Joint Source/Relay Precoder Design in Nonregenerative Cooperative Systems Using an MMSE Criterion

Fan-Shuo Tseng, *Student Member, IEEE,* Wen-Rong Wu, *Member, IEEE,* and Jwo-Yuh Wu, *Member, IEEE*

***Abstract*—This paper considers transmitter precoding in an amplify-and-forward cooperative system where multiple antennas are equipped at the source, the relay, and the destination. Existing methods for the problem only consider the design of the relay precoder. To further improve the performance, we include the source precoder into the design. Using a minimum-mean-square-error (MMSE) criterion, we propose a joint source/relay precoder design method, taking both the direct and relay links into account. It is shown that the MMSE is a highly nonlinear function of the precoding matrices, and a direct minimization is not feasible. To facilitate analysis, we propose to design the precoders toward first diagonalizing the MSE matrix of the relay link. This imposes certain structural constraints on both precoders that allow us to derive an analytically tractable MSE upper bound. By conducting minimization with respect to this bound, the solution can be obtained by an iterative water-filling technique.**

***Index Terms*—Precoder, amplify-and-forward (AF), cooperative transmission schemes, channel state information (CSI), multiple-input multiple-output (MIMO), mean-square-error (MSE).**

I. Introduction

Cooperative communications can realize spatial diversity in a distributed manner and has attracted considerable attention in the past few years [2]-[11]. Most of the existing works focused on devising, and analyzing, different cooperative transmission protocols such as amplify-and-forward (AF) and decode-and-forward (DF) [2]. Also, there have been many proposals which leverage the traditional MIMO processing techniques, e.g., beamforming and space-time block coding, and develop the corresponding distributed realizations for enhancing the link quality [3]-[10]. In the study of conventional cooperative systems, each user terminal is commonly assumed to be equipped with a single antenna. To further increase the spatial degrees-of-freedom, one simple approach is to place multiple antennas at each node [11]-[14]. Current research on such MIMO cooperative networks mainly focuses on precoder designs under the AF protocol, either for boosting capacity [12],[13], or for improving link reliability [14]. All of these proposals, however, consider precoders only at the relays. Some of them even neglect the direct (source-to-destination) link in the problem formulation so as to ease the relay precoder design [12], [14]. Hence, the available link resources are not yet fully exploited in the existing schemes.

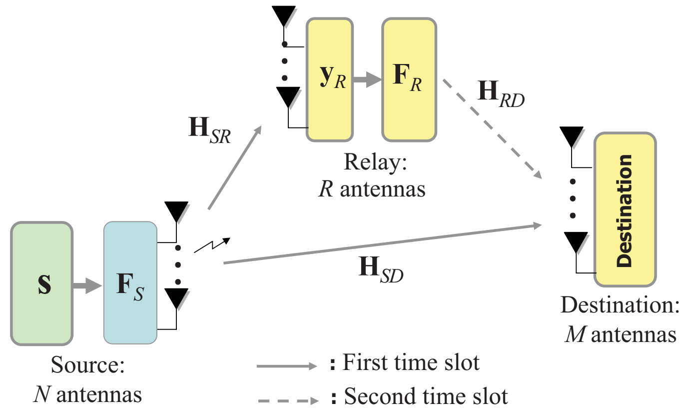

Fig. 1. Three-node AF MIMO relay system.

As far as we know, the joint source-relay precoder design for AF MIMO relay networks which take both the direct and relay links into account has not been studied before. This work aims to provide a solution to this problem in the typical three-node system scenario [12]-[14]. Using the minimum-mean-square-error (MMSE) criterion and individual power constraints at the source and the relay, we formulate the design as a joint optimization problem. However, it is found that the MMSE is a complicated function of precoding matrices. Direct minimization with the cost function is not feasible. To overcome the difficulty, we propose to design the precoders toward first diagonalizing the MSE matrix of the relay link. This imposes certain structural constraints on the precoding matrices that allow us to derive a tractable MSE upper bound. Minimization with this upper bound, instead of the original MMSE, then becomes feasible and a suboptimal closed-form solution is then obtained. The proposed precoders can be computed via an iterative water-filling technique. The rest of this paper is organized as follows. Section 2 introduces the system model and problem formulation. The main results are given in Section 3. Finally, Section 4 concludes this paper.

