



## Innovative reliability allocation using the maximal entropy ordered weighted averaging method<sup>☆</sup>

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### ABSTRACT

Reliability allocation is one of the most important factors to consider when determining the reliability and competitiveness of a product. The feasibility-of-objectives (FOO) technique has become the current standard for assessing reliability designs for military mechanical–electrical systems, whereas the average weighting allocation method is widely used for commercial applications. However, assessment results are biased because these methods share two fundamental problems. The first problem is the measurement scale, while the second problem is that the system allocation factors are not equally weighted to one another. Both problems represent serious flaws from a technical perspective. To address these issues, we propose the use of the maximal entropy ordered weighted averaging (ME-OWA) method, which efficiently resolves the shortcomings of the FOO technique and the average weighting allocation method. As a comparative case study between the ME-OWA method and the two standards used in the military and commercially, this study evaluates reliability allocation in the context of a fighter aircraft airborne radar system. The results from this comparison show that the proposed method is both accurate and flexible.

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### 1. Introduction

Reliability allocation is one of the most important factors to consider when determining the reliability and competitiveness of a product, and is also an important analytical tool that can be used for improving system reliability. In recent years, attention to system reliability has risen because of the increase in sophistication of engineering systems used in high-tech industrial processes.

Reliability allocation is a top-down method for apportioning accuracy goals in a system. The Advisory Group on Reliability of Electronic Equipment (Advisory Group of Reliability of Electronic Equipment (AGREE), 1957) developed a method for reliability apportionment. This method is based on unit or subsystem complexity and criticality rather than failure rates. In contrast to this method, Aeronautical Radio Inc. (Alven, 1964) published the AR-INC apportionment technique, which is based on the failure rates of units or subsystems. In addition to these methods, Bracha (1964) introduced an allocated reliability method using four factors: state-of-the-art, subsystem complexity as estimated by number of parts, environmental conditions, and relative operating time, whereas Karmiol (1965) evaluated the complexity, state-of-the-art, operational profile, and criticality of the system to mission objectives to apportion subsystem reliability. More recently, the engineering design guide, Reliability Design Handbook

(Anderson, 1976), featured the feasibility-of-objectives (FOO) technique, which was incorporated into the Mil-hdbk-338B handbook (United States Department of Defense, 1988), an established standard for military reliability design. The FOO technique specifically provides a detailed reliability allocation procedure for mechanical–electrical systems, which Smedley (1992) employed to perform reliability analysis among low energy booster (LEB) ring magnet power systems in a superconducting supercollider. In addition, Kuo (1999) created an average weighting allocation method as a guide for commercial reliability allocation design, while Falcone, Silvestri, and Di Bona (2003) used the integrated factors method (IFM) for reliability allocation for an aerospace prototype project. This method evaluates four factors, criticality (C), complexity (K), functionality (F), and effectiveness (O), to calculate system reliability. Unfortunately, the Karmiol method, the FOO technique, the average weighting allocation method, and the IFM method all share a common weakness in their measurement scale. The first problem with these methods is system factors are evaluated according to discrete ordinal scales of measure; in particular, multiplication is not meaningful and in fact misleading. The second problem is that the system factors are not equally weighted, thereby creating problems with analysis and interpretation of the results.

In resolving the FOO technique and average weighting allocation problems, the proposed approach is based on the traditional reliability allocation method, which uses Yager's OWA (1988) and the ME-OWA (Fuller & Majlender, 2001; Cheng & Chang, 2006; Cheng, Cheng, & Chang, 2008) operators. Yager (1988) first introduced the

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concept of OWA operators to solve the problems described here with the FOO technique. Additionally, Fuller and Majlender (2001) used Lagrange multipliers on Yager's OWA equation to derive a polynomial equation, which determines the optimal weighting vector under maximal entropy (ME-OWA operator). The proposed approach thus determines the optimal weighting vector under maximal entropy, and the OWA operator ascertains the optimal reliability allocation rating after an aggregation process. This method is both a simple and effective approach that can efficiently resolve the shortcomings of the FOO technique and average weighting allocation.

The remainder of this paper is organized as follows: Section 2 introduces conventional reliability allocation methods, Section 3 introduces ME-OWA operations and applications, Section 4 proposes the ME-OWA method, and in Section 5, an example is drawn from an aircraft airborne radar system using the proposed approach for reliability allocation assessment. Section 6 is the conclusion.

## 2. Conventional reliability allocation methods

In a large complex system, it is necessary to translate reliability requirements into subsystems. The allocation technique is essential when different design teams, subcontractors, or manufacturers are involved. Currently, many reliability allocation techniques are available, including the equalization allocation method (Department of Defense of USA, 1988), ARINC (Alven, 1964), Advisory Group of Reliability of Electronic Equipment (AGREE, 1957), pair comparison allocation (Kuo, 1999), the FOO technique (Department of Defense of USA, 1988), the minimization of effort algorithm (Department of Defense of USA, 1988), and the average weighting allocation method (Kuo, 1999). Because the FOO technique and average weighting allocation method are important methods in reliability allocation design, the basic definition and procedure of the FOO technique and average weighting allocation method are reviewed in this section, including specific shortcomings of the FOO technique and average weighting allocation method.

