Modeling the Slotted Nonpersistent CSMA Protocol for Wireless Access Networks with Multiple Packet Reception

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***Abstract*—In this paper, we propose probability models for performance analysis of the slotted nonpersistent CSMA protocol in wireless networks with multiple packet reception. Variants of the CSMA protocol are well-known and play important roles in medium access control. Most of the previous works on CSMA use the conventional $(0, 1, e)$ channel model, which prohibits an access point from concurrently receiving multiple packets from distinct nodes. In this paper, we use the Poisson random traffic model to study the performance of the slotted nonpersistent CSMA protocol when the access point could simultaneously receive two or more packets from different nodes in wireless access networks. We show that in terms of the network throughput, our analytical results are consistent with simulation results.**

***Index Terms*—Carrier sense multiple access, multiple packet reception, wireless access network, performance analysis.**

I. INTRODUCTION

IN this paper, we analyze the performance of the slotted nonpersistent CSMA (Carrier Sense Multiple Access) protocol [1] [2] for distributed random access in wireless access networks with multiple packet reception (MPR) [4]. We assume that there are an access point and M nodes in the wireless access network. The CSMA mechanism plays an essential role in many well-known medium access control protocols such as the IEEE 802.11 standard and the IEEE 802.15.4 standard. Most of the previous works on medium access control used the conventional $(0, 1, e)$ channel model [2], which prohibits the access point from successfully receiving/decoding two or more packets from distinct nodes in a time slot. In contrast, in a wireless network with multiple packet reception, the access point could receive/decode a number of packets from different nodes simultaneously. MPR technologies could increase the network throughput without increasing the transmission power or the bandwidth requirement. In addition to capture effects, multiple packet reception could be realized by CDMA multiuser detection techniques and successive interference cancellation. Let $r_{i,j}$ be the probability that the access point will successfully receive/decode j packets given that i nodes simultaneously transmit packets. Note that $r_{i,0} = 1 - \sum_{j=1}^{i} r_{i,j}$, $\forall i$. The multiple packet reception capability at the access point is fully characterized by the $M \times M$ channel matrix R such that $[R]_{i,j} = r_{i,j}$, $\forall 1 \leq i, j \leq M$ [4].

Zhao and Tong [4] proposed the MQSR protocol, which is a polling-based medium access control scheme for wireless networks with multiple packet reception. Gau and Chen [8] proposed a predictive polling algorithm for medium access control in wireless networks with multiple packet reception and queueing. Naware, Mergen, and Tong [7] studied the impact of multiple packet reception on the stability and delay of slotted Aloha. Lotfinezhad, Liang, and Sousa [9] derived the optimal retransmission probabilities for slotted Aloha in wireless sensor networks with multiple packet reception. Gau and Chen [10] proposed probability models to derive the network throughput and the average packet delay for the classic tree/stack splitting algorithm in wireless networks with multiple packet reception. Kleinrock and Tobagi [1] analyzed the performance of CSMA protocols based on the conventional $(0, 1, e)$ channel model. Chan and Berger [5] investigated the impact of multiple packet reception on CSMA, when each node always has packets to be transmitted. In contrast, we use the Poisson random traffic model in this paper. Chan, Berger, and Tong [6] used drift analysis techniques to evaluate the performance of the slotted nonpersistent CSMA protocol in wireless networks with multiple packet reception. While they focused on the value of the maximum stable throughput, we derive the complete throughput function, which is a mapping from the network traffic load to the network throughput. Our work is the first in the literature that extends and applies the analytical techniques developed in [1] to CSMA protocols in wireless networks with MPR.

II. PROBABILITY MODELS AND ANALYTICAL RESULTS

Except for the channel model, we adopt all the system models in [1]. We assume that each packet is of constant length. We choose the time unit to be the packet transmission time. Let a be the maximum propagation delay. Namely, when a node starts packet transmission at time s, all other nodes in the network will sense the packet transmission before or on time $s + a$. A minislot equals a time units. According to the slotted nonpersistent CSMA protocol, when a packet arrives at a node within a minislot, the node will not perform carrier sense until the beginning of the next minislot. Without loss of essential generality, it is assumed that $\frac{1}{a}$ is an integer.