II. Proposed System Model and Problem Formulation

A. System Model

We consider a three-node precoded AF system over flat fading channels as depicted in Figure 1, in which the source, the relay, and the destination are equipped with N, R, M antennas. Using the typical two-phase transmission scheme

Manuscript received August 27, 2008; revised January 11, 2009 and May 18, 2009; accepted July 2, 2009. The associate editor coordinating the review of this letter and approving it for publication is B. S. Rajan.

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This work is sponsored jointly by the National Science Council under grant NSC-97-2221-E-009-001-MY3, by the Ministry of Education of Taiwan under the MoE ATU Program, and by MediaTek Research Center at National Chiao Tung University, Taiwan.

Digital Object Identifier 10.1109/TWC.2009.081152

[12], [13], we can express the received signal at the destination in a vector form as

$$\mathbf{y}_D := \underbrace{\begin{bmatrix} \mathbf{H}_{SD}\mathbf{F}_S \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}\mathbf{F}_S \end{bmatrix}}_{:=\mathbf{H}} \mathbf{s} + \underbrace{\begin{bmatrix} \mathbf{n}_{D,1} \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{n}_R + \mathbf{n}_{D,2} \end{bmatrix}}_{:=\mathbf{w}}. \quad (1)$$

In (1) $\mathbf{s} \in \mathbb{C}^L$ is the transmitted signal vector with zero mean and $E\left[\mathbf{s}\mathbf{s}^H\right] = \sigma_s^2\mathbf{I}$; $\mathbf{y}_D \in \mathbb{C}^{2M\times 1}$ is the received signal vector at the destination; $\mathbf{F}_S \in \mathbb{C}^{N\times L}$ and $\mathbf{F}_R \in \mathbb{C}^{R\times R}$ are the precoding matrices at the source and the relay, respectively; $\mathbf{H}_{SR} \in \mathbb{C}^{R\times N}$, $\mathbf{H}_{SD} \in \mathbb{C}^{M\times N}$ and $\mathbf{H}_{RD} \in \mathbb{C}^{M\times R}$ are the channel matrices of the source-to-relay, the source-to-destination, and the relay-to-destination links, respectively; $\mathbf{n}_{D,1} \in \mathbb{C}^{M\times 1}$, $\mathbf{n}_R \in \mathbb{C}^{R\times 1}$, and $\mathbf{n}_{D,2} \in \mathbb{C}^{M\times 1}$ are the received noise vectors at the destination in the first-phase, at the relay, and at the destination in the second-phase. Here, we assume $L \leq \{N, R, M\}$ to provide sufficient degrees of freedom for transmission.

To recover the source signal, a linear equalizer is adopted at the destination. The equalizer, specified with a matrix $\mathbf{G} \in \mathbb{C}^{M\times M}$, is designed by minimizing the MSE

$$J = E\left\{\|\mathbf{G}\mathbf{y}_D - \mathbf{s}\|^2\right\}. \quad (2)$$

The optimal solution is easily shown to be [1]

$$\mathbf{G}_{opt} = \sigma_s^2\mathbf{H}^H\left(\sigma_s^2\mathbf{H}\mathbf{H}^H + \mathbf{R}_w\right)^{-1}, \quad (3)$$

where $\mathbf{R}_w = E\left[\mathbf{w}\mathbf{w}^H\right]$ is the covariance matrix of the combined noise vector $\mathbf{w}$. At the equalizer output, the detection process is done by a symbol-by-symbol based method. Denote the variance of the noise components at the destination as $\sigma_{n,d}^2$, and that at the relay as $\sigma_{n,r}^2$. Based on (1), (2), and (3), we can have the MMSE as

$$J_{min} = tr\{\underbrace{\left(\sigma_s^{-2}\mathbf{I}_L + \mathbf{E}_S + \mathbf{E}_R\right)^{-1}}_{:=\mathbf{E}}\}, \quad (4)$$

with

$$\mathbf{E}_S = \sigma_{n,d}^{-2}\mathbf{F}_S^H\mathbf{H}_{SD}^H\mathbf{H}_{SD}\mathbf{F}_S \quad (5)$$

and

$$\mathbf{E}_R = \mathbf{F}_S^H\mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H \cdot \left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}\mathbf{F}_S. \quad (6)$$

We note that $\mathbf{E}_R$ and $\mathbf{E}_S$ account for the MSE components which arise due to the relay and the direct communication links, respectively.

