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Abstract. The characteristics of guided waves scattered by nonuniform Bragg waveguide gratings are systematically investigated by using a staircase approximation, which is a combination of the building blocks of the multimode network theory and a rigorous mode-matching procedure. The bent dielectric waveguides, periodically with Bragg gratings of rectangular profile and chirped waveguide gratings, are taken as an example of the presented approach. Extensive numerical data are obtained to develop guidelines for integrated optical applications.

Keywords: Bent waveguide gratings, mode-matching method

Analyse numérique de réseaux de Bragg non uniformes

Résumé. Les caractéristiques des ondes diffusées par un réseau de Bragg guide d'onde sont systématiquement étudiées en utilisant une approximation par segments qui est une combinaison d'éléments de base d'une théorie de réseaux multimodes et d'une procédure d'adaptation de modes rigoureuse. Des guides d'onde diélectriques courbés périodiquement avec des réseaux de Bragg de profil rectangulaire et des réseaux déformés sont pris comme pour illustrer la présente approche. Un très grand nombre de données numériques sont exemples pour proposer des directives pour las applications à l'optique intégrée.

Mots clés: Réseaux guides d'ondes courbés, méthode d'adaptation de modes

1. Introduction

Aperiodic and nonuniform Bragg gratings have wide applications in optical devices, such as the dispersion compensator [1], Bragg reflector, wavelength-selective couplers, filters and sensor design [2]. It is well known that recently the linear chirped Bragg gratings have been employed in dispersion compensation in optical communications for long distance and high bit rate data transmission. In general, chirped Bragg gratings, tapered gratings and bent waveguide gratings, may be regarded as nonuniform gratings. Nonuniformities may be introduced into a periodic guided-wave structure artificially by improving the characteristics of photonics components or due to imperfection in a manufacturing process. We can obtain a nonuniform Bragg grating as a certain optical device by several approaches such as bending a uniform Bragg grating, varying the pitch length of each period along the waveguide or tapering the corrugate depth or index contrast along the grating. Therefore, a rigorous analysis of these nonuniform Bragg gratings is very important for designing integrated optical devices. In this paper, we present an analysis of a variety of nonuniform waveguide gratings for integrated optical applications.

The electromagnetic characteristics associated with the operation of a uniform periodic structure for using an individual component have been analysed by various methods, such as coupled mode formulation [3], finite element method [4] and rigorous full wave analysis [5]. With these basic analyses, the properties of uniform Bragg gratings can be considered to be well understood. Among these methods, the simpler approach is the coupled mode formulation, which can often provide physical insight but only approximate quantitative information. This method is also widely used for a mathematical analysis of the properties of nonuniform Bragg gratings [6, 7]. Although this method is simpler, an approximation has been made by assuming that the grating periodicity acts only as a small perturbation in the dielectric waveguide. Thus, using this method may cause calculation deviation in many practical

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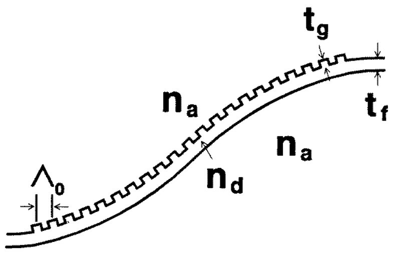

Figure 1. Generalized configuration structure of nonuniform waveguide grating.