As in [1] [3], it is assumed that the network has a large number of nodes, which collectively generate a Poisson traffic with mean aggregate rate of G channel requests/packets per time unit. The time interval $[0, \infty)$ is partitioned into regenerative cycles [11]. Let Z_n be the beginning of the n-th regenerative cycle, $\forall n \geq 1$. Note that $Z_1 = 0$. Each regenerative cycle is composed of an idle period and a subsequent busy period [1]. Let A_n be the time instance when the first channel

Manuscript received June 22, 2009. The associate editor coordinating the review of this letter and approving it for publication was S. Gupta.

This work was supported in part by the National Science Council, Taiwan, R.O.C. under grant number NSC-97-2221-E-110-052-MY3.

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Digital Object Identifier 10.1109/LCOMM.2009.091306

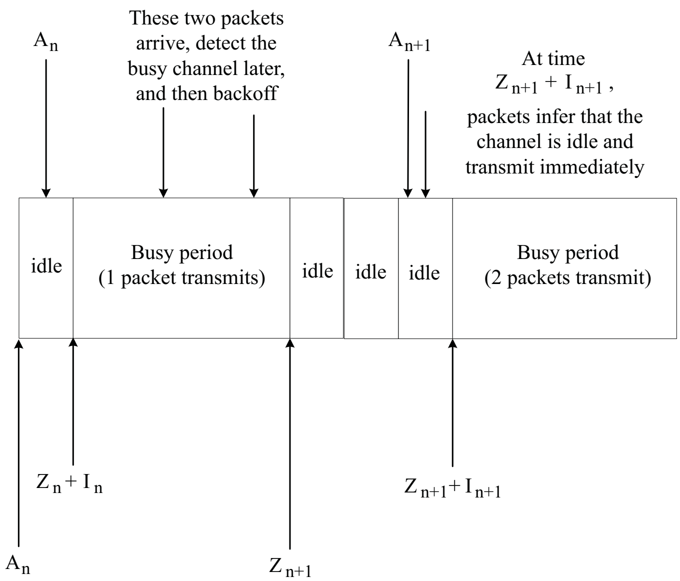

Fig. 1. An illustration of network evolution ($a = 0.25$).

request/packet in the n-th regenerative cycle arrives. The packet could be either a new packet or a retransmitted packet. Denote the ceiling function by $\lceil \cdot \rceil$. According to the slotted nonpersistent CSMA protocol, the packets that arrive within the time interval $(A_n, \lceil A_n/a \rceil \cdot a)$ will not sense the channel until time $\lceil A_n/a \rceil \cdot a$. Thus, the idle period in the n-th regenerative cycle is defined to be the time interval $[Z_n, \lceil A_n/a \rceil \cdot a)$. Let $I_n = \lceil A_n/a \rceil \cdot a - Z_n$ be the length of the idle period in the n-th regenerative cycle. The packet arriving at time A_n will learn that the channel is idle at time $\lceil A_n/a \rceil \cdot a$ and will be transmitted during the time interval $[\lceil A_n/a \rceil \cdot a, \lceil A_n/a \rceil \cdot a + 1]$. On the other hand, packets arriving during the time interval $(\lceil A_n/a \rceil \cdot a, \lceil A_n/a \rceil \cdot a + 1)$ will learn that the channel is busy and therefore postpone their transmissions. Therefore, the busy period in the n-th regenerative cycle is defined to be the time interval $[\lceil A_n/a \rceil \cdot a, \lceil A_n/a \rceil \cdot a + 1)$. Note that the first packet that arrives after time $\lceil A_n/a \rceil \cdot a + 1$ will learn that the channel is idle. Therefore, $Z_{n+1} = \lceil A_n/a \rceil \cdot a + 1$. An example of system evolution is shown in Fig. 1.

