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Chaos synchronization and chaotization of complex chaotic systems in series form by optimal control

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ABSTRACT

By the method of quadratic optimum control, a quadratic optimal regulator is used for synchronizing two complex chaotic systems in series form. By this method the least error with less control energy is achieved, and the optimization on both energy and error is realized synthetically. The simulation results of two Quantum-CNN chaos systems in series form prove the effectiveness of this method. Finally, chaotization of the system is given by optimal control.

1. Introduction

Chaos synchronization has been widely investigated and many effective methods have been presented recently. Thus, as a key technique of secret communication, chaos synchronization has become a very important goal. Since Pecora and Corrall discovered the synchronization of chaotic systems [1–5], many synchronization methods have been developed [6–9]. For chaos synchronization of practical engineering systems, the control cost must be taken into account. Optimal control method is preferable in such cases [10–13].

In this paper, a quadratic optimal regulator is used for chaos synchronization. In practical system, it is difficult to obtain the precise mathematical model, so in practical applications the investigators would like to employ simple and efficient controllers. Therefore, how to design a simple controller with limited information of a chaotic system is still an open problem [20-26].

As numerical example, recently developed Quantum Cellular Neural Network (Quantum-CNN) chaotic oscillator in series form is used. Quantum-CNN oscillator equations are derived from a Schrödinger equation taking account of quantum dots cellular automata structures to which in the last decade a wide interest has been devoted, with particular attention towards quantum computing [19].

Furthermore, chaotization is studied. Chaotization aims at creating or enhancing the system complexity. Chaotization of Quantum-CNN system is accomplished by an optimal control method.

This paper is organized as follows. In Section 2, a linearly coupled chaos synchronization scheme by optimum control is given. In Section 3, numerical results of the synchronization of two Quantum-CNN oscillator systems by unidirectional and by mutual linear coupling are presented, respectively. In Section 4, chaotization of Quantum-CNN chaotic system and simulation results are obtained. Finally, conclusions are given in Section 5.

2. Linearly coupled chaos synchronization scheme by optimum control

The optimum control is defined as a method by which the specified performance index of a system has optimum value when the desired control assignment is fulfilled.

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The state equation of a linear system is

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $x(t)$ is an n-dimensional state variable of the system, A is an $n \times n$ dimensional constant matrix and B is an appropriate $n \times r$ dimensional constant matrix. The matrix $[A\ B]$ is controllable entirely and $u(t)$ is an r-dimensional control input of the system. Assuming that $u(t)$ has no restriction and $u(0) = 0$, the performance index is

$$J = \int_0^\infty (x^T Qx + u^T Ru)dt. \tag{2}$$

In Eq. (2), Q is an $n \times n$ dimensional positive semidefinite real symmetric constant matrix; R is an $r \times r$ dimensional positive definite real symmetric constant matrix. The choice of the weighting matrix Q or R is based on eclectic considerations which can enhance the control performance and reduce the control energy consumption. The aim of the optimum control is to get $u(t) = Kx(t)$ and then make the performance index Eq. (2) to be minimum, where Kalman gain K is an $r \times n$ dimensional matrix.

So the design of the optimum control system is simplified to get the elements of matrix K. By stability theory, the optimization of the quadratic performance index indicated by Eq. (2) can be solved.

The feedback gain matrix K of the quadratic optimal regulator is obtained as follows [29]:

$$K = R^{-1}B^T S. \tag{3}$$

The matrix S in Eq. (3) is a positive definite matrix and must satisfy the following Riccati equation [9]:

$$A^T S + SA - SBR^{-1}B^T S + Q = 0. \tag{4}$$

Then the following nonlinear chaotic system is considered:

$$\dot{x}(t) = Ax(t) + F(t,x) + Bu_1(t), \tag{5}$$

where A is an $n \times n$ dimensional constant matrix, $x = (x_1, x_2, \ldots, x_n) \in R^n$ is the state variable of the system, $F(x) = (F_1, F_2, \ldots, F_n)^T$ is the nonlinear terms of the chaotic system and $u_1(t) = k_a(y(t) - x(t))$ is an r-dimensional control input where k_a is a constant vector. The second chaotic system is

$$\dot{y}(t) = Ay(t) + F(t,y) + Bu_2(t), \tag{6}$$

where B is an appropriate constant matrix, $u_2(t) = k_s(x(t) - y(t))$ is an r-dimensional control input where k_s is also a constant vector.

