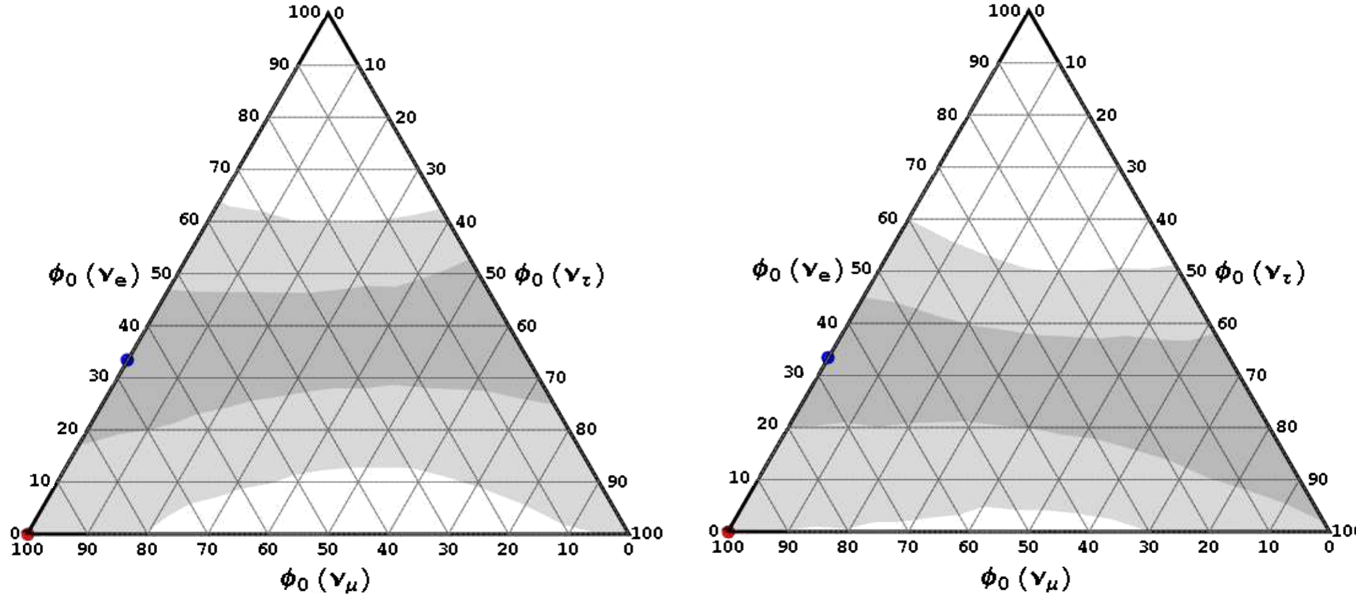

FIG. 3 (color online). The reconstructed ranges for the neutrino flavor ratios for an input pion source with $\Delta R_\pi/R_\pi = 10\%$ and $\Delta S_\pi/S_\pi$ related to the former by the Poisson statistics. The gray and light gray areas in the left (right) panel denote the reconstructed 1σ and 3σ ranges with parameter set 1 (2).

1. $(\sin^2\theta_{13})_{\mathbf{best\ fit}} = 0$

We begin our analysis with parameter sets 1 and 2, where $(\sin^2\theta_{13})_{\text{best fit}} = 0$ and $(\sin^2\theta_{23})_{\text{best fit}} = 0.45$ and 0.55, respectively. Figures 2 and 3 show the reconstructed neutrino flavor ratios for an input muon-damped source and an input pion source, respectively. The reconstructed initial flavor ratios are seen to include the region with significant ν_τ fractions. It has been shown in Sec. II that