Note that in the considered system the source and the destination need to know all link channel matrices for the precoder design and the MMSE receiver implementation. While each point-to-point link channel can be estimated via the conventional training scheme, the acquisition of the "far-end" channel, e.g., the estimation of the source-to-relay channel at the destination, can be done by using the technique reported in [15].

B. Problem Formulation

As we can see from (4), the MMSE is a function of the precoder matrices $\mathbf{F}_S$ and $\mathbf{F}_R$. Hence the optimal precoders can be obtained as the solution to the following optimization problem:

$$\begin{aligned} &\min_{\mathbf{F}_S,\mathbf{F}_R} tr\left\{\left(\sigma_s^{-2}\mathbf{I}_L + \mathbf{E}_S + \mathbf{E}_R\right)^{-1}\right\} \\ &s.t. \\ &tr\left\{E\left[\mathbf{F}_R\mathbf{y}_R\mathbf{y}_R^H\mathbf{F}_R^H\right]\right\} = \\ &tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R + \sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\} \leq P_{R,T} \\ &tr\left\{\mathbf{F}_S E\left[\mathbf{s}\mathbf{s}^H\right]\mathbf{F}_S^H\right\} = \sigma_s^2 tr\left\{\mathbf{F}_S\mathbf{F}_S^H\right\} \leq P_{S,T} \end{aligned} \quad (7)$$

where $P_{S,T}$ and $P_{R,T}$ are the maximal available powers at the source and the relay, respectively. By ignoring the direct link and simultaneously adopting a precoder only at the relay, the MMSE design criterion was also considered in [14] for the AF MIMO system. In this paper, we further incorporate the precoder at the source and consider both the direct and relay link signals to enhance the performance of the AF MIMO system.

III. Joint Source/Relay Precoder Design

Taking a closer look at (4), we readily find that the minimum MSE involves a series of matrix multiplications and inversions and is thus a complicated function of $\mathbf{F}_S$ and $\mathbf{F}_R$. Exact solution to (4), therefore, is difficult to derive. In this section, we propose an effective method for finding a suboptimal solution. The main idea is to use a constrained precoder structure, derive an MMSE upper bound having a simple expression, and conduct minimization with this upper bound. The resultant optimization problem can be solved by using the standard Lagrange technique followed by an iterative water-filling procedure.

A. Proposed Approach

For the special case that the direct link is absent and only a relay precoder is used, the optimal MMSE precoder can be analytically obtained through MSE matrix diagonalization [14]. Motivated by this fact and toward an analytical procedure for the joint source/relay precoder design, we propose to find a solution via a similar matrix diagonalization technique. Indeed, if the error matrix $\mathbf{E}$ in (4) can be diagonalized, the trace operation can be easily conducted, and the whole problem can be greatly simplified. To do that, we firstly consider singular value decomposition (SVD) for channel matrices in all links as:

$$\begin{aligned} \mathbf{H}_{SD} &= \mathbf{U}_{sd}\mathbf{\Sigma}_{sd}\mathbf{V}_{sd}^H; \\ \mathbf{H}_{SR} &= \mathbf{U}_{sr}\mathbf{\Sigma}_{sr}\mathbf{V}_{sr}^H; \\ \mathbf{H}_{RD} &= \mathbf{U}_{rd}\mathbf{\Sigma}_{rd}\mathbf{V}_{rd}^H. \end{aligned} \quad (8)$$

For later use, we define $\sigma_{sd,i}$, $\sigma_{sr,i}$, and $\sigma_{rd,i}$ as the ith diagonal element of $\mathbf{\Sigma}_{sd} \in \mathbb{R}^{M\times N}$, $\mathbf{\Sigma}_{sr} \in \mathbb{R}^{R\times N}$, and $\mathbf{\Sigma}_{rd} \in \mathbb{R}^{M\times R}$, respectively.