Define error vector $e = x - y$. From Eqs. (5) and (6), the error system is

$$\dot{e}(t) = [A - B(k_s + k_a)]e + F(t,x) - F(t,y). \tag{7}$$

In current schemes of chaos synchronization, maximum values of states must be determined by simulation [15–18]. They are half analytic method but not pure analytic method. In [14] $F(t,x) - F(t,y)$ nonlinear item is ignored. This is incorrect since there exist linear terms of e in $F(t,x) - F(t,y)$, which cannot be ignored. In this paper, the series expansion analysis offers a correct method.

The series expansion form of Eq. (7) is

$$\dot{e} = [A + M(x(t), y(t)) - B(k_s + k_a)]e + H(x(t), y(t), e), \tag{8}$$

where $M(x(t),y(t))e + H(x(t),y(t),e) = F(t,x) - F(t,y)$. The elements of $M(x(t),y(t))$ depend on state vectors x, y, and all of them are bounded convergent infinite series of x, y. $H(x(t),y(t),e)$ contains higher degree terms of e only.

If we choose appropriate k_a and k_s to make $A + M(x(t),y(t)) - B(k_s + k_a)$ asymptotically stable, then by first approximation theory, the zero solution $e = 0$ of Eq. (8) is asymptotically stable, i.e., systems (5) and (6) are synchronized.

Now we construct an optimal regulator, which is used to synchronize chaotic systems according to the theory of the quadratic optimal regulator, respectively, and the aim is to get the feedback gain matrices k_a and k_s of system (5) and of system (6), respectively. The steps to get matrices k_a and k_s are: (a) selecting an $n \times n$ dimensional positive semidefinite real symmetric constant matrix Q, an $r \times r$ dimensional positive definite real symmetric constant matrix R and a constant matrix B, with the constant matrix A we can get a Riccati equation as shown in Eq. (4). Then, we should solve this equation to get matrix S. If the positive definite matrix S exists, the matrix $A + M(x(t),y(t)) - B(k_s + k_a)$ is asymptotically stable and the design of control for the synchronization of two systems is successful. Otherwise we should reselect Q, R and B and calculate again. (b) Putting the matrix S in Eq. (3), we can get the gain matrices k_a and k_s of the regulators. After getting the matrices k_a and k_s according to the above steps, we put k_a, k_s and the matrix B in Eqs. (5) and (6). Then we get two synchronized systems.

3. Numerical results of the synchronization of two Quantum-CNN oscillator systems by unidirectional and by mutual linear coupling

Case I. The synchronization by unidirectional linear coupling.

For a two-cell Quantum-CNN, the following differential equations are obtained [19]:

$$\begin{cases} \dot{x}_1 = -2a_1\sqrt{1-x_1^2}\sin x_2, \\ \dot{x}_2 = -\omega_1(x_1-x_3) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2, \\ \dot{x}_3 = -2a_2\sqrt{1-x_3^2}\sin x_4, \\ \dot{x}_4 = -\omega_2(x_3-x_1) + 2a_2\frac{x_3}{\sqrt{1-x_3^2}}\cos x_4, \end{cases} \tag{9}$$

where x_1 and x_3 are polarizations, x_2 and x_4 are quantum phase displacements, a_1 and a_2 are proportional to the inter-dot energy inside each cell and ω_1 and ω_2 are parameters that weigh effects on the cell of the difference of the polarization of neighboring cells, like the cloning templates in traditional CNNs. Let $a_1 = a_2 = 2.47$, $\omega_1 = 1$, $\omega_2 = 1$, chaos is obtained for this system [20,23,24].