### 2.1. The FOO technique

The FOO technique was first introduced in 1976 and is included in the Mil-hdbk-338B Electronic Reliability Design Handbook (Department of Defense of USA, 1988) as a method for developing and implementing sound reliability programs for all types of military products. Since its introduction, the Mil-hdbk-338B has gained recognition as a standard for evaluating reliability of military-related products. With the FOO method, subsystem allocation factors are computed as a function of a numerical rating of system intricacy ( $I$ ), state-of-the-art ( $S$ ), performance time ( $P$ ), and environment ( $E$ ). Each rating is based on a scale from 1 to 10, and is estimated using design engineering and expert judgments. The four respective rating values are then multiplied to derive the *ISPE*, i.e.  $ISPE = I \times S \times P \times E$ , so that the final product results in a value ranging from 1 to 10,000. They may also be determined by a group of engineers using a voting method, such as the Delphi technique. For the  $I$  factor, the least intricate system is rated as 1, and the most highly intricate system is rated as 10. For the  $S$  factor, the least developed design or method is assigned a value of 10, and the most highly developed is assigned a value of 1. For the  $P$  factor, the element that operates for the entire mission time is rated 10, and the element that operates the least time during the mission is rated as 1. For the  $E$  factor, elements expected to experience harsh and very severe environments during their operation are rated as 10, and those expected to encounter the least severe environments are rated as 1. The subsystem ratings are then normalized so that their sum is equal to 1.

Suppose a system is composed of  $N$  subsystems. Let  $\lambda_s$  be the system failure rate and let  $T$  be the mission duration. Also, let  $\bar{\lambda}_k$

be the failure rate allocated to the  $k$ th subsystem,  $C'_k$  be the complexity of the  $k$ th subsystem, and  $w'_k$  be the rating for the  $k$ th subsystem –  $\forall k$ .  $W'$  denotes the sum of the rated products, and  $r'_{ik}$  is used to represent the rating for each of the four factors for the  $k$ th subsystem,  $\forall k$  and  $\forall i \in \{I, S, P, E\}$ .

The reliability allocation weighing factor is determined by equations Eqs. (1)–(5).

$$\lambda_s T = \bar{\lambda}_k T \quad (1)$$

$$\bar{\lambda}_k = C'_k \lambda_s, \quad \forall k \quad (2)$$

$$C'_k = \frac{W'_k}{W'}, \quad \forall k \quad (3)$$

$$W'_k = r'_{Ik} \times r'_{Sk} \times r'_{Pk} \times r'_{Ek}, \quad \forall k \quad (4)$$

$$W' = \sum_{k=1}^N W'_k \quad (5)$$

### 2.2. Average weighting allocation method

Kuo (1999) created an average weighting allocation method as a guide for reliability allocation designs. The method uses a questionnaire investigation approach to select the most influential system reliability factors, such as complexity, state-of-the-art, system criticality, environment, safety, and maintenance, to determine the subsystem reliability allocation ratings. Each rating, on a scale from 1 to 10, is estimated using design engineering and expert judgments to obtain the subsystem reliability ratio.

Suppose a system is composed of  $m$  subsystems. Let  $R_s$  be the system allocated rating,  $R_i$  be the allocated rating to the  $i$ th subsystem,  $n$  be the number of system factors, and  $p$  be the number of experts. Let  $Y_{ij}$  denote the  $j$ th rating for subsystem  $i$ .  $X_{Lij}$  is the  $j$ th rating for subsystem  $i$  set by the  $L$ th expert;  $w_i$  is used to represent the composite rating for the subsystem  $i$ .

The reliability allocation weighing factor is determined by Eqs. (6)–(10).

$$Y_{ij} = \left( \sum_{L=1}^p X_{Lij} \right) / p, \quad \forall i, j \quad (6)$$

Two different models can be used to allocate weighting factors  $w_i$ ,  $\forall i$ :

(1) Geometric model:

$$w_i = \prod_{j=1}^n Y_{ij} / \sum_{i=1}^m \prod_{j=1}^n Y_{ij} \quad (7)$$

(2) Arithmetic model:

$$w_i = \sum_{j=1}^n Y_{ij} / \sum_{i=1}^m \sum_{j=1}^n Y_{ij} \quad (8)$$

According to the weighting factor  $w_i$ , the rating allocation to the  $i$ th subsystem  $R_i$  can be calculated as:

$$R_i = (R_s)^{w_i} = 1 - w_i \times [1 - R_s] \quad (9)$$

Assuming the failure rates are exponentially distributed, the failure rate for  $i$ th subsystem can be calculated by Eq. (10).

$$\lambda_i = w_i \times \lambda_s \quad (10)$$

### 2.3. Shortcomings of the FOO technique and average weighting allocation method

The FOO technique and the average weighting allocation method have been widely adopted in reliability allocation. However, these methods have been criticized for their two fundamental

shortcomings. The first problem with this method is that the four system factors *I*, *S*, *P* and *E* are evaluated according to discrete ordinal scales of measure; in particular, multiplication is not meaningful and in fact misleading. The second problem is that the four system factors are not equally weighted, thereby creating problems with analysis and interpretation of the results. For example, for two components with ISPE values of  $8 \times 2 \times 2 \times 2 = 64$  and  $6 \times 3 \times 2 \times 2 = 72$ , respectively, the former should have had a higher reliability allocation overall rating than the latter, even though it has a lower ISPE value. Detailed descriptions of these problems are provided in the following subsections.

