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The worst-practice DEA model with slack-based measurement

Fuh-hwa Franklin Liu *, Cheng-Li Chen

Department of Industrial Engineering and Management, National Chiao Tung University, 1001 University Road, Hsin Chu 300, Taiwan, ROC

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Abstract

An original data envelopment analysis (DEA) model is to evaluate each decision-making unit (DMU) with a set of most favorable weights of performance indices. The efficient DMUs obtained from the original DEA construct an efficient (best-practice) frontier. The original DEA can be considered to identify good (efficient) performers in the most favorable scenario. For the purpose of identifying bad performers such as bankrupt firms in the most unfavorable (worst-case) scenario, radial worst-practice frontier DEA (WPF–DEA) model in which the "worst efficient" DMUs construct a worst-practice frontier has been proposed. To identify bad performers together with the slack values we formulate another model called WPF–SBM. Then we develop the HypoSBM model to distinguish the worst performers from the bad ones. Finally, a solution approach is suggested to fully rank worst efficiencies in the worst-case scenario.

Keywords: Data envelopment analysis; Worst-case scenario; Worst-practice frontier; Worst efficiency; Slack-based efficiency measure

1. Introduction

In capital-intensive industries such as high-tech or large-scale manufacturing industry, technology getting more advanced usually means a vast amount of money (capital investment) involved. In risk-taking industries such as insurance industry or banking industry, high profit will come along with high risk. For financial institutions or individual investors in the business of investing in risk-taking or capital-intensive industries, investment risk evaluation becomes a significant issue. A type I error occurs if the investment is bad and can not be identified. Type I error is an indication of investment risk which has to be minimized. It is obvious that the cost of a type I error (the loss resulting from a failed investment) is much higher than that of a type II error (the loss represented by the revenue which the financial institutions or individual investors would have been received if they had made the successful investment). The identification and quantification of investment risk is therefore very important.

The financial institutions or individual investors surely have to evaluate the performance of those companies in the industry before they invest. Data envelopment analysis (DEA) has been proven as an excellent

* Corresponding author. Tel.: +886 0932010022; fax: +886 35731739.
E-mail address: fliu@mail.nctu.edu.tw (F.F. Liu).

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data-oriented performance evaluation method when multiple inputs and outputs are present in a set of peer decision-making units (DMUs). The original DEA model which is referred as CCR (Charnes, Cooper and Rhodes) model establishes an efficient frontier among the units based on a comparison process in which the ratio scales of the weighted sum of the outputs to that of the inputs are evaluated. The efficient DMUs obtained from the original DEA construct an efficient (best-practice) frontier. The DMUs not on this frontier are deemed inefficient. Cooper, Seiford, and Tone (2000) pointed out that the set of optimal weights for the DMU_o, the decision-making unit to be evaluated, is actually the set of most favorable weights for the DMU_o in the sense that it maximizes the efficiency ratio scale. The original DEA can be considered to identify good (efficient) performers in the most favorable scenario. Therefore, the original DEA is called "best-practice frontier" DEA (BPF–DEA) in this paper. Most of DEA-based papers are in the category of BPF–DEA.

There are few BPF–DEA papers such as the works of Simak (1997), Pendharkar (2002), Pille and Paradi (2002), Cielen, Peeters, and Vanhoof (2004) and Sueyoshi (2006) have focused on the use of DEA in corporate failure or bankruptcy prediction. However, BPF–DEA models select potentially distressed companies by measuring how inefficient they are in the most favorable scenario, which is not suitable in the real world. In vulnerable and competitive business climate, the potential companies who will go out of business first are usually the ones of least competitiveness in comparison with others as the scenario is getting worse (more unfavorable), especially when an economic depression or financial crisis such as the Asia financial crisis occurred in 1997. Therefore, for the problem of investment risk evaluation or bankruptcy prediction, we believe that it should be more meaningful to propose an appropriate model formulation for evaluating and ranking units for the purpose of identifying bad performers in the most unfavorable (worst-case) scenario.

