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Sliding Abrikosov lattice in a superconductor with a regular array of artificial pinning centers: AC conductivity and criticality at small frequencies

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ABSTRACT

Dynamics of the flux lattice in the mixed state of strongly type-II superconductor near the upper critical field subjected to AC field and interacting with a periodic array of short range pinning centers is considered. The superconductor in a magnetic field in the absence of thermal fluctuations on is described by the time-dependent Ginzburg–Landau equations. For a special case of the δ-function shaped pinning centers and for pinning array commensurate with the Abrikosov lattice (so that vortices outnumber pinning centers) an analytic expression or the AC conductivity is obtained. It is found that below a certain critical pinning strength and for sufficiently low frequencies there exists a sliding Abrikosov lattice, which vibrates nearly uniformly despite interactions with the pinning centers. At very small frequencies the conductivity diverges at the critical pinning strength.

The great interest in the problem of magnetic flux pinning in type-II superconductors stems from its relevance to technological applications as well as with its implications to the general problem of complex nonlinear dynamics with tunable parameters. An important challenge in applications of type-II superconductors is in achieving optimal critical currents under given magnetic fields. This requires preventing depinning of Abrikosov vortices during formation of the resistive state under the applied current. Recently there have been advances in the study of vortex pinning by fabricating periodic arrays of pinning sites where each pinning site may be either magnetic or normal inclusion effectively trapping vortices. Pinning arrays with triangular, square, and rectangular geometries have been fabricated using either microholes or blind holes arrays of magnetic dots and periodic array of columnar defects [1]. The resulting critical current is enhanced when vortex lattice is commensurate with the periodic array of pinning sites. In addition this system is a convenient experimental tool to study the general problem of interacting periodic system moving in periodical potential like dislocations in crystals or charge density waves. Theory of dynamics of the pinned vortex matter by a random distribution of pins is very complicated. However in the absence of significant thermal fluctuations, the problem simplifies considerably. It was studied theoretically, mostly in 2D systems, using either numerical methods within a model of interacting points-like particles representing vortices subject to pinning potential and driving force [2] or within the framework of elasticity theory, in which the vortex matter is treated as an elastic manifold subject to both the pinning stress and a driving force [3].

Theory of the Abrikosov lattice subjected to an AC field and periodic pinning is simpler, but so far has been treated either numerically using molecular dynamics approach or by means of the elastic manifold approach in London approximation. On the other hand this approach completely ignores the contribution of the vortex cores essentially important when the distances between vortices and artificial pinning sites are not much larger than the size of the coherence length. In fact there is still no an analytical theory describing AC properties of a type-II superconductor with periodic pinning array subjected to a strong magnetic field. Here we present a theory of AC conductivity in the time-dependent Ginzburg–Landau (TDGL) approximation describing superconductor in a strong magnetic field. In the absence of thermal fluctuations, an exact solution for the linear response in the case of a δ-function model for the periodical array of the pinning centers in which it is commensurate with the Abrikosov lattice (vortices outnumber pinning centers) is obtained for the first time.

Let us consider a type-II superconductor under a constant external magnetic field H parallel to a system of pinning centers directed along z axis and carrying electric current along the y axis, see Fig. 1.

The simplest relaxation dynamics of a superconductor in the presence of electric field is described by TDGL [4] and Maxwell equations (in the dimensionless variables)

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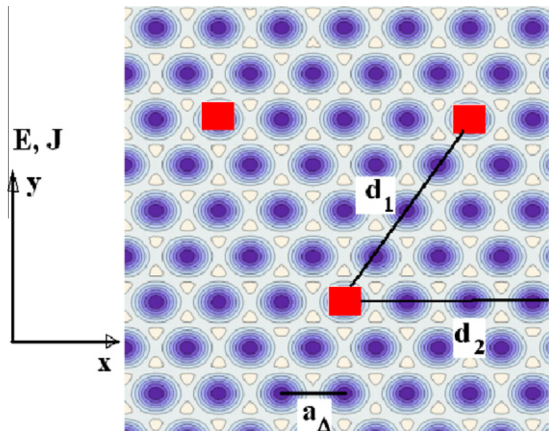

