Comparison of p control charts for low defective rate

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ARTICLE INFO

ABSTRACT

It is well known that the conventional p control chart constructed by the normal approximation for the binomial distribution suffers a serious inaccuracy in the monitor process when the true rate of nonconforming items is small. A similar problem also arises in the binomial confidence interval estimation. Adjusted confidence intervals are established in the literature to improve the coverage probability when the binomial proportion is small. In this paper, a new p control chart based on an adjusted confidence interval is established, which can substantially improve the existing control charts when the nonconforming rate is small.

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1. Introduction

The use of attribute control charts arises when items are compared with some standard and then are classified as to whether they meet that standard or not. The p control chart is used to determine if the rate of nonconforming product is stable, and it will detect when a deviation from stability has occurred.

Suppose that the fraction of nonconforming items is p. Let X_i denote the number of nonconforming items found when n_i items are inspected. If the inspection is done independently, X_i has a binomial distribution $B(n_i, p)$. Let x_i denote the observation of X_i when the same inspection procedure is carried out. The process can be monitored by plotting the value $x_i/n_i, i = 1, \ldots,$ on a p chart, which is a type of control chart that is used to monitor the sample proportion on nonconforming items. The evaluation of a p chart can be based on the type I error, which is the probability that x_i/n_i does not fall between the upper and the lower limits of the chart. When p is known, the widely used p control chart with type I error 0.0027 is the control chart with 3-sigma control limits

$$UCL = p + 3\sqrt{\frac{p(1-p)}{n}},$$
$$CL = p$$

and

$$LCL = p - 3\sqrt{\frac{p(1-p)}{n}}. \tag{1}$$

When p is unknown, the widely used p control chart with type I error 0.0027 is the control chart with control limits

$$UCL = \frac{x}{n} + 3\frac{\sqrt{x(1-x/n)}}{n},$$
$$CL = \frac{x}{n}$$

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and

$$LCL = \frac{x}{n} - 3\frac{\sqrt{x(1 - x/n)}}{n}, \tag{2}$$

where $x = x_1 + \cdots + x_i$ and $n = n_1 + \cdots + n_i$, $i = 1, \ldots$.

For the case when p is known, the type I error of the control chart (1) is not far away from the nominal level when n is large. However, when p is unknown, the real type I error of the conventional control chart (2) not only depends on n, but also depends heavily on the value of the true p. Since the control chart is constructed by a normal approximation, it may not monitor the process well when n is not large enough (see Wetherill and Brown, 1991; Xie and Goh, 1993). If the sample size n is large enough, we expect that a feasible control chart should monitor the process well for p being small because the nonconforming fraction is usually small for high-quality processes. It is well known that this conventional control chart (2) has serious drawbacks in detecting when p is small. The true type I error of (2) is far away from the nominal normal value 0.0027 when p is small. Some related references are Ryan (1989), Quesenberry (1991, 1995)), Woodall (1997) and Shore (2000).

In addition, the problem for a low-nonconformity and high-yield process has been extensively studied in the literature (see Chan et al., 1997; Goh and Xie, 1995; Kuralmani et al., 2002; McCool and Joyner-Motley, 1998; Xie and Goh, 1993). A good survey for the control charts of high-quality processes can be found in Xie et al. (2002).

Ryan (1989) used an arcsine transformation to construct a chart to monitor p, and Chen (1998) used the Cornish–Fisher expansion studied by Winterbottom (1993) to construct a p chart which can achieve better normal approximation than the conventional chart. Although these proposed charts can increase the monitor accuracy, they still lack achievement of desirable accuracy when the true p is small. Some other modifications can be found in Quesenberry (1997), Ryan and Schwertman (1997), Acosta-Mejia (1999), Shore (2000) and Kanji and Arif (2001).

Chan et al. (2002) proposed a CPC chart based on the CCC and CQC charts proposed by Chan et al. (2000) to overcome the difficulty of the poor performance when the defect rate of the process is low. The CCC chart was first developed in Calvin (1983) to monitor zero-defect (ZD) processes. The use of a CCC type control chart has been further studied by Xie and Goh (1993), Ermer (1995), Wu et al. (2000).

Chan et al. (2002) use the fact that if the defective items follow a binomial $B(n, p)$ distribution, the number x of items inspected until a defective item is observed follows a geometric distribution with the density function $p(1 - p)^{x-1}$, $x = 1, 2, \ldots$, to construct a chart. This chart basically has better performance when p is small, but has worse performance than the above charts when p is not close to 0. Besides, the information of the number of items inspected until a defective item is observed mainly needed for this chart is different from the information of the proportion of the defective items to all items used for the other charts. It may be not convenient to obtain the information of the number of the first defective item that has occurred. Chan et al. (2007) has further investigation based on this chart.

A similar problem also occurs in statistical interval estimation, such as in the confidence interval, tolerance interval or prediction interval estimation for the binomial distribution, namely that the coverage probability of the standard interval constructed directly from the normal approximation is much lower than the nominal level when p is small; see Wang (2007, 2008, 2009)), Wang and Tsung (2009) and Cai and Wang (2009). Usually the conventional confidence interval built up solely based on the normal approximation does not possess a desirable accomplishment for small p in the interval estimation. In the literature, there are several approaches proposed to modify the conventional confidence interval, the Wald interval, such that the performance of the proposed intervals can attain a desirable achievement for small p (see Agresti and Coull, 1998; Brown et al., 2002).

