

# Fault-Tolerant Hamiltonian Connectivity and Fault-Tolerant Hamiltonicity of the Fully Connected Cubic Networks

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Many papers on the fully connected cubic networks have been published for the past several years due to its favorite properties. In this paper, we consider the fault-tolerant hamiltonian connectivity and fault-tolerant hamiltonicity of the fully connected cubic network. We use  $FCCN_n$  to denote the fully connected cubic network of level  $n$ . Let  $G = (V, E)$  be a graph. The *fault-tolerant hamiltonian connectivity*  $H_f^l(G)$  is defined to be the maximum integer  $l$  such that  $G - F$  remains hamiltonian connected for every  $F \subset V(G) \cup E(G)$  with  $|F| \leq l$ . The *fault-tolerant hamiltonicity*  $H_f(G)$  is defined to be the maximum integer  $l$  such that  $G - F$  remains hamiltonian for every  $F \subset V(G) \cup E(G)$  with  $|F| \leq l$ . We prove that  $H_f^l(FCCN_n) = 0$  and  $H_f(FCCN_n) = 1$  if  $n \geq 2$ .

**Keywords:** hamiltonian, hamiltonian connected, fault-tolerant hamiltonian, fault-tolerant hamiltonian connected, fully connected cubic network

## 1. INTRODUCTION

As is customary in structure studies of parallel architectures, we focus on a set of identical processors and view the architectures of the underlying interconnection networks as graphs. The vertices of a graph represent the processors of architecture, and the edges of the graph represent the communication links between processors. Network topology is a crucial factor for the interconnection networks since it determines the performance of the networks. Many interconnection network topologies have been proposed in the literature for the purpose of connecting thousands of processing elements. The hypercube networks have received much attention over the past few years since they offer a rich interconnection structure with large bandwidth, logarithmic diameter, high degree of fault tolerance, and embedding of various interconnection. The hypercube network has been chosen as the interconnection topology in several commercially available parallel computers [11]. However, the hypercube networks are not truly expandable because we have to change the hardware configuration of all the vertices whenever the number of vertices grows exponentially, as the vertices have to be provided with additional ports. Fortunately, hierarchical interconnection networks (HINs) provide excellent expandability to meet this need.

For decades, two kinds of HINs were proposed addressing the issue of constructing

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hierarchical interconnection networks. One is the HINs consisting of exactly two levels [3, 5, 8, 12-14], and another is recursively defined HIN [2, 15, 17, 18]. Moreover, the *fully connected cubic networks (FCCNs)* proposed in [15] are defined recursively by taking the 3-dimensional cube as the basis graph. It indicates that fully connected cube networks are a class of newly proposed hierarchical networks for multisystems, which enjoy the strengths of constant vertex degree and good expandability. Some interesting properties about FCCNs are discussed in [15]. In particular, Yang *et al.* [16] presented the shortest-path routing algorithm. Fault-tolerant hamiltonicity and fault-tolerant hamiltonian connectivity are other important issues in the design and the analysis of interconnection networks [4, 6, 7, 9, 10]. In this paper, we discuss these two parameters for FCCNs.

The remaining of this paper is organized as follows: Section 2 provides the formal definition of FCCNs and some preliminaries. In section 3, we discuss the fault-tolerant hamiltonian connectivity for FCCNs. In section 4, we discuss the fault-tolerant hamiltonicity for FCCNs.

## 2. PRELIMINARIES

An interconnection network is represented by a graph  $G = (V, E)$  with vertices and edges symbolizing the processors and communication links between processors, respectively. For graph definitions and notations we follow [1]. An *n-dimensional cube*,  $Q_n$ , is a graph with  $2^n$  vertices that can be one-to-one labeled with 0-1 binary strings so that two vertices are adjacent if and only if their labels differ in exactly one position. A *path*,  $\langle v_0, v_1, \dots, v_k \rangle$ , is an ordered list of distinct vertices such that  $v_i$  and  $v_{i+1}$  are adjacent for  $0 \leq i \leq k-1$ . We also write the path  $\langle v_0, v_1, \dots, v_k \rangle$  as  $\langle v_0, \dots, v_i, Q, v_j, \dots, v_k \rangle$  where  $Q$  is a path from  $v_i$  to  $v_j$ . A path is a *hamiltonian path* if its vertices are distinct and span  $V$ . A *cycle*,  $\langle v_0, v_1, \dots, v_k, v_0 \rangle$ , is a path with at least three vertices such that the first vertex is the same as the last vertex. A cycle is a *hamiltonian cycle* if it traverses every vertex of  $G$  exactly once. A graph is *hamiltonian* if it has a hamiltonian cycle. A graph  $G$  is *hamiltonian connected* if there exists a hamiltonian path joining any two vertices of  $G$ . A hamiltonian connected graph with at least three vertices is hamiltonian.