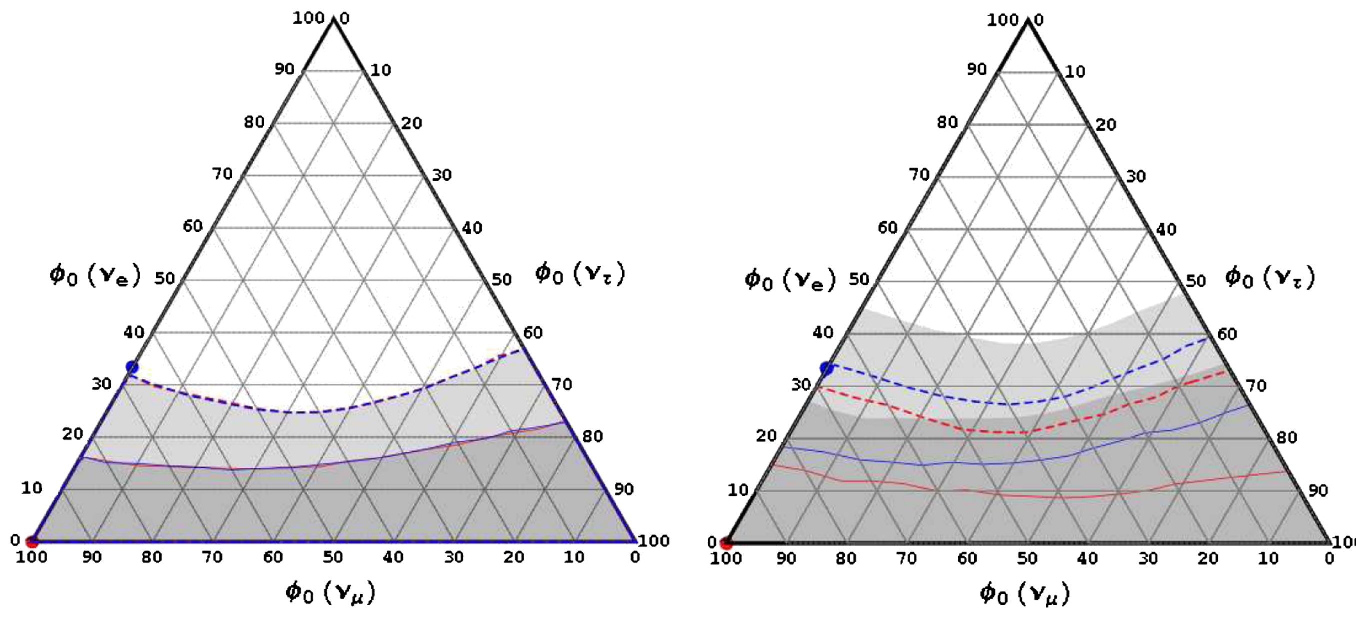

FIG. 4 (color online). The reconstructed ranges for the neutrino flavor ratio at the source for an input muon-damped source with $\Delta R_\mu/R_\mu = 10\%$ and $\Delta S_\mu/S_\mu$ related to the former by the Poisson statistics. The left panel is obtained with θ_{13} and θ_{23} taken from parameter set 1 and the input *CP* phase taken to be 0, $\pi/2$ and π, respectively. The right panel is obtained with parameter sets 3a, 3b, and 3c. In the right panel, the light gray area, the first dashed line (counted from the top of the triangle), and the second dashed line correspond to the 3σ ranges for the reconstructed neutrino flavor ratio at the source for $\cos\delta = 1$, $\cos\delta = 0$, and $\cos\delta = -1$, respectively. The gray area, the first solid line, and the second solid line correspond to the 1σ ranges for the reconstructed neutrino flavor ratio at the source for $\cos\delta = 1$, $\cos\delta = 0$, and $\cos\delta = -1$, respectively. The left panel shows that the *CP* phase δ does not affect the confidence regions.