In this paper, we utilize an adjusted confidence interval of the binomial proportion to construct an improved p control chart. From a simulation study, this new chart can successfully improve the monitor accuracy when the true nonconforming rate is small.

This paper is organized as follows. The existing control charts for the nonconforming rate when the true rate is known or unknown are discussed in Section 2. The performances of these charts are presented in terms of the type I error criterion. A new chart is proposed in Section 3 which can substantially improve the existing charts by reducing the type I error when the true nonconforming rate is small. In Section 4, the proposed chart is compared with the existing methods by evaluating their expected widths. The performances of these charts are illustrated by an example in Section 5. Section 6 concludes the paper with a concluding remark.

2. Existing methods

Besides the widely used control chart (2), the three existing charts with nominal type I error 0.0027 discussed in Chen (1998), Ryan (1989), Winterbottom (1993), and Chan et al. (2000, 2002) are introduced as follows.

The arcsine p chart for p known

Define

$$w_i = 2\sqrt{n_i} \left[\sin^{-1} \left(\sqrt{\frac{x_i + 3/8}{n_i + 3/4}} \right) - \sin^{-1} \left(\sqrt{\bar{p}} \right) \right].$$

By Ryan (1989), since w_i is approximately a standard normal random variable, we can plot $w_2, w_3, \ldots$ on a chart with control limits at $UCL = 3$, $CL = 0$, and $LCL = -3$.

The arcsine p chart for p unknown

Define

$$\hat{p}_i = \frac{(x_1 + \cdots + x_i)}{(n_1 + \cdots + n_i)},$$

and for $i = 2, 3, \ldots$ define

$$w_i = 2\sqrt{n_i} \left[\sin^{-1}\left(\sqrt{\frac{x_i + 3/8}{n_i + 3/4}} \right) - \sin^{-1}\left(\sqrt{\hat{p}_{i-1}} \right) \right].$$

We can plot $w_2, w_3, \ldots$ on a chart with control limits at $UCL = 3$, $CL = 0$, and $LCL = -3$.

The p chart modified by Cornish–Fisher expansion for p known

Define $y_i = x_i/n_i$. Plot y_i for $i = 2, 3, \ldots$ on a chart with control limits at

$$UCL = p + 3\sqrt{\frac{p(1-p)}{n_i}} + \frac{4(1-2p)}{3n_i},$$

$$CL = p,$$

and

$$LCL = p - 3\sqrt{\frac{p(1-p)}{n_i}} + \frac{4(1-2p)}{3n_i}.$$

The p chart modified by Cornish–Fisher expansion for p unknown

Define $y_i = x_i/n_i$ and

$$\hat{p}_i = \frac{(x_1 + \cdots + x_i)}{(n_1 + \cdots + n_i)}.$$

Plot y_i for $i = 2, 3, \ldots$ on a chart with control limits at

$$UCL = \hat{p}_{i-1} + 3\sqrt{\frac{\hat{p}_{i-1}(1 - \hat{p}_{i-1})}{n_i}} + \frac{4(1 - 2\hat{p}_{i-1})}{3n_i},$$

$$CL = \hat{p}_{i-1},$$

and

$$LCL = \hat{p}_{i-1} - 3\sqrt{\frac{\hat{p}_{i-1}(1 - \hat{p}_{i-1})}{n_i}} + \frac{4(1 - 2\hat{p}_{i-1})}{3n_i}.$$

The CPC chart for p known

Define y_i to be the number of items inspected until a defective item is observed. Since the estimate of p is $1/y_i$, for plotting the chart for p, it is equivalent to plot the chart for y_i.

Plot y_i for $i = 2, 3, \ldots$ on a chart with control limits at

$$UCL = \ln(\alpha/2)/\ln(1 - p)$$
$$CL = \ln(1/2)/\ln(1 - p)$$

and

$$LCL = \ln(1 - \alpha/2)/\ln(1 - p).$$

The CPC chart for p unknown

Define y_i to be the number of items inspected until a defective item is observed and