The *fault-tolerant hamiltonian connectivity* of a graph  $G$ ,  $H_f^k(G)$ , is defined to be the maximum integer  $l$  such that  $G - F$  remains hamiltonian connected for every  $F \subset V(G) \cup E(G)$  with  $|F| \leq l$  if  $G$  is hamiltonian connected and is undefined otherwise. A hamiltonian graph  $G$  is *k fault-tolerant hamiltonian* if  $G - F$  remains hamiltonian for every  $F \subset V(G) \cup E(G)$  with  $|F| \leq k$ . The *fault-tolerant hamiltonicity* of a graph  $G$ ,  $H_f(G)$ , is defined to be the maximum integer  $k$  such that  $G$  is *k fault-tolerant hamiltonian* if  $G$  is hamiltonian and is undefined otherwise.

Let  $Z_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ . For  $m \geq 1$  and  $a \in Z_8$ , let  $a^m = \underbrace{aa\dots a}_m$ .

For  $n \geq 1$ , an *n-level FCCN*,  $FCCN_n$ , is a graph defined recursively as follows:

- (1)  $FCCN_1$  is a graph with  $V(FCCN_1) = Z_8$  and  $E(FCCN_1) = \{(0, 1), (0, 2), (1, 3), (2, 3), (4, 5), (4, 6), (5, 7), (6, 7), (0, 4), (1, 5), (2, 6), (3, 7)\}$ . Obviously,  $FCCN_1$  is isomorphic to  $Q_3$ .
- (2) When  $n \geq 2$ ,  $FCCN_n$  is built from eight vertex-disjoint copies of  $FCCN_{n-1}$  by adding 28 edges. For  $0 \leq k \leq 7$ , we let  $kFCCN_{n-1}$  denote a copy of  $FCCN_{n-1}$  with each vertex

being prefixed with  $k$ , then  $FCCN_n$  is defined by

$$V(FCCN_n) = \bigcup_{k=0}^7 V(kFCCN_{n-1}),$$

$$E(FCCN_n) = \left( \bigcup_{k=0}^7 E(kFCCN_{n-1}) \right) \cup \{(pq^{n-1}, qp^{n-1}) \mid 0 \leq p < q \leq 7\}.$$

Let  $n \geq 2$ . A vertex  $v$  in  $FCCN_n$  is a *boundary vertex* if it is of the form  $p^n$  and  $v$  is an *intercubic vertex* if it is of the form  $pq^{n-1}$  with  $p \neq q$ . An *intercubic edge* is an edge joining two intercubic vertices.

Figs. 1 (a)-(c) show  $FCCN_1$ ,  $FCCN_2$  and  $FCCN_3$ . In essence, each vertex of an  $FCCN$  has four links, with each boundary vertex having one I/O channel link that is not counted in the vertex degree. Obviously,  $kFCCN_{n-1}$  have 7 intercubic vertices and 1 boundary vertex for  $0 \leq k \leq 7$  and  $n \geq 2$ .