Fig. 1. The hexagonal Abrikosov vortex lattice (distribution of the superconducting density $|\psi(r)|^2$) and pinning centers. Zeroes of order parameter fall on the locations of the columnar defects (red squares), so that vortices outnumber the pins. Vectors $\mathbf{d}_1$ and $\mathbf{d}_2$ are lattice vectors of pinning array. Distance between nearest neighbors of the Abrikosov lattice is a_Δ. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$$
\begin{aligned}
&f_{GL} = \psi^* \widehat{H}\psi - a_h\psi^*\psi + \frac{1}{2}(\psi^*\psi)^2; \\
&-\partial_t\psi = \widehat{H}\psi - a_h\psi + \psi^*\psi^2 - i\Phi\psi; \qquad (1)\\
&\widehat{H}_p = -\frac{D^2}{2} - \frac{h}{2} + U_0\sum_a \delta(\mathbf{r}-\mathbf{r_a}); \\
&a_h = \frac{1-t_h-h}{2} - u_0; \quad t_h = \frac{T}{T_c}; \qquad (2)\\
&\widehat{H} = \widehat{H}_p - u_0; \quad U_0 = \frac{\pi w^2\varepsilon}{T_c}; \\
&j = -\nabla\Phi + \frac{i}{2}[\psi^*\mathbf{D}\psi - c.c.]; \quad \nabla\mathbf{j} = 0 \qquad (3)
\end{aligned}
$$

Here $\mathbf{D}$ are the covariant derivatives, ψ is the order parameter and Φ the electric potential, h the magnetic field in units of H_{c2}, f_{GL} is GL energy density, w the radius of the single pinning potential and $u_0 = 2\pi U_0 n_p h$ is dimensionless pinning strength. In our units $\sigma_n = \rho_n = 1$.

This set of the equations can be simplified near the coexistence line where $a_h \ll 1$ and for small electric fields. Expanding the order parameter in first order in the form $\psi(\mathbf{r},t) = \varphi(\mathbf{r}) + \theta(\mathbf{r},t)$ one obtains

$$
\begin{aligned}
&\partial_t\theta = -\widehat{H}\theta + i\Phi\theta; \quad E = -\nabla\Phi; \\
&E = j - i[\theta^*\mathbf{D}\phi - \theta(\mathbf{D}\phi)^*]
\end{aligned} \qquad (4)
$$

here $\phi(\mathbf{r}) = \left(\frac{a_h}{\beta_A}\right)^{1/2}\varphi_0(\mathbf{r}) + O(a_h^{3/2})$ where $\varphi_0(\mathbf{r})$ is the Lowest Landau level order parameter. Defining the retarded Green function by

$$(\partial_t + \widehat{H})G(\mathbf{r},\mathbf{r}',t-t') = \delta(\mathbf{r}-\mathbf{r}',t-t') \qquad (5)$$

one obtains the following relation between the current density and the electric field

$$E(\mathbf{r},t) = j(t) - \int_{t'=-\infty}^{t} j(t')\int_{r'} G(\mathbf{r},\mathbf{r}',t-t')y'\varphi(r')(\mathbf{D}_y\phi)^* + c.c. \qquad (6)$$

where $E = j + O(a_h)$.

For a uniform AC density $j(t) = j_0\cos\omega t$ one obtains after average over volume of the sample for complex conductivity

$$\rho(\omega) = \lim_{T\to\infty}\frac{1}{Tj_0}\int_0^T dt e^{-i\omega t}\langle E(\mathbf{r},t)\rangle_r \qquad (7)$$

Performing integrations one obtains

$$
\begin{aligned}
&\rho(\omega) = \tfrac{1}{1+\sigma_s(\omega)} \approx 1 - \sigma_s(\omega) \\
&\sigma_s(\omega) = -\int_{r'} y'\langle\phi(r')(\mathbf{D}_y\phi)^* G(\mathbf{r},\mathbf{r}',\omega) + \phi^*(r')(\mathbf{D}_y\phi)G^*(\mathbf{r},\mathbf{r}',-\omega)\rangle_r
\end{aligned} \qquad (8)
$$

Eq. (1) allows to relate the dynamic conductivity in the superconductor with the Green function (GF) of the quantum mechanical Hamiltonian $\widehat{H}_p$ of a charged particle in magnetic field in the presence of periodic potential. Representing Green function in the integral form, one obtains the Dyson equation

$$G(\mathbf{r},\mathbf{r}',\omega) = G_{cl}(\mathbf{r},\mathbf{r}',\omega) - U_0\sum_a G_{cl}(\mathbf{r},\mathbf{r}_a,\omega)G_{cl}(\mathbf{r}_a,\mathbf{r}',\omega) \qquad (9)$$

where $G_{cl}(\mathbf{r},\mathbf{r}',\omega)$ is the Green function of the clean superconductor

$$
\begin{aligned}
&G_{cl}(\mathbf{r},\mathbf{r}',t) = e^{\frac{ih}{2}(xy'-yx')}g_{cl}(\mathbf{r},\mathbf{r}',t) \\
&g_{cl}(\mathbf{r},\mathbf{r}',t) = C(t)e^{-\frac{r^2}{2\eta(t)}} \\
&C(t) = \tfrac{h}{4\pi}e^{\frac{ht}{2}}\sinh^{-1}\left(\tfrac{ht}{2}\right) \\
&\eta(t) = \tfrac{2}{h}\tanh\left(\tfrac{ht}{2}\right)
\end{aligned} \qquad (10)
$$