$$\hat{p}_i = (1/y_1 + \cdots + 1/y_i)/i.$$

Plot y_i for $i = 1, 2, 3, \ldots$ on a chart with control limits at

$$UCL = \ln(\alpha/2)/\ln(1 - \hat{p}_i)$$
$$CL = \ln(1/2)/\ln(1 - \hat{p}_i)$$

and

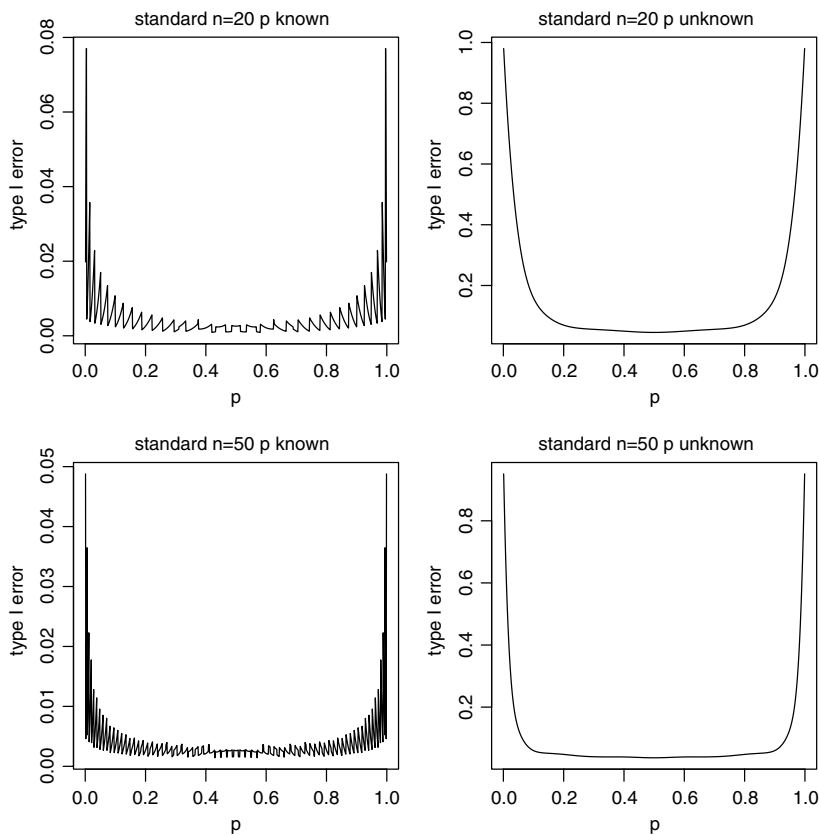$$LCL = \ln(1 - \alpha/2)/\ln(1 - \hat{p}_i).$$

Fig. 1. Type I errors of the conventional charts (2) when p is known and unknown for $n = 20$ and $n = 50$.

We will evaluate the existing methods in terms of their performances of the type I error. Note that for calculating the type I error, to simplify the calculation, we approximate it by the type I error calculated by assuming that $\hat{p}_{i-1}$ and $\hat{p}_i$ follow the same distribution $B(n, p)$. Thus, for the standard chart and arcsine charts, the type I error of a chart with limits LCL and UCL at $p = p_0$ is computed by calculating the probability

$$1 - (F(nUCL) - F(nLCL)) = 1 - (Pr_{p_0}(X \leq nUCL) - Pr_{p_0}(X \leq nLCL)). \tag{3}$$

Since $nUCL$ and $nLCL$ are not always integers, we use $1 - (P_{p_0}(X \leq [nUCL] + 1) - P_{p_0}(X \leq [nLCL]))$ to approximate (3), where $[x]$ denotes the largest integer less than or equal to x. The type I errors of the chart modified by Cornish–Fisher expansion and the CPC chart can be calculated in a similar way. The type I errors for the charts derived by the four existing methods with respect to p known and p unknown cases are shown in Figs. 1–4. For the standard control chart with known p case, Fig. 1 shows that the type I error oscillates and has a decreasing trend in p for $p \in (0, 0.5)$. Although when the true p is small, the type I error is not very close to the nominal level 0.0027, the bias is less than 0.1, which is not very large. However, for the p unknown case, the type I error goes to 1 as p goes to 0. In real applications, the situation of small p is important because the true p may be very small. Fig. 1 shows that the standard chart may be acceptable if p is known, but the chart (2) is not satisfactory when p is unknown.

For the arcsine control chart, Fig. 2 shows that when p is known, the bias of the type I error is also not very large; however, when p is unknown, the type I error is always zero. This indicates that the arcsine chart is not suitable to be used to monitor $\hat{p}_i$ because it is too conservative and it may lead to $\hat{p}_i$ always falling into the limits even when the process is out of control.

For the chart modified by Cornish–Fisher expansion, the type I error for the p known case goes to 1 as p goes to 0 or 1. The type I error for the p unknown case is too large when p is small, which indicates that it is also unsatisfactory for small p.

For the CPC chart, the type I error for the p known case is close to the nominal level 0.0027 when p is small. However, for the p unknown case, although the type I error is relatively small for small p compared with the situation when p is large, it is greater than 0.1, which is far away from the nominal level 0.0027. Note that, since for this chart the random variable is the number x of the first defect item inspected, in the plots in Fig. 4, we do not need to consider the cases for different sample sizes.