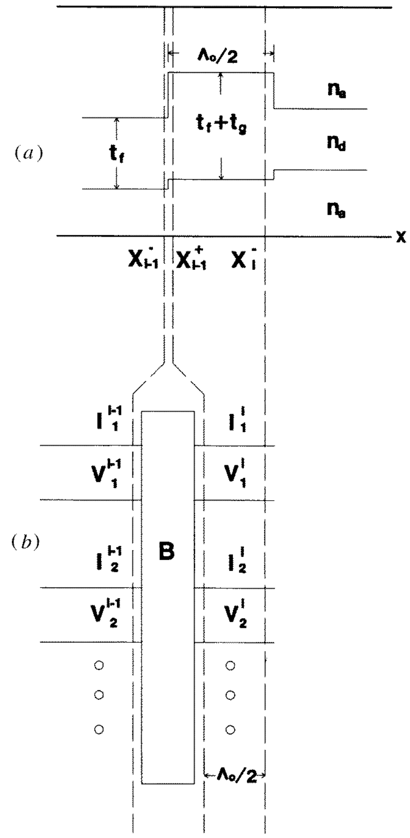

Figure 2. An equivalent network for a staircase structure. (*a*) Staircase approximation of a nonuniform Bragg grating in the neighbourhood of x_i. (*b*) Equivalent network.

cases, such as thick corrugated gratings. In this paper, we present a modal transmission line formulation, which can accurately analyse arbitrarily thick or higher index contrast gratings which are placed on an arbitrary bent waveguide relevant to current application. This method has been shown to be particularly powerful for analysing the bent dielectric waveguide [8]. Because of the mathematical difficulties involved in the analysis of a nonuniform structure even for simple geometrical profiles, conventional approaches must resort to approximate analysis. In this approach, the first approximation is a staircase approximation of the nonuniform region; this discretization in geometry. Secondly, we will employ the technique of discretizing the continuous spectrum of radiation modes into a complete set of discrete-type modes, this is done by enclosing the whole structure inside an oversized parallel-plate waveguide, as customarily done in the past [8]. The scattered power (including the reflected and transmitted power of the fundamental mode) varies, and part of the power may be carried by higher-order modes which can be viewed as radiation in the continuous spectrum in the case of an open structure. Thus, in this approach, the radiated power is presented by the total power of the higher modes in the partially filled parallel-plate waveguide. Finally, some numerical results are presented to quantify the scattered characteristics of nonuniform Bragg gratings.

2. Method of analysis

Figure 1 shows the geometrical configuration of the generalized bent Bragg grating under investigation. The staircase approximation of a bent Bragg grating in the neighbourhood of x_i by a piecewise constant one is as shown in figure 2. This stepped structure can be viewed as a combination of basic units, each consisting of a step discontinuity and a uniform waveguide with finite length. The number of discontinuity steps in each period must be large enough to guarantee the accuracy of the numerical calculation when the curvature becomes large. On the basis of literature [8], an equivalent network of the bent Bragg gratings can be developed by cascading all the transformer banks, which can accurately describe the scattering characteristics of waveguide discontinuity between uniform transmission line sections. Thus, the scattering of surface waves by the stepped structure can be analysed in terms of the voltage and current in this equivalent network.

According to the procedure of the rigorous mode matching [9], the electromagnetic field in each uniform region of the structure can be represented in terms of a complete set of mode functions ϕ in that region. For the TE-polarized incident case, the tangential field components can be represented by

$$E_z^i = \sum_n V_n^i(x)\phi_n^i(y) \tag{1}$$

$$H_y^i = \sum_n I_n^i(x)\phi_n^i(y) \tag{2}$$

where V_n and I_n are the voltage and current of the nth mode, respectively. By matching the tangential fields at the waveguide discontinuities, it can easily be deduced that the input impedance matrix $Z(x_i^-)$ at the $x = x_i^-$ plane looking to the right satisfies

$$Z(x_i^-) = Q_i(Z(x_i^+))Q_i^t \tag{3}$$

where t stands for transpose, and the Q-matrix characterizes the coupling of modes at the step discontinuity between two uniform waveguides, and their elements are defined by scalar products or overlap integrals of mode functions on each of the two sides of a discontinuity, as follows:

$$(Q_i)_{mn} = \langle \phi_m^i(x_i^-)|\phi_n^i(x_i^+)\rangle. \tag{4}$$

From the impedance transform matrix formula (3), the reflection coefficient matrix $\Gamma(x_i^-)$ at the $x = x_i^-$ plane and the impedance matrix $Z(x_{i-1}^-)$ at the $x = x_{i-1}^+$ plane looking to the right is obtained respectively as,