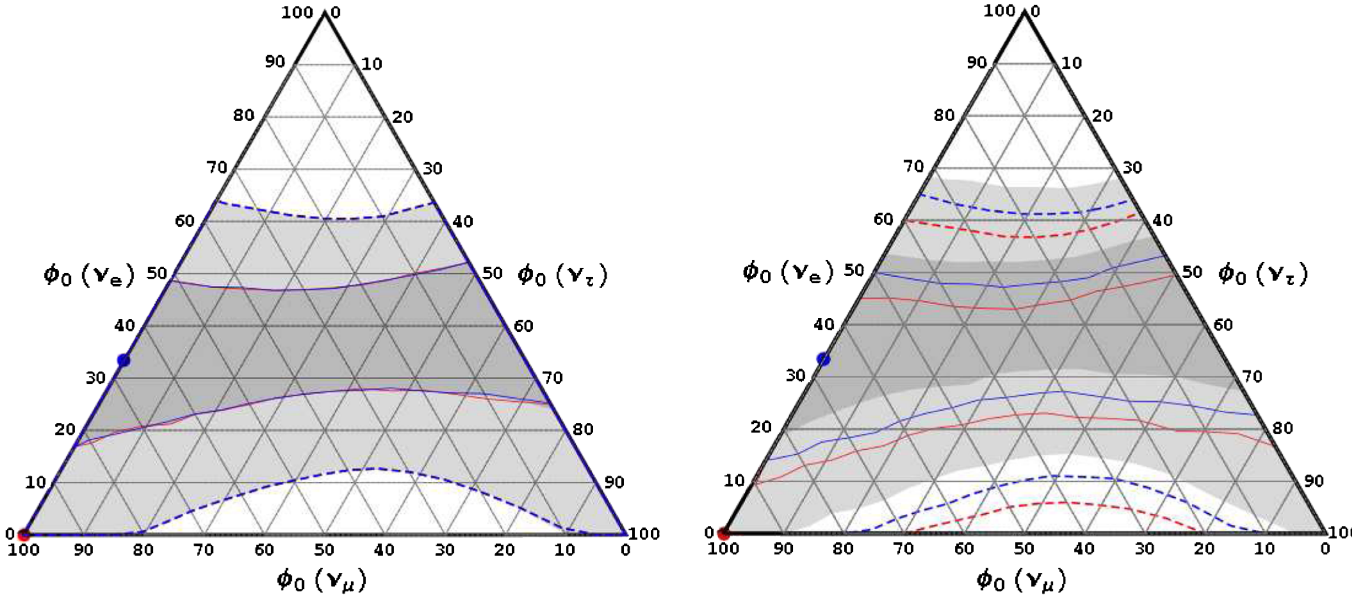

FIG. 5 (color online). The reconstructed 1σ and 3σ ranges for the neutrino flavor ratio at the source for an input pion source with $\Delta R_\pi/R_\pi = 10\%$ and $\Delta S_\pi/S_\pi$ related to the former by the Poisson statistics. The choices of parameter sets are identical to those of Fig. 4. In the right panel, the light gray area, the area bounded by the first and the third dashed lines (counted from the top of the triangle), and the area bounded by the second and the fourth dashed lines correspond to the 3σ ranges for the reconstructed neutrino flavor ratio at the source for $\cos\delta = 1$, $\cos\delta = 0$, and $\cos\delta = -1$, respectively. The corresponding solid lines describe the 1 σ ranges. Once more, the effect from the CP phase δ only appears in the right panel.

the flux combination $(1+4r\Delta)\phi_0(\nu_\mu)-(1-2r\Delta)\phi_0(\nu_\tau)$ is poorly constrained due to the smallness of eigenvalue λ_b' associated with V'^b [see Eqs. (7) and (8)]. This then leads to an extension in the reconstructed range of the initial neutrino flavor ratio along the V'^b direction. In the limit $\Delta \equiv \cos 2\theta_{23} = 0$, i.e., $\sin^2\theta_{23} = 0.5$, V'^b reduces to V^b [see Eq. (2)] which is exactly parallel to the ν_e-less side of the flavor-ratio triangle. The direction of V'^b deviates slightly from that of V^b in opposite ways depending on the sign of Δ. This is seen by comparing the left and right panels of

