Computers & Industrial Engineering 57 (2009) 913–919

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

An optimization algorithm for cutting stock problems in the TFT-LCD industry

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ARTICLE INFO

Article history:
Received 27 August 2008
Received in revised form 31 December 2008
Accepted 13 March 2009
Available online 24 March 2009

Keywords:
Cutting stock problem
Optimization
Production

ABSTRACT

Due to lack of efficient approaches of mixed production, the present production approach of the TFT-LCD industry is batch production that each glass substrate is cut into LCD plates of one size only. This study proposes an optimization algorithm for cutting stock problems of the TFT-LCD industry. The proposed algorithm minimizes the number of glass substrates required to satisfy the orders, therefore reducing the production costs. Additionally, the solution of the proposed algorithm is a global optimum which is different from a local optimum or a feasible solution that is found by the heuristic algorithm. Numerical examples are also presented to illustrate the usefulness of the proposed algorithm.

1. Introduction

This study considers the cutting stock problem (CSP) of TFT-LCD (thin-film transistor liquid–crystal display) industry which aims to seek an optimal production schedule of cutting several LCD panels with different sizes from a given glass substrate to meet the orders. The glass substrate is one of important raw materials in the whole manufacturing process of LCD. If an enterprise designs a production scheme using the minimum number of glass substrates based on the orders received, it can reduce the manufacturing costs and increase the product's competitiveness in the market.

The CSP attempts to plan the optimal production schedule for minimizing the production costs, i.e. minimum trim loss. Different variants of CSP are available. An important variant of the CSP is the one-dimensional CSP. Many approaches for this problem have been proposed. For instance, Holthaus (2002) considered the integer one-dimensional cutting stock problem with different types of standard lengths and the objective of cost minimization. Umetani, Yagiura, and Ibaraki (2003) designed an approach which is based on meta-heuristics, and incorporates an adaptive pattern generation technique. Gradisar and Trkman (2005) also proposed a combined method for the solution to the general one-dimensional cutting stock problem (G1D-CSP).

Another variant of CSP is the two-dimensional CSP. In this variant, a set of order pieces is cut from a large supply of rectangular stock sheets of fixed size in a way that minimizes the total cost. Based on this goal, we are interested in finding 'cutting patterns' that minimize the unused area (trim loss). The problem is called two-dimensional cutting problem. Cutting and Packing problems belong to an old and very well-known family, called CP in Dyckhoff (1990) and Sweeney and Paternoster (1992). This is a family of natural combinatorial optimization problems.

The two-dimensional cutting and packing problem is widely applied in optimally cutting raw materials such as glass, steel and paper, in two-dimensional bin packing, and in layout designing problems. Many scholars have devoted themselves to developing many methods one after another to solve the problem; these methods can be grouped into two major types.

(i) *Deterministic:* Deterministic methods take advantage of analytical properties of the problem to generate a sequence of points that converge to a global solution. For example, Chen, Sarin, and Balasubramanian (1993) presented a mixed integer programming model for a class of assortment problems. Li and Tsai (2001) proposed a new method which finds the optimum of cutting problems by solving few linear mixed 0–1 problems. Li, Chang, and Tsai (2002) developed an approach using the piecewise linearization technique of the quadratic objective function to improve an approximate model for two-dimensional cutting problems.
(ii) *Heuristic:* Heuristic algorithms can obtain a solution quickly, but the quality of the solution cannot be guaranteed. G and Kang (2001) developed a heuristic that finds efficient layouts with low complexity for two-dimensional pallet loading problems of large size. Wu, Huang, Lau, Wong, and Young (2002) introduced an effective deterministic heuristic, Less Flexibility First, for solving the classical NP-complete rectangle-packing problem. Leung, Chan, and Troutt (2003) proposed an application of a mixed simulated annealing–

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doi:10.1016/j.cie.2009.03.006

genetic algorithm heuristic for the two-dimensional orthogonal packing problem. Beasley (2004) also presented a heuristic algorithm for the constrained two-dimensional non-guillotine cutting problem. The main defect of these heuristic algorithms is that they fail to claim the solution obtained is a global optimum unless the whole solution space is completely searched. Toward TFT-LCD industry involving mass production, the costs can be further reduced substantially if a global optimum can be derived instead of a local optimum or a feasible solution. For more detailed articles about the cutting optimization problem, readers can consult Lodi, Martello, and Monaci (2002) and Valério de Carvalho (2002).