The applications of DEA-based approaches for particularly evaluating performance in the category of worst-case scenario can only be found in the works of Paradi, Asmild, and Simak (2004), Shuai and Li (2005) and Liu and Chen (submitted for publication). After Paradi et al. (2004) introduced the concept of worst practice DEA (but without specific mathematical expression of model), the concept of worst practice DEA was employed in conjunction with rough set theory in the work of Shuai and Li (2005) for dealing with imprecise data which is discussed in a specific category of DEA. Then Liu and Chen (submitted for publication) developed a radial model of the worst-practice frontier DEA (WPF–DEA). The WPF–DEA picks out struggling companies based on how bad they perform in the worst-case scenario. This concept is a fit for the investment risk evaluation problem, where it is the worst (potentially failed) companies that need to be identified. The approaches in the works of Paradi et al. (2004) and Liu and Chen (submitted for publication) both get perfect results for bankruptcy prediction through a classification procedure.

However, radial WPF–DEA models basically have an inconvenience that the slack values need to be calculated in an indirect way and by a more complicated procedure. To evaluate the worst efficiency directly together with the slack values in the worst-case scenario, a slack-based measure (SBM) of efficiency is taking into account. Therefore, the main purpose of this paper is to formulate a new model for incorporating slack-based measure into the WPF–DEA. Moreover, to meet the need of fully ranking instead of classifying all units in the worst-case scenario, this paper will propose full ranking technique in combination with WPF–DEA models.

The rest of this paper is organized as follows. In Section 2, a short introduction of the worst-practice frontier CCR (WPF–CCR) model is provided. In Section 3, a model formulation of the worst-practice frontier SBM (WPF–SBM) is presented. In Section 4, the hypo-efficiency evaluated by WPF–SBM and a solution approach of full ranking are proposed. In Section 5, an illustrative example is described and the results are discussed. Finally, Section 6 gives conclusions and future directions.

2. The worst-practice frontier CCR (WPF–CCR) model

In the real world, the companies who have more potential of going out of business are usually those of least competitiveness in comparison with others in a scenario which is unfavorable. The WPF–CCR model is formulated based on this concept in identifying the bad performers in the worst-case scenario, in contrast to the BPF–CCR which evaluates DMU_o in the most favorable scenario. While BPF–CCR establishes a best-practice frontier based on the best observed performance and evaluates the efficiency of each DMU relative to this frontier, WPF–CCR establishes a worst-practice frontier based on the worst observed performance, and the efficiency score of a DMU that does not lie on the frontier is evaluated relative to a linear combination of the worst efficient DMUs.

To illustrate the difference between the best-practice frontier and the worst-practice frontier we use an example of two inputs and one output data as shown in Table 1. All inputs are normalized to 1 for simplicity. The best-practice and worst-practice frontiers of the example are presented in Fig. 1.

In BPF–CCR, *C*, *D*, and *E* are evaluated as efficient DMUs which construct a best-practice frontier (the dotted line), and *A*, *B*, *F*, *G*, and *H* are less efficient in comparison with the efficient DMUs. In WPF–CCR, *B*, *C*, and *F* are evaluated as the worst efficient DMUs which construct a worst-practice frontier (the solid line) and *A*, *D*, *E*, *G*, and *H* are more efficient relative to the worst efficient DMUs.

To see how bad a company's performance could possibly be in the worst-case scenario, the objective is to minimize the measure of efficiency. Since the company's performance is evaluated in the worst-case scenario, it can be considered as the "worst efficiency". Therefore, in WPF–CCR, we call the units on the worst-practice frontier as the "worst efficient" DMUs and the units not on the frontier the "more efficient" DMUs. To form the worst-practice frontier, the constraints in WPF–CCR should construct a piece-wise concave hull. Therefore, we formulate WPF–CCR into the following fractional program to assess DMU_o

$$\begin{aligned} \min \quad & h_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ \text{s.t.} \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \geqslant 1, \quad j = 1, 2, \ldots, n \\ & u_r, v_i \geqslant 0; \quad r = 1, 2, \ldots, s; \; i = 1, 2, \ldots, m. \end{aligned} \tag{1}$$

where x_{ij} and y_{rj} are the ith input and rth output of DMU j ($j = 1, \ldots, n$), respectively. The constraints mean that the ratio scales of the weighted sum of the outputs to that of the inputs should exceed 1 for every DMU.