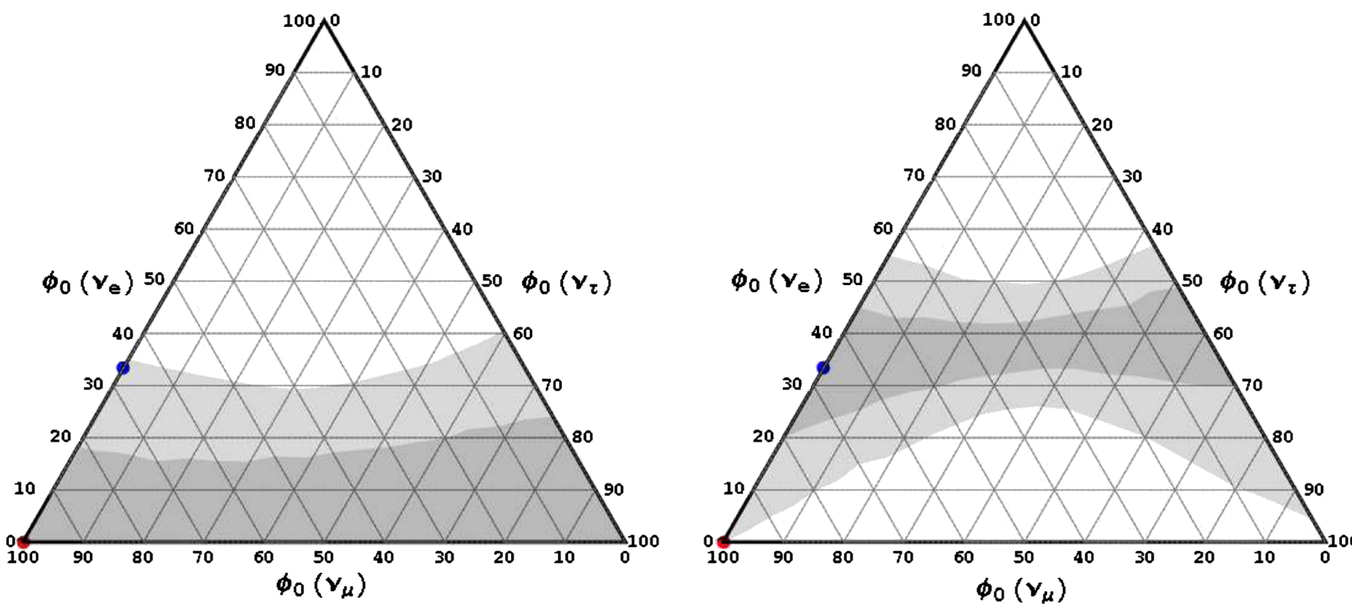

FIG. 6 (color online). Critical accuracies needed to distinguish between the pion source and the muon-damped source. In the left panel where the muon-damped source is the true source, the reconstructed 3σ range for the neutrino flavor ratio just touches the pion source at $\Delta R_\mu/R_\mu = 13\%$. In the right panel where the pion source is the true source, the reconstructed 3σ range for the neutrino flavor ratio just touches the muon-damped source at $\Delta R_\pi/R_\pi = 6\%$. We choose parameter set 1 for this analysis.

both Figs. 2 and 3. Because of uncertainties of neutrino mixing parameters, we note that the boundaries for 1σ and 3σ regions are not straight lines. For an input muon-damped source, the pion source can be ruled out at the 3σ level as shown in Fig. 2. However, the converse is not true as seen from Fig. 3. Finally, as shown in the right panel of Fig. 2, an astrophysical hidden source with $\Phi_{0,\mathrm{ah}} = (1/3, a, 2/3 - a)$ [31,51] can be ruled out at the 3σ level for an input muon-damped source with $(\sin^2\theta_{23})_{\mathrm{best\,fit}} = 0.55$.

2. $(\sin^2\theta_{13})_{\mathbf{best\ fit}} > 0$

A nonzero θ_{13} introduces the CP phase contribution to every element of matrix P, except P_{ee}. We study the effect of the CP phase δ on the reconstruction of neutrino flavor ratio at the source. We choose parameter sets 3a, 3b, and 3c for performing the statistical analysis. The results are shown in the right panels of Figs. 4 and 5. For comparisons, we also perform the analysis with θ_{13} and θ_{23} taken from parameter set 1 and the input CP phase taken to be 0, $\pi/2$ and π, respectively. The results are shown in the left panels of Figs. 4 and 5.

The left panels of Figs. 4 and 5 indicate that the reconstructed ranges for initial neutrino flavor ratios are independent of the input CP phase for $(\sin^2\theta_{13})_{\mathrm{best\,fit}} = 0$. The dependencies on the CP phase only appear in the right panels. For an input muon-damped source (see Fig. 4), the allowed 1σ and 3σ ranges for initial neutrino flavor ratios are the smallest for $\cos\delta = -1$, i.e., $\delta = \pi$. In this case, the pion source and the astrophysical hidden source mentioned earlier can both be ruled out at the 3σ level [51]. The allowed ranges become the largest (denoted by gray areas) for $\cos\delta = 1$, i.e., $\delta = 0$. For an input pion source with different CP phases, the allowed 3σ ranges for the initial neutrino flavor ratio always cover the muon-damped source.

C. Critical accuracies needed for distinguishing astrophysical sources

It is important to identify critical accuracies in the measurement needed to distinguish between the pion source and the muon-damped source. Choosing parameter set 1 for the analysis, we present the results in Figs. 6 and 7 where two different approaches for determining $\Delta S_i/S_i$ are used. In Fig. 6, we determine $\Delta S_i/S_i$ by applying Poisson statistics. In the left panel of Fig. 6, which has the muon-damped source as the true source, the reconstructed 3σ range for the neutrino flavor ratio just touches the pion source at $\Delta R_\mu/R_\mu = 13\%$ and $\Delta S_\mu/S_\mu = 16\%$. In the right panel of this figure, which has the pion source as the true source, the reconstructed 3σ range for the neutrino flavor ratio just touches the muon-damped source at $\Delta R_\pi/R_\pi = 6\%$ and $\Delta S_\pi/S_\pi = 7\%$. In Fig. 7, we fix $\Delta S_\mu/S_\mu = 25\%$ for the left panel and fix $\Delta S_\pi/S_\pi = 15\%$ for the right panel. The result in the left panel is for $\Delta R_\mu/R_\mu = 2\%$. We find that the pion source can be ruled out at the 3σ level if $\Delta R_\mu/R_\mu$ is lowered to 1%. The result in the right panel is for $\Delta R_\pi/R_\pi = 1.5\%$. If $\Delta R_\pi/R_\pi$ is raised to 2%, we find that the muon-damped source cannot be ruled out at the 3σ level. We have also investigated the case $\Delta S_\pi/S_\pi = 25\%$. In this case the reconstructed 3σ