2.3.1. Misleading measurement scale

The essence of measurement is to separate elements into categories based on the property of measurement. If the source of data is equipped with biases, ambiguities, or other types of flaws, even the most sophisticated statistical method cannot produce accurate information. Bowles (2003) and Evie (2008) indicate that ordinal measurement scales are frequently used; but operations of multiplication and division are not meaningful on ordinal numbers, and addition and subtraction, while sometimes meaningful, must be done carefully because they assume an equal interval between category labels. As presented by Reaves (1992), the basic four types of measurement scales are:

- (1) Nominal measurements: a nominal scale classifies data into distinct categories in which no ranking is implied. Nominal measurements are the most basic type of measurements, but nominal scaling is the weakest form of measurement because ranking across the various categories cannot be specified. For example, the property of the type of stocks owned can be represented by such names as “growth”, “value”, “other”, “none”, etc. Questions of “more” or “less” have no meaning to nominal measurements.
- (2) Ordinal measurements: the ordinal measurement is a stronger measurement compared to the nominal. It specifies the order in which categories are to be placed from first to last. Its weakness is the absence of a measurement of how far apart the various categories are to each other; using ordinal measurements, the numbers, and sometimes letters of the alphabet or words are often used as symbols.
- (3) Equal interval measurements: an equal interval measurement is an ordered scale in which the difference between measurements is a meaningful quantity. Its weakness is that measurements on interval scales often have negative values, which means that the numbers cannot be meaningfully multiplied or divided.
- (4) Ratio measurements: a ratio scale is an ordered scale in which the difference between the measurements involves true zero points, as in height, weight, age, or salary measurements, for example. It is the only scale for which the numbers can be meaningfully multiplied and divided.

The FOO technique uses four system factors to rate and assess the reliability allocation. A subsystem with a higher ISPE is assumed to be allocated a higher rating than one with a lower ISPE. The fundamental problem of the FOO technique is that the four parameters *I*, *S*, *P* and *E* are evaluated according to discrete ordinal scales of measure. Because multiplication is not meaningful, the results are in fact misleading. Since an ISPE value is obtained by multiplying the *I*, *S*, *P* and *E* factors, the multiplication magnifies the misleading effect.

2.3.2. Unequal weighting of *I*, *S*, *P* and *E*

Although as many as 10,000 numbers are possible products of *I*, *S*, *P* and *E*, only six of them are unique (only six ISPE values are

formed by a single, unique combination of *I*, *S*, *P* and *E*). Most ISPE values are non-unique, some being recycled as many as 180 times (ISPE = 360). For example, as shown in Table 1, ISPE = 1280 can result from 36 different combinations of *I*, *S*, *P* and *E*. It is difficult to accept that the 36 different combinations of *I*, *S*, *P* and *E* have the same reliability allocation overall rating. As a result, some (*I*, *S*, *P*, *E*) scenarios produce an ISPE value that is lower than other combinations but potentially produce a higher reliability allocation overall rating. For example, the scenario with ISPE value  $9 \times 5 \times 2 \times 2 = 180$  is lower than the scenario with ISPE value  $7 \times 5 \times 3 \times 2 = 210$  even though it should have a higher reliability allocation overall rating. Therefore, *I*, *S*, *P* and *E* are not equally weighted with respect to one another in terms of overall rating.

3. ME-OWA operators and its operations

3.1. ME-OWA operators

Yager (1988) first introduced the concept of OWA operators, which are important aggregation operators within the class of weighted aggregation methods. It has the ability to derive optimal weights of the attributes based on the rating of the weighting vectors after an aggregation process (see Definition 1).

**Definition 1.** An OWA operator of dimension *n* is mapped  $F: R^n \rightarrow R$ , which has an associated *n* weighting vector  $W = [w_1, w_2, \dots, w_n]^T$  of the properties  $\sum_i w_i = 1, \forall w_i \in [0, 1], i = 1, \dots, n$ , such that

$$f(a_1, a_2, \dots, a_n) = \sum_{i=1}^n w_i b_i \tag{11}$$

where  $b_i$  is the *i*th largest element in the vector  $(a_1, a_2, \dots, a_n)$ , and  $b_1 \geq b_2 \geq \dots \geq b_n$ .

Yager (1988) also introduced two important characterizing measurements with respect to the weighting vector *W* of the OWA operator. One of these two measures is orness of the aggregation, which is defined in Definition 2.

**Definition 2.** Assume *F* is an OWA aggregation operator with a weighting function  $W = [w_1, w_2, \dots, w_n]$ . The degree of orness associated with this operator is defined as:

$$Orness(W) = \frac{1}{n-1} \sum_{i=1}^n (n-i)w_i \tag{12}$$

where *orness* (*W*) =  $\alpha$  is a situation parameter.

It is clear that *orness* (*W*)  $\in [0, 1]$  holds for any weighting vector.

The second characterizing measurement introduced by Yager (1988) is a measure of dispersion of the aggregation, which is defined in Definition 3.