Combining the above results, when p is known, some of the existing charts can monitor $\hat{p}_i$ well when the true p is not very small, and it is still acceptable when p is small. When p is unknown, the four charts are not satisfactory for p being small. Therefore, a better chart which can reduce the type I error for small p is proposed in the next section.
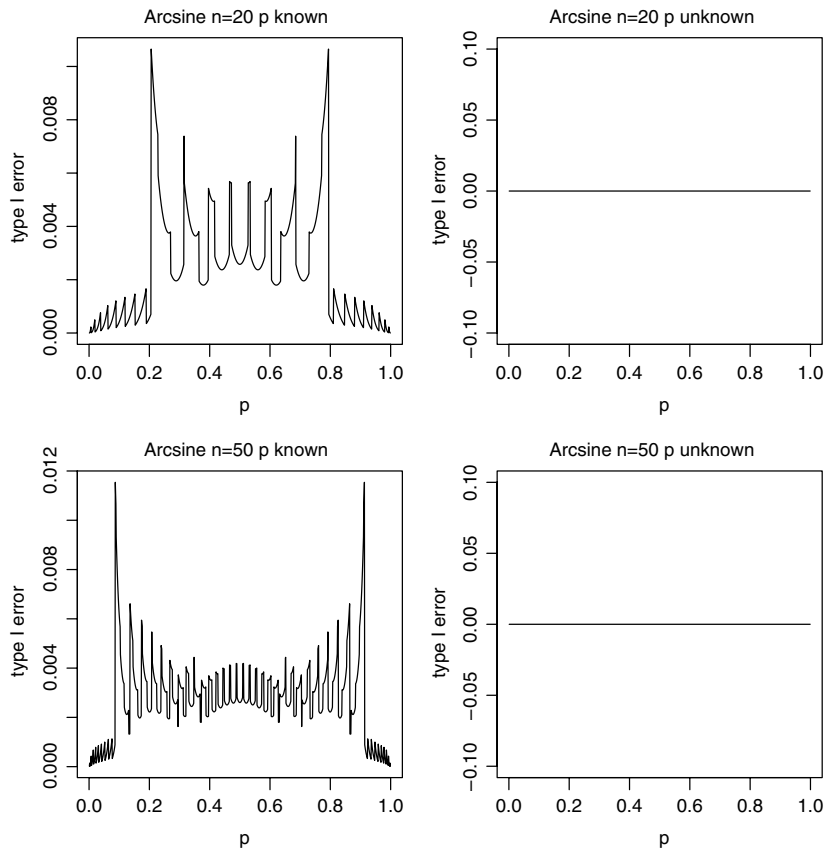
Fig. 2. Type I errors of the arcsine p charts when p is known and unknown for $n = 20$ and $n = 50$.

3. Improved p chart

We first introduce a binomial confidence interval in the literature, then construct an approximated control chart based on this interval. Let X be a random variable following a binomial distribution $B(n, p)$. For estimating p, it is well known that the coverage probability of the Wald interval $X/n \pm k\sqrt{X/n(1 - X/n)/n}$ goes to 0 as p goes to 0 or 1. The following interval constructed by Agresti and Coull (1998) can successfully increase the coverage probability for small p.

The Agresti–Coull interval. Let $\tilde{X} = X + k^2/2$ and $\tilde{n} = n + k^2$. Let $\tilde{p} = \tilde{X}/\tilde{n}, \tilde{q} = 1 - \tilde{p}, \hat{p} = X/n$ and $\hat{q} = 1 - \hat{p}$. Here k is the $1 - \alpha/2$ upper cutoff point of the standard normal distribution. The $1 - \alpha$ interval for p has the form

$$CI_{AC}(X) = \left(\tilde{p} - k \left(\tilde{p}\tilde{q} \right)^{1/2} \tilde{n}^{-1/2}, \ \tilde{p} + k \left(\tilde{p}\tilde{q} \right)^{1/2} \tilde{n}^{-1/2} \right).$$

The Agresti–Coull interval is an adjusted interval from the usual standard interval, which has higher coverage probabilities for small p than the Wald interval; see Brown et al. (2002) and Wang (2007). Based on this interval, we propose a new control chart. The limits of the proposed p chart with type I error $1 - \alpha$ are

$$UCL_A = \frac{\tilde{x}}{\tilde{n}} + k \frac{\sqrt{\tilde{x}(1 - \frac{\tilde{x}}{\tilde{n}})}}{\tilde{n}},$$

$$CL_A = \frac{\tilde{x}}{\tilde{n}}$$

and

$$LCL_A = \frac{\tilde{x}}{\tilde{n}} - k \frac{\sqrt{\tilde{x}(1 - \frac{\tilde{x}}{\tilde{n}})}}{\tilde{n}} \tag{4}$$

where $\tilde{x} = x_1 + \cdots + x_i + k^2/2, n = n_1 + \cdots + n_i + k^2, i = 1, \ldots$ and $k = z_{1-\alpha/2}$, where $z_{1-\alpha/2}$ denotes the $(1 - \alpha/2)$th quantile of the standard normal distribution.
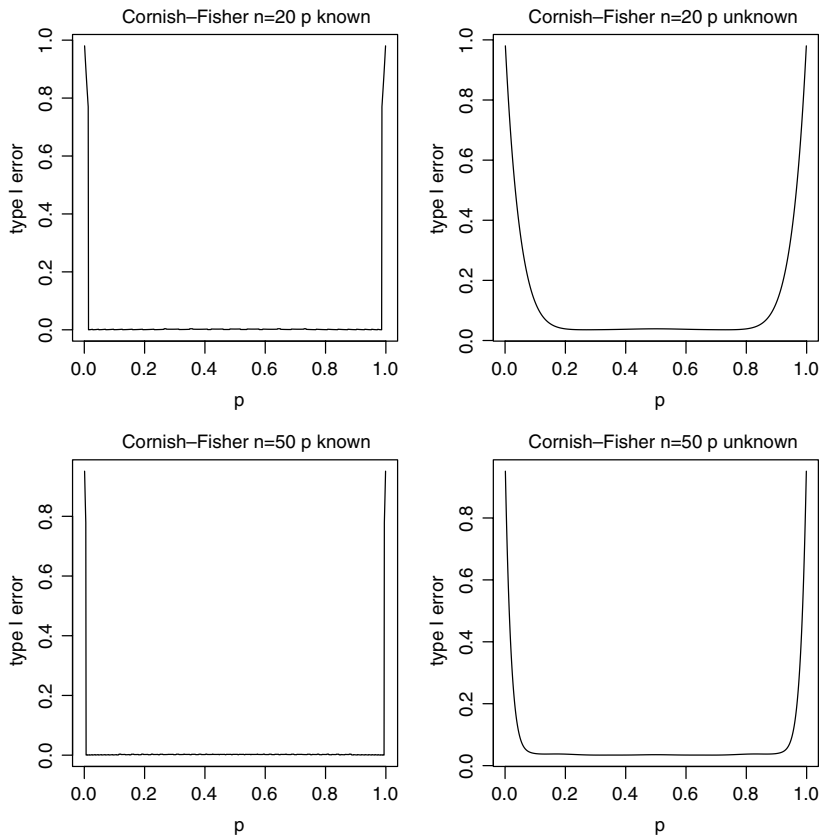
Fig. 3. Type I errors of the p charts modified by Cornish–Fisher expansion when p is known and unknown for $n = 20$ and $n = 50$.