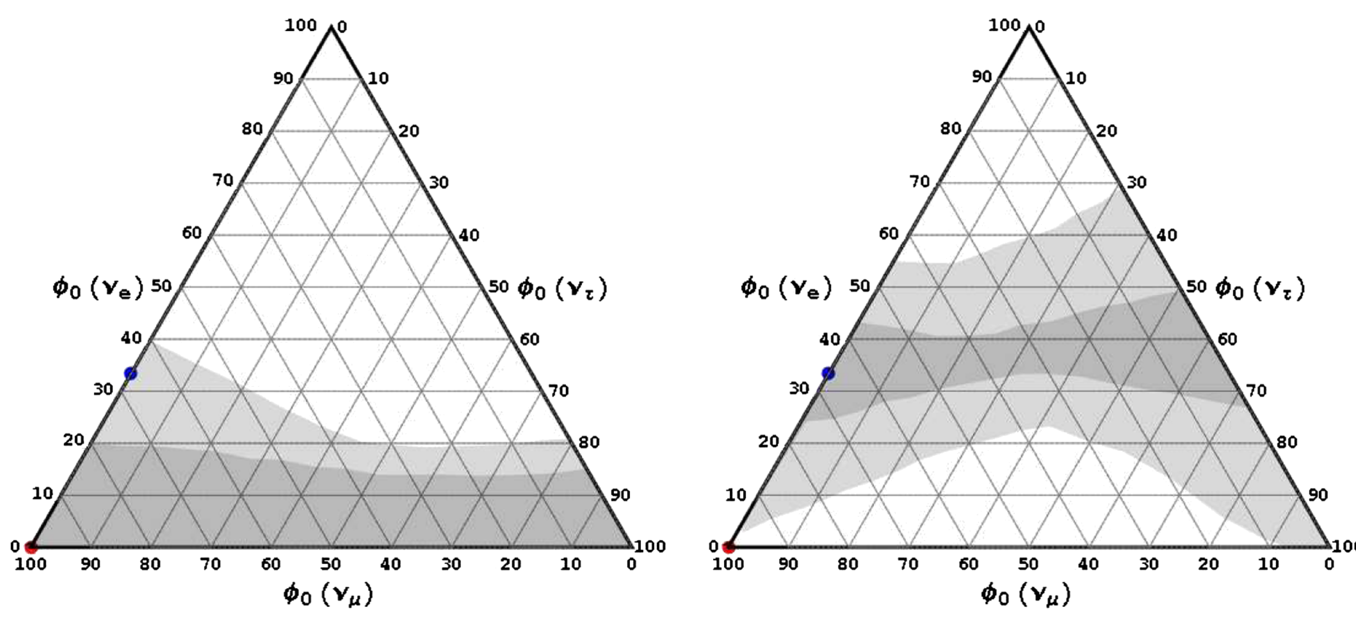

FIG. 7 (color online). Left panel: the reconstructed 1σ and 3σ ranges for the neutrino flavor ratio at the source for an input muon-damped source with $\Delta S_\mu/S_\mu = 25\%$ and $\Delta R_\mu/R_\mu = 2\%$. Right panel: the reconstructed 1σ and 3σ ranges for the neutrino flavor ratio at the source for an input pion source with $\Delta S_\pi/S_\pi = 15\%$ and $\Delta R_\pi/R_\pi = 1.5\%$. We choose parameter set 1 for this analysis.

range of the neutrino flavor ratio covers the entire physical region unless $\Delta R_\pi/R_\pi$ is smaller than 1%.

IV. DISCUSSION AND CONCLUSION

The structure of the oscillation probability matrix P (singular in the limit $\theta_{23} = \pi/4$ and $\theta_{13} = 0$) makes it difficult to constrain a flux combination approximately like the difference between $\phi_0(\nu_\mu)$ and $\phi_0(\nu_\tau)$. This then leads to an extension in the reconstructed range for the initial neutrino flavor ratio along the direction of $V^{\prime b}$.