$$\Gamma(x_i^-) = [Z(x_i^-) + Z_{0i}]^{-1}[Z(x_i^-) - Z_{0i}] \quad (5)$$

and

$$Z(x_{i-1}^+) = Z_{0i}[I + H_i\Gamma_i H_i][I - H_i\Gamma_i H_i]^{-1} \quad (6)$$

where Z_{0i} and H_i are the characteristic impedance and the phase matrices of the ith step discontinuity and their elements are defined respectively as follows,

$$(Z_{0i})_{mn} = \delta_{mn} Z_{0in} \quad (7)$$

$$(H_{0i})_{mn} = \delta_{mn} \exp(-j\kappa_{xin} l_i) \quad (8)$$

where κ_{xin} and Z_{0in} are, respectively, the wavenumber in the x direction and the characteristic impedance for the nth mode in the ith dielectric waveguide section, and l_i is the length of this waveguide. It is obvious that the staircase approximation is very simple and straightforward, and it has been shown that it can generally be applied directly to the nonuniform integrated optical devices.

3. Results

We present here some numerical results to demonstrate the scattering characteristics of several types of nonuniform periodic structures. The data of our numerical method agree well with literature [4, 10, 11] for the analysis of the scattering characteristics of single dielectric waveguide step discontinuity and uniform periodic dielectric waveguide. Basically, there are many types of nonuniform waveguide gratings with various bent sharp curves, and the complex curved waveguide grating will lead to more difficult analyses. For the sake of simplicity, several kinds of simpler curved waveguide gratings have been analysed in this paper. Although these bent sharp curved waveguide gratings are simpler, they can be used as a basis to analyse more complicated structures.

Figure 3(b) shows the Bragg reflection spectrum of the TE_0 mode, for which the Bragg grating was bent around a circle, as shown in figure 3(a). The bent Bragg gratings that we analyse have total periods $N_p = 25$ and their physics parameters are depicted in figure 3(b). In this figure we also show the reflective spectrum for a Bragg grating without bending (represented by the dotted curve). It can be seen that the reflective spectrum between the bent Bragg grating and the normal Bragg grating has the same tendency, a stopband with a bandwidth of about 1.5–1.6 μm, and sidelobes around it. Although their behaviour seems very similar, there are obvious differences between them. When the normal Bragg grating begins to bend along the circle, the sidelobes' intensity drops drastically, and the bandwidth of the stopband begins to become narrower. For example, the bending radius of curvature $R_a = 25$ μm, the first sidelobe near the stopband at wavelength 1.478 μm is -12.9 dB; $R_a = 20$ μm is -17 dB and normal gratings is -5.4 dB, respectively. Therefore, an obvious effect for symmetrically bent Bragg grating is the decrease of the intensity of the sidelobes. Thus, bending the Bragg grating