**Definition 3.** Assume *W* is a weighting vector with elements  $w_1, \dots, w_n$ ; then the measure of dispersion of *W* is defined as:

**Table 1**  
Thirty-six combinations of *I*, *S*, *P* and *E* yield an ISPE of 1280.

<i>I</i>	<i>S</i>	<i>P</i>	<i>E</i>	<i>I</i>	<i>S</i>	<i>P</i>	<i>E</i>	<i>I</i>	<i>S</i>	<i>P</i>	<i>E</i>	<i>I</i>	<i>S</i>	<i>P</i>	<i>E</i>
10	2	8	8	8	8	10	2	8	2	10	8	4	10	4	8
10	8	4	4	8	4	5	8	8	8	2	10	4	8	10	4
10	8	8	2	8	5	4	8	8	8	5	4	4	10	8	4
10	4	4	8	8	10	8	2	5	4	8	8	4	8	4	10
10	4	8	4	8	4	10	4	5	8	4	8	4	8	8	5
10	8	2	8	8	4	4	10	5	8	8	4	4	4	10	8
8	10	4	4	8	5	8	4	4	5	8	8	2	10	8	8
8	8	4	5	8	10	2	8	4	4	8	10	2	8	8	10
8	2	8	10	8	4	8	5	4	8	5	8	2	8	10	8

$$Dispersion (W) = - \sum_{i=1}^n w_i \ln w_i \tag{13}$$

O'Hagan (1988) combined the principle of maximum entropy and OWA operators to propose a particular OWA weight that has maximum entropy with a given level of orness. This approach is based on the solution of the following mathematical programming problem:

$$\text{Maximize } - \sum_{i=1}^n w_i \ln w_i \tag{14}$$

$$\text{Subject to: } \frac{1}{n-1} \sum_{i=1}^n (n-i)w_i = \alpha, \quad 0 \leq \alpha \leq 1, \tag{15}$$

$$\sum_{i=1}^n w_i = 1, \quad 0 \leq w_i \leq 1 \leq 1, \quad i = 1, \dots, n, \tag{16}$$

### 3.2. Determination of ME-OWA weights

Fuller and Majlender (2001) used the method of Lagrange multipliers on Yager's OWA equation to derive a polynomial equation, which can determine the optimal weighting vector under the maximal entropy. By their method, the associated weighting vector is easily obtained by Eqs. (17)–(19).

$$\ln w_j = \frac{j-1}{n-1} \ln w_n + \frac{n-j}{n-1} \ln w_1 \Rightarrow w_i = \sqrt[n-1]{w_1^{n-j} w_n^{j-1}} \tag{17}$$

$$\text{and } w_n = \frac{((n-1)\alpha - n)w_1 + 1}{(n-1)\alpha + 1 - nw_1} \tag{18}$$

$$\text{then } w_1[(n-1)\alpha + 1 - nw_1]^n = ((n-1)\alpha)^{n-1} [(n-1)\alpha - n)w_1 + 1] \tag{19}$$

where  $w$  is the weight vector,  $n$  is the number of attributes, and  $\alpha$  is the situation parameter.

## 4. Proposed ME-OWA method

### 4.1. Advantages of the ME-OWA method

To resolve the misleading problems resulting from the FOO technique and average weighting allocation method, this paper proposes an approach to include ME-OWA operators in the FOO technique. The major advantage of the proposed ME-OWA method is that it uses the OWA operator to derive a polynomial equation to determine the optimal weighting vector under maximal entropy. In addition, it can overcome fundamental shortcomings of the FOO technique and average weighting allocation method. The proposed approach can determine the optimal weighting vector under maximal entropy, and the OWA operator has the ability to ascertain the optimal reliability allocation rating after an aggregation process. The proposed ME-OWA method provides a conditional parameter  $\alpha$  ( $\alpha = 0.5, 0.6, 0.7, 0.8, 0.9, 1$ ) to flexibly compute the reliability allocation value;  $\alpha = 1$  is used to represent the situation when the decision-maker is maximally optimistic (a pure optimistic), and  $\alpha = 0.5$  is used when the decision-maker faces a moderate assessment. The conditional parameter is particularly useful when available information is imprecise, incomplete, or uncertain in the reliability allocation design phase.

With the optimal weighting vector under maximal entropy with respect to different  $\alpha$  values, sensitivity analysis enables the identification of different  $\alpha$  values to evaluate their impact on the reliability allocation rating using Eqs. (17)–(19) with  $n = 4$ . Results from this analysis are presented in Table 2.

### 4.2. Procedures of the ME-OWA method

The procedure of the proposed ME-OWA reliability allocation method is organized into nine steps and is described as follows:

Step 1. List the structure of systems and subsystems.

Step 2. Define the system reliability and mission time.

Step 3. Determine the scales for  $I, S, P$  and  $E$ , respectively, for each subsystem.

Subsystem allocation factors are computed as a function of numerical ratings of  $I, S, P$  and  $E$ .

Step 4. Compute the allocation rating  $r'$  for each subsystem and derive the overall rating  $w'_k$  for the  $k$ th subsystem.

$r'$  is used to represent the rating for each of the four factors for the  $k$ th subsystem.

Based on Table 2 and Eq. (11), the aggregated value by OWA weights is calculated.

Step 5. Compute the complexity  $C'_k$  for the  $k$ th subsystem,  $\forall k$ .