From (4), it is seen that the error matrix $\mathbf{E}$ can be diagonalized if $\mathbf{F}_S$ and $\mathbf{F}_R$ can be chosen to simultaneously diagonalize $\mathbf{E}_S$ and $\mathbf{E}_R$. Such a task, however, appears quite difficult to achieve mainly because $\mathbf{E}_R$ depends on the relay precoder $\mathbf{F}_R$ through the matrix inversion $(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M)^{-1}$. This thus motivates us to first choose $\mathbf{F}_R$ to diagonalize $\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M$ so that the inverse can be easily tackled. Such an approach, though suboptimal, will considerably simplify the analysis; more importantly, it allows us to derive a tractable MSE upper bound which leads to a water-filling based solution. With the aid of the SVD of the channel matrix $\mathbf{H}_{RD}$ in (8), an immediate choice for $\mathbf{F}_R$ to diagonalize $\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M$ is

$$\mathbf{F}_R = \mathbf{V}_{rd}\boldsymbol{\Sigma}_r\mathbf{U}_r, \tag{9}$$

where the diagonal $\boldsymbol{\Sigma}_r \in \mathbb{R}^{R\times R}$ and the unitary $\mathbf{U}_r \in \mathbb{C}^{R\times R}$ are to be determined. With (9) we have

$$\begin{aligned}&\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\\ &= \mathbf{U}_{rd}\left(\sigma_{n,r}^2\boldsymbol{\Sigma}_{rd}\boldsymbol{\Sigma}_r^2\boldsymbol{\Sigma}_{rd}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{U}_{rd}^H \end{aligned}\tag{10}$$

and the matrix $\mathbf{E}_R$ can be rearranged into

$$\begin{aligned}\mathbf{E}_R = {} & \mathbf{F}_S^H\mathbf{H}_{SR}^H\mathbf{U}_r^H\boldsymbol{\Sigma}_r^H\boldsymbol{\Sigma}_{rd}^H\cdot\\ & \left(\sigma_{n,r}^2\boldsymbol{\Sigma}_{rd}\boldsymbol{\Sigma}_r^2\boldsymbol{\Sigma}_{rd}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\boldsymbol{\Sigma}_{rd}\boldsymbol{\Sigma}_r\mathbf{U}_r\mathbf{H}_{SR}\mathbf{F}_S.\end{aligned}\tag{11}$$

Equation (11) allows us to simplify the expression for $\mathbf{E}_R$ even further. In particular, $\mathbf{E}_R$ can be diagonalized if $\mathbf{U}_r$ and the source precoder $\mathbf{F}_S$ are chosen so that $\mathbf{U}_r\mathbf{H}_{SR}\mathbf{F}_S$ is diagonal. Again with SVD of the $\mathbf{H}_{SR}$ in (8), this can be done by setting

$$\mathbf{U}_r = \mathbf{U}_{sr}^H, \tag{12}$$

and

$$\mathbf{F}_S = \mathbf{V}_{sr}\boldsymbol{\Sigma}_s, \tag{13}$$

where $\boldsymbol{\Sigma}_s \in \mathbb{R}^{N\times L}$ is a diagonal matrix yet to be specified. Combining (9) and (13), the diagonalization of $\mathbf{E}_R$ imposes the following structural constraint on the relay precoder

$$\mathbf{F}_R = \mathbf{V}_{rd}\boldsymbol{\Sigma}_r\mathbf{U}_{sr}^H. \tag{14}$$

With (13) and (14), and after some straightforward manipulations, the MSE cost function in (4) can be expressed as (15), in which $\mathbf{V} = \mathbf{V}_{sd}^H\mathbf{V}_{sr}$ is a constant matrix depending on the channels. We make the following key observations regarding the alternative MSE expression (15): i) Since (15) is obtained based on the particular precoding matrices $\mathbf{F}_S$ in (13) and $\mathbf{F}_R$ in (14), it serves as an upper bound of the true minimal MSE; ii) Compared with the original MSE formula (4), the expression (15) is more appealing because the unknowns involved are $\boldsymbol{\Sigma}_r$ and $\boldsymbol{\Sigma}_s$, which are diagonal matrices and are more amenable to handle; iii) The matrix $\mathbf{E}_S$ cannot be diagonized. However, starting from (15) and exploiting the diagonal nature of $\mathbf{E}_R$, we can further derive a more tractable MSE upper bound that will be used as the design cost function, as shown next.