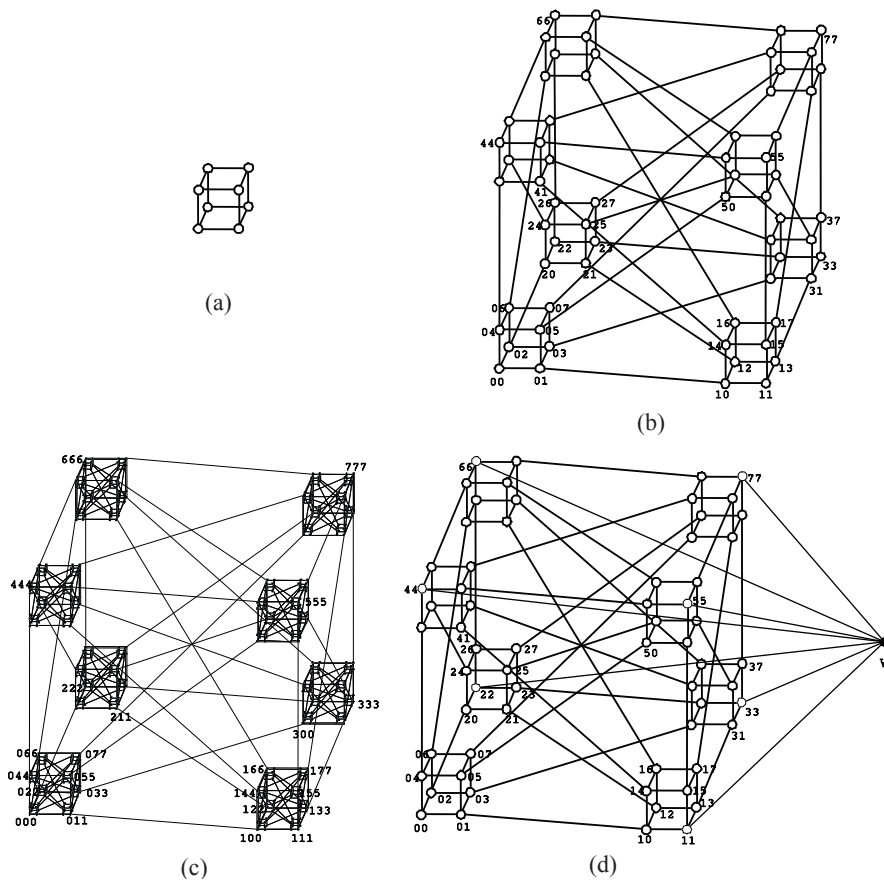


Fig. 1. The graphs (a)  $FCCN_1$ , (b)  $FCCN_2$ , (c)  $FCCN_3$ , and (d)  $FCCN_2^0$ .

In order to discuss the fault-tolerant hamiltonicity and fault-tolerant hamiltonian connectivity of fully connected cubic networks, we need to introduce the extended fully connected cubic networks. For every  $t \in Z_8$ , the *extended fully connected cubic network*  $FCCN_n^t$  is the graph obtained from  $FCCN_n$  by joining the vertices in the set  $\{p^n \mid p \in Z_8 - \{t\}\}$  to an extra vertex  $w$ . For example,  $FCCN_2^0$  is illustrated in Fig. 1 (d). Note that  $FCCN_n^i$  is isomorphic to  $FCCN_n^j$  for every  $i, j$  in  $Z_8$ . We only discuss  $FCCN_n^0$ .

Since  $FCCN_2$  has 64 vertices and  $FCCN_2^0$  has 65 vertices, we can check the following two lemmas by brute force.

**Lemma 1**  $FCCN_2$  and  $FCCN_2^0$  are hamiltonian connected.

**Lemma 2**  $FCCN_2 - f$  is hamiltonian for any  $f \in V(FCCN_2) \cup E(FCCN_2)$  with  $|f| = 1$ . Moreover,  $FCCN_2^0 - f^*$  is hamiltonian for any  $f^* \in V(FCCN_2^0) \cup E(FCCN_2^0)$  with  $|f^*| = 1$ .

### 3. FAULT-TOLERANT HAMILTONIAN CONNECTIVITY

**Lemma 3** Both  $FCCN_n$  and  $FCCN_n^0$  are hamiltonian connected for  $n \geq 2$ .

**Proof:** We prove this lemma by induction. With Lemma 1, the statement holds for  $n = 2$ . We assume the statement holds for  $FCCN_l$  with  $2 \leq l \leq n$ .

First, we prove that  $FCCN_{n+1}$  is hamiltonian connected. Let  $u$  and  $v$  be any two distinct vertices in  $FCCN_{n+1}$ . We need to find a hamiltonian path of  $FCCN_{n+1}$  joining  $u$  to  $v$ .