Two Quantum-CNN chaotic systems using the unidirectional linear coupling can be written as

$$\begin{cases} \dot{x}_1 = -2a_1\sqrt{1-x_1^2}\sin x_2, \\ \dot{x}_2 = -\omega_1(x_1-x_3) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2, \\ \dot{x}_3 = -2a_2\sqrt{1-x_3^2}\sin x_4, \\ \dot{x}_4 = -\omega_2(x_3-x_1) + 2a_2\frac{x_3}{\sqrt{1-x_3^2}}\cos x_4 \end{cases} \tag{10}$$

and

$$\begin{cases} \dot{y}_1 = -2a_1\sqrt{1-y_1^2}\sin y_2 + k_1(x_1-y_1), \\ \dot{y}_2 = -\omega_1(y_1-y_3) + 2a_1\frac{y_1}{\sqrt{1-y_1^2}}\cos y_2 + k_2(x_2-y_2), \\ \dot{y}_3 = -2a_2\sqrt{1-y_3^2}\sin y_4 + k_3(x_3-y_3), \\ \dot{y}_4 = -\omega_2(y_3-y_1) + 2a_2\frac{y_3}{\sqrt{1-y_3^2}}\cos y_4 + k_4(x_4-y_4). \end{cases} \tag{11}$$

The initial values for these linearly coupled Quantum-CNN systems are taken as $x_1(0) = 0.8$, $x_2(0) = -0.77$, $x_3(0) = -0.72$, $x_4(0) = 0.57$, $y_1(0) = -0.2$, $y_2(0) = 0.41$, $y_3(0) = 0.25$ and $y_4(0) = -0.81$.

Expand the right hand sides of Eqs. (10) and (11) into power series:

$$\begin{cases} \dot{x}_1 = -2a_1\left(-\frac{1}{2}x_1^2x_2 + \frac{1}{12}x_1^2x_2^3 - \frac{1}{8}x_1^4x_2 + x_2 - \frac{1}{6}x_2^3 + \frac{1}{120}x_2^5 + \cdots\right), \\ \dot{x}_2 = -\omega_1(x_1-x_3) + 2a_1\left(x_1 - \frac{1}{2}x_1x_2^2 + \frac{1}{24}x_1x_2^4 + \frac{1}{2}x_1^3 - \frac{1}{4}x_1^3x_2^2 + \frac{5}{8}x_1^5 + \cdots\right), \\ \dot{x}_3 = -2a_2\left(-\frac{1}{2}x_3^2x_4 + \frac{1}{12}x_3^2x_4^3 - \frac{1}{8}x_3^4x_4 + x_4 - \frac{1}{6}x_4^3 + \frac{1}{120}x_4^5 + \cdots\right), \\ \dot{x}_4 = -\omega_2(x_3-x_1) + 2a_2\left(x_3 - \frac{1}{2}x_3x_4^2 + \frac{1}{24}x_3x_4^4 + \frac{1}{2}x_3^3 - \frac{1}{4}x_3^3x_4^2 + \frac{5}{8}x_3^5 + \cdots\right) \end{cases} \tag{12}$$

and

$$\begin{cases} \dot{y}_1 = -2a_1\left(-\frac{1}{2}y_1^2y_2 + \frac{1}{12}y_1^2y_2^3 - \frac{1}{8}y_1^4y_2 + y_2 - \frac{1}{6}y_2^3 + \frac{1}{120}y_2^5 + \cdots\right) + k_1(x_1-y_1), \\ \dot{y}_2 = -\omega_1(y_1-y_3) + 2a_1\left(y_1 - \frac{1}{2}y_1y_2^2 + \frac{1}{24}y_1y_2^4 + \frac{1}{2}y_1^3 - \frac{1}{4}y_1^3y_2^2 + \frac{5}{8}y_1^5 + \cdots\right) + k_2(x_2-y_2), \\ \dot{y}_3 = -2a_2\left(-\frac{1}{2}y_3^2y_4 + \frac{1}{12}y_3^2y_4^3 - \frac{1}{8}y_3^4y_4 + y_4 - \frac{1}{6}y_4^3 + \frac{1}{120}y_4^5 + \cdots\right) + k_3(x_3-y_3), \\ \dot{y}_4 = -\omega_2(y_3-y_1) + 2a_2\left(y_3 - \frac{1}{2}y_3y_4^2 + \frac{1}{24}y_3y_4^4 + \frac{1}{2}y_3^3 - \frac{1}{4}y_3^3y_4^2 + \frac{5}{8}y_3^5 + \cdots\right) + k_4(x_4-y_4). \end{cases} \tag{13}$$