To proceed, let us use the matrix inversion lemma [16] to rewrite (15) as

$$\begin{aligned}&tr(\mathbf{E}) = \\ &tr\left(\left[\underbrace{\left(\sigma_s^{-2}\mathbf{I}_L + \mathbf{E}_R\right)}_{:=\mathbf{A}} + \boldsymbol{\Sigma}_s^H\underbrace{\left(\sigma_{n,d}^{-2}\mathbf{V}^H\boldsymbol{\Sigma}_{sd}^H\boldsymbol{\Sigma}_{sd}\mathbf{V}\right)}_{:=\mathbf{B}}\boldsymbol{\Sigma}_s\right]^{-1}\right)\\ &= tr\left(\mathbf{A}^{-1}\right) - \\ &\quad tr\left(\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H\left(\mathbf{B}^{-1} + \boldsymbol{\Sigma}_s\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H\right)^{-1}\boldsymbol{\Sigma}_s\mathbf{A}^{-1}\right).\end{aligned}\tag{16}$$

Based on (16), the MSE upper bound can be obtained with the aid of the next lemma.

Lemma: Let $\mathbf{Z}$ is a positive definite matrix, then we have

$$\left(\mathbf{Z}\right)^{-1}(i,i) \geq \frac{1}{\mathbf{Z}(i,i)}. \tag{17}$$

Proof: Considering eigen-decomposition of $\mathbf{Z} := \mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^H$, we have

$$\begin{aligned}\mathbf{Z} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^H &= \underbrace{\mathbf{U}\boldsymbol{\Sigma}^{1/2}\mathbf{U}^H}_{:=\mathbf{Z}^{1/2}}\underbrace{\mathbf{U}\boldsymbol{\Sigma}^{1/2}\mathbf{U}^H}_{:=\mathbf{Z}^{1/2}};\\ \mathbf{Z}^{-1} = \mathbf{U}\boldsymbol{\Sigma}^{-1}\mathbf{U}^H &= \underbrace{\mathbf{U}\boldsymbol{\Sigma}^{-1/2}\mathbf{U}^H}_{:=\mathbf{Z}^{-1/2}}\underbrace{\mathbf{U}\boldsymbol{\Sigma}^{-1/2}\mathbf{U}^H}_{:=\mathbf{Z}^{-1/2}}.\end{aligned}\tag{18}$$

Since $1 = \mathbf{e}_i^T\mathbf{I}\mathbf{e}_i = \mathbf{e}_i^T\mathbf{Z}^{1/2}\mathbf{Z}^{-1/2}\mathbf{e}_i$, where $\mathbf{e}_i$ is the ith unit standard vector, we have

$$\begin{aligned}&1 = \|\mathbf{e}_i^T\mathbf{Z}^{1/2}\mathbf{Z}^{-1/2}\mathbf{e}_i\|_2^2 \leq \underbrace{\|\mathbf{e}_i^T\mathbf{Z}^{1/2}\|_2^2}_{:=\mathbf{Z}(i,i)}\underbrace{\|\mathbf{Z}^{-1/2}\mathbf{e}_i\|_2^2}_{:=\mathbf{Z}^{-1}(i,i)},\\ &\text{equivalently } \mathbf{Z}^{-1}(i,i) \geq \frac{1}{\mathbf{Z}(i,i)},\end{aligned}\tag{19}$$

where the inequality in (19) follows from the sub-multiplicative property of the matrix norm [16]. QED.

By setting $\mathbf{Z} = \mathbf{B}^{-1} + \boldsymbol{\Sigma}_s\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H$ and applying the lemma to (16), we can obtain the following key result

$$tr(\mathbf{E}) \leq \sum_{i=1}^{L}\frac{1}{\sigma_s^{-2} + \frac{\sigma_{s,i}^2\sigma_{r,i}^2\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{n,r}^2\sigma_{r,i}^2\sigma_{rd,i}^2 + \sigma_{n,d}^2} + \sigma_{n,d}^{-2}\frac{\sigma_{s,i}^2}{\mathbf{B}^{-1}(i,i)}} \tag{20}$$