We have illustrated the reconstruction of the neutrino flavor ratio at the source from the measurements of energy-independent ratios $R \equiv \phi(\nu_\mu)/(\phi(\nu_e) + \phi(\nu_\tau))$ and $S \equiv \phi(\nu_e)/\phi(\nu_\tau)$ among the integrated neutrino flux. The ranges of neutrino mixing parameters used in this analysis are summarized in Eq. (11). By just measuring R alone from either an input pion source or an input muon-damped source with a precision $\Delta R/R = 10\%$, the reconstructed 3σ range for the initial neutrino flavor ratio is almost as large as the entire physical range for the above ratio. By measuring both R and S from an input muon-damped source, the pion source can be ruled out at the 3σ level for parameter sets 1 and 2 with $\Delta R_\mu/R_\mu = 10\%$ and $\Delta S_\mu/S_\mu$ related to the former by the Poisson statistics. With a pion source as the input true source and the choice of parameter set 1 for our analysis, the muon-damped source cannot be ruled out at the 3σ level until $\Delta R_\pi/R_\pi$ and $\Delta S_\pi/S_\pi$ reach to 6% and 7%, respectively. In the case $(\sin^2\theta_{13})_{\text{best fit}} > 0$ as suggested by Ref. [47], the CP phase δ is seen to affect the reconstructed range for the neutrino flavor ratio at the source. We have also presented results for $\Delta S_\mu/S_\mu$ and $\Delta S_\pi/S_\pi$ fixed at 25% and 15%, respectively. To distinguish the pion source and the muon-damped source in this case, both $\Delta R_\mu/R_\mu$ and $\Delta R_\pi/R_\pi$ should be of the order 1% or smaller.

We have also performed a statistical analysis with the errors of θ_{23} and θ_{12} both reduced and the limit of θ_{13} improved to $\sin^2\theta_{13} < 0.0025$ (i.e., $\sin^2 2\theta_{13} < 0.01$). The result of this analysis can be best described by the modification to the left panel of Fig. 3. With $\sin^2\theta_{13} < 0.0025$, it is possible to rule out the muon-damped source at the 3σ level for an input pion source by reducing the errors of both θ_{23} and θ_{12} to 70% of their original values.

We like to point out that our analysis has been based upon the ideal scenario that the true astrophysical neutrino source is either a pure pion source or a pure muon-damped source. In practice, the neutrino flavor ratio in a single astrophysical source can depend on neutrino energies such that it behaves like the one from a pion source at the low energy and gradually makes a transition to the one from a muon-damped source as the neutrino energy increases [28]. Hence the reconstruction of the source flavor ratio ought to be carried out separately for low and high energy portions of the data. Furthermore, an analysis on the diffuse neutrino flux is challenging since such a flux arises from astrophysical sources with different neutrino flavor ratios. Nevertheless, the very high energy part of the flux spectrum is possibly dominated by the cosmogenic neutrino flux [7] arising from Greisen-Zatsepin-Kuz'min [52,53] interactions. The cosmogenic neutrino flux is a typical example of neutrino fluxes due to the pion source. Therefore it is sensible to reconstruct the source flavor ratio with respect to the highest energy part of the diffuse neutrino spectrum, provided there are a sufficient number of events.

The accuracy $\Delta R_i/R_i = 10\%$ $(i = \mu, \pi)$ frequently used in our discussions requires $\mathcal{O}(100)$ neutrino events for each flavor. Furthermore, the above accuracy requires an improved understanding on the background atmospheric neutrino flux to a level better than 10% in the future. Taking a neutrino flux upper bound $E^2\phi(\nu_\alpha) = 10^{-7}\,\text{GeV}\,\text{cm}^{-2}\,\text{s}^{-1}$ $(\alpha = e,\ \mu,\ \tau)$ derived by Waxman and Bahcall [34], we estimate by a simple rescaling of the result in Ref. [54] that it takes the IceCube detector about a decade to accumulate $\mathcal{O}(100)$ ν_μ events. We stress that the above bound is for the diffuse neutrino flux. The flux from the individual point source is smaller. Hence it takes even a longer period to accumulate the same number of events. The IceRay [6] detector is expected to accumulate neutrino events in a much faster pace. However the efficiency of flavor identification in this detector still requires further studies.

In summary, we have demonstrated that it is challenging to reconstruct the neutrino flavor ratio at the astrophysical source, requiring a lot more than a decade of data taking in a neutrino telescope such as IceCube for distinguishing between the pion source and the muon-damped source. We stress that the large uncertainty in the flavor ratios of astrophysical neutrinos should be taken into account as one uses these neutrinos as a beam source to extract the neutrino mixing parameters.

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Note added.—As we were writing up this paper, we became aware of a paper by A. Esmaili and Y. Farzan [arXiv:0905.0259] which also discusses the initial flavor composition of cosmic neutrinos with an approach different from ours.