Suppose that  $u \in bFCCN_n$  and  $v \in eFCCN_n$  with  $b \neq e$ . Let  $k_0, k_1, \dots, k_7$  be any permutation of  $Z_8$  such that  $k_0 = b, k_7 = e, k_0k_1^n \neq u$ , and  $k_7k_6^n \neq v$ . Let  $x_i = k_i k_{i-1}^n$  for  $1 \leq i \leq 7, y_j = k_j k_{j+1}^n$  for  $0 \leq j \leq 6, x_0 = u$ , and  $y_7 = v$ . By induction, there exists a hamiltonian path  $P_i$  of  $k_i FCCN_n$  joining  $x_i$  to  $y_i$  for  $0 \leq i \leq 7$ . Obviously,  $P = \langle u, P_0, P_1, \dots, P_7, v \rangle$  forms a hamiltonian path of  $FCCN_{n+1}$  joining  $u$  to  $v$ . See Fig. 2 (a) for an illustration.

Suppose that  $\{u, v\} \subset bFCCN_n$ . By induction hypothesis, there is a hamiltonian path  $P_b$  of  $bFCCN_n^b$  joining  $u$  and  $v$ . Obviously,  $P_b$  can be written as  $\langle u, P_{b1}, bk_1^n, w, bk_7^n, P_{b2}, v \rangle$  where  $w$  is an extra vertex of  $bFCCN_n^b$ . Clearly,  $b \notin \{k_1, k_7\}$ . Let  $k_2, k_3, \dots, k_6$  be any permutation of  $Z_8 - \{b, k_1, k_7\}$ . We set  $k_8 = k_0 = b$ . Let  $x_i = k_i k_{i-1}^n$  and  $y_i = k_i k_{i+1}^n$  for  $1 \leq i \leq 7$ . By induction, there exists a hamiltonian path  $P_i$  of  $k_i FCCN_n$  joining  $x_i$  to  $y_i$  for  $1 \leq i \leq 7$ . Obviously,  $P = \langle u, P_{b1}, P_1, \dots, P_7, P_{b2}, v \rangle$  forms a hamiltonian path of  $FCCN_{n+1}$  joining  $u$  to  $v$ . See Fig. 2 (b) for an illustration.

Second, we prove that  $FCCN_{n+1}^0$  is hamiltonian connected. Let  $u$  and  $v$  be any two distinct vertices in  $FCCN_{n+1}^0$ . We need to find a hamiltonian path of  $FCCN_{n+1}^0$  joining  $u$  to  $v$ .

Suppose that  $u \in bFCCN_n$  and  $v$  is the extra vertex  $w$ . Let  $k_0, k_1, \dots, k_7$  be any permutation of  $Z_8$  such that  $k_0 = b, k_0k_1^n \neq u$ , and  $k_7 \neq 0$ . Let  $x_i = k_i k_{i-1}^n$  for  $1 \leq i \leq 7, y_j = k_j k_{j+1}^n$  for  $0 \leq j \leq 6, x_0 = u$ , and  $y_7 = k_7^{n+1}$ . By induction, there exists a hamiltonian path  $P_i$  of  $k_i FCCN_n$  joining  $x_i$  to  $y_i$  for  $0 \leq i \leq 7$ . Obviously,  $P = \langle u, P_0, P_1, \dots, P_7, v = w \rangle$  forms a hamiltonian path of  $FCCN_{n+1}^0$  joining  $u$  to  $v$ . See Fig. 2 (c) for an illustration.

Suppose that  $u \in bFCCN_n$  and  $v \in eFCCN_n$  with  $b \neq e$ . Let  $k_0, k_1, \dots, k_7$  be any permutation of  $Z_8$  such that  $k_0 = b, k_7 = e, k_0k_1^n \neq u, k_7k_6^n \neq v$ , and  $0 \notin \{k_1, k_2\}$ . Let  $x_i = k_i k_{i-1}^n$

for  $3 \leq i \leq 7$ ,  $y_j = k_j k_{j+1}^n$  for  $2 \leq j \leq 6$ ,  $x_0 = u$ ,  $x_1 = k_1 k_0^n$ ,  $x_2 = k_2^{n+1}$ ,  $y_0 = k_0 k_1^n$ ,  $y_1 = k_1^{n+1}$ , and  $y_7 = v$ . By induction, there exists a hamiltonian path  $P_i$  of  $k_i FCCN_n$  joining  $x_i$  to  $y_i$  for  $0 \leq i \leq 7$ . Obviously,  $P = \langle u, P_0, P_1, w, P_2, \dots, P_7, v \rangle$  forms a hamiltonian path of  $FCCN_{n+1}^0$  joining  $u$  to  $v$ . See Fig. 2 (d) for an illustration.