From Eqs. (12) and (13), the error dynamics is:

$$\dot{e} = [A + M(x(t), y(t)) - Bk_s]e + H(x, y, e), \tag{14}$$

where $e = (y_1 - x_1, y_2 - x_2, y_3 - x_3, y_4 - x_4)^T$ and

$$M(x(t), y(t)) = \begin{pmatrix} M_{11} & -2a_1 + M_{21} & 0 & 0 \\ 2a_1 + M_{12} & M_{22} & 0 & 0 \\ 0 & 0 & M_{33} & -2a_2 + M_{43} \\ 0 & 0 & 2a_2 + M_{34} & M_{44} \end{pmatrix}$$

in which

$$M_{11} = a_1\left[2x_1y_2 - \frac{1}{6}x_1y_2^3 + \frac{1}{4}(x_1y_1^2y_2 + 3x_1^2y_1y_2) + \cdots\right]$$

$\ldots$

and $H(x, y, e)$ contains higher degree terms of e only.

The infinite power series of the first element of M, i.e., M_{11} is

$$2x_1y_2 - \frac{1}{6}x_1y_2^3 + \frac{1}{4}(x_1y_1^2y_2 + 3x_1^2y_1y_2) + \cdots \tag{15}$$

It is well-known [28] that a necessary and sufficient condition for the convergence of the infinite series

$$u_1 + u_2 + \cdots + u_n + \cdots$$

is that for any previously assigned positive ε there exists an N such that, for any $n > N$ and for positive p,

$$|u_{n+1} + u_{n+2} + \cdots u_{n+p}| < \varepsilon. \tag{16}$$

From Fig. 1, we know that

$$|x_i| < 1, \quad |y_i| < 1 \qquad (i = 1,2,3,4). \tag{17}$$

Therefore, M_{11} and series contained in other elements of $M(x(t),y(t))$ are convergent series and they have bounded sums.

We can get the optimum gain $k_s = [k_1,k_2,k_3,k_4]^T$ by the method of constructing a quadratic optimal regulator. With

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\omega_1 & 0 & \omega_1 & 0 \\ 0 & 0 & 0 & 0 \\ \omega_2 & 0 & -\omega_2 & 0 \end{bmatrix}$$

we choose

$$B = [0 \quad 0 \quad 0 \quad 1]^T; \quad R = [1]; \quad Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}. \tag{18}$$

After solving the corresponding Riccati equation, we get the gain matrix $k_s = [k_1,k_2,k_3,k_4]^T = [0,1,0,1]^T$.

From the simulation results of Fig. 1, it is shown that master system and slave system reach the synchronization state after they are controlled by the quadratic optimal regulator. It is noticed that the synchronization effect is good.

Case II. The synchronization by mutual linear coupling.

Two Quantum-CNN systems with mutual linear coupling are given:

$$\begin{cases} \dot{x}_1 = -2a_1\sqrt{1-x_1^2}\sin x_2 + k_{11}(y_1 - x_1), \\ \dot{x}_2 = -\omega_1(x_1 - x_3) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2 + k_{12}(y_2 - x_2), \\ \dot{x}_3 = -2a_2\sqrt{1-x_3^2}\sin x_4 + k_{13}(y_3 - x_3), \\ \dot{x}_4 = -\omega_2(x_3 - x_1) + 2a_2\frac{x_3}{\sqrt{1-x_3^2}}\cos x_4 + k_{14}(y_4 - x_4) \end{cases} \tag{19}$$

and

$$\begin{cases} \dot{y}_1 = -2a_1\sqrt{1-y_1^2}\sin y_2 + k_{21}(x_1 - y_1), \\ \dot{y}_2 = -\omega_1(y_1 - y_3) + 2a_1\frac{y_1}{\sqrt{1-y_1^2}}\cos y_2 + k_{22}(x_2 - y_2), \\ \dot{y}_3 = -2a_2\sqrt{1-y_3^2}\sin y_4 + k_{23}(x_3 - y_3), \\ \dot{y}_4 = -\omega_2(y_3 - y_1) + 2a_2\frac{y_3}{\sqrt{1-y_3^2}}\cos y_4 + k_{24}(x_4 - y_4). \end{cases} \tag{20}$$