APPENDIX: THE EXACT OSCILLATION PROBABILITIES OF ASTROPHYSICAL NEUTRINOS

The exact analytic expressions for the components of P are given by

$$
\begin{aligned}
P_{ee} &= (1 - \tfrac{1}{2}\omega)(1 - D^2)^2 + D^4, \\
P_{e\mu} &= \tfrac{1}{4}(1 - D^2)[\omega(1 + \Delta) + (4 - \omega)(1 - \Delta)D^2 + 2\sqrt{\omega(1 - \omega)(1 - \Delta^2)}D\cos\delta], \\
P_{e\tau} &= \tfrac{1}{4}(1 - D^2)[\omega(1 - \Delta) + (4 - \omega)(1 + \Delta)D^2 - 2\sqrt{\omega(1 - \omega)(1 - \Delta^2)}D\cos\delta], \\
P_{\mu\mu} &= \tfrac{1}{2}[(1 + \Delta^2) - (1 - \Delta)^2D^2(1 - D^2)] - \tfrac{1}{8}\omega[(1 + \Delta)^2 + (1 - \Delta)^2D^4 - (1 - \Delta^2)D^2(2 + 4\cos^2\delta)] \\
&\quad - \tfrac{1}{2}\sqrt{\omega(1 - \omega)(1 - \Delta^2)}[(1 + \Delta) - (1 - \Delta)D^2]D\cos\delta, \\
P_{\mu\tau} &= \tfrac{1}{2}(1 - \Delta^2)(1 - D^2 + D^4) - \tfrac{1}{8}\omega[(1 - \Delta^2)(1 + 4D^2\cos^2\delta + D^4) - 2(1 + \Delta^2)D^2] \\
&\quad + \tfrac{1}{2}\sqrt{\omega(1 - \omega)(1 - \Delta^2)}\Delta(1 + D^2)D\cos\delta, \\
P_{\tau\tau} &= \tfrac{1}{2}[(1 + \Delta^2) - (1 + \Delta)^2D^2(1 - D^2)] - \tfrac{1}{8}\omega[(1 - \Delta)^2 + (1 + \Delta)^2D^4 - (1 - \Delta^2)D^2(2 + 4\cos^2\delta)] \\
&\quad + \tfrac{1}{2}\sqrt{\omega(1 - \omega)(1 - \Delta^2)}[(1 - \Delta) - (1 + \Delta)D^2]D\cos\delta, \qquad \text{(A1)}
\end{aligned}
$$

where $\omega \equiv \sin^2 2\theta_{12}$, $\Delta \equiv \cos 2\theta_{23}$, $D \equiv \sin\theta_{13}$, and δ the CP phase.