Suppose that  $\{u, v\} \subset bFCCN_n$ . By induction hypothesis, there is a hamiltonian path  $P_b$  of  $bFCCN_n^b$  joining  $u$  and  $v$ . Obviously,  $P_b$  can be written as  $\langle u, P_{b1}, bk_1^n, w, bk_7^n, P_{b2}, v \rangle$ . Clearly,  $b \notin \{k_1, k_7\}$ . Obviously, at least one of  $k_1$  and  $k_7$  is not 0. Without loss of generality, we assume  $k_1 \neq 0$ . Let  $k_2, k_3, \dots, k_6$  be any permutation of  $Z_8 - \{b, k_1, k_7\}$  such that  $k_2 \neq 0$ . We set  $k_8 = b$ . Let  $x_i = k_i k_{i-1}^n$  for  $3 \leq i \leq 7$ ,  $y_i = k_i k_{i+1}^n$  for  $2 \leq i \leq 7$ ,  $x_1 = k_1 b^n$ ,  $x_2 = k_2^{n+1}$ , and  $y_1 = k_1^{n+1}$ . By induction, there exists a hamiltonian path  $P_i$  of  $k_i FCCN_n$  joining  $x_i$  to  $y_i$  for  $1 \leq i \leq 7$ . Obviously,  $P = \langle u, P_{b1}, P_1, w, P_2, \dots, P_7, P_{b2}, v \rangle$  forms a hamiltonian path of  $FCCN_{n+1}^0$  joining  $u$  to  $v$ . See Fig. 2 (e) for an illustration.  $\square$

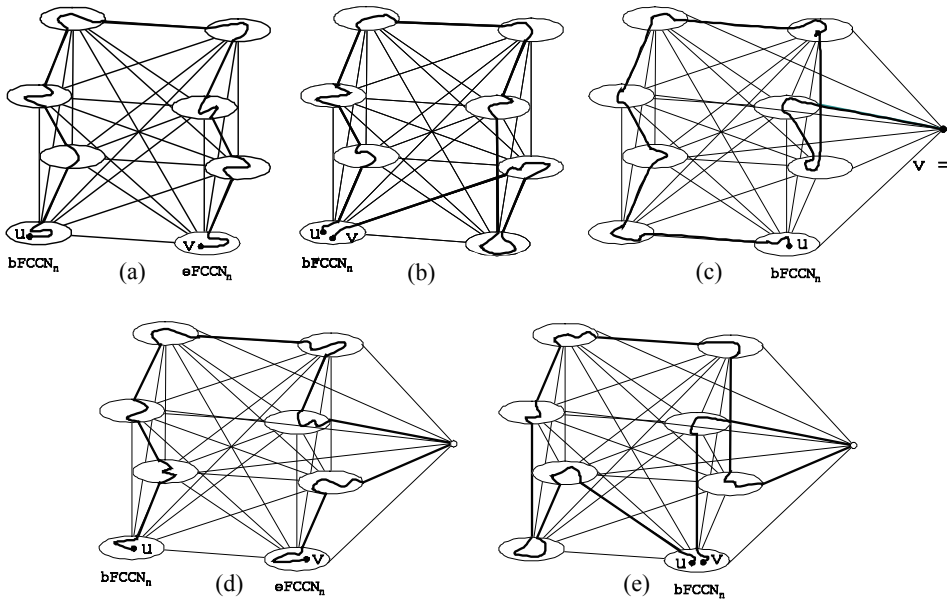


Fig. 2. Illustration for Lemma 3.