Compared with the original MSE function (4), the upper bound (20) admits a simple rational form and is analytically tractable. Hence, we propose to design the precoders by minimizing the MSE upper bound (20). By setting $p_{s,i} = \sigma_{s,i}^2$ and $p_{r,i} = \sigma_{r,i}^2$ in (20), the optimization problem is reformulated as

$$\begin{aligned}&\min_{\substack{p_{s,i},p_{r,i},\\ i=1,\cdots,L}}\sum_{i=1}^{L}\frac{1}{\sigma_s^{-2} + \frac{p_{s,i}p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{n,r}^2p_{r,i}\sigma_{rd,i}^2 + \sigma_{n,d}^2} + \sigma_{n,d}^{-2}\frac{p_{s,i}}{\mathbf{B}^{-1}(i,i)}}\\ &s.t.\\ &tr\left\{\boldsymbol{\Sigma}_r\left(\sigma_{n,r}^2\mathbf{I}_R + \sigma_s^2\boldsymbol{\Sigma}_{sr}\boldsymbol{\Sigma}_s\boldsymbol{\Sigma}_s^H\boldsymbol{\Sigma}_{sr}^H\right)\boldsymbol{\Sigma}_r^H\right\} = \\ &\sum_{i=1}^{L}p_{r,i}\left(\sigma_{n,r}^2 + \sigma_s^2p_{s,i}\sigma_{sr,i}^2\right) \leq P_{R,T}\\ &\sigma_s^2tr\{\boldsymbol{\Sigma}_s\boldsymbol{\Sigma}_s^H\} = \sigma_s^2\sum_{i=1}^{L}p_{s,i} \leq P_{S,T},\\ &p_{s,i} \geq 0,\ p_{r,i} \geq 0,\ \forall i.\end{aligned}\tag{21}$$

$$tr\{\mathbf{E}\} = tr\left\{\left(\sigma_s^{-2}\mathbf{I}_L + \underbrace{\mathbf{\Sigma}_s^H\mathbf{\Sigma}_{sr}^H\mathbf{\Sigma}_r^H\mathbf{\Sigma}_{rd}^H\left(\sigma_{n,r}^2\mathbf{\Sigma}_{rd}\mathbf{\Sigma}_r^2\mathbf{\Sigma}_{rd}^H+\sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{\Sigma}_{rd}\mathbf{\Sigma}_r\mathbf{\Sigma}_{sr}\mathbf{\Sigma}_s}_{=\mathbf{E}_R} + \underbrace{\sigma_{n,d}^{-2}\mathbf{\Sigma}_s^H\mathbf{V}^H\mathbf{\Sigma}_{sd}^H\mathbf{\Sigma}_{sd}\mathbf{V}\mathbf{\Sigma}_s}_{=\mathbf{E}_S}\right)^{-1}\right\}, \quad (15)$$

$$p_{r,i} = \frac{1}{(\sigma_s^2 p_{s,i}\sigma_{sr,i}^2+\sigma_{n,r}^2)} \times \left[\frac{\mu_r\sigma_{n,d}\sqrt{p_{s,i}}\sigma_{sr,i}\sigma_{rd,i}\left(\sigma_s^2p_{s,i}\sigma_{sr,i}^2+\sigma_{n,r}^2\right)^{1/2} - \left(\sigma_s^2p_{s,i}\sigma_{sr,i}^2+\sigma_{n,r}^2\right)\sigma_{n,r}^2\left(\sigma_s^{-2}+\sigma_{n,d}^{-2}p_{s,i}(\mathbf{B}^{-1}(i,i))^{-1}\right)}{\sigma_{rd,i}^2\left(\sigma_{n,r}^2\left(\sigma_s^{-2}+\sigma_{n,d}^{-2}p_{s,i}(\mathbf{B}^{-1}(i,i))^{-1}\right)+p_{s,i}\sigma_{sr,i}^2\right)}\right]^+ \quad (22)$$

$$p_{s,i} = \left[\frac{\mu_s\sqrt{\beta_i}-\sigma_s^{-2}(\sigma_{n,d}^2+p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2)}{\left(\sigma_{n,d}^{-2}(\mathbf{B}^{-1}(i,i))^{-1}(\sigma_{n,d}^2+p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2)+p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2\right)}\right]^+ \quad (23)$$

The problem (21) can be resolved by using the standard Lagrange technique followed by an iterative water-filling procedure [18]. After some manipulations, the resultant solution is expressed as (22), in which $[y]^+ = max[0,y]$, and $\mu_r = \lambda_r^{-1/2}$ is the water level which should be chosen to satisfy the power constraint at the relay. Similarly, we can obtain $p_{s,i}$ as (23), where μ_s is the water level which is chosen to meet the total power constraint $\sum_{i=1}^{L} p_{s,i} = P_{S,T}$ at the source node, and

$$\beta_i = \left(\sigma_{n,d}^2+p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2\right) \times \left(\sigma_{n,d}^{-2}(\mathbf{B}^{-1}(i,i))^{-1}\left(\sigma_{n,d}^2+p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2\right)+p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2\right). \quad (24)$$

The computation of the proposed precoders mainly involves the SVD of the link channel matrices in (8) and inversion of the matrix $\mathbf{B}$ in (16). The computational complexity of the proposed algorithm measured in terms of flop point operations is summarized in Table I.