We now derive the probability mass function of I_n. First, since $\frac{1}{a}$ is an integer, $\frac{Z_n}{a}$ is also an integer. Therefore, $\lceil \frac{A_n}{a} \rceil \cdot a - Z_n = \lceil \frac{A_n - Z_n}{a} \rceil \cdot a$. Since the arrivals of channel requests/packets form a Poisson process with rate G, the random variable $A_n - Z_n$ is exponentially distributed with mean equals $\frac{1}{G}$. Then,

$$\begin{aligned} P\{I_n = ka\} &= P\{\lceil \frac{A_n}{a} \rceil \cdot a - Z_n = ka\} \\ &= P\{\lceil \frac{A_n - Z_n}{a} \rceil \cdot a = ka\} \\ &= P\{(k-1)a < A_n - Z_n \le ka\} \\ &= e^{-Ga(k-1)} \times (1 - e^{-Ga}) \end{aligned} \tag{1}$$

Denote the expected value of a random variable X by $E[X]$. Since $I_1, I_2, I_3, ...$ are IID (independent and identically distributed) random variables, we abbreviate $E[I_n]$ by $E[I]$. We now derive the value of $E[I]$ as follows.

$$\begin{aligned} E[I] &= \sum_{k=1}^{\infty} ka \times P\{I_n = ka\} \\ &= \sum_{k=1}^{\infty} ka \times e^{-Ga(k-1)} \times (1 - e^{-Ga}) \\ &= a(1 - e^{-Ga}) \times \sum_{k=1}^{\infty} k \cdot (e^{-Ga})^{k-1} \\ &= \frac{a}{1 - e^{-Ga}} \end{aligned} \tag{2}$$

The second equality is due to the previous equation. The fourth equality is due to that $\sum_{k=1}^{\infty} k \cdot r^{k-1} = \frac{1}{(1-r)^2}$, $\forall r \in (0, 1)$.

Let Q_β be the probability that β nodes will simultaneously transmit packets in a busy period. Let X_n be a random variable that represents the total number of arrivals of channel requests/packets in the last minislot of the n-th regenerative cycle. Then, $Q_\beta = P\{X_n = \beta\}$. We now derive the values of Q_β's. By definition, $Q_0 = 0$. Let Y be a random variable that represents the total number of arrivals of channel requests/packets in a minislot. Since $\{N(t), t \ge 0\}$ is a Poisson counting process with rate G, Y is a Poisson-distributed random variable with mean equals Ga. Then, $\forall \beta \in \{1, 2, .., M\}$,

$$\begin{aligned} Q_\beta &= P\{X_n = \beta\} \\ &= P\{Y = \beta | Y \ge 1\} \\ &= \frac{P\{Y = \beta, Y \ge 1\}}{P\{Y \ge 1\}} \\ &= \frac{P\{Y = \beta\}}{P\{Y \ge 1\}} \\ &= \frac{(Ga)^\beta e^{-Ga}}{\beta!(1 - e^{-Ga})} \end{aligned} \tag{3}$$

The second equality is due to that at least one channel request/packet arrives in the last minislot of an idle period. The fourth equality is due to that $\beta \ge 1$.

Let U_n be a random variable that represents the total number of packets that are successfully received by the access point in the n-th regenerative cycle. Let $P_k = P\{U_n = k\}$ be the probability that the access point will successfully receive/decode k packets in a busy period. Then,

$$\begin{aligned} & P\{U_n = k\} \\ =& \sum_{\beta=1}^{M} P\{U_n = k, X_n = \beta\} \\ =& \sum_{\beta=1}^{M} P\{X_n = \beta\} \times P\{U_n = k | X_n = \beta\} \\ =& \sum_{\beta=1}^{M} Q_\beta \cdot r_{\beta,k} \end{aligned} \tag{4}$$

The last equality is based on the MPR channel model. Since $U_1, U_2, U_3, ...$ are IID random variables, we abbreviate U_n by U whenever appropriate.