Many approaches for two-dimensional cutting stock problem have also been proposed. Hifi (1997) discussed one of the best-known exact algorithms, due to Viswanathan and Bagchi (1993), for solving constrained two-dimensional cutting stock problem optimally. They proposed a modification of this algorithm in order to improve the computational performance of the standard version. Cung, Hifi, and Cun (2000) developed a new version of the algorithm in Hifi (1997) for solving exactly some variants of (un)-weighted constrained two-dimensional cutting stock problems. Leung, Yung, and Troutt (2001) applied a genetic algorithm and a simulated annealing approach to the two-dimensional non-guillotine cutting stock problem and carried out experimentation on several test cases. Vanderbeck (2001) developed a nested decomposition approach for two-dimensional cutting stock problem. Burke, Kendall, and Whitwell (2004) presented a new best-fit heuristic for the two-dimensional rectangular stock-cutting problem and demonstrated its effectiveness.

The two-dimensional cutting stock problem considered in this study is to derive the minimum number of glass substrates based on the available cutting combinations to meet the order demands. Due to lack of efficient approaches of mixed production, the present production approach is batch production that each glass substrate is cut into LCD plates of one size only. However, the computational result shows that the mixed production has a higher utilization of a glass substrate. Moreover, the proposed optimization algorithm can significantly reduce production costs to enhance the competitiveness of products.

The main advantages of the proposed method are listed as follows:

1. In comparison with the batch production, the proposed method presents a mixed production approach that generates LCD plates of various sizes in a glass substrate to increase the utilization of glass substrates (i.e. total area of produced products/ total area of glass substrates).
2. The proposed method provides the optimal production scheme according to the quantities of the orders.
3. The proposed method is able to find out all the alternative solutions with the same optimal objective value (i.e., different cutting combinations under the same utilization of a material substrate).

The rest of this paper is organized as follows: in Section 2, the mathematical models of a cutting optimization problem are formulated. In Section 3, a cutting stock optimization algorithm is proposed. Section 4 presents numerical examples to illustrate the proposed method and concluding remarks are included in Section 5.

2. Mathematical models

Since the mixed production has a higher utilization of a glass substrate than batch production, herein we construct some models to find out all cutting combinations in a glass substrate. To facilitate the discussion, the following notations are introduced first:

(l_0, w_0): The length and width of the glass substrate.
Z: Number of LCD products with different sizes which have to be produced.
(l_z, w_z): The length and width of the zth LCD product, $l_z w_z \le l_{z+1} w_{z+1}$, $z = 1,2,\ldots,Z$.
J: Index of multiple solutions with the same objective value.
T: Number of possible cutting combinations with different objective values.
c_{zj}^t: The cutting quantity of the zth product of the jth alternative solution with the same objective value in the tth iteration, $z = 1,2,\ldots,Z$, $t = 1,2,\ldots,T$, $j = 1,2,\ldots,J$.

First, let $Obj(0) = l_0 w_0$, consider the following model:

2.1. Model 1.1

$$\text{Max} \quad Obj(t) = \sum_{z=1}^{Z} c_{z1}^t (l_z w_z) \tag{1}$$

subject to

$$Obj(t) \le Obj(t-1). \tag{2}$$

This model aims to find the possible cutting combinations with different objective values. For $t = 1$, the constraint $Obj(1) \leqslant Obj(0)$ represents to use the maximum portion of a glass substrate. For $t = 2,3,\ldots T$, the constraint $Obj(t) \leqslant Obj(t-1)$ represents that the utilization of a glass substrate decreases as t increases. After each t iteration, we can obtain a solution and an objective value. Because distinct solutions may exist under the same objective value, we develop the following model for finding multiple solutions:

2.2. Model 1.2

$$\text{Max} \sum_{z=1}^{Z} c_{zj}^t (l_z w_z) \tag{3}$$

subject to

$$\sum_{z=1}^{Z} \left| c_{zj}^t - c_{z,j-1}^t \right| \geqslant 1, \quad \text{for all } j. \tag{4}$$

If the objective value obtained of Model 1.2 is equal to $Obj(t)$ deriving from Model 1.1 in the tth iteration, then alternative solutions exist. The purpose of constraint (4) is to search for an alternative solution. For example, let $Obj(0) = 200$, $(l_1, w_1) = (5,5)$, $(l_2, w_2) = (5,10)$ and $(l_3, w_3) = (10,10)$. The possible cutting combinations can be obtained by using Models 1.1 and 1.2 as follows:

Step 1: Let $t = 1$, $j = 1$, and $Obj(0) = 200$. Using Model 1.1 to find the possible cutting combination, we have the following model:

$$\text{Max} \quad Obj(1) = c_{11}^1(5*5) + c_{21}^1(5*10) + c_{31}^1(10*10)$$
$$\text{s.t. } Obj(1) \le Obj(0), \quad c_{11}^1, c_{21}^1, c_{31}^1 \in \text{integer}.$$

Solving the above model, we can obtain a possible cutting combination $(c_{11}^1, c_{21}^1, c_{31}^1) = (8, 0, 0)$, and the objective value is $Obj(1) = 200$.