Expand the right hand sides of Eqs. (19) and (20) into power series:

$$\begin{cases} \dot{x}_1 = -2a_1\left(-\frac{1}{2}x_1^2x_2 + \frac{1}{12}x_1^2x_2^3 - \frac{1}{8}x_1^4x_2 + x_2 - \frac{1}{6}x_2^3 + \frac{1}{120}x_2^5 + \cdots\right) + k_{11}(y_1 - x_1), \\ \dot{x}_2 = -\omega_1(x_1 - x_3) + 2a_1\left(x_1 - \frac{1}{2}x_1x_2^2 + \frac{1}{24}x_1x_2^4 + \frac{1}{2}x_1^3 - \frac{1}{4}x_1^3x_2^2 + \frac{5}{8}x_1^5 + \cdots\right) + k_{12}(y_2 - x_2), \\ \dot{x}_3 = -2a_2\left(-\frac{1}{2}x_3^2x_4 + \frac{1}{12}x_3^2x_4^3 - \frac{1}{8}x_3^4x_4 + x_4 - \frac{1}{6}x_4^3 + \frac{1}{120}x_4^5 + \cdots\right) + k_{13}(y_3 - x_3), \\ \dot{x}_4 = -\omega_2(x_3 - x_1) + 2a_2\left(x_3 - \frac{1}{2}x_3x_4^2 + \frac{1}{24}x_3x_4^4 + \frac{1}{2}x_3^3 - \frac{1}{4}x_3^3x_4^2 + \frac{5}{8}x_3^5 + \cdots\right) + k_{14}(y_4 - x_4) \end{cases} \tag{21}$$

and

$$\begin{cases} \dot{y}_1 = -2a_1\left(-\frac{1}{2}y_1^2y_2 + \frac{1}{12}y_1^2y_2^3 - \frac{1}{8}y_1^4y_2 + y_2 - \frac{1}{6}y_2^3 + \frac{1}{120}y_2^5 + \cdots\right) + k_{21}(x_1 - y_1), \\ \dot{y}_2 = -\omega_1(y_1 - y_3) + 2a_1\left(y_1 - \frac{1}{2}y_1y_2^2 + \frac{1}{24}y_1y_2^4 + \frac{1}{2}y_1^3 - \frac{1}{4}y_1^3y_2^2 + \frac{5}{8}y_1^5 + \cdots\right) + k_{22}(x_2 - y_2), \\ \dot{y}_3 = -2a_2\left(-\frac{1}{2}y_3^2y_4 + \frac{1}{12}y_3^2y_4^3 - \frac{1}{8}y_3^4y_4 + y_4 - \frac{1}{6}y_4^3 + \frac{1}{120}y_4^5 + \cdots\right) + k_{23}(x_3 - y_3), \\ \dot{y}_4 = -\omega_2(y_3 - y_1) + 2a_2\left(y_3 - \frac{1}{2}y_3y_4^2 + \frac{1}{24}y_3y_4^4 + \frac{1}{2}y_3^3 - \frac{1}{4}y_3^3y_4^2 + \frac{5}{8}y_3^5 + \cdots\right) + k_{24}(x_4 - y_4). \end{cases} \tag{22}$$