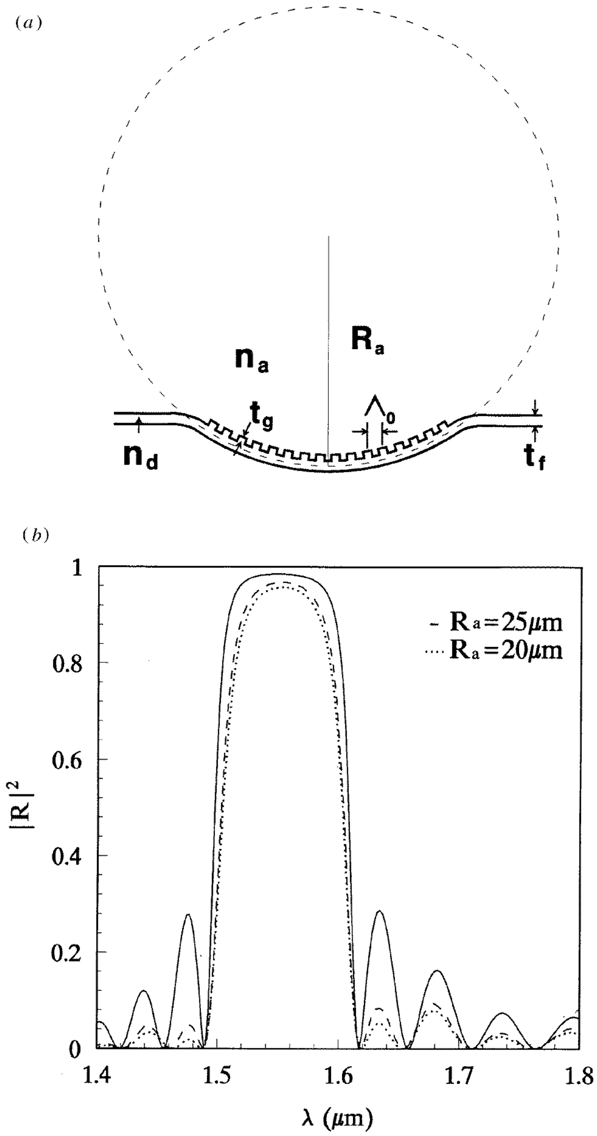

Figure 3. (a) Configuration structure of a Bragg grating bent around a circle. (b) Reflected power of a fundamental TE mode, for which a Bragg grating was bent around a circle: n_d = 3.4, n_a = 1.0, t_f = 0.15 μm, t_g = 0.06 μm, Λ_0 = 0.304 μm and N_p = 25.

can provide a degree of freedom for designing a Bragg reflector with a narrow bandwidth and low sidelobes.

Consider the second case of bent Bragg gratings, as shown in figure 1. The profile of the bending function is characterized by

$$y = a_0 \tanh(x/T) \quad (9)$$

where a_0 is a modulation constant and T represents the bending region in the x direction. The variation of the reflective spectrum of the fundamental TE mode occurs with a respective wavelength as shown in figure 4. It is found that increasing the modulation constant a_0 (i.e. the Bragg grating is bent more seriously), could broaden the reflective bandwidth, and also increase the intensity of the sidelobes. Therefore, according to the numerical results shown in figures 3(b) and 4, the reflective spectrum can be

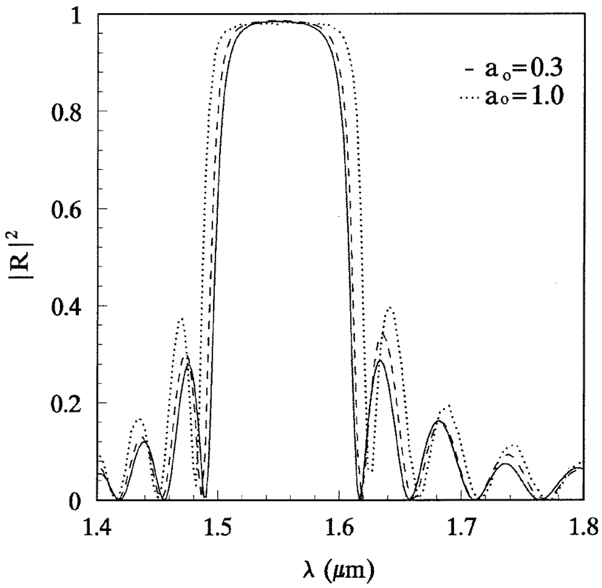

Figure 4. Reflective spectrum of fundamental TE mode of a bent Bragg grating, the bending function is described by $y = a_0 \tanh(x/T)$: $n_d = 3.4$, $n_a = 1.0$, $t_f = 0.15$ μm, $t_g = 0.06$ μm, $\Lambda_0 = 0.304$ μm and $N_p = 25$.

adjusted by bending the Bragg grating for various types of bending functions. These results also provide an approach for designing integrated optics devices.