Step 2: Using Model 1.2 to find the alternative solutions, we have the following model:

$$\text{Max} \quad c_{12}^1(5*5) + c_{22}^1(5*10) + c_{32}^1(10*10)$$
$$\text{s.t. } |c_{12}^1 - 8| + |c_{22}^1 - 0| + |c_{32}^1 - 0| \ge 1, \quad c_{12}^1, c_{22}^1, c_{32}^1 \in \text{integer}.$$

Solving the above model, we can find an alternative solution $(c_{12}^1, c_{22}^1, c_{32}^1) = (6, 1, 0)$ with the same objective value 200. By performing this process iteratively, all the alternative solutions can be found. However, the absolute terms in constraint (4) must be linearized so that Model 1.2 can be transferred into a linear programming problem. Consider the following propositions for linearizing an absolute term contained in a constraint:

Proposition 1. Let $\alpha \in \{0,1\}, \beta \geq 0$ then:

$$|x-a| = x-a+2a\alpha-2\beta \Longleftrightarrow \begin{cases} \text{(i)} -M\alpha \leq x-a \leq M(1-\alpha) \\ \text{(ii)}\ M(\alpha-1)+x \leq \beta \leq M(1-\alpha)+x \\ \text{(iii)}\ \beta \leq M\alpha \end{cases}$$

Proof 1.

(i) If $x - a \geqslant 0$, then $\alpha = 0$, $\beta = 0$ based on (i) and (iii); which results in $x - a + 2a\alpha - 2\beta = x - a$.
(ii) If $x - a \leqslant 0$, then $\alpha = 1$, $\beta = x$ based on (i) and (ii); which results in $x - a + 2a\alpha - 2\beta = a - x$. □

By Proposition 1, Model 1.2 is equivalently transformed into another linear program formulated as below.

2.3. Model 1.3

$$\text{Max} \sum_{z=1}^{Z} c_{zj}^t (l_z w_z)$$

subject to

$$\sum_{z=1}^{Z} (c_{zj}^t - c_{z,j-1}^t + 2c_{z,j-1}^t \alpha_z - 2\beta_z) \geq 1, \quad \text{for all} \quad j, \tag{5}$$

$$-M\alpha_z \leq c_{zj}^t - c_{z,j-1}^t \leq M(1-\alpha_z), \quad \text{for all} \quad j \text{ and } z, \tag{6}$$

$$0 \leq \beta_z \leq M\alpha_z, \quad \text{for all} \quad j \text{ and } z, \tag{7}$$

$$M(\alpha_z - 1) + c_{zj}^t \leq \beta_z \leq M(1-\alpha_z) + c_{zj}^t, \quad \text{for all} \quad j \text{ and } z, \tag{8}$$

where α_z are 0–1 variables, M is a large constant and the other variables are defined as before.

Model 1.3 is a linear programming problem solvable to obtain a global optimum and capable of finding out all possible cutting combinations even there are multiple solutions.

According to the above discussions, for the iteration of t ($t = 1,2,\ldots,T$), Model 1.1 is applied to solve the possible cutting combinations with different utilizations of a glass substrate. For the iteration of j, Model 1.3 is applied to solve the multiple solutions under a certain utilization derived from Model 1.1. Therefore some possible cutting combinations can be acquired by Model 1.1 and Model 1.3. However Models 1.1 and 1.3 only check the total measure of the LCD plates is less than that of a glass substrate. Next this study aspires to verify all rectangles of each cutting combination can be allocated into a glass substrate. Suppose the lengths and widths of n rectangles are given. A two-dimensional cutting optimization problem is to allocate all of these rectangles within an enveloping rectangle on x-axis and y-axis which occupies minimum area. The concept of the problem is stated as follows:

2.4. Model 2.1

Min $\quad xy$

subject to

1. All of the n rectangles are non-overlapping.
2. All of the n rectangles are within the range of x and y.
3. $0 < x \leqslant l_0$ and $0 < y \leqslant w_0$.