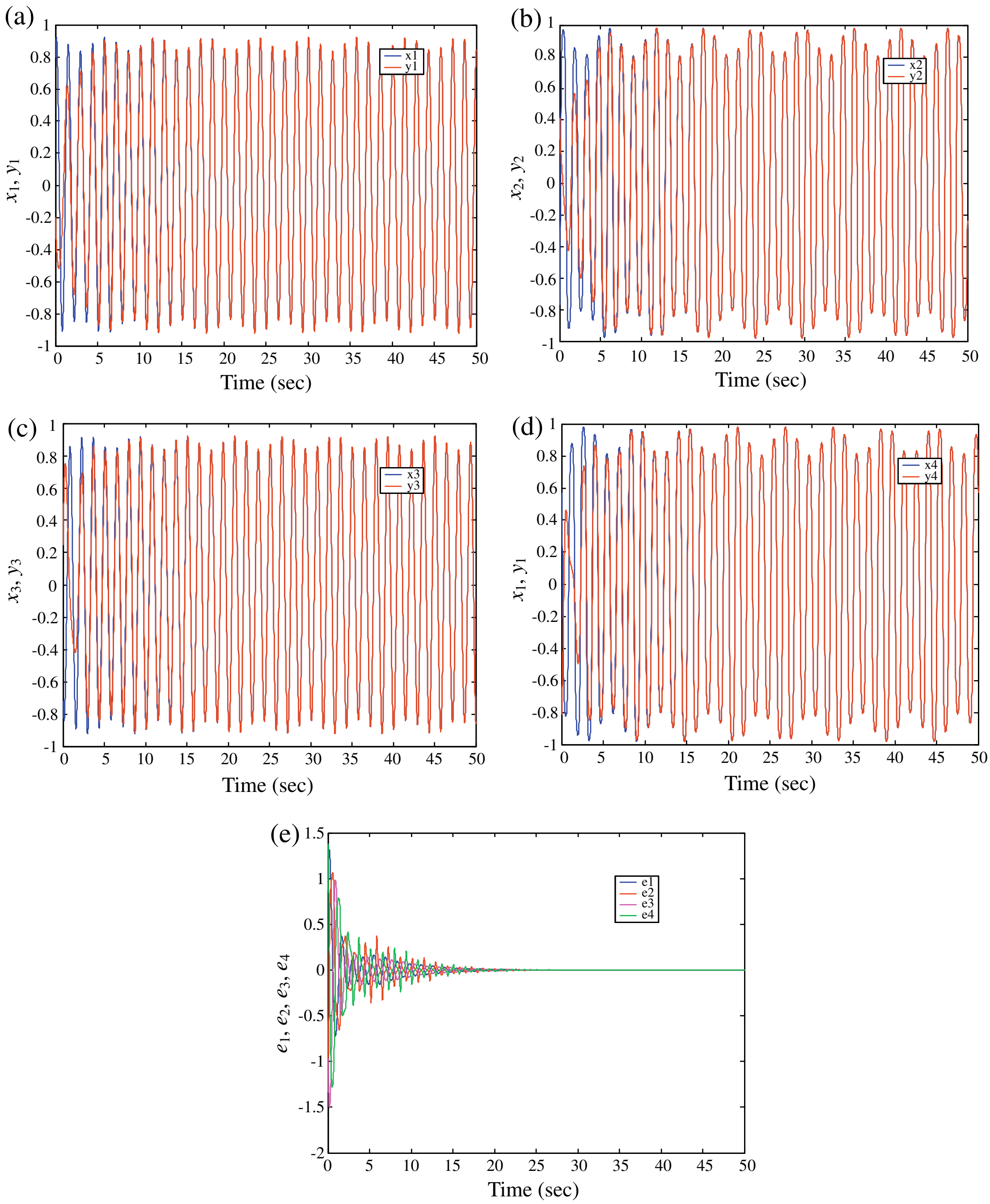

Fig. 1. Time histories of states, state errors for unidirectional linear coupling case.

From Eqs. (21) and (22), the error dynamics is:

$$\dot{e} = [A + M(x(t), y(t) - B(k_s + k_a))]e + H(x, y, e), \tag{23}$$

where $e = (y_1 - x_1, y_2 - x_2, y_3 - x_3, y_4 - x_4)^T$ and

$$M(x(t), y(t)) = \begin{pmatrix} M_{11} & -2a_1 + M_{21} & 0 & 0 \\ 2a_1 + M_{12} & M_{22} & 0 & 0 \\ 0 & 0 & M_{33} & -2a_2 + M_{43} \\ 0 & 0 & 2a_2 + M_{34} & M_{44} \end{pmatrix}$$

in which

$$M_{11} = a_1\left[2x_1y_2 - \frac{1}{6}x_1y_2^3 + \frac{1}{4}(x_1y_1^2y_2 + 3x_1^2y_1y_2) + \cdots\right]$$
$$\ldots$$

Similar to Case I, from Fig. 2, $|x_i| < 1$, $|y_i| < 1$ $(i = 1,2,3,4)$, the infinite power series elements of $M(x(t),y(t))$ are all convergent and have bounded sums [27,28].