Remark 1. It may be possible that the proposed UCL_A is less than 0 and LCL_A is greater than 1 for some situations. Since the nonconforming rate is between 0 and 1, we can modify LCL_A to 0 when the LCL_A is less than 0, and modify UCL_A to 1 when the UCL_A is greater than 1. However, since the existing charts also have this problem, to make a fair comparison with the existing charts, we still use the form of (4) for investigation in the rest of the paper.

This new chart can successfully reduce the type I error when the true p is small; see Fig. 4. Compared with the CPC chart, it has smaller type I error, and the type I error is less than a tolerance bound for all p.

In addition, we also investigate the performance of the new chart when the sample size increases. Compared with Fig. 5, it is shown in Fig. 6 that the type I errors for the cases of $n = 80$ and $n = 100$ are less than 0.04 for all p, which are less than the type I errors for the case of $n = 20$ and $n = 50$.

There are other improved confidence intervals except the Agresti–Coull interval proposed in the literature to improve the coverage probability of the Wald interval, such as the Wilson interval and the likelihood ratio interval; see Brown et al. (2002) and Wang (2007). However, chart limits based on the Wilson interval failed to reduce the type I error for small p in a simulation study, and the likelihood ratio test approach cannot derive a chart with a closed form. The chart based on the Agresti–Coull interval does not have the above disadvantage and possess a simple closed form.

4. Width comparison

In this section, we will compare the expected width of the new chart with those of the existing charts. Note that, for the case of p known, the width does not depend on the observation, which can be directly derived by taking the difference of the upper limit and the lower limit. For the case of p unknown, the width is dependent on the observation. Hence we make a comparison of their expected widths for the p unknown case. The expected width of a chart is defined as the expected value of the upper limit minus the expected value of the lower limit, which can be calculated using the formula

$$\sum_{i=0}^{n} (UCL(i) - LCL(i)) \binom{n}{i} p^i (1-p)^{n-i}.$$

The expected widths for the standard chart and the new chart are shown in Fig. 7.
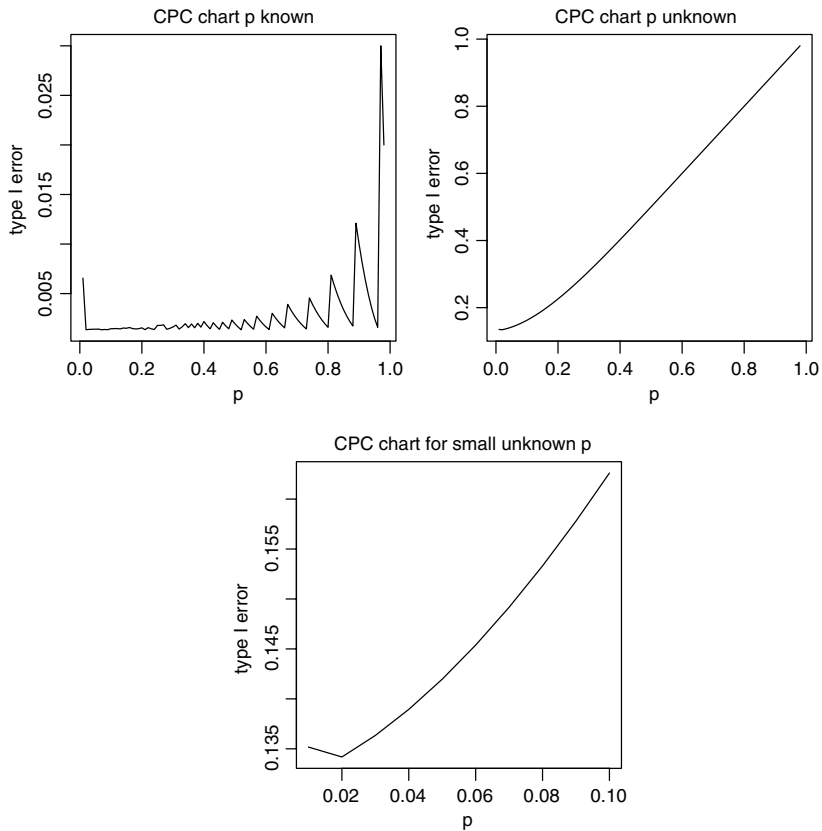
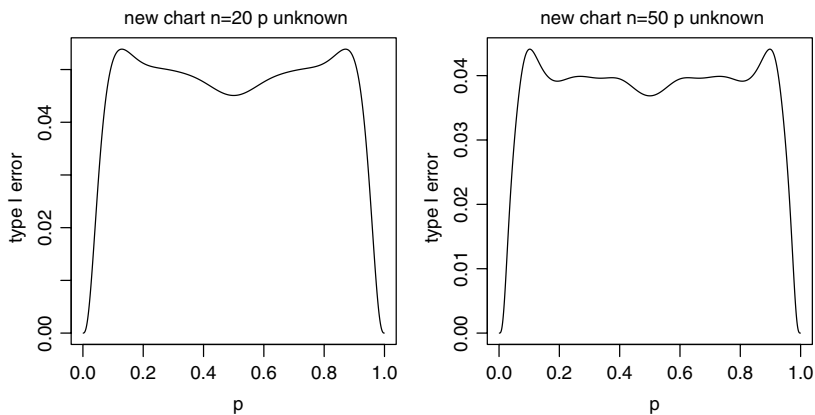**Fig. 4.** Type I errors of the CPC chart when *p* is known and unknown.