In particular at pinning points $\mathbf{r} = \mathbf{r}_b$, assuming commensurability with the vortex lattice, one obtains

$$G(\mathbf{r}_a,\mathbf{r}',\omega) = \sum_a M_{ab}^{-1}(\omega)G_{cl}(\mathbf{r}_b,\mathbf{r}',\omega) \qquad (11)$$

where a symmetric matrix $M_{ba}(\omega)$ is defined by

$$
\begin{aligned}
&M_{ab}^{-1}(\omega) = \tfrac{1}{S_{BZ}}\int_{q\subset BZ} e^{iq(\mathbf{r}_b-\mathbf{r}_a)}\Pi_{q,\omega} \\
&\Pi_{q,\omega} = \tfrac{1}{1+U_0 n_p G_{cl}(q,\omega)}
\end{aligned} \qquad (12)
$$

Substituting it into the expression for full GF with arbitrary positions one obtains

$$
\begin{aligned}
&G(\mathbf{r},\mathbf{r}',\omega) = G_{cl}(\mathbf{r},\mathbf{r}',\omega) - \tfrac{U_0}{S_{BZ}}\sum_{a,b}\int_q e^{iq(\mathbf{r}_a-\mathbf{r}_b)}\Pi_{q,\omega}K(\mathbf{r},\mathbf{r}_a,\mathbf{r}_b,,\mathbf{r}',\omega) \\
&K(\mathbf{r},\mathbf{r}_a,\mathbf{r}_b,\mathbf{r}',\omega) = G_{cl}(\mathbf{r},\mathbf{r}_a,\omega)G_{cl}(\mathbf{r}_b,\mathbf{r}',\omega)
\end{aligned} \qquad (13)
$$

To determine the operator GF for operator $\widehat{H}$ one has to subtract the constant u_0 from $\widehat{H}_p$. In the ω space such transformation is equivalent to a shift of frequency by the imaginary number $i\omega$ in the GF. Substituting the full GF into expression for conductivity one obtains two contributions in terms of "clean" GF:

$$
\begin{aligned}
&\sigma_s(\omega) = \sigma_I(\omega) + \sigma_{II}(\omega) \qquad (14)\\
&\sigma_I(\omega) = -\frac{2}{L_xL_y}\int y'\varphi(r')(\mathbf{D}_y\phi)^*G_{cl}(\mathbf{r},\mathbf{r}',\omega+iu_0) \\
&\sigma_{II}(\omega) = \frac{2U}{S_{BZ}}\sum_{a,b}\Sigma_a^1(\omega)\Sigma_b^2(\omega)\int_q e^{iq(\mathbf{r}_b-\mathbf{r}_a)}\Pi_{q,\omega+iu_0}
\end{aligned}
$$

where

$$
\begin{aligned}
&\Sigma_a^1(\omega) = \frac{1}{\sqrt{L_xL_y}}\int_r(\mathbf{D}_y\phi(\mathbf{r}))^*G(\mathbf{r},\mathbf{r}_a,\omega+iu_0) \\
&\Sigma_a^1(\omega) = \frac{1}{\sqrt{L_xL_y}}\int_{r'} y'\phi(\mathbf{r}')G(\mathbf{r}_b,\mathbf{r},\omega+iu_0)
\end{aligned}
$$

Integrations result in:

$$
\begin{aligned}
&\sigma_s(\omega) = \sigma_{FF}\left[1 + \frac{3.75U_0 n_p h(i\omega+h-u_0)^{-1}}{\frac{i\omega}{i\omega-u_0} + u\Theta\left(1+\frac{i\omega}{h}-u\right)}\right] \qquad (15)\\
&\sigma_{FF} = \frac{a_h}{\beta_A}\frac{1}{i\omega+h-u_0}; \quad \Theta(X) = \log\left(\frac{K_{\max}^2}{2h}\right) - \Psi(X)
\end{aligned}
$$

Here $\Psi(X)$ is digamma function and $K_{\max}$ is maximal vector of the reciprocal pinning centers' lattice.

Let us consider some limiting cases important for experiment.