B. Simulation Results

We consider an AF MIMO relay system with $L = M = R = N = 4$. The elements of each channel matrix are assumed to be i.i.d. complex Gaussian random variables with zero mean and unity variance. Let SNR_{sr}, SNR_{rd}, and SNR_{sd} denote, respectively, the signal-to-noise ratio per receive antenna of the source-to-relay, relay-to-destination, and source-to-destination links. Here, we let $\text{SNR} = \text{SNR}_{sr} = \text{SNR}_{rd}$, $\text{SNR}_{sd} = 5$ dB. Also, we assume that the transmit symbols are drawn from QPSK constellation. We compare the performance of one un-precoded and two precoded systems. For precoded systems, one uses the optimal relay precoder in [14], and the other uses the proposed source/relay precoders. To further understand the behavior of the proposed precoders, we also consider the scenarios where the zero forcing (ZF) and the maximum likelihood (ML) receivers, instead of MMSE receivers, are adopted at the destination. Figure 2 shows the bit-error-rate (BER) versus SNR result. From the figure we can see that: (i) The proposed precoded system does outperform the un-precoded one; (ii) The proposed method also outperforms that in [14];

TABLE I
COMPUTATIONAL COMPLEXITY OF PROPOSED PRECODED SCHEME.

Operation	SVD (8)	$\mathbf{B}^{-1}$ (16)	$p_{s,i}$ and $p_{r,i}$ (22), (23)	$\mathbf{F}_S$ and $\mathbf{F}_R$ (13), (14)
Flops	$(14MN^2+8N^3)+(14RN^2+8N^3)+(14MR^2+8R^3)$	$2MN^2+2MN+2N^3+13/4N^2+N^2$	$(21LI_r+14LI_s)I_i$	$2RL+2R^2L+2NL$
Total	$(14MN^2+8N^3)+(14RN^2+8N^3)+(14MR^2+8R^3)$ + $2MN^2+2MN+2N^3+13/4N^2+N^2$ + $(21LI_r+14LI_s)I_i$ + $2RL+2R^2L+2NL$			

N: number of transmit antennas
R: number of relay antennas
M: number of receive antennas
L: number of transmitted symbol streams
I_r: number of iteration for computing $p_{r,i}$
I_s: number of iteration for computing $p_{s,i}$
I_i: number of iteration of the water-filling process

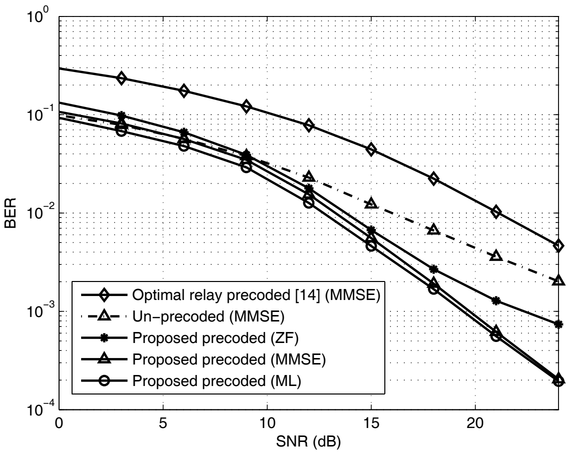

Fig. 2. BER performance comparison for non-precoded and proposed precoded in MIMO relay systems (with different receivers).

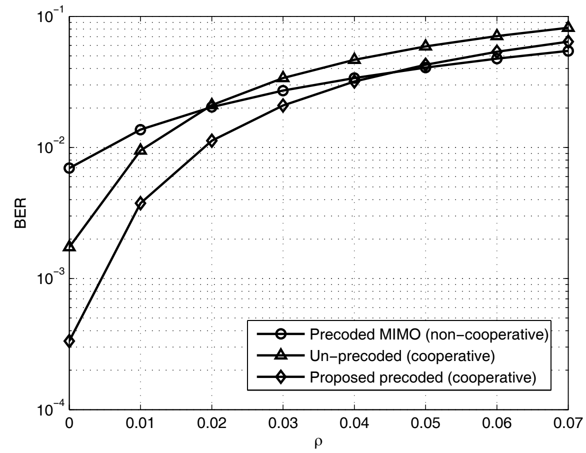

Fig. 4. BER comparison of precoded MIMO [1], un-precoded MIMO relay, and proposed precoded MIMO relay systems (SNR_{sr} = SNR_{rd} = SNR_{sd} = 20 dB).