Chirped distributed feedback Bragg gratings are attractive and widely applied in optical communications. The properties of photonic components can be improved by choosing specific chirping functions. In general, a chirped Bragg grating with a specific chirping function can be fabricated directly on the dielectric waveguide by varying the interval of every period. This kind of chirped Bragg grating has been widely analysed with coupled mode formulation and practically applied to optical fibre communications and integrated optical applications. In this paper we analysed one type of bent chirped Bragg grating which can be achieved by contacting a Bragg grating with a bent dielectric waveguide, as shown in figure 5(*a*). The projection of the interval between every period on the x-axis is constant. Figure 5(*b*), which illustrates the calculated reflection response of a bent chirped grating with 100 periods by the present method and the bending function, is characterized by

$$y(x) = b_0 x^n \tag{10}$$

where b_0 is the tunable constant and n is the power of (10). The characteristics of bent chirped Bragg gratings can be adjusted by modulation of various types of bending functions, so that the performance of optoelectric devices can be optimally designed. Please note, our staircase approximation can also accurately analyse the reflective response shift of the Bragg grating sensor which is bound on a structure and monitors the structure's deformation.

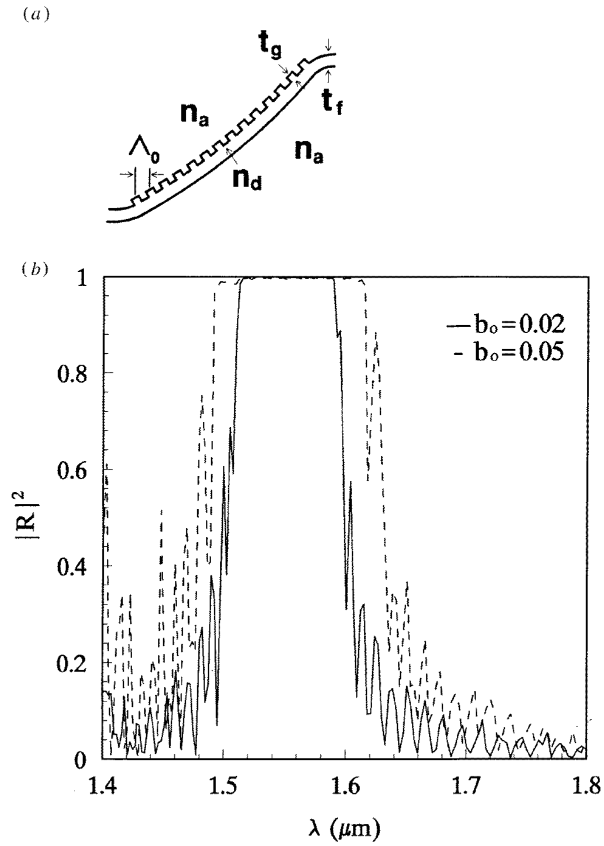

Figure 5. (*a*) Structure configuration of a bent chirped DFB grating. (*b*) Reflective spectrum of fundamental TE mode, the bending function of the Bragg grating is $y = b_0 x^{3/2}$: $n_d = 1.47$, $n_a = 1.0$, $t_f = 0.42$ μm, $t_g = 0.18$ μm, $\Lambda_0 = 0.618$ μm and $N_p = 100$.

4. Conclusion

We have presented a theoretical procedure that is well suited to design nonuniform Bragg gratings for integrated optical applications. This procedure utilizes the staircase approximation, which combines the building blocks of the multimode network theory with the rigorous mode-matching method. The electromagnetic characteristics of the nonuniform Bragg gratings were analysed from the viewpoint of scattering of the incident surface guided waves; the effects of higher-order modes were also taken into account. Particular attention was given to the reflection characteristics of bent Bragg gratings and chirped Bragg gratings. Extensive numerical data were obtained to develop the guidelines for the improved and tailored operation of photonic components for integrated optics application.

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