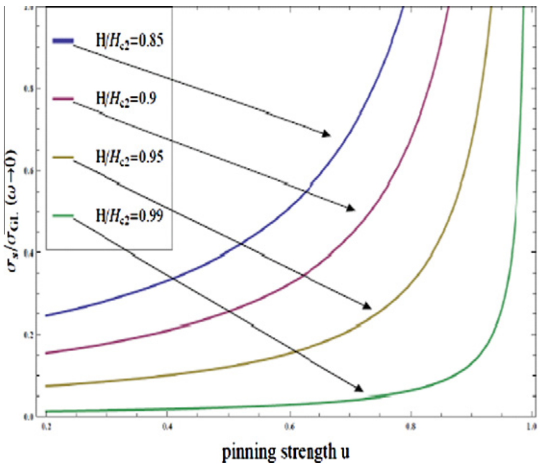

Fig. 2. Real part of conductivity at $\omega \to 0$ as function of the pinning strength u for magnetic field in the h = 0.85–0.99 range. When the pinning strength approaches the critical value the conductivity diverges.

(i) No pinning $U_0 = 0$

$$\sigma_{FF} = \frac{a_h}{\beta_A}\frac{1}{i\omega + h} \tag{16}$$

If ω = 0 when Eq. (18) gives a well known Bardeen–Stephen result for flux flow conductivity.

(ii) Criticality near the critical pinning strength $h = u_0$ for small frequency. This means that the vortex lattice is pinned and electric field cannot penetrate the superconductor despite persistent current flow in it at least when the current is not large. In this case the real part of the conductivity diverges. Near this line the conductivity reads (see Figs. 2 and 3):

$$\sigma_s(\omega \to 0) \approx \frac{a_h}{2\pi n_p \beta_A h}\frac{1}{U_0^c - U_0} \tag{17}$$

where $U_0^c = (2\pi n_p)^{-1}$. Therefore the pinning strength is only factor determining the transition into the pinned state. The critical value is independent of the magnetic induction.

(iii) AC conductivity at the critical line ($u = u_c$)

In this case the conductivity at small frequencies $\omega \ll h$ has only the imaginary part

$$\sigma_s \approx \frac{a_h}{\beta_A}\frac{1}{i\omega} \tag{18}$$

(iv) AC for subcritical pinning strength

In this case $u \ll 1$ and AC conductivity reads

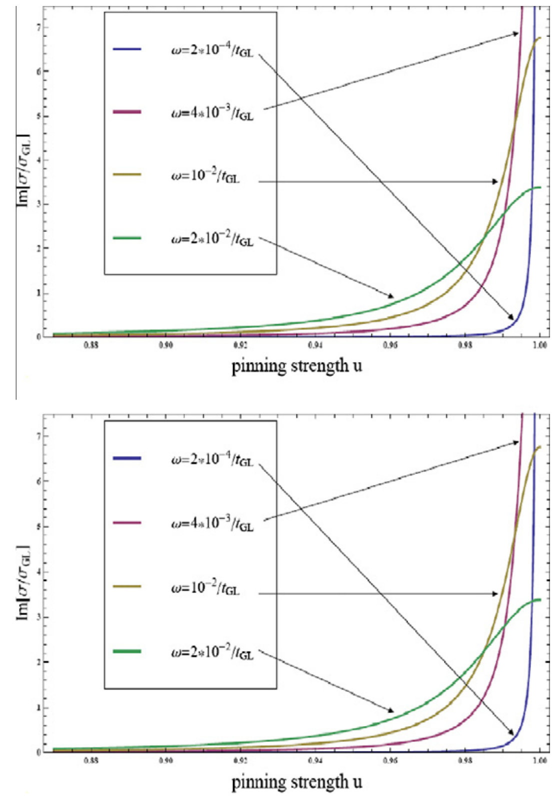

Fig. 3. Real and imaginary parts of conductivity at $\omega \to 0$ as function of the pinning strength u for magnetic field h = 0.95. When the pinning strength approaches the critical u the conductivity diverges at small frequencies.

$$\frac{\sigma_s}{\sigma_{FF}} = 1 + \frac{0.6uh}{(i\omega + h)\left[\frac{i\omega}{i\omega - h} + u\Theta\left(1 + \frac{i\omega}{h}\right)\right]}$$

where the second term in this expression describes pinning correction to usual Bardeen–Stephen conductivity.

In summary, we developed the theory of AC conductivity for a superconductor with periodic pinning array in the Ginzburg–Landau approximation and predicted that above some pinning strength, the AC conductivity in the limit of small frequency shows typical for ideal superconductor behavior.

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