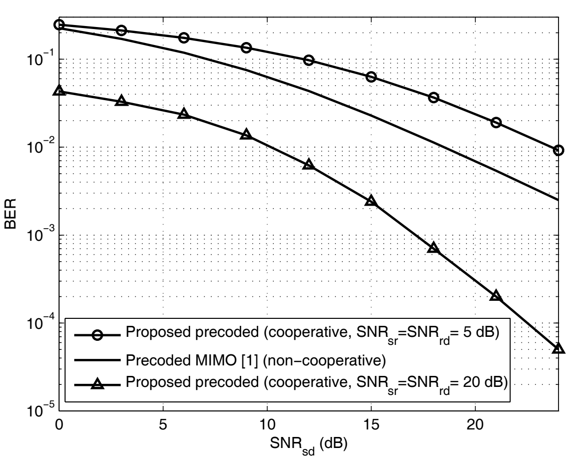

Fig. 3. BER performance comparison for precoded MIMO [1] and proposed precoded MIMO relay systems.

this is because the proposed method takes both the source and relay link resources into consideration, simultaneously; (iii) The proposed precoders combined with the ML receiver yields slight performance improvement over the linear MMSE receiver; this is because the proposed precoders are specifically designed for the MMSE equalizer, not for the ML receiver. We further compare the proposed cooperative system with the conventional (non-cooperative) MIMO system in which the MMSE precoder [1] is used at the transmitter. For fair comparison (the same spectral efficiency), the former uses the 16-QAM whereas the latter uses 4-QAM modulation schemes. For two different relay link conditions SNR_{sr} = SNR_{rd}=5 dB and SNR_{sr} = SNR_{rd}=20 dB, Figure 3 depicts the resultant BER curves when the direct-link SNR, denoted by SNR_{sd}, varies from 0 dB to 24 dB. As we can see, the cooperative scheme outperforms the non-cooperative one when the relay link SNR is high. However, for low relay link SNR the non-cooperative system yields a better performance. With SNR_{sd} = SNR_{rd} = 20 dB, Figure 4 further compares the precoded MIMO system with the proposed cooperative scheme when channel estimation error occurs (the system parameters are likewise set as those in Figure 3). As in previous works (e.g., [19]) the estimated channel $\hat{\mathbf{H}}$ is related to the true channel $\mathbf{H}$ via the equation $\hat{\mathbf{H}} = \sqrt{(1-\rho)}\mathbf{H} + \sqrt{\rho}\Delta\mathbf{H}$, where $\Delta\mathbf{H}$ models the channel mismatch with elements being i.i.d. Gaussian distributed with zero mean and same variance, and the coefficient ρ ($0 \leq \rho \leq 1$) characterizes the channel estimation quality. The figure shows that the cooperative systems yield a better performance when ρ is small (say, $\rho \leq 0.04$). As ρ increases, the performance of all system degrades; however, the performance of the cooperative systems, either precoded or un-precoded, degrades faster than that of the precoded MIMO system. This is because, for the cooperative systems, signal distortion due to the severe channel mismatch effect will further deteriorate by the amplify-and-forward transmission mechanism.

IV. Conclusions

In this paper we propose a joint source/relay precoder design method for an AF MIMO cooperative system. With a linear equalizer adopted at the receiver, the procoders are designed to minimize the MSE. It is seen that the MSE is a complicated function of precoding matrices, and a direct minimization is not feasible. To solve the problem, we propose an MSE-matrix-diagonalization-based approach, and this leads to a constrained precoder structure, which then facilitates the derivation of a tractable MSE upper bound. By conducting minimization with respect to this upper bound, a suboptimal solution can be obtained via the standard Lagrange technique followed by an iterative water-filling procedure. The proposed approach exploits all the available link resources, and can outperform the existing relay precoding scheme.

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