The related terminologies used in the model, referring to Li and Tsai (2001) are stated below:

(x,y): The top right corner coordinates of the enveloping rectangle.
(p_i,q_i): The dimension of rectangle i, p_i is the long side and q_i is the short side, p_i and q_i are constants, $i \in K$, K is the set of given rectangles.
x_i': Distance between center of rectangle i and original point along the x-axis.
y_i': Distance between center of rectangle i and original point along the y-axis.
s_i: An orientation indicator for rectangle i, $i \in K$. $s_i = 1$ if p_i is parallel to the x-axis; $s_i = 0$ if p_i is parallel to the y-axis.

The conditions of non-overlapping between rectangles i and k can be reformulated by introducing two binary variables u_{ik}, v_{ik} as follows:

Condition 1: $(u_{ik}, v_{ik}) = (0,0)$, rectangle i is at the right of rectangle k.
Condition 2: $(u_{ik}, v_{ik}) = (1,0)$, rectangle i is at the left of rectangle k.
Condition 3: $(u_{ik}, v_{ik}) = (0,1)$, rectangle i is at above of rectangle k.
Condition 4: $(u_{ik}, v_{ik}) = (1,1)$, rectangle i is at below of rectangle k.

Model 2.1 is a nonlinear programming problem which is difficult to solve for finding an optimal solution. By referring to the linearization technique of Li and Tsai (2001), we can formulate the original problem as a linear programming problem below:

2.5. Model 2.2

$$\text{Min} \quad x + y \tag{9}$$

subject to

$$(x_i' - x_k') + u_{ik} l_0 + v_{ik} l_0 \geq \frac{1}{2}[p_i s_i + q_i(1-s_i) + p_k s_k + q_k(1-s_k)], \quad \forall i,\ k \in K, \tag{10}$$

$$(x_k' - x_i') + (1-u_{ik}) l_0 + v_{ik} l_0 \geq \frac{1}{2}[p_i s_i + q_i(1-s_i) + p_k s_k + q_k(1-s_k)], \quad \forall i,\ k \in K, \tag{11}$$

$$(y_i' - y_k') + u_{ik} w_0 + (1-v_{ik}) w_0 \geq \frac{1}{2}[p_i(1-s_i) + q_i s_i + p_k(1-s_k) + q_k s_k], \quad \forall i,\ k \in K, \tag{12}$$

$$(y_k' - y_i') + (1-u_{ik}) w_0 + (1-v_{ik}) w_0 \geq \frac{1}{2}[p_i(1-s_i) + q_i s_i + p_k(1-s_k) + q_k s_k], \quad \forall i,\ k \in K, \tag{13}$$

$$l_0 \geq x \geq x_i' + \frac{1}{2}[p_i s_i + q_i(1-s_i)], \quad \forall i \in K, \tag{14}$$

$$w_0 \geq y \geq y_i' + \frac{1}{2}[p_i(1-s_i) + q_i s_i], \quad \forall i \in K, \tag{15}$$

$$x_i' - \frac{1}{2}[p_i s_i + q_i(1-s_i)] \geq 0, \quad \forall i \in K, \tag{16}$$

$$y_i' - \frac{1}{2}[p_i(1-s_i) + q_i s_i] \geq 0, \quad \forall i \in K, \tag{17}$$

where u_{ik}, v_{ik}, s_i, s_k are 0–1 variables, and $x, y, x_i', x_k', y_i', y_k'$ are bounded continuous variables.

Constraints (10)–(13) are non-overlapping conditions and constraints (14)–(17) ensure that all rectangles are within the enveloping rectangle. Model 2.2 can be solved efficiently to obtain the global optimum which is an upper bound of Model 2.1.

By Model 2.2, we know whether we are able to use the least square measure of the lager rectangle to produce the needed smaller rectangles or not. In addition, we try our best to make the surplus area centralized for cutting even smaller products once again.