Fig. 5. Type I errors of the new *p* chart when *p* is unknown for $n = 20$ and $n = 50$.

Note that the expected width of the chart modified by Cornish–Fisher expansion is the same as that of the standard chart because its upper and lower limits are the upper and lower limits of the standard interval adding the same value $4(1 - 2\hat{p})/(3n)$, which can be canceled when calculating the width.

From Fig. 7, the expected width of the proposed chart is lower than that of the standard chart for *p* being in an interval centered at $p = 0.5$ and higher than that of the standard chart for *p* being outside the interval. The difference of the two expected widths decreases as *n* increases. The expected width of the standard chart goes to 0 as *p* goes to 0 or 1, which may cause the poor performance of the chart for small *p*. The expected width of the proposed chart is in an accepted region because its maximum value is lower than the maximum value of the standard chart.

For the arcsine chart, since the chart is used to detect w_i, which approximates the standard normal distribution (Ryan (1989), Chen (1998)), the upper and lower limits are the upper 0.00135th quantile 3 and the lower 0.00135th quantile -3 of the standard normal distribution. Unlike the other three charts whose limits are used to detect $\hat{p}$, the arcsine chart detects
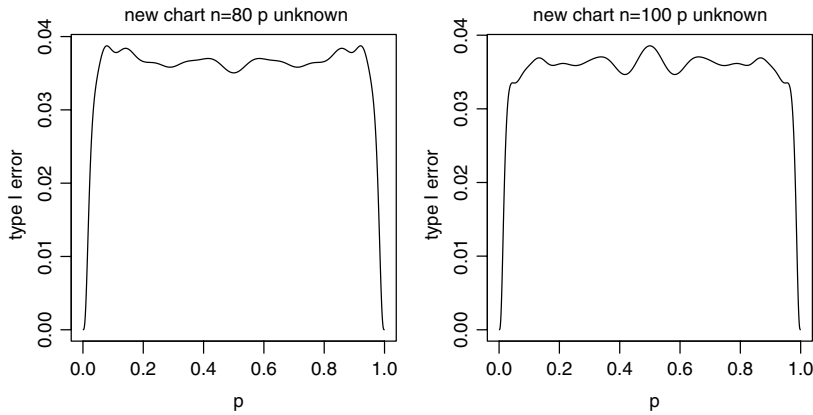
Fig. 6. Type I errors of the new p chart when p is unknown for $n = 80$ and $n = 100$.
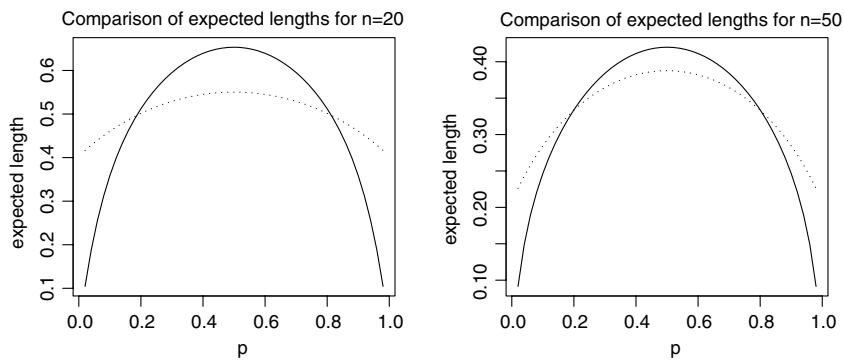

Fig. 7. The solid and the dotted lines denote the expected widths of the standard chart and the new chart respectively for the cases of $n = 20$ and $n = 50$.

another random variable w_i. Thus, we cannot directly compare the expected width of the arcsine chart with those of the other three charts. By investigation of the type I error of the arcsine chart, the normal approximation approach is satisfactory for the p known case, but it is not for the p unknown case because the probability that w_i is outside the limits is zero.