The optimum gains $k_a = [k_{11},k_{12},k_{13},k_{14}]^T$ and $k_s = [k_{21},k_{22},k_{23},k_{24}]^T$ can be obtained by the method of constructing a quadratic optimal regulator. With

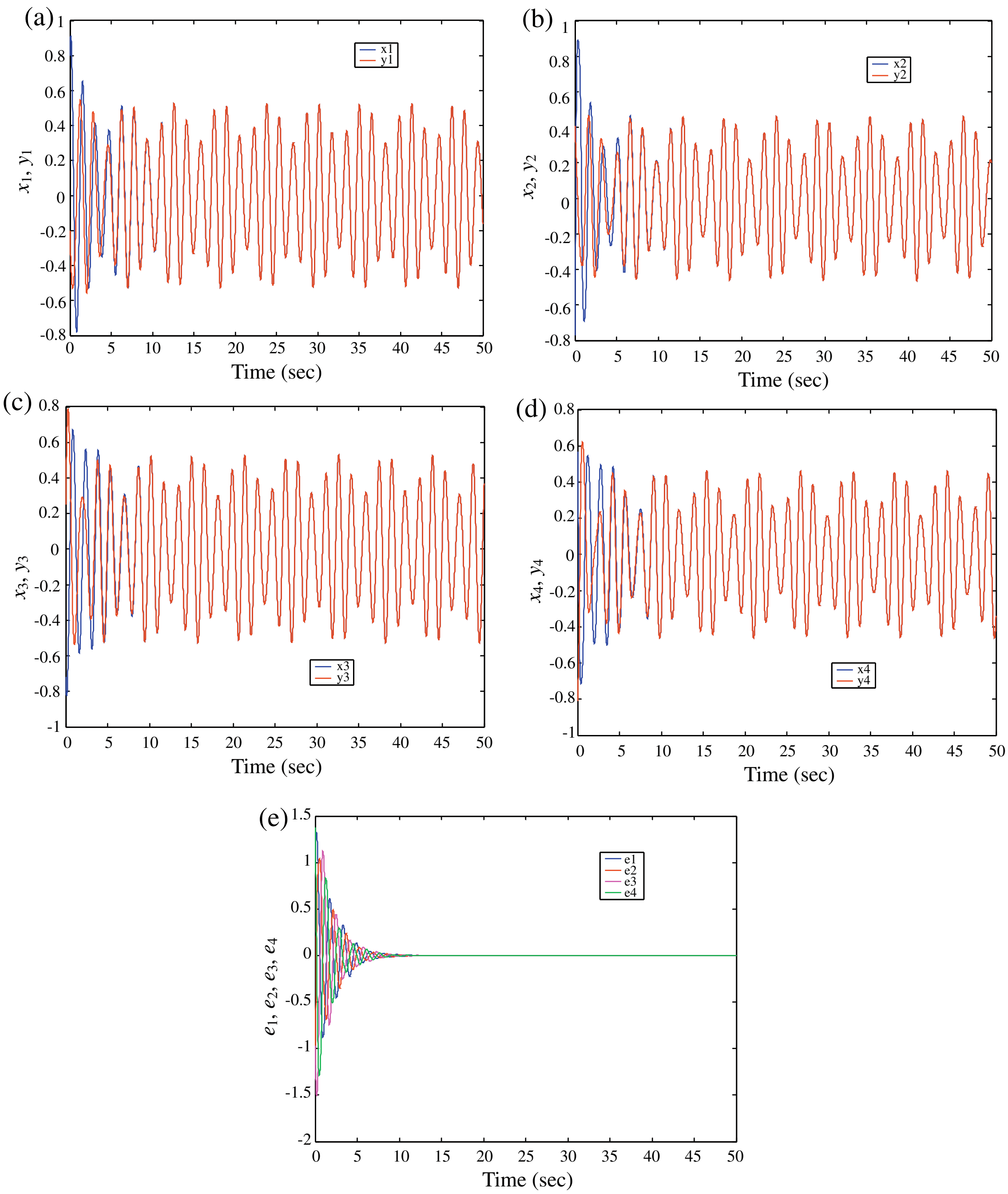

Fig. 2. Time histories of states, state errors for mutual linear coupling case.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\omega_1 & 0 & \omega_1 & 0 \\ 0 & 0 & 0 & 0 \\ \omega_2 & 0 & -\omega_2 & 0 \end{bmatrix}$$

we choose

$$B = [0 \quad 0 \quad 0 \quad 1]^T; \quad R = [1]; \quad Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}. \tag{24}$$

After solving the corresponding Riccati equation, we then get two gain matrices $k_a = [k_{11}, k_{12}, k_{13}, k_{14}]^T = [0, 0.5, 0, 0.5]^T$ and $k_s = [k_{21}, k_{22}, k_{23}, k_{24}]^T = [0, 0.5, 0, 0.5]^T$.