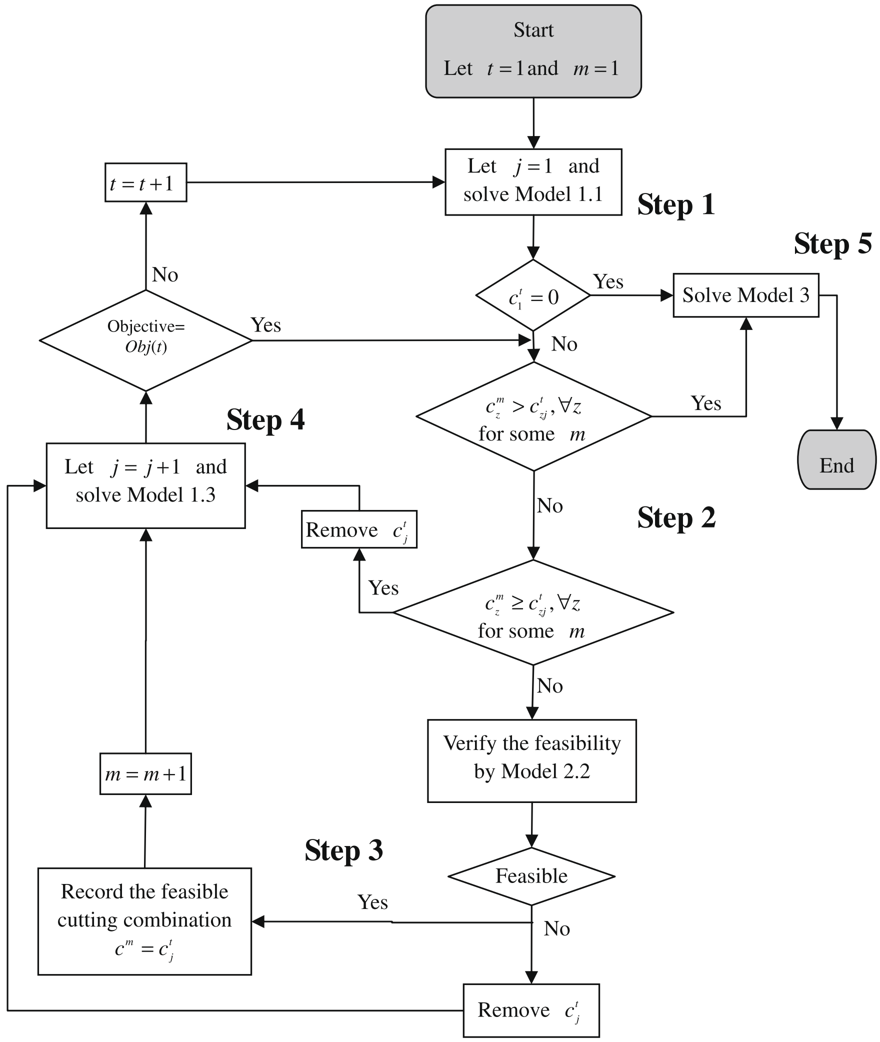

Fig. 1. Flowchart of the proposed algorithm.

By using Model 2.2 to examine all possible cutting combinations, we can get the feasible cutting combinations for designing the optimal production plan. Assume that $\mathbf{c}^m = (c_1^m, c_2^m, \ldots, c_Z^m)$ denotes the mth feasible cutting combination, o_z represents the quantity ordered of the zth product and g_m denotes the number of glass substrates that have to be cut by the mth feasible cutting combination. In order to minimize the total quantity of glass substrates required for fulfilling the demand of orders, the optimal production model is formulated as follows:

2.6. Model 3

$$\text{Min } \sum_{m=1}^{M} g_m \tag{18}$$

subject to

$$\sum_{m=1}^{M} c_z^m g_m \geqslant o_z, \quad \text{for } z = 1, 2, \ldots, Z, \tag{19}$$

where $g_m \in$ integer.

According to the solution of Model 3, we can know the minimal number of the required glass substrates and how many glass substrates are cut by each feasible cutting combination, respectively.

3. Solution algorithm

The algorithm of cutting stock problems is described as follows:

Input:{(l_0, w_0), (l_z, w_z), o_z, $t = 1$, and $m = 1$}
Processes:
{
Step 1: Let $j = 1$ and do Model 1.1
    if $(c_{zj}^t = 0)$ for all z then go to Step 5
Step 2: Compare c_z^m with c_{zj}^t
    if $(c_z^m > c_{zj}^t$ for some m or $c_{zj}^t = 0)$ then go to Step 5
    if $(c_z^m \geq c_{zj}^t$ for some m) then go to Step 4
Step 3: Verify the feasibility of combination c_{zj}^t
    do Model 2.2
    if (feasible) then $(c_1^m, c_2^m, \ldots, c_Z^m) = (c_{1j}^t, c_{2j}^t, \ldots, c_{Zj}^t)$ and $m = m + 1$
Step 4: Find possible cutting combination in the same utilization
    $j = j + 1$ and do Model 1.3
    if (Objective of Model 1.3 = Obj(t)) then go to Step 2
    else $t = t + 1$ and go to Step 1
Step 5: Find the optimal production combination
    if $(m \geqslant 2)$ do Model 3
}
Output:
{
    if $(m \geqslant 2)$ output (the optimal production combination $(g_1, g_2, \ldots, g_M)$)
    else output (no feasible production combination)
}

According to the above algorithm, we can obtain the optimal production scheme to produce plates most efficiently. The process of the algorithm is depicted in Fig. 1.