Use Eq. (3) to calculate complexity  $C'_k, \forall k$ .

Step 6. Compute the system failure rate from system specifications. Let  $R$  be the system reliability and  $T$  be the mission duration; the system failure rate  $\lambda_s$  is computed as  $\lambda_s = -\ln(R)/T$ .

Step 7. Compute the allocated subsystem failure rate.

Use Eq. (2) to calculate the allocated subsystem failure rate  $\lambda_k, \forall k$ .

Step 8. Compute reliability allocation values by assigning different values to the conditional parameter  $\alpha$  ( $\alpha = 0.5, 0.6, 0.7, 0.8, 0.9, 1$ ).

Based on Table 2 and Eq. (11), calculate the aggregated values by OWA weights with respect to different values of  $\alpha$  ( $\alpha = 0.5, 0.6, 0.7, 0.8, 0.9, 1$ ).

Step 9. Analyze the results and select the optimal reliability allocation decision.

## 5. A case study of the ME-OWA method

A case study of an airborne radar system for a fighter aircraft drawn from an aircraft company in Taiwan was used to demonstrate the proposed approach. An airborne radar system is a modern, digital, computer-controlled system that provides a full range of air-to-air capabilities, which include look-up and look-down range with search and situation awareness capability. The radar system also provides a full range of air-to-surface modes, such as ground mapping with expansion, Doppler beam sharpening, freezing over land/sea, moving target indication, and tracking. The airborne radar consists of five major line replaceable units (LRUs), an equipment rack, and ancillary installation materials. The LRUs are the antenna (ANT), transmitter (TRAN), radar target data processor (RTDP), radar data computer (RDC) and the filter. The structure of the airborne radar system is shown in Fig. 1.

Based on the design requirements and the system operational environment, the system reliability of the fighter aircraft airborne radar system is set as 0.9971429 and the mission time as 2.4 h. For ease of comparing the three methods described in this paper, we use the same four system factors as allocation reliability weighting:  $I, S, P$ , and  $E$ . The results of the proposed ME-OWA method are compared with the FOO technique and the average weighting allocation methods below.

**Table 2**  
The optimal weighting vector under maximal entropy ( $n = 4$ ).

Weight	$w_1$	$w_2$	$w_3$	$w_4$
$\alpha = 0.5$	0.250000	0.250000	0.250000	0.250000
$\alpha = 0.6$	0.416657	0.233398	0.130859	0.073547
$\alpha = 0.7$	0.493805	0.237305	0.113770	0.054918
$\alpha = 0.8$	0.596466	0.251953	0.106445	0.045018
$\alpha = 0.9$	0.764099	0.182129	0.043457	0.010365
$\alpha = 1.0$	1.000000	0.000000	0.000000	0.000000

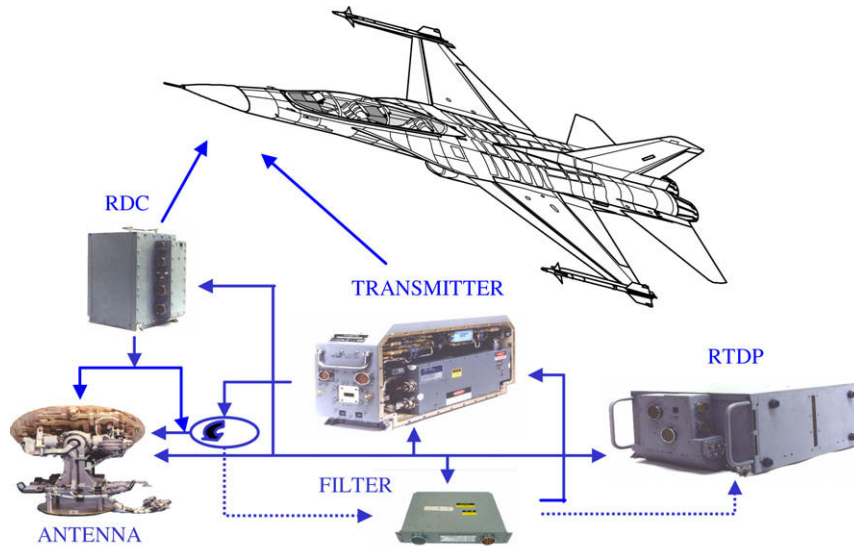


Fig. 1. The structure of the airborne radar system.

5.1. FOO technique analysis

The FOO technique is a detailed reliability allocation procedure for mechanical–electrical systems. Using design engineering expert judgments, the function of numerical ratings of *I*, *S*, *P* and *E* conditions were estimated; the estimated ratings are shown in Table 3, columns (2) through (5), using Eqs. (1)–(5). We computed the overall rating  $w'_k$ , and complexity factors  $C'_k$ , and then ascertained the allocated failure rate.

If the system reliability is 0.9971429 and the mission time is 2.4 h, then the failure rate for RTDP can be calculated by

$$\lambda_s = -\ln(R)/T = -\ln(0.9971429)/2.4 = 0.0011922$$

$$w'_k = 8 \times 10 \times 8 \times 7 = 4480$$

$$C'_k = \frac{w'_k}{W'} = \frac{4480}{10752} = 0.417$$

Allocated failure rate is  $119.22 \times 0.417 = 49.675$  per  $10^5$  h.