The situation is the same for the CPC chart, in that it does not directly detect $\hat{p}$, but the number of the first defective item that has occurred. Thus, we cannot directly compare the expected width of the CPC chart with the width of the proposed chart.

5. Chart with required maximum type I error

Although the new chart has a better performance than the existing charts, it is constructed from the large sample theory and its type I error is not exactly equal to the nominal level. For example, from Fig. 5, the range of the type I error of the new chart is from 0 to 0.055 for $n = 20$. Despite the small type I error when the true p is close to zero, the type I error is higher than the nominal level 0.0027 for p belonging to a region. Therefore, we may be interested in constructing a more satisfactory p chart with a desirable type I error. However, the type I error is a function of p. In the p unknown case, since we do not know the point of p in the parameter space at which the maximum type I error occurs, it is hard to directly construct a chart with maximum type I error close to the nominal level.

In spite of the above difficulty, we can use a looser criterion to construct a desirable control chart. When p is unknown, since the lower limit and upper limit of a p chart depend on the observations, the type I error of a chart conditional on a sample is a variable function of the sample. By this fact, we can view the chart limits as tolerance limits and use a criterion on the tolerance interval to construct a desirable p chart. By definition, $(L(X), U(X))$ is a $1 - \beta$ content and $1 - \alpha$ confidence tolerance interval for F if it satisfies

$$Pr_p\{[F(U(X)) - F(L(X))] \geq 1 - \beta\} = 1 - \alpha, \tag{5}$$

where F is a binomial cumulative distribution $B(n, p)$, which can be written as

$$Pr_p\{[1 - (F(U(X)) - F(L(X)))] \leq \beta\} = 1 - \alpha. \tag{6}$$

Note that if $U(X)$ and $L(X)$ are the upper and lower limits of a chart, respectively, $1 - (F(U(X)) - F(L(X)))$ is the type I error conditional on observation x. Under this criterion, we require that the probability for an observation such that the type I error based on the observation less than β is close to $1 - \alpha$.

Table 1
The k values in (4) for different sample sizes n such that it has at least near 0.9 confidence that the type I error is less than 0.0027 for $p \in (0, 1)$.

n	k	Minimum confidence	n	k	Minimum confidence
10	6	0.9020	60	5.2	0.8997
20	7	0.8861	70	5.3	0.9154
30	6	0.9149	80	5.2	0.8948
40	5.9	0.9104	90	5.2	0.9072
50	5.4	0.8957	100	5	0.9048

Table 2
The data for the numbers of misplaced components and PCB and the rates.

Misplaced	0	1	0	2	0	1
PCB	250	200	200	150	200	250
Rate	0	0.005	0	0.0133	0	0.004
Misplaced	0	0	1	0	0	1
PCB	100	300	100	150	200	200
Rate	0	0	0.01	0	0	0.005

Under this criterion, we aim to modify the proposed chart such that it has $1 - \alpha$ confidence that the type I error is less than β. By Wang and Tsung (2009), a procedure, which is modified from the procedure of constructing a desirable tolerance interval, to construct the desirable chart limits is provided.

Procedure 1: Derive an appropriate k in the proposed chart (4) such that it has $1 - \alpha$ confidence that the type I error is less than β for p in a region (a, b).

Step 1. Choose a value of k for UCL_A and LCL_A in (4).

Step 2. For each observation, x, $x = 0, \ldots, n$, make $g_x(p) = 1 - \sum_{i=LCL_A(x)}^{UCL_A(x)} \binom{n}{i} p^i (1-p)^{n-i}$.

Step 3. Calculate the solutions of $g_x(p) = \beta$ for each x. There may be zero, one or two solutions. The number of solutions depends on β and x. Assume that there are totally m solutions belonging to (a, b) for all $x = 0, \ldots, n$.

Step 4. Rank all m solutions in **Step 3**. Let v_i be the ith smallest solution in **Step 3**.

Step 5. Compute the probability, $Pr_{v_i}(1 - (F(UCL_A(X)) - F(LCL_A(X))) \le \beta)$, for each i. The smallest value among these probabilities, $Pr_{v_i}(1 - (F(UCL_A(X)) - F(LCL_A(X))) \le \beta)$, $v_i = 1, \ldots, m$, is the minimum confidence of the control chart for $p \in (a, b)$.

Step 6. Adjust the k value such that the minimum confidence derived in **Step 5** can be close to a specific level. Then the chart of (4) with this k value is the chart such that it has $1 - \alpha$ confidence that the type I error is less than β for p in a region (a, b).

Using the procedure, we list the k value in (4) for different sample sizes such that it has 0.9 confidence that its type I error is less than 0.0027 for $p \in (0, 1)$ (see Table 1).

6. Example

We will illustrate the new chart by an example with a small nonconforming rate.