**Theorem 1**  $H_f^k(FCCN_n) = 0$  if  $n \geq 2$  and is undefined if  $n = 1$ .

**Proof:** Assume  $n \geq 2$ . With Lemma 3, we have proved that  $H_f^k(FCCN_n) \geq 0$ . Obviously,  $0^n$  is a boundary vertex of  $FCCN_n$  with exactly three neighbors, say  $x, y$  and  $z$ . It is easy to see that there is no hamiltonian path of  $FCCN_n - \{x\}$  joining  $y$  to  $z$ . Thus,  $FCCN_n - \{x\}$  is not hamiltonian connected. Thus,  $H_f^k(FCCN_n) = 0$  if  $n \geq 2$ . Note that  $FCCN_1$  is isomorphic to  $Q_3$ . Since  $Q_3$  is a bipartite graph with 8 vertices, there is no hamiltonian path joining two vertices in the same partite set. Thus,  $FCCN_1$  is not hamiltonian connected. Therefore,  $H_f^k(FCCN_1)$  is undefined.  $\square$

### 4. FAULT-TOLERANT HAMILTONICITY

**Lemma 4** Both  $FCCN_n$  and  $FCCN_n^0$  are 1 fault-tolerant hamiltonian for  $n \geq 2$ .

**Proof:** We prove this lemma by induction. It is sufficient to prove a graph  $G$  is 1 fault-tolerant hamiltonian by proving  $G - f$  is hamiltonian for any  $f \in V(G) \cup E(G)$  with  $|f| = 1$ . By Lemma 2, the statement holds for  $n = 2$ . We assume the statement holds for  $FCCN_l$  and  $FCCN_l^0$  with  $2 \leq l \leq n$ .

First, we prove that  $FCCN_{n+1} - f$  is hamiltonian.

Suppose that  $f \in V(bFCCN_n) \cup E(bFCCN_n)$ . By induction, there is a hamiltonian cycle  $\langle w, bk_7^n, P_b, bk_1^n, w \rangle$  of  $bFCCN_n - f$ . Clearly,  $b \notin \{k_1, k_7\}$ . Let  $k_2, k_3, \dots, k_6$  be any permutation of  $Z_8 \setminus \{b, k_1, k_7\}$ . We set  $k_0 = k_8 = b$ . Let  $x_i = k_i k_{i-1}^n$  and  $y_i = k_i k_{i+1}^n$  for  $1 \leq i \leq 7$ . By Theorem 1, there exists a hamiltonian path  $P_i$  of  $k_i FCCN_n$  joining  $x_i$  to  $y_i$  for  $1 \leq i \leq 7$ . Obviously,  $\langle bk_7^n, P_b, P_1, \dots, P_7, bk_1^n \rangle$  forms a hamiltonian cycle for  $FCCN_{n+1} - f$ .

Suppose that  $f$  is an intercubic edge between  $bFCCN_n$  and  $eFCCN_n$ . Thus,  $f = (be^n, eb^n)$ . Let  $k_0, k_1, \dots, k_7$  be any permutation of  $Z_8$  such that  $k_0 = b$  and  $k_2 = e$ . Let  $x_i = k_i k_{i-1}^n$  for  $1 \leq i \leq 7$ ,  $y_j = k_j k_{j+1}^n$  for  $0 \leq j \leq 6$ ,  $x_0 = k_0 k_7^n$ , and  $y_7 = k_7 k_0^n$ . By Theorem 1, there exists a hamiltonian path  $P_i$  of  $k_i FCCN_n$  joining  $x_i$  to  $y_i$  for  $0 \leq i \leq 7$ . Obviously,  $\langle bk_7^n, P_0, P_1, \dots, P_7, bk_1^n \rangle$  forms a hamiltonian cycle for  $FCCN_{n+1} - f$ .

Second, we prove that  $FCCN_{n+1}^0 - f$  is hamiltonian.

Suppose that  $f$  is the extra vertex  $w$ . Obviously,  $FCCN_{n+1}^0 - \{w\} = FCCN_{n+1}$ . By Theorem 1,  $FCCN_{n+1}^0 - \{w\}$  is hamiltonian connected. Therefore,  $FCCN_{n+1}^0 - f$  is hamiltonian.