The allocated failure rates for RTDP, RDC, TRAN, ANT, and FILTER can therefore be calculated as 49.675, 15.967, 51.094, 2.218, and 0.266, respectively. The results are summarized in Table 3.

5.2. Average weighting allocation method analysis

The average weighting allocation method is popularly used in reliability design. This method uses a questionnaire investigation approach to select the most influential system reliability factors, such as complexity, state-of-the-art, system criticality, environment, safety, and maintenance, to decide the subsystem reliability allocation rating. In order to compare the different method capabilities, the same influential system reliability factors as the ones

used by the FOO technique were selected: *I*, *S*, *P* and *E*. Also, the same estimated rating derived from design engineering and expert judgment (using Eqs. (6)–(10)) was used to compute the overall rating  $W'_k$ , and complexity factors  $C'_k$ . Following these calculations, the allocated failure rates were determined. Using the geometric model, the results obtained are equal to those obtained by the FOO technique (summarized in Table 3). Using the arithmetic model, the results obtained are shown in column (8) of Table 4.

5.3. Proposed ME-OWA method analysis

The proposed approach uses maximal entropy OWA for weight calculation. A sensitivity analysis using different values of  $\alpha$  is presented to evaluate their impact on the reliability allocation rating. Based on Table 2, the optimal weighting under maximal entropy ( $n = 4$ ), and Eq. (11), the failure rate of RTDP allocated is calculated as follows.

For  $\alpha = 0.5$  (used when the decision-maker faces a moderate assessment), if *I*, *S*, *P* and *E* for subsystem RTDP are 8, 10, 8 and 7, respectively, and the weighting vector contains  $w_1 = 0.25$ ,  $w_2 = 0.25$ ,  $w_3 = 0.25$  and  $w_4 = 0.25$ , then

$$w'_k = (10 \times 0.25) + (8 \times 0.25) + (8 \times 0.25) + (7 \times 0.25) = 8.25$$

$$C'_k = 8.25/29 = 0.28448$$

As a result, the failure rate RTDP (for  $\alpha = 0.5$ ) =  $0.28448 \times 119.22 = 33.91603$  per  $10^5$  h  $\alpha = 1$  is used to represent the situation when the decision-maker is maximally optimistic (a pure optimistic). For  $\alpha = 1$ , the  $OWA(a_1, a_2, a_3) = \text{Max}(a_1, a_2, a_3)$  if *I*, *S*, *P* and *E* for subsystem RTDP are 8, 10, 8 and 7, respectively. The weighting vector contains  $w_1 = 1$ ,  $w_2 = 0$ ,  $w_3 = 0$ , and  $w_4 = 0$ ; consequently,

Table 3 The reliability allocation results for the airborne radar system (FOO technique).

(1) Subsystem	(2) Intricacy ( <i>I</i> )	(3) State-of-the-art ( <i>S</i> )	(4) Performance time ( <i>P</i> )	(5) Environment ( <i>E</i> )	(6) Overall Rating $w'_k$	(7) Complexity $C'_k$	(8) Allocated failure rate (per $10^5$ h)
RTDP	8	10	8	7	4480	0.417	49.675
RDC	6	6	8	5	1440	0.134	15.967
TRAN	8	8	8	9	4608	0.428	51.094
ANT	4	5	5	2	200	0.019	2.218
FILTER	2	2	3	2	24	0.002	0.266
Total	28	31	32	25	10752	1.000	119.220

**Table 4**  
Comparison of airborne radar system failure rate using three methods.

Method	ME-OWA						Average weighting		FOO technique
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1.0$	Geometric	Arithmetic	
RTDP	33.91603	33.81277	33.81353	33.82047	33.88289	34.06286	49.67500	33.91603	49.67500
RDC	25.69397	26.20514	26.32981	26.42644	26.76296	27.25029	15.96696	25.69397	15.96696
TRAN	33.91603	32.28534	31.98839	31.78152	31.19976	30.65657	51.09429	33.91603	51.09429
ANT	16.44414	17.45429	17.55104	17.59214	17.53444	17.03143	2.21763	16.44414	2.21763
FILTER	9.24983	9.46246	9.53723	9.59943	9.83996	10.21886	0.26612	9.24983	0.26612

$$w'_k = (10 \times 1) + (8 \times 0) + (8 \times 0) + (7 \times 0) = 10$$

$$C'_k = 10/35 = 0.285714$$

As a result, the failure rate RTDP (for  $\alpha = 1$ ) =  $0.285714 \times 119.22 = 34.06286$  per  $10^5$  h.

Following the calculation above, the aggregated values of OWA weights by different values of  $\alpha$  ( $\alpha = 0.5, 0.6, 0.7, 0.8, 0.9, 1$ ) are calculated for subsystem RTDP. The resulting failure rates are 33.91603, 33.81277, 33.81353, 33.82047, 33.88289, and 34.06286, respectively. The failure rates for RDC, TRAN, FILTER, and ANT are also calculated, and the results are summarized in Table 4, columns (1) through (6). The comparison of airborne radar system failure rate with respect to the three methods is also shown in Table 4.