Table 1
Example data

Index	Notation	DMU							
		A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Input 1	x_1	4	7	8	4	1	4	6	3
Input 2	x_2	3	3	1	2	5	6	3	5
Output	y	1	1	1	1	1	1	1	1

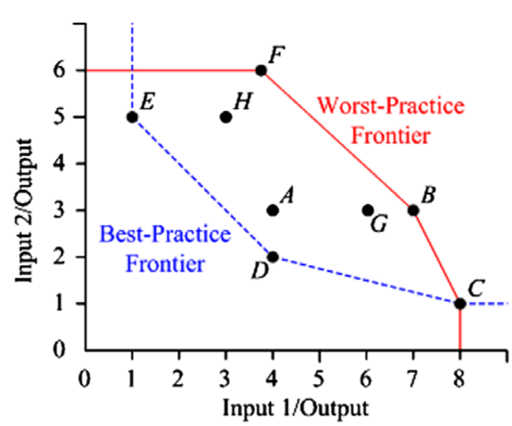

Fig. 1. Illustration of the worst-practice frontier.

The $(\mathbf{v}^*, \mathbf{u}^*)$ obtained as an optimal solution for (1) results in a set of most unfavorable weights for the DMU_o in the sense of minimizing the ratio scale. Each DMU is assigned a set of most unfavorable weights with values that may vary from one DMU to another. The fractional program (1) can be replaced by the following (multiplier form) linear program:

$$\begin{aligned} \min \quad & h_o = \sum_{r=1}^{s} u_r y_{ro} \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{io} = 1 \\ & -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} \geqslant 0, \quad j = 1,2,\ldots,n \\ & u_r, v_i \geqslant 0; \quad r = 1,2,\ldots,s; \; i = 1,2,\ldots,m. \end{aligned} \tag{2}$$

The dual envelopment form of (2) is expressed with a real variable θ_o and a set of nonnegative variables $\boldsymbol{\lambda} = (\lambda_j; j = 1,\ldots,n)$ as follows:

$$\begin{aligned} \max \quad & \theta_o \\ \text{s.t.} \quad & x_{io}\theta_o - \sum_{j=1}^{n} x_{ij}\lambda_j \leqslant 0, \quad i = 1,2,\ldots,m \\ & \sum_{j=1}^{n} y_{rj}\lambda_j \leqslant y_{ro}, \quad r = 1,2,\ldots,s \\ & \lambda_j \geqslant 0,\ldots, \quad j = 1,2,\ldots,n; \; \theta_o \text{ free.} \end{aligned} \tag{3}$$

The worst efficiency θ_o^* is not less than 1. The unit with higher efficiency score is considered more efficient. If an optimal solution obtained satisfies $\theta_o^* = 1$ and is zero-slack, then the DMU_o is WPF–CCR worst efficient. However, radial WPF–DEA models like WPF–CCR and WPF–BCC basically have an inconvenience that the slack values need to be calculated in an indirect way and by a more complicated procedure.

3. The worst-practice frontier SBM model

To evaluate the worst efficiency directly together with the slack values, a slack-based measure (SBM) of efficiency (Tone, 2001) is taken into account. According to the concept of worst-practice frontier, the WPF–SBM model can be developed based on the production possibility set of SBM. The production possibility set of WPF–SBM is defined as:

$$P = \left\{ (\mathbf{x}, \mathbf{y}) | \mathbf{x} \leqslant \sum_{j=1}^{n} \lambda_j \mathbf{x}_j, \; \mathbf{y} \geqslant \sum_{j=1}^{n} \lambda_j \mathbf{y}_j, \boldsymbol{\lambda} \geqslant \mathbf{0} \right\}. \tag{4}$$