From the simulation results of Fig. 2 two systems reach the synchronization state after they are controlled by the quadratic optimal regulator. It is noticed that the synchronization effect is also satisfactory.

4. Chaotization of Quantum-CNN chaotic system scheme and simulation

Optimal control is a well-established engineering control strategy, and is useful for both linear and nonlinear system with linear or nonlinear controllers [3]. Now, we use a typical optimal control for the chaotization of Quantum-CNN system. Consider the system (9) with a controller u and define the Hamilton function:

$$\begin{aligned} &H(x_1, x_2, x_3, x_4, u, p) = \boldsymbol{p}^T \boldsymbol{F}(x_1, x_2, x_3, x_4, u, p); \\ &\boldsymbol{p}^T = [p_1 \ p_2 \ p_3 \ p_4], \end{aligned} \tag{25}$$

where $\boldsymbol{p}$ is a Lagrange multiplier, called a co-state vector, $\boldsymbol{F}$ is the right hand side of Eq. (9). Following the variational principle of optimal control, we can obtain

$$p_2\left(-\omega_1(x_1 - x_3) + 2a_1\frac{x_1}{\sqrt{1 - x_1^2}}\cos x_2\right) + p_3\left(-2a_2\sqrt{1 - x_3^2}\sin x_4\right) + p_4\left(-\omega_2(x_3 - x_1) + 2a_2\frac{x_3}{\sqrt{1 - x_3^2}}\cos x_4\right) = 0, \tag{26}$$

$$p_2\frac{-2a_1}{\sqrt{1 - x_1^2}}\sin x_2 = 0. \tag{27}$$

This yield a non-trivial solution for (p_2, p_3, p_4) if and only if

$$\frac{-2a_1}{\sqrt{1 - x_1^2}}\sin x_2 = 0. \tag{28}$$

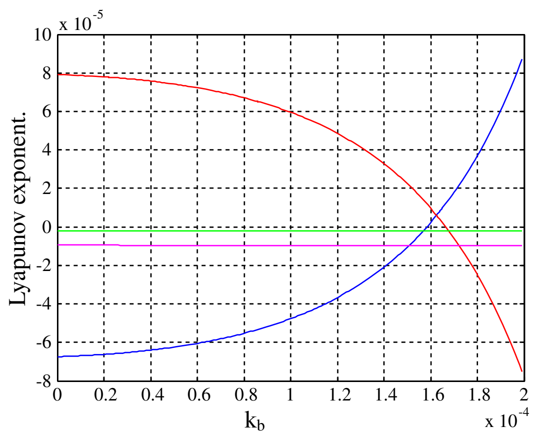

Fig. 3. Lyapunov exponents of controlled Quantum-CNN system.

It gives an optimal surface singularly in the state space. This type of control assumes values on the two allowable boundaries (27) and (28) alternatively according to a switching surface. Locating system trajectories on the surface, a typical feedback control in the form

$$u = -k_b \mathrm{sgn}\left[\frac{-2a_1}{\sqrt{1-x_1^2}}\sin x_2\right] \tag{29}$$

can be used. By adjusting the value of k_b from zero initial value to $k_b = 1.6 \times 10^{-4}$ in the above controller with the signum function

$$\mathrm{sgn}[v] = \begin{cases} 1 & \text{if } v > 0, \\ 0 & \text{if } v = 0, \\ -1 & \text{if } v < 0 \end{cases} \tag{30}$$

the chaotic motion with one positive Lyapunov exponent can be controlled to chaotic motion with two positive Lyapunov exponents as shown by the simulation result in Fig. 3.

5. Conclusions

Two chaotic Quantum-CNN systems are synchronized in two cases: unidirectional linear coupling by optimum control, mutual linear coupling by optimum control. The number of controllers for optimum control is less than that for synchronization only by linear couplings. This results in lower cost. In chaos synchronization cases, by a theorem of convergent series, we prove that all infinite power series elements of $A + M(x(t),y(t)) - B(k_s + k_a)$ are convergent and have bounded sums. This synchronization of chaos systems can be used to increase the security of communication. Next, the optimum control is used for chaotization, i.e., to enhance original chaotic state to more complex chaotic state. Numerical simulations are used to verify the effectiveness of the proposed scheme.

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