[1] P. Berghaus (IceCube Collaboration), Nucl. Phys. B, Proc. Suppl. **190**, 127 (2009).
[2] The KM3NeT Collaboration, http://www.km3net.org.
[3] For a recent review, see V. S. Berezinsky, arXiv:0901.1428.
[4] P. Gorham *et al.* (ANITA Collaboration), arXiv:0812.2715.
[5] T. P. A. Collaboration, arXiv:0903.3385.
[6] P. Allison *et al.*, arXiv:0904.1309.
[7] V. S. Berezinsky and G. T. Zatsepin, Phys. Lett. **28B**, 423 (1969); R. Engel, D. Seckel, and T. Stanev, Phys. Rev. D **64**, 093010 (2001).
[8] J. F. Beacom, N. F. Bell, D. Hooper, S. Pakvasa, and T. J. Weiler, Phys. Rev. D **69**, 017303 (2004).
[9] S. Pakvasa, Yad. Fiz. **67**, 1179 (2004) [Mod. Phys. Lett. A **19**, 1163 (2004)].
[10] M. L. Costantini and F. Vissani, Astropart. Phys. **23**, 477 (2005); F. Vissani, Astropart. Phys. **26**, 310 (2006); arXiv: astro-ph/0609575.
[11] P. Bhattacharjee and N. Gupta, arXiv:hep-ph/0501191.
[12] P. D. Serpico and M. Kachelriess, Phys. Rev. Lett. **94**, 211102 (2005).
[13] P. D. Serpico, Phys. Rev. D **73**, 047301 (2006).
[14] Z. Z. Xing and S. Zhou, Phys. Rev. D **74**, 013010 (2006).
[15] W. Winter, Phys. Rev. D **74**, 033015 (2006).
[16] Z. Z. Xing, Phys. Rev. D **74**, 013009 (2006).
[17] D. Majumdar and A. Ghosal, Phys. Rev. D **75**, 113004 (2007).
[18] W. Rodejohann, J. Cosmol. Astropart. Phys. 01 (2007) 029.
[19] D. Meloni and T. Ohlsson, Phys. Rev. D **75**, 125017 (2007).
[20] K. Blum, Y. Nir, and E. Waxman, arXiv:0706.2070.
[21] G. R. Hwang and S. Kim, Phys. Rev. D **78**, 093008 (2008).
[22] S. Pakvasa, W. Rodejohann, and T. J. Weiler, J. High Energy Phys. 02 (2008) 005.
[23] S. Choubey, V. Niro, and W. Rodejohann, Phys. Rev. D **77**, 113006 (2008).
[24] M. Maltoni and W. Winter, J. High Energy Phys. 07 (2008) 064.
[25] J. G. Learned and S. Pakvasa, Astropart. Phys. **3**, 267 (1995).
[26] H. Athar, M. Jezabek, and O. Yasuda, Phys. Rev. D **62**, 103007 (2000); L. Bento, P. Keranen, and J. Maalampi, Phys. Lett. B **476**, 205 (2000).
[27] J. P. Rachen and P. Meszaros, Phys. Rev. D **58**, 123005 (1998).
[28] T. Kashti and E. Waxman, Phys. Rev. Lett. **95**, 181101 (2005).
[29] M. Kachelriess, S. Ostapchenko, and R. Tomas, Phys. Rev. D **77**, 023007 (2008).
[30] L. A. Anchordoqui, H. Goldberg, F. Halzen, and T. J. Weiler, Phys. Lett. B **593**, 42 (2004).
[31] O. Mena, I. Mocioiu, and S. Razzaque, Phys. Rev. D **75**, 063003 (2007).
[32] For an exception, see Ref. [14] where the authors discuss the implications of varying the initial neutrino flavor ratio.
[33] J. F. Beacom, N. F. Bell, D. Hooper, S. Pakvasa, and T. J. Weiler, Phys. Rev. D **68**, 093005 (2003); **72**, 019901(E) (2005).
[34] E. Waxman and J. N. Bahcall, Phys. Rev. D **59**, 023002 (1998).
[35] T. K. Gaisser and M. Honda, Annu. Rev. Nucl. Part. Sci. **52**, 153 (2002).
[36] G. D. Barr, S. Robbins, T. K. Gaisser, and T. Stanev, Phys. Rev. D **74**, 094009 (2006).
[37] R. Abbasi *et al.* (IceCube Collaboration), Phys. Rev. D **79**, 102005 (2009).
[38] C. G. S. Costa, F. Halzen, and C. Salles, Phys. Rev. D **66**, 113002 (2002).
[39] E. V. Bugaev *et al.*, Phys. Rev. D **58**, 054001 (1998).
[40] A. D. Martin, M. G. Ryskin, and A. M. Stasto, Acta Phys.

Pol. B **34**, 3273 (2003).

[41] For comparisons between conventional and prompt atmospheric neutrino flux, see, for example, J. F. Beacom and J. Candia, J. Cosmol. Astropart. Phys. 11 (2004) 009; H. Athar, F. F. Lee, and G. L. Lin, Phys. Rev. D **71**, 103008 (2005).

[42] See, for example, A. Achterberg *et al.* (IceCube Collaboration), Phys. Rev. D **75**, 102001 (2007); **76**, 042008 (2007); **77**, 089904(E) (2008).

[43] M. C. Gonzalez-Garcia and M. Maltoni, Phys. Rep. **460**, 1 (2008), and references therein.

[44] M. C. Gonzalez-Garcia, M. Maltoni, and A. Y. Smirnov, Phys. Rev. D **70**, 093005 (2004), and references therein.

[45] S. Choubey and P. Roy, Phys. Rev. D **73**, 013006 (2006).

[46] P. Huber, M. Lindner, M. Rolinec, T. Schwetz, and W. Winter, Phys. Rev. D **70**, 073014 (2004).

[47] G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo, and A. M. Rotunno, Phys. Rev. Lett. **101**, 141801 (2008).

[48] H. Athar, G. Parente, and E. Zas, Phys. Rev. D **62**, 093010 (2000).

[49] F. Halzen and D. Saltzberg, Phys. Rev. Lett. **81**, 4305 (1998).

[50] J. F. Beacom, P. Crotty, and E. W. Kolb, Phys. Rev. D **66**, 021302(R) (2002).

[51] It is important to clarify that we focus on the energy range $E_\nu > 10^4$ GeV when we refer to the class of sources discussed in Ref. [31]. This is to match the assumed neutrino energy range for the pion source and the muon-damped source. However, Mena *et al.* focused on the energy range $E_\nu < 10^4$ GeV for identifying astrophysical hidden sources from the neutrino energy spectra.

[52] K. Greisen, Phys. Rev. Lett. **16**, 748 (1966).

[53] G. T. Zatsepin and V. A. Kuz'min, JETP Lett. **4**, 78 (1966).

[54] J. Ahrens *et al.* (IceCube Collaboration), Astropart. Phys. **20**, 507 (2004).