We consider an expression to describe some DMU $(\mathbf{x}_o, \mathbf{y}_o)$ as

$$\mathbf{x}_o = \sum_{j=1}^{n} \lambda_j \mathbf{x}_j - \mathbf{s}^+, \quad \mathbf{y}_o = \sum_{j=1}^{n} \lambda_j \mathbf{y}_j + \mathbf{s}^-$$

with $\boldsymbol{\lambda} \geqslant \mathbf{0}$, $\mathbf{s}^+ \geqslant \mathbf{0}$, and $\mathbf{s}^- \geqslant \mathbf{0}$. The vectors $\mathbf{s}^+$ and $\mathbf{s}^-$ indicate the input and output deteriorations of this expression, and are also called slacks. From the conditions $\mathbf{y}_j \geqslant \mathbf{0}$ and $\boldsymbol{\lambda} \geqslant \mathbf{0}$, it holds $\mathbf{y}_o \geqslant \mathbf{s}^-$. Using $\mathbf{s}^+$ and $\mathbf{s}^-$, the following index ρ

$$\rho = \frac{1 + 1/m\sum_{i=1}^{m} s_i^+/x_{io}}{1 - 1/s\sum_{r=1}^{s} s_r^-/y_{ro}} \tag{5}$$

is defined in terms of the amount of slack. From $\mathbf{y}_o \geqslant \mathbf{s}^-$, we have $\rho \geqslant 1$. The WPF–SBM worst efficiency is obtained from the following fractional program

$$\begin{aligned}
\max \quad & \rho = \frac{1 + 1/m\sum_{i=1}^{m} s_i^+/x_{io}}{1 - 1/s\sum_{r=1}^{s} s_r^-/y_{ro}} \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} - s_i^+ = x_{io}, \quad i = 1 \ldots m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} + s_r^- = y_{ro}, \quad r = 1 \ldots s \\
& \lambda_j \geqslant 0, \quad s_i^+ \geqslant 0,\ s_r^- \geqslant 0, \quad \forall j, i, r.
\end{aligned} \tag{6}$$

Model (6) can be transformed into a linear program using the scale transformation in the similar way as the CCR model. Let us multiply a scalar variable $t(>0)$ to both the denominator and the numerator of (6). We adjust t so that the denominator becomes 1. Then this term is moved to constraints. The objective is to maximize the numerator. Thus we have

$$\begin{aligned}
\max \quad & \tau = t + \frac{1}{m}\sum_{i=1}^{m} ts_i^+ \Big/ x_{io} \\
\text{s.t.} \quad & 1 = t - \frac{1}{s}\sum_{r=1}^{s} ts_r^- \Big/ y_{ro} \\
& \sum_{j=1}^{n} t\lambda_j x_{ij} - ts_i^+ = tx_{io}, \quad i = 1 \ldots m \\
& \sum_{j=1}^{n} t\lambda_j y_{rj} + ts_r^- = ty_{ro}, \quad r = 1 \ldots s \\
& \lambda_j \geqslant 0, \quad s_i^+ \geqslant 0,\ s_r^- \geqslant 0, \quad \forall j, i, r; t > 0.
\end{aligned} \tag{7}$$

Model (7) still contains nonlinear terms. We can transform (7) into a linear program as follows. Let us define $S_i^+ = ts_i^+$, $S_r^- = ts_r^-$, and $\Lambda_j = t\lambda_j$. Then (7) becomes the following linear program in t, S_i^+, S_r^-, and Λ_j:

$$\begin{aligned}
\max \quad & \tau = t + \frac{1}{m}\sum_{i=1}^{m} S_i^+ \Big/ x_{io} \\
\text{s.t.} \quad & 1 = t - \frac{1}{s}\sum_{r=1}^{s} S_r^- \Big/ y_{ro} \\
& \sum_{j=1}^{n} \Lambda_j x_{ij} - S_i^+ = tx_{io}, \quad i = 1 \ldots m \\
& \sum_{j=1}^{n} \Lambda_j y_{rj} + S_r^- = ty_{ro}, \quad r = 1 \ldots s \\
& \Lambda_j \geqslant 0, \quad S_i^+ \geqslant 0,\ S_r^- \geqslant 0, \quad \forall j, i, r; t > 0.
\end{aligned} \tag{8}$$

Let an optimal solution of (8) be $(\tau^*, t^*, \mathbf{\Lambda}^*, \mathbf{S}^{+*}, \mathbf{S}^{-*})$. Then we have an optimal solution of the WPF–SBM model as defined by $\rho^* = \tau^*$, $\boldsymbol{\lambda}^* = \mathbf{\Lambda}^*/t^*$, $\mathbf{s}^{+*} = \mathbf{S}^{+*}/t^*$, $\mathbf{s}^{-*} = \mathbf{S}^{-*}/t^*$.