Example 1. The data in this example are about the misplaced components for a particular board. In electronics, printed circuit boards, or PCBs, are used to mechanically support and electrically connect electronic components using conductive pathways, or traces, etched from copper sheets laminated onto a non-conductive substrate. There are 12 records for the misplaced components in PCBs, which are listed in Table 2. The average rate for the 12 records is 0.0031, and the average number of PCBs is 200. We may assume that the true p is near 0.0031. Under this assumption, then misplaced numbers 0, 1 and 2 in the data are less than $1 - 0.0027 = 0.9973$ quantile of the distribution, which is 4. Thus, we expect that the 12 points can fall between the upper and lower limits of a chart. Note that we do not consider the CPC chart for this example because we do not have the data of the number of the first defect item occurred.

From Figs. 8 and 9, there are several points out of the control region for the conventional chart and the chart modified by Cornish–Fisher expansion. Figs. 10 and 11 show that all of the points can fall between the upper and lower limits for the arcsine chart and the new chart. Since, as mentioned above, we expected that the 12 rates would fall between the limits, the conventional chart and the chart modified by Cornish–Fisher expansion are not satisfactory in this case. The arcsine chart may be conservative from the argument in Section 2. The proposed chart can meet the requirement for this example.

7. Conclusion

In this paper, we evaluate the four existing charts, the standard chart, the arcsine chart, the chart modified by Cornish–Fisher expansion and the CPC chart, in terms of the type I error and expected width criteria. The existing methods can perform well when the nonconforming rate is known, but they are unsatisfactory when the nonconforming rate is unknown. Therefore, we propose a new chart in this paper for the nonconforming rate unknown case, which is based on the form of the Agresti–Coull confidence interval.
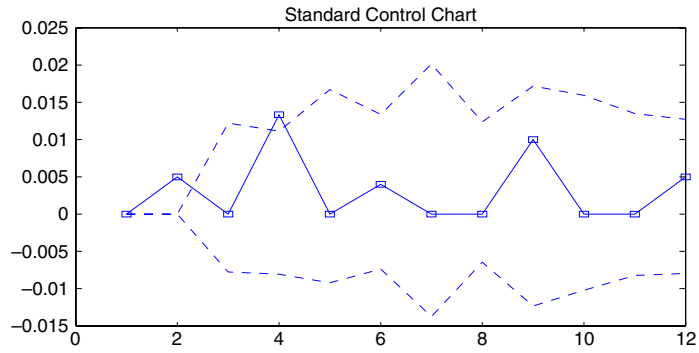
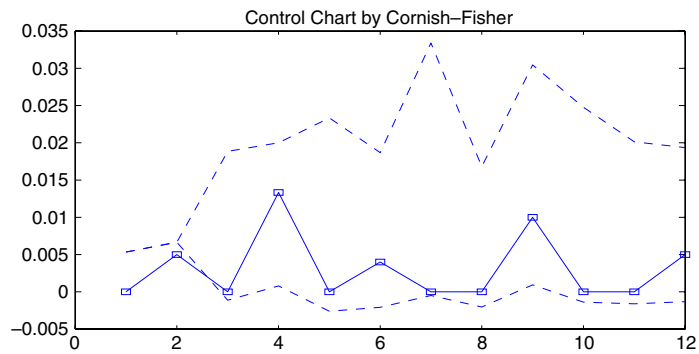**Fig. 8.** The dashed lines are the upper and lower control limits for the conventional chart.

Fig. 9. The dashed lines are the upper and lower control limits for the chart modified by Cornish–Fisher expansion.
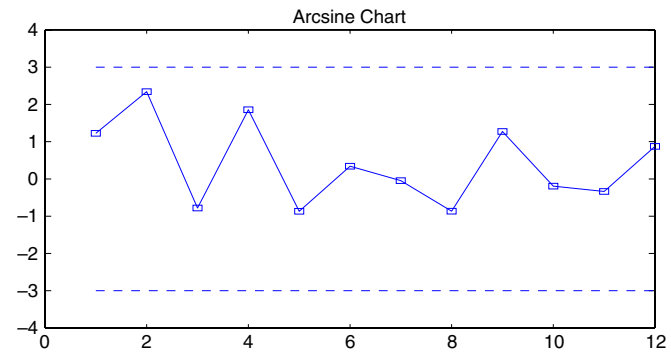

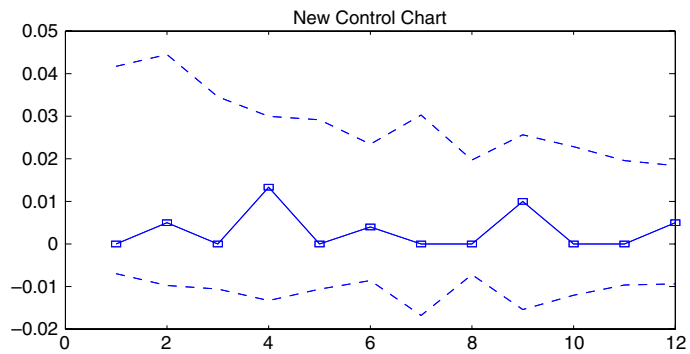**Fig. 10.** The dashed lines are the upper and lower control limits for Arcsine chart.

Fig. 11. The dashed lines are the upper and lower control limits for the new chart.

Compared with the existing charts, the new chart can successfully reduce the type I error when the nonconforming rate is small, and its expected width is in an accepted region. In addition, it has a simple closed form which can be easily adopted in real applications. Combining these merits, the proposed chart is a competitive one compared with the existing charts.

Acknowledgements

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