Suppose that  $f \in V(bFCCN_n) \cup E(bFCCN_n)$ . By induction hypothesis, there is a hamiltonian cycle  $C$  of  $bFCCN_n - f$ . Since  $C$  can be traversed forward and backward, we can assume that  $C = \langle w, bk_7^n, P_b, bk_1^n, w \rangle$  with  $k_1 \neq 0$  and  $k_7 \neq 0$ . Let  $k_2, k_3, \dots, k_6$  be any permutation of  $Z_8 - \{b, k_1, k_7\}$  such that  $k_2 \neq 0$ . We set  $k_8 = b$ . Let  $x_i = k_i k_{i-1}^n$  for  $3 \leq i \leq 7$ ,  $y_j = k_j k_{j+1}^n$  for  $2 \leq j \leq 7$ ,  $x_1 = k_1 b^n$ ,  $x_2 = k_2^{n+1}$ , and  $y_1 = k_1^{n+1}$ . By Theorem 1, there exists a hamiltonian path  $P_i$  of  $k_i FCCN_n$  joining  $x_i$  to  $y_i$  for  $1 \leq i \leq 7$ . Obviously,  $\langle bk_7^n, P_b, P_1, w, P_2, \dots, P_7, bk_1^n \rangle$  forms a hamiltonian cycle for  $FCCN_{n+1}^0 - f$ .

Suppose that  $f$  is an edge of the form  $(r^{n+1}, w)$ . Let  $b$  and  $e$  be two indices in  $Z_8$  with  $\{b, e\} \cap \{0, r\} = \emptyset$ . By Theorem 1, there is a hamiltonian path  $P$  of  $FCCN_{n+1}$  joining  $b^{n+1}$  to  $e^{n+1}$ . Obviously,  $\langle w, b^{n+1}, P, e^{n+1}, w \rangle$  forms a hamiltonian cycle for  $FCCN_{n+1}^0 - f$ .

Suppose that  $f$  is an intercubic edge between  $bFCCN_n$  and  $eFCCN_n$ . Thus,  $f = (be^n, eb^n)$ . Let  $k_0, k_1, \dots, k_7$  be any permutation of  $Z_8$  such that  $k_0 = b$ ,  $k_1 \neq 0$ ,  $k_2 \neq 0$ , and  $k_3 = e$ . Let  $x_i = k_i k_{i-1}^n$  for  $3 \leq i \leq 7$ ,  $y_j = k_j k_{j+1}^n$  for  $2 \leq j \leq 6$ ,  $x_0 = k_0 k_7^n$ ,  $x_1 = k_1 k_0^n$ ,  $x_2 = k_2^{n+1}$ ,  $y_0 = k_0 k_1^n$ ,  $y_1 = k_1^{n+1}$ , and  $y_7 = k_7 k_0^n$ . By Theorem 1, there exists a hamiltonian path  $P_i$  of  $k_i FCCN_n$  joining  $x_i$  to  $y_i$  for  $0 \leq i \leq 7$ . Obviously,  $\langle bk_7^n, P_0, P_1, w, P_2, \dots, P_7, bk_1^n \rangle$  forms a hamiltonian cycle for  $FCCN_{n+1}^0 - f$ . □

**Theorem 2**  $H_f(FCCN_n) = 1$  if  $n \geq 2$  and  $H_f(FCCN_1) = 0$ .

**Proof:** Assume  $n \geq 2$ . With Lemma 4, we have proved that  $H_f^k(FCCN_n) \geq 1$ . Obviously,  $0^n$  is a boundary vertex of  $FCCN_n$  with exactly three neighbors, say  $x, y$  and  $z$ . Since there is only one vertex  $z$  adjacent to  $0^n$  in  $FCCN_n - \{x, y\}$ ,  $FCCN_n - \{x, y\}$  is not hamiltonian.

Thus,  $H_f(FCCN_n) = 1$  if  $n \geq 2$ . Note that  $FCCN_1$  is isomorphic to  $Q_3$ . It is easy to check that  $Q_3$  is hamiltonian but  $Q_3 - \{0\}$  is not hamiltonian. Therefore,  $H_f(FCCN_1) = 0$ .  $\square$

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