A DMU $(\mathbf{x}_o, \mathbf{y}_o)$ is WPF–SBM worst efficient if and only if $\rho^* = 1$, because $\rho^* = 1$ implies that all slacks are zero and the DMU is located on the worst-practice frontier. The slack-based worst efficiency score is units invariant.

4. Hypo-efficiency evaluated by WPF–SBM

In most models of BPF–DEA, the best performers have the "efficient status" denoted by unity, and usually plural DMUs have this efficient status. The WPF–SBM models would also result in plural DMUs which have the "worst efficient status". In the real world, however, there is only a small portion of the public companies filing for bankruptcy in an average year. Discriminating among these worst efficient DMUs becomes an interesting subject. We call this problem as the "hypo-efficiency" problem.

Hypo-efficiency can be considered as the efficiency worse than the worst efficiency. We discuss this hypo-efficiency issue based on the SuperSBM model (Tone, 2002). We develop the HypoSBM model as a method for ranking the worst performers which are identified by using WPF–SBM.

Similarly, we begin with defining a production possibility set $P\backslash(\mathbf{x}_o, \mathbf{y}_o)$ spanned by (X, Y) excluding $(\mathbf{x}_o, \mathbf{y}_o)$, i.e.

$$P \setminus (\mathbf{x}_o, \mathbf{y}_o) = \left\{ (\bar{\mathbf{x}}, \bar{\mathbf{y}}) | \bar{\mathbf{x}} \leqslant \sum_{j=1,\neq o}^{n} \lambda_j \mathbf{x}_j, \bar{\mathbf{y}} \geqslant \sum_{j=1,\neq o}^{n} \lambda_j \mathbf{y}_j, \bar{\mathbf{x}} \geqslant 0, \boldsymbol{\lambda} \geqslant 0 \right\}. \tag{9}$$

Further, we define a subset $\overline{P} \setminus (\mathbf{x}_o, \mathbf{y}_o)$ of $P\backslash(\mathbf{x}_o, \mathbf{y}_o)$ as

$$\overline{P} \setminus (\mathbf{x}_o, \mathbf{y}_o) = P \setminus (\mathbf{x}_o, \mathbf{y}_o) \cap \{\bar{\mathbf{x}} \leqslant \mathbf{x}_o, \bar{\mathbf{y}} \geqslant \mathbf{y}_o\}. \tag{10}$$

As a weighted l_1 distance from $(\mathbf{x}_o, \mathbf{y}_o)$ to $(\bar{\mathbf{x}}, \bar{\mathbf{y}}) \in \overline{P} \setminus (\mathbf{x}_o, \mathbf{y}_o)$, we employ the index as δ defined by

$$\delta = \frac{\frac{1}{m}\sum_{i=1}^{m} \frac{\bar{x}_i}{x_{io}}}{\frac{1}{s}\sum_{i=1}^{s} \frac{\bar{y}_r}{y_{ro}}}. \tag{11}$$

From (10), this distance is not greater than 1 and attains 1 if and only if $(\mathbf{x}, \mathbf{y}) \in \overline{P} \setminus (\mathbf{x}_o, \mathbf{y}_o)$, i.e. exclusion of the DMU $(\mathbf{x}_o, \mathbf{y}_o)$ has no effect on the original production possibility set P.