5.4. Method comparison

As shown in Table 3, using the FOO technique, the ISPE value of subsystem RTDP ( $8 \times 10 \times 8 \times 7 = 4480$ ) is lower than the ISPE value of subsystem TRAN ( $8 \times 8 \times 8 \times 9 = 4608$ ), even though subsystem RTDP should have a higher reliability allocation overall rating than subsystem TRAN. Furthermore, using the average weighting allocation method (arithmetic model), the ISPE value of the RTDP subsystem ( $8 + 10 + 8 + 7 = 33$ ) is equal to the ISPE value of subsystem TRAN ( $8 + 8 + 8 + 9 = 33$ ). From columns (9) and (10) in Table 5, it can be seen that when using the ME-OWA method (using  $\alpha = 0.9$ ), the ISPE value of subsystem RTDP is 9.518, which is higher than the corresponding ISPE value of subsystem TRAN, 8.764. This result shows that the ME-OWA method obtains a more reasonable reliability allocation rating than the conventional FOO technique and the average weighting allocation method.

A comparison of the conventional FOO technique, the weighting average allocation method, and the proposed ME-OWA approach is summarized in Table 6. “O” indicates that the related factor is applicable, whereas “X” indicates that the related factor is not applicable. Based on this comparison, we have identified a number of issues including:

- (1) A misleading measurement scale problem: from the above results, we can ascertain that the results obtained by the conventional FOO technique equals that obtained when using the geometric model of the average weighting allocation method. However, since the ordinal scale operations of multiplication and division are not only meaningless, but also misleading, the FOO technique cannot help the manager or designer make reasonable decisions. The arithmetic model of the average weighting allocation method assumes an equal interval between the category labels; hence the results are meaningful and reasonable. However, this method does not address the ordered weighted problem.
- (2) Ordered weighted problem: the results from Table 5 show that the results obtained by the arithmetic model of the average weighting allocation method are the same as that obtained by the proposed ME-OWA method when  $\alpha = 0.5$ . However, the proposed ME-OWA method can adjust the conditional parameter  $\alpha$  to calculate OWA weights in order to derive broader information for decision-making. In the presented case study, the results obtained by the arithmetic model of the average weighting allocation method represent a special case of the ME-OWA method. Therefore, the proposed ME-OWA method, as opposed to the conventional FOO technique and the average weighting allocation method, can better help managers or designers make correct decisions.

**Table 6**  
Comparison of the three methods.

Method	Consider factor	
	Measurement scale	Order weight
Proposed method	O	O
FOO technique	X	X
Average weighting allocation (Geometric)	X	X
Average weighting allocation (Arithmetic)	O	X

Note: “O” represents that the factor is applicable, and “X” represents that the factor is not applicable.

**Table 5**  
The reliability allocation results for an airborne radar system.

Method	(1)	(2)	(3)	(4)	FOO technique		Average weighting		ME-OWA ( $\alpha = 0.9$ )	
	Intricity (I)	State-of-the-art (S)	Performance time (P)	Environment (E)	(5)	(6)	(7)	(8)	(9)	(10)
					Overall rating $w/k$	Allocated failure rate (per $10^5$ h)	Overall rating $w/k$	Allocated failure rate (per $10^5$ h)	Overall rating $w/k$	Allocated failure rate (per $10^5$ h)
RTDP	8	10	8	7	4480	49.675	33	33.916	9.518	33.883
RDC	6	6	8	5	1440	15.967	25	25.694	7.518	26.763
TRAN	8	8	8	9	4608	51.094	33	33.916	8.764	31.200
ANT	4	5	5	2	200	2.218	16	16.444	4.926	17.534
FILTER	2	2	3	2	24	0.266	9	9.250	2.764	9.840
Total	28	31	32	25	10752	119.220	116	119.220	33.491	119.220

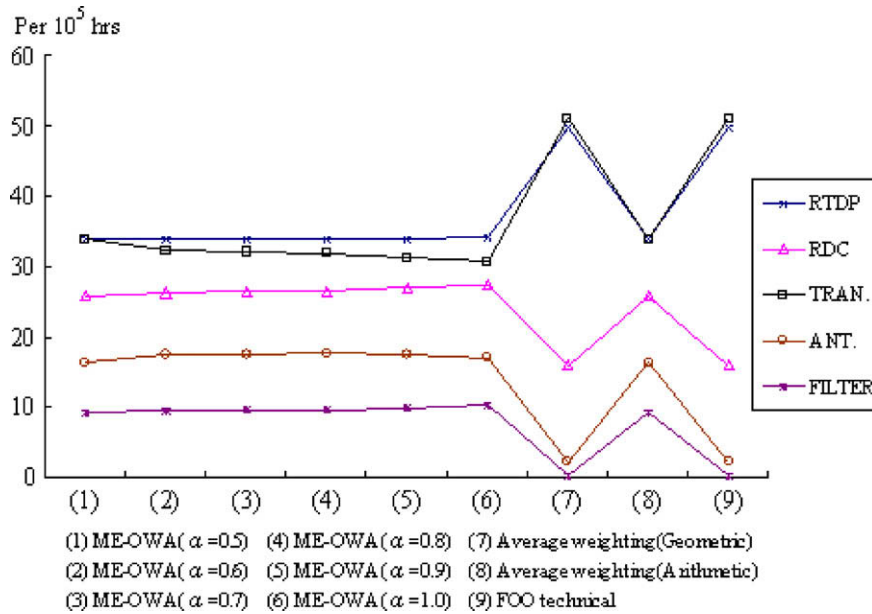


Fig. 2. Comparison of failure rate with different methods.