4. Example

In a TFT-LCD plant, assume the dimension of the glass substrate is 150 cm $*$ 180 cm. This plant wants to produce three kinds of products (40 in., 42 in. and 46 in.). The ratio of length to width of the LCD plates is 16:10. The information of these products is listed in Table 1.

To enhance the understanding of the proposed algorithm, the following illustrates the solution process of the example problem step by step.

Initial: Let $t = 1$ and $m = 1$.

Step 1-1: Let $j = 1$. By using Model 1.1, we find the solution (2,1,2) with the objective value 26,620.
Step 1-2: Because current feasible cutting combination set is empty, proceed straight to Step 3.
Step 1-3: Verify the possible cutting combination (2,1,2) by model 2.2. The result reveals that (2,1,2) is infeasible. Therefore, remove (2,1,2).
Step 1-4: Let $j = 2$ and add the constraint $|c^1_{12} - 2| + |c^1_{22} - 1| + |c^1_{32} - 2| \geq 1$ to Model 1.2 for finding alternative solutions. By solving Model 1.3, we find that the objective obtained is no equal to 26,620. Then let $t = 2$ and go to Step 1.
Step 2-1: Let $j = 1$ and solve Model 1.1. We find another possible cutting combination (0,4,1) with the objective value 26,360.
Step 2-2: The feasible cutting combination set is empty, then proceed to Step 3.
Step 2-3: Verify the feasibility of (0,4,1) by Model 2.2. The result shows that it is infeasible. Therefore, remove (0,4,1).
Step 2-4: Let $j = 2$ and add the constraint $|c^2_{12} - 0| + |c^2_{22} - 4| + |c^2_{32} - 1| \geq 1$ to Model 1.2 for finding alternative solutions. By solving Model 1.3, the objective value found is not equal to 26,360. Then let $t = 3$ and go to Step 1.
Step 3-1: Let $j = 1$ and solve Model 1.1. We find another possible cutting combination (0,5,0) with the objective value 25,200.
Step 3-2: The feasible cutting combination set is empty, then proceed to Step 3.
Step 3-3: Verify the feasibility of (0,5,0) by Model 2.2. The result indicates that (0,5,0) is feasible. Then record the feasible cutting combination $(c^1_1, c^1_2, c^1_3) = (0, 5, 0)$ and let $m = 2$.
Step 3-4: Let $j = 2$ and add the constraint $|c^3_{12} - 0| + |c^3_{22} - 5| + |c^3_{32} - 0| \geq 1$ to Model 1.2 for finding alternative solutions. By solving Model 1.3, the objective value obtained is not equal to 25,200. Then let $t = 4$ and go to Step 1.
Step 4-1: Let $j = 1$ and solve Model 1.1. We find another possible cutting combination (3,1,1) with the objective value 25,010.
Step 4-2: (3,1,1) does not meet the stop condition and there is no feasible cutting combination satisfies the condition, $c^m_1 \geq 3, c^m_2 \geq 1, c^m_3 \geq 1$ for some m. Then proceed to Step 3.
Step 4-3: Verify the feasibility of (3,1,1) by Model 2.2. The result shows that (3,1,1) is infeasible. Therefore, remove (3,1,1).
Step 4-4: Let $j = 2$ and add the constraint $|c^4_{12} - 3| + |c^4_{22} - 1| + |c^4_{32} - 1| \geq 1$ to Model 1.2 for finding alternative solutions. By solving Model 1.3, the objective value acquired is not equal to 25,010. Then let $t = 5$ and go to Step 1.

Continuing the solution process, we find that the possible cutting combination (0,0,3) is also a feasible cutting combination. Then add (0,0,3) into the feasible cutting combination set. After several iterations, we get the cutting combination (0,0,2). This cutting combination satisfies the termination condition $c^m_z \geq c^t_{zj},\ z = 1, 2$ and $c^m_3 > c^t_{3j}$ (i.e., 3 > 2) for some m. Therefore, stop the solution process.