We define the hypo-efficiency of $(\mathbf{x}_o, \mathbf{y}_o)$ as the optimal objective function values δ^* of the following fractional program:

[HypoSBM]

$$\begin{aligned}
&\delta^* = \max\ \delta = \frac{\frac{1}{m}\sum_{i=1}^{m}\bar{x}_i/x_{io}}{\frac{1}{s}\sum_{r=1}^{s}\bar{y}_r/y_{ro}} \\
&\text{s.t. } \bar{x}_i \leqslant \sum_{j=1,\neq o}^{n} \lambda_j x_{ij}, \quad i = 1, 2, \ldots, m \\
&\qquad \bar{y}_r \geqslant \sum_{j=1,\neq o}^{n} \lambda_j y_{rj}, \quad r = 1, 2, \ldots, s \\
&\qquad \bar{x}_i \leqslant x_{io}, \bar{x}_i \geqslant 0, \quad i = 1, 2, \ldots, m \\
&\qquad \bar{y}_r \geqslant y_{ro}, \quad r = 1, 2, \ldots, s \\
&\qquad \lambda_j \geqslant 0, \quad j = 1, 2, \ldots, n.
\end{aligned} \tag{12}$$

The fractional program [HypoSBM] can be transformed into a linear program using the scale transformation in the similar way as the WPF–SBM model. Let us multiply a scalar variable t(>0) to both the denominator and the numerator of objective in (12). We adjust t so that the denominator becomes 1. Then this term is moved to constraints. The objective is to maximize the numerator. Thus, we have

[HypoSBMt]

$$
\begin{aligned}
&\tau^* = \max \ \tau = \frac{1}{m}\sum_{i=1}^{m} t\bar{x}_i/x_{io} \\
&\text{s.t. } 1 = \frac{1}{s}\sum_{r=1}^{s} t\bar{y}_r/y_{ro} \\
&\qquad \bar{x}_i \leqslant \sum_{j=1,\neq o}^{n} \lambda_j x_{ij}, \quad i = 1,2,\ldots,m \\
&\qquad \bar{y}_r \geqslant \sum_{j=1,\neq o}^{n} \lambda_j y_{rj}, \quad r = 1,2,\ldots,s \\
&\qquad \bar{x}_i \leqslant x_{io}, \bar{x}_i \geqslant 0, \quad i = 1,2,\ldots,m \\
&\qquad \bar{y}_r \geqslant y_{ro}, \quad r = 1,2,\ldots,s \\
&\qquad \lambda_j \geqslant 0, \quad j = 1,2,\ldots,n \\
&\qquad t > 0.
\end{aligned} \tag{13}
$$

We can transform the nonlinear term $t\bar{y}_r$ $(r = 1,\ldots,s)$ and $t\bar{x}_i$ $(i = 1,\ldots,m)$ by defining $\tilde{x}_i = t\bar{x}_i$, $\tilde{y}_r = t\bar{y}_r$ and $\Lambda_j = t\lambda_j$ $(j = 1,\ldots,n)$. Then [HypoSBMt] becomes the following linear program in t, $\tilde{x}_i$, $\tilde{y}_r$ and Λ_j:

[LP]

$$
\begin{aligned}
&\tau^* = \max \ \tau = \frac{1}{m}\sum_{i=1}^{m} \tilde{x}_i/x_{io} \\
&\text{s.t. } 1 = \frac{1}{s}\sum_{r=1}^{s} \tilde{y}_r/y_{ro} \\
&\qquad \tilde{x}_i \leqslant \sum_{j=1,\neq o}^{n} \Lambda_j x_{ij}, \quad i = 1,2,\ldots,m \\
&\qquad \tilde{y}_r \geqslant \sum_{j=1,\neq o}^{n} \Lambda_j y_{rj}, \quad r = 1,2,\ldots,s \\
&\qquad \tilde{x}_i \leqslant tx_{io}, \tilde{x}_i \geqslant 0, \quad i = 1,2,\ldots,m \\
&\qquad \tilde{y}_r \geqslant ty_{ro}, \quad r = 1,2,\ldots,s \\
&\qquad \Lambda_j \geqslant 0, \quad j = 1,2,\ldots,n \\
&\qquad t > 0.
\end{aligned} \tag{14}
$$

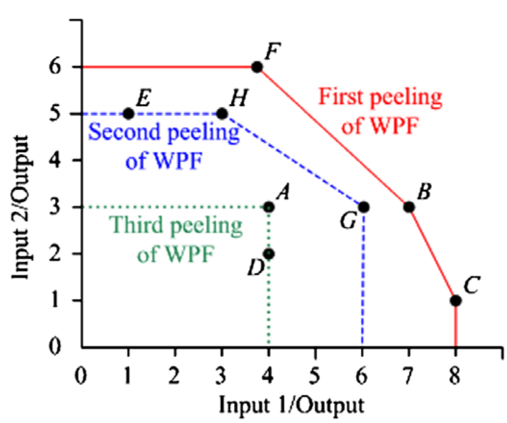

Fig. 2. Illustration of three layers of the worst-practice frontier.