In addition, $E[U] = \sum_{k=1}^{M} P_k \times k$. Let S be the network throughput of the nonpersistent CSMA protocol. Recall that the length of a busy period is always one time unit. According

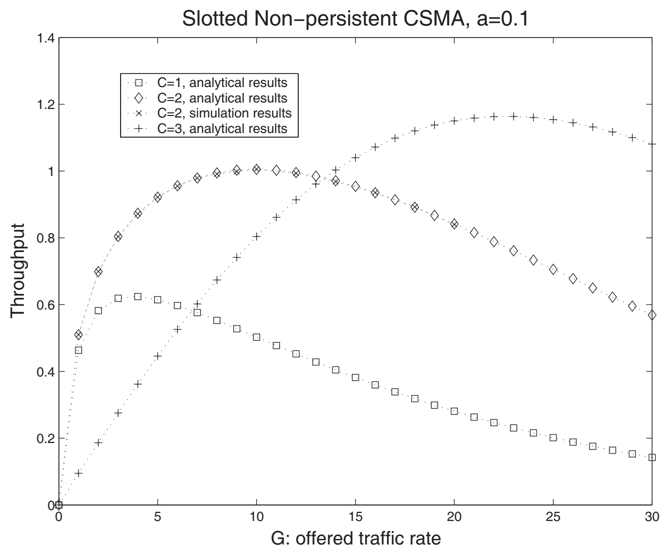

Fig. 2. The throughput function.

to renewal theory [1] [11], $S = \frac{E[U]}{E[I]+1}$. Based on the above equations, we have

$$S = \frac{\sum_{k=1}^{M} k \cdot \sum_{\beta=k}^{M} \frac{(Ga)^{\beta} e^{-Ga}}{\beta!(1-e^{-Ga})} \cdot r_{\beta,k}}{\frac{a}{1-e^{-Ga}} + 1} \tag{5}$$

III. Numerical and Simulation Results

We wrote a Matlab program to obtain numerical results based on the equations in this paper. In addition, we wrote a C program to perform discrete event simulation. Let C be a positive integer that represents the maximum number of successfully received packets in a time slot. As in [4] [10], we study on the case in which $r_{i,i} = 1$, $\forall 1 \le i \le C$, while all other elements in the matrix R are zeros. Note that $r_{C+1,C+1} = r_{C+2,C+2} = .. = r_{M,M} = 0$. Namely, if more than C nodes transmit packets simultaneously, the access point receives none of them for sure. For each value of G, we simulate 100000 regenerative cycles to obtain the network throughput. In Fig. 2, we show that our analytical results are consistent with simulation results when $C = 2$, $a = 0.1$, and $G \in [0, 20]$. For example, when $G = 1.0$, the analytical result is 0.5100, while the simulation result is 0.5098. When $G = 10.0$, the analytical result is 1.0050, while the simulation result is 1.0057.

IV. Conclusion

In this paper, we have proposed probability models for performance analysis of the slotted nonpersistent CSMA protocol in wireless networks with multiple packet reception. Most of the previous works on CSMA use the conventional $(0, 1, e)$ channel model, which prohibits an access point from concurrently receiving multiple packets from distinct nodes. In this paper, we have used the Poisson random traffic model to study the performance of the slotted nonpersistent CSMA protocol when the access point could simultaneously receive two or more packets from different nodes. We have shown that in terms of the network throughput, our analytical results are consistent with simulation results. Since multiple packet reception techniques could increase the throughput without increasing the bandwidth requirement, they could be applied to wireless networks in which bandwidth is relatively scarce. As the value of C increases, the hardware complexity to realize the corresponding multiple packet reception capability also increases. Our studies shows that the increasing rate of the maximum throughput tends to be a decreasing function of C. Thus, we recommend to select $C \in \{2, 3\}$ for hardware implementation in practice.

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