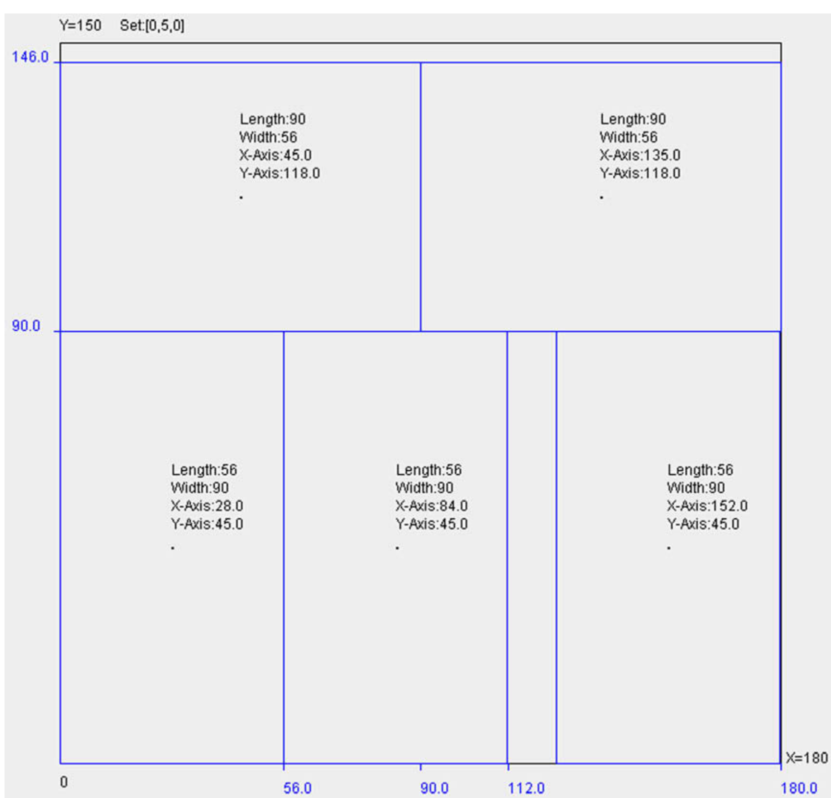

Fig. 2. The graphic solution of the optimal cutting combination (0,5,0).

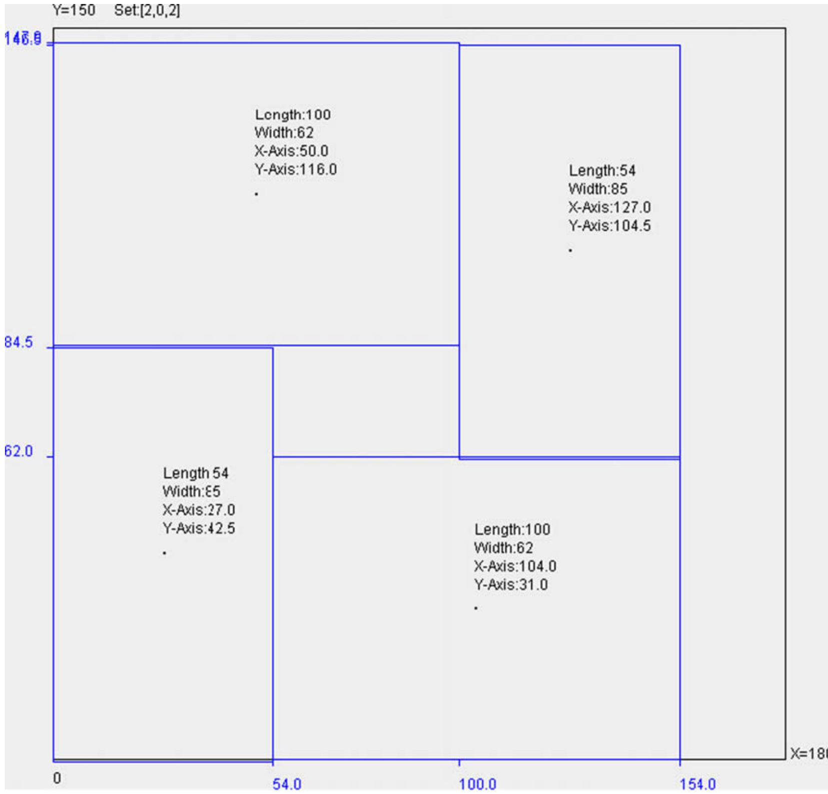

Fig. 3. The graphic solution of the optimal cutting combination (2,0,2).

Table 1
Dimensions and orders of the products.