Let an optimal solution of [LP] be $(\tau^*, \tilde{\mathbf{x}}^*, \tilde{\mathbf{y}}^*, \mathbf{\Lambda}^*, t^*)$. Then we have an optimal solution of [HypoSBM] as expressed by $\delta^* = \tau^*, \bar{\mathbf{x}}^* = \tilde{\mathbf{x}}^*/t^*, \bar{\mathbf{y}}^* = \tilde{\mathbf{y}}^*/t^*, \boldsymbol{\lambda}^* = \mathbf{\Lambda}^*/t^*$.

Furthermore, we can partition the whole set of DMUs into several layers of the worst-practice frontiers using the peeling technique (Thanassoulis, 1999; Paradi et al., 2004). Using this approach, the DMUs on the first worst-practice frontier resulting from the WPF–SBM are removed, after which the WPF–SBM model is run again resulting in a new set of frontier units, which are then removed before the model is run a third time and so on. Three layers of the worst-practice frontier for the example data in Table 1 are illustrated in Fig. 2.

When DMUs on a specific frontier obtained from WPF–SBM are viewed as having "equal worst performance", the HypoSBM model allows us to differentiate the equal worst performance based upon the same specific evaluation context. Consequently, the combined use of WPF–SBM, hypo-efficiency measures, and peeling technique can comprehensively characterize the worst performance of DMUs. This approach is a fit for the investment risk evaluation problem faced by financial institutions or individual investors in the business of investing in high-tech or capital-intensive industry, especially when there is a need of full ranking in the worst-case scenario.

5. Illustrative example

In this section, we illustrate the methods we have developed using an example of firms in the banking industry. The banking industry has the same property of being capital intensive as the high-tech industry. The cost of type I error is of course the greatest concern. This example is a set of real data of 24 commercial banks in the work of Kao and Liu (2004). The data on 24 commercial banks in Taiwan were acquired from their financial statements of Year 2000 as shown in Table 2. Three inputs are considered in evaluating a bank's performance: total deposits, interest expenses, and noninterest expenses. There are also three output factors: total loans, interest income, and noninterest income.

Of the 24 commercial banks, banks 8 and 23 were later taken over by government, which is considered as bankrupts. For the purpose of this paper we use the proposed approach to investigate how model combi-

Table 2
Illustrative data set, in million Taiwan dollars

Bank	Total deposits	Interest expenses	Noninterest expenses	Total loans	Interest income	Noninterest income
1	824,107	42,494	12,473	741,433	62,898	7240
2	980,038	46,845	16,936	806,429	68,820	13,292
3	938,205	42,377	13,645	823,782	61,386	12,505
4	480,609	31,277	6563	447,144	47,438	6057
5	246,441	8253	2953	181,108	12,222	1731
6	268,353	8900	1218	214,366	12,015	3044
7	113,919	5677	1472	85,624	8395	323
8	80,816	4200	2578	51,235	5321	267
9	401,634	28,829	5605	337,616	36,626	5647
10	531,555	23,779	3904	426,360	40,224	3454
11	177,809	8828	3792	151,727	12,109	1788
12	191,038	8717	1628	163,439	12,397	760
13	452,867	21,992	2821	373,837	31,029	5133
14	751,438	33,965	5287	633,021	48,663	3228
15	106,054	5750	5160	82,183	7938	619
16	132,952	7354	3208	130,663	9783	1816
17	159,400	8307	6263	131,733	12,846	810
18	156,493	8483	2931	135,488	12,651	2531
19	199,135	9746	693	155,295	14,017	2168
20	227,120	9962	2001	159,535	13,266	5092
21	137,386	6803	4034	103,615	9220	1643
22	224,620	11,170	4871	185,694	20,969	3268
23	159,180	9479	14,765	100,249	11,699	1138
24	164,146	8185	1696	146,802	11,778	927

Source: Kao and Liu (2004).