The comparison of failure rates allocated by the three different methods is shown in Fig. 2. From Fig. 2, the results from the FOO technique and average weighting allocation method (geometric model) indicate that the RTDP and TRAN have higher failure rates, while the ANT and FILTER have lower failure rates. Note that by the conventional FOO technique method, the failure rate for the FILTER is extremely low and is very close to zero (0.266 per 10<sup>5</sup> h). However, in reality, to design and manufacture a subsystem with such an extremely low failure rate would consume a considerable amount of resources. Despite the cost issue, the necessity of requiring such an extremely low failure rate device is disputable. Furthermore, the geometric model indicates that the RTDP and TRAN have higher failure rates. Such rates translate into low reliability as a result of frequent failures and costly repairs, as well as safety-of-flight issues should such failure occur in flight or combat. Such extremely high failure rates are certainly noncompliant with their respective performance specifications. Using the proposed ME-OWA method (take  $\alpha = 0.9$  as an example), the failure rate of the FILTER is the lowest, 9.840 per 10<sup>5</sup> h. The resources that are required to build a subsystem with a failure rate of 9.840 per 10<sup>5</sup> h is considerably less than those needed to build one with a failure rate of 0.266 per 10<sup>5</sup> h (37 times failure rate improvement), which would translate into a cost increase to 37 times higher than the original FILTER cost.

The mean time between failures (MTBF) obtained by the three methods is summarized in Table 7. As shown in this table, the ANT and FILTER have longer MTBFs, while the RTDP and TRAN have shorter MTBFs. A longer MTBF indicates higher reliability, whereas

a shorter MTBF indicates lower reliability. While the RTDP and TRAN have higher overall reliability ratings, results from the FOO technique are misleading due to measurement scale problems, which magnify the apportionment results. The irrational allocation rating used by the FOO technique will negatively affect allocation of the entire system service life.

In order to verify the performance of the proposed approach, we consulted with reliability engineer and manager to verify the results of the reliability allocation rating. These experts indicated that the ME-OWA method is correct and flexible in real-world applications, and could thereby provide a structured arrangement for reliability allocation.

### 6. Conclusion

Though widely used, the Mil-hdbk-338B handbook, which features the FOO technique, contains a severely flawed rating technique. The fundamental problem is that the measurement scales are ordinal, rendering multiplication of the factors *I*, *S*, *P* and *E* meaningless and misleading. This paper proposes a novel ME-OWA method to resolve this problem, which also occurs with the average weighting allocation method. Using a real case study of a fighter aircraft airborne radar system, we demonstrated the proposed approach and compared this approach with the conventional FOO technique and the average weighting allocation method. Our findings show that the results obtained by the geometric model of the average weighting allocation method are the same as that obtained by using the FOO technique. Moreover, the

Table 7 Comparison of airborne radar system MTBF across the three methods.

Method	ME-OWA						Average weighting allocation		FOO technique
	(1) $\alpha = 0.5$	(2) $\alpha = 0.6$	(3) $\alpha = 0.7$	(4) $\alpha = 0.8$	(5) $\alpha = 0.9$	(6) $\alpha = 1.0$	(7) Geometric	(8) Arithmetic	
RTDP	2948.46	2957.46	2957.40	2956.79	2951.34	2935.75	2013.09	2948.46	2013.09
RDC	3891.96	3816.05	3797.98	3784.09	3736.51	3669.69	6262.93	3891.96	6262.93
TRAN	2948.46	3097.38	3126.13	3146.48	3205.15	3261.94	1957.17	2948.46	1957.17
ANT	6081.19	5729.25	5697.67	5684.36	5703.06	5871.50	45093.11	6081.19	45093.11
FILTER	10811.01	10568.07	10485.22	10417.28	10162.64	9785.83	375775.88	10811.01	375775.88

arithmetic model of the average weighting allocation method is a special case of the proposed ME-OWA method. When the value of  $\alpha$  is used in the ME-OWA approach, the results obtained by both methods are the same. The main advantages of the proposed ME-OWA method are described as follows: (1) the proposed method efficiently solves the problem with the equally weighted problem of the measurement scale for factors  $I$ ,  $S$ ,  $P$  and  $E$  of the conventional FOO technique and average weighting allocation method, (2) the results obtained from the case study described in this paper show that the proposed method can accurately and efficiently allocate reliability ratings throughout reasonably assigned reliability levels in subsystems, meet customer needs, control reasonable support costs, and decrease maintenance costs, (3) the conditional parameter ( $\alpha$ ) in the proposed method provides valuable information about the subsystem and can thus be used to better assist designers in making correct decisions for reliability allocation, and (4) the proposed method provides an organized approach and a more flexible structure for combining subsystem allocation ratings – the validity of these ratings depend upon the reliability of the designer's assessment and selection of applicable variables, such as system intricacy, state-of-the-art, cost, and maintenance. The ME-OWA method can also be used in a wide variety of different fields and industries. This method provides more flexible assignments and, thus, helps managers to accurately and precisely determine the appropriate allocation of resources in a system.

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