Product (in.)	Length (cm)	Width (cm)	Area	Order
40	85	54	4590	1000
42	90	56	5040	1000
46	100	62	6200	1000

According to the orders of products and the obtained set of feasible cutting combination as shown in Table 2, we solve Model 3 with LINGO (2004) to find the best objective value 700 and the optimal solution $g_1 = 200$, $g_7 = 500$ and the other variables are zero. The solution process of this problem takes 546 s by using a Pentium 4 CPU 3.2 G PC. The graphic solutions of the optimal solutions are shown in Figs. 2 and 3.

If the plant adopts the production way suggested by the proposed algorithm, it can use g_1 and g_7 to achieve the highest utilization of the glass substrate and only consume 700 glass substrates to accomplish the orders. By the batch production approach, the number of glass substrates needed to fulfill the orders is 734. Additionally, the difference of the order quantities does not change the result that the proposed method is superior to the batch production. Table 3 lists results compared between the proposed method and the batch production of four examples with different order quantities. Since the computational time is not affected by the order quantities very much, the computation times of these four problems are all about 550 s. The results demonstrate that the proposed algorithm can solve the cutting stock problems of big size products in the TFT-LCD industry effectively.

In the proposed algorithm, the process of finding all feasible cutting combinations is the most time-consuming according to our testing. The computational time mainly depends on how many various LCD plates can be cut from a glass substrate. Therefore, the proposed method is suitable for the cutting stock problem of big size LCD products. The problems with more than 30 various small LCD plates that have to be cut from a glass substrate may integrate some heuristic or distributed algorithms to reduce the computational time for finding the feasible cutting combinations and that is an interesting issue for further research. The extended problems with the same size of glass substrate 180 cm $*$ 150 cm and eight products listed in Table 4 are solved within 10 min. The results shown in Table 5 reveal that the proposed method utilizes fewer glass substrates than the batch method and has significant saving ratios.

Table 2
Feasible cutting combinations.

Feasible cutting combinations (c_1^m, c_2^m, c_3^m)			
$m = 1$: (0,5,0)	$m = 2$: (1,4,0)	$m = 3$: (2,3,0)	$m = 4$: (3,2,0)
$m = 5$: (4,1,0)	$m = 6$: (5,0,0)	$m = 7$: (2,0,2)	$m = 8$: (0,3,1)
$m = 9$: (1,2,1)	$m = 10$: (2,1,1)	$m = 11$: (3,0,1)	$m = 12$: (0,0,3)
$m = 13$: (0,1,2)			

Table 3
Differences between the mixed production and the batch production.

Order quantities (40 in., 42 in., 46 in.)	Number of glass substrates		Saving	
	Proposed method (m)	Batch method (b)	Quantities	Ratio (%)
(1000,2000,3000)	1567	1600	33	2.06
(2000,1000,3000)	1534	1600	66	4.13
(3000,2000,1000)	1300	1334	34	2.55
(1000,1000,1000)	1400	1467	34	4.57

Saving ratio = $1 - (m/b)$.

Table 4
Dimensions and orders of the products.

LCD panels		Order #			
Product #	(Length, width)	1	2	3	4
1	(90,56)	5500	8500	9000	9900
2	(93,60)	7700	8500	9000	10,000
3	(99,63)	8100	8500	8500	8950
4	(95,66)	8500	8700	8750	9550
5	(98,64)	9000	9900	9950	10,500
6	(100,62)	9500	9650	10,500	11,550
7	(108,72)	13,000	13,500	15,550	17,000
8	(126,82)	15,000	15,500	20,500	21,550

Table 5
Comparison results of the proposed method and batch method.

Order #	Number of glass substrates		Saving	
	Proposed method (m)	Batch method (b)	Quantities	Ratio (%)
1	24,715	34,702	9987	28.78
2	26,350	36,835	10,485	28.46
3	30,042	43,052	13,010	30.22
4	32,268	46,049	13,781	29.93

Saving ratio = $1 - (m/b)$.

5. Conclusions

This study proposes a cutting stock optimization method for TFT-LCD industry, which can find the optimal cutting way according to the quantities of orders. The optimization techniques for finding alternative solutions and the approach for linearizing absolute terms are also presented. The results of numerical examples illustrate the usefulness of the proposed method. Especially to the TFT-LCD industry which needs mass production, an effective method can reduce production costs and promote the competitiveness of products. The directions for further research are to take more production situations and factors into consideration, such as the costs, the defect rate and the time of delivery and to integrate heuristic or distributed algorithms to enhance the computational efficiency. Using column generation techniques to solve this problem is also a very interesting topic for further investigation.

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