Table 3
The efficiency scores of WPF–SBM and HypoSBM in three layers of peeling

Bank	Rank	Scores in the 1st peeling		Scores in the 2nd peeling		Scores in the 3rd peeling	
		WPF–SBM	HypoSBM	WPF–SBM	HypoSBM	WPF–SBM	HypoSBM
1	1	3.344		2.101		1.096	
2	4	2.988		1.609		**1.000**	**0.995**
3	5	2.893		1.313		**1.000**	**0.987**
4	10	4.378		2.627		**1.000**	**0.957**
5	20	**1.000**	**0.971**				
6	21	**1.000**	**0.966**				
7	22	**1.000**	**0.950**				
8	**23**	**1.000**	**0.899**				
9	18	4.108		**1.000**	**0.940**		
10	9	4.938		3.097		**1.000**	**0.976**
11	11	2.254		1.182		**1.000**	**0.912**
12	16	2.258		**1.000**	**0.968**		
13	2	5.917		2.544		**1.000**	**0.999**
14	13	2.689		**1.000**	**0.997**		
15	19	1.278		**1.000**	**0.881**		
16	6	2.738		1.170		**1.000**	**0.985**
17	15	1.368		**1.000**	**0.979**		
18	3	3.392		2.107		**1.000**	**0.998**
19	8	10.395		4.523		**1.000**	**0.979**
20	17	2.927		**1.000**	**0.958**		
21	14	1.818		**1.000**	**0.997**		
22	7	3.173		2.025		**1.000**	**0.984**
23	**24**	**1.000**	**0.755**				
24	12	3.279		1.960		**1.000**	**0.880**

nation can effectively identify the worst performers as potentially failed banks. The results are represented in Table 3.

Bank 5, 6, 7, 8, and 23 are first identified as the worst efficient units using WPF–SBM model as shown in the third column of Table 3, which are candidates for further identification using HypoSBM model. The scores with 1.000 are typed in boldfaced. The results of HypoSBM model are shown in the fourth column of Table 3. Of the worst efficient units, bank 23 is identified as the worst one. Since the cost of type I error is most concerned, it is reasonable to classify a small (but not too small) group of units as potentially failed companies. We suggest the portion of the small group should be greater than the ratio of failed to healthy companies in the industry in an average year. Since we can only find the ratio of failed to healthy companies for small and medium businesses in Taiwan, which is about 7%, we take this ratio as a criterion. Therefore, bank 8 (the second worst one) should be included. The identified potentially failed banks are exactly the bankrupt firms.

Furthermore, using the peeling technique twice we can get a full ranking in the worst-case scenario. The results of full ranking are shown in the second column of Table 3.

6. Conclusions and future directions

We presented the model formulations of WPF–SBM and HypoSBM based on the concept of worst-practice frontier. The application of HypoSBM model not only discriminates between those worst performers obtained from the WPF–SBM model, but also fits the real situation that there is only a small portion of companies filing for bankruptcy in an average year. The ratio of failed to healthy companies should depend on the region or country where the industry is located and on the economic cycle (depression or boom). Then we proposed an approach of full ranking for the worst efficiencies in the worst-case scenario. The results of the numerical illustration on investment risk evaluation validate the WPF–SBM and HypoSBM models. The best combination of layered WPF–SBM and HypoSBM models yields an impressive bankruptcy prediction.

We would like to mention that there does not exist a perfect model or approach for performance evaluation, risk evaluation or bankruptcy prediction. Our study is particularly interested in evaluating corporate performance in the worst-case scenario through DEA models. The proposed approach could be limited by the insufficient information about the objects under study. Choosing a set of adequate performance indices would be one of the critical issues. The proposed approach provides a new wide avenue for future researches. For the cases with imprecise data, new worst-practice DEA models are expected. Many existing best-practice DEA models that are employed to deal with problems could be revised into worst-practice DEA models to examine the problem from a different point of view. The model can be used for other risk-taking industries such as insurance industry. Other appropriate formulations of extended WPF–DEA models, along with other approaches and applications, are also important to illustrate its practicality.

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