Transmit Equal Gain Precoding in Rayleigh Fading Channels

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***Abstract*—Precoding with limited feedback information can achieve satisfactory performance while the amount of feedback information is kept small. In this paper, we analyze the theoretical performance of equal gain precoder and find that its performance is at most 1.049 dB worse than the optimal precoder no matter how the number of transmit antennas increases. Moreover, we analyze the performance degradation of the equal gain precoder due to scalar quantization theoretically. The result shows that 2–3 bits per transmit antenna (excluding the first antenna) can achieve 0.5–0.25-dB performance gap compared with the same scheme without quantization. Furthermore, we found that the equal gain precoder in general can achieve comparable performance with the Grassmannian precoder in the same moderate feedback bits. Simulation results are provided to corroborate the theoretical results.**

***Index Terms*—Beamforming, equal gain precoding, limited feedback, MIMO, precoding, scalar quantization.**

I. Introduction

MIMO techniques are widely used in current wireless communication standards such as IEEE 802.11n and IEEE 802.16. Among the MIMO skills, precoding/beamforming can provide full diversity order and additional precoding gain. Such nice properties can greatly improve system performance.

If complete channel formation is known to the transmitter, we can jointly design precoders and decoders by optimizing several parameters such as MMSE, maximizing information rate, or maximizing SNR (see [2], [11], and [13]). Although such precoding schemes can achieve optimal performance using different design criterion, the hardware complexity and the amount of feedback information are high.

To overcome the drawbacks of the optimal precoders, research has been directed to the precoding schemes with limited feedback recently. The precoder design with the channel mean and the covariance matrix available at the transmitter was studied in [8] and [15] from the view point of channel capacity. In [6], the precoding schemes that achieve the minimum outage probability was studied. Examples of codebook construction were also given. In [3], equal gain precoders with different combining methods were shown to achieve full diversity order. In [4], Grassmannian precoding was proposed. The Grassmannian precoder has been shown to have good performance in practical communication systems. The Grassmannian precoding was extended to space–time block code by the same authors in [5]. In [7], the authors analyzed the capacity loss of equal gain precoder due to both vector and scalar quantization. An optimal bit allocation for equal gain precoder with scalar quantization was proposed in [19].

In this paper, we analyze the theoretical performance of the equal gain precoder in multiple-input single-output (MISO) channel environments and found several interesting results as follows.

First, the bit error probability (BEP) performance gap between the equal gain precoder and the optimal precoder varies from 0.5 to 1.049 dB as the number of transmit antennas grows from 2 to infinity. This is interesting since the performance of equal gain precoder is at most 1.049 dB worse than the optimal precoder, no matter how the number of transmit antennas increases.

Second, we analyze the performance loss due to scalar quantization for the equal gain precoder theoretically. We found that in the interested range of the number of transmit antennas, e.g., four antennas with RF (radio frequency) for Wi-Fi and Wi-MAX in current standards, performance loss is within 0.5 dB by using six total feedback bits in the scalar quantized equal gain precoder.

Finally, we found from simulation results that the scalar quantized equal gain precoder can achieve comparable performance with the Grassmannian precoder in the same moderate number of feedback bits. This result shows several advantages of the equal gain precoder. First, the codeword determination of the equal gain precoder is simple, since in MISO channel environments the optimal solution for the equal gain precoder is actually the channel vector. Hence, there is no need to perform exhaustive search to determine the codeword as Grassmannian does. Second, for the equal gain precoder, when the feedback bits are less than two per transmit antenna, there is no need to perform multiplications in the transmitter side since the codeword elements in this case are ± 1 or $\pm j$. Please note that for small size Grassmannian precoder, operation without multiplications is also possible. Third, the equal gain precoder can be easily extended to arbitrary transmit antenna number since it does not need to construct the codebook in advance.

Notations: $\mathbb{E}\{x\}$ is the expectation of x. $\mathbf{A}^*$ and $\mathbf{A}^t$ are the conjugate and transpose of $\mathbf{A}$, respectively. $\mathbf{A}^\dagger$ is the conjugate-transpose of $\mathbf{A}$. $\Re\{x\}$ denotes the real part of variable x. σ_x^2 is the variance of x.

II. System Model and Performance Analysis

Let the number of transmit antennas be N_t. First, one symbol x is sent to N_t branches and each symbol in different branch is multiplied by a different phase rotation, i.e., $e^{j\theta_i}/\sqrt{N_t}$. After the precoding, the symbol vector to be transmitted is $\mathbf{s} = (s_1\ s_2\ \cdots s_{N_t})^t = 1/\sqrt{N_t}\mathbf{p}x$, where $\mathbf{p}$ is a $N_t \times 1$ vector and its $i\text{th}$ element is $e^{j\theta_i}$. Then, $\mathbf{s}$ is transmitted to the channel. At the receive, the received symbol r is $r = \mathbf{h}^t\mathbf{s} + n$, where $\mathbf{h}$ is a $N_t \times 1$ channel vector and its $i\text{th}$ coefficient is h_i, and n is a noise scalar. To achieve the best performance, we use MRC [3] in the receiver and this leads to

$$z = \mathbf{p}^\dagger\mathbf{h}^* r = \frac{1}{\sqrt{N_t}}\gamma x + \mathbf{p}^\dagger\mathbf{h}^* n \quad (1)$$

where $\gamma = \mathbf{p}^\dagger\mathbf{h}^*\mathbf{h}^t\mathbf{p}$ is a gain effect (including diversity gain and precoding gain) due to the precoding.

Now, we analyze the average SNR performance and it terms out that this average SNR performance is highly correlated to the bit error probability performance (see [1] and [4]). From (1), for a given channel realization (channel is deterministic), the instantaneous SNR of the equal gain precoder is

$$\rho_e = \frac{\gamma}{N_t}\frac{\sigma_x^2}{\sigma_n^2}. \quad (2)$$

From (2), for a given channel realization and σ_x^2/σ_n^2, the instantaneous SNR is determined by γ. Let us look at γ more detailed. From (1) and due to phase mismatch, γ can be upper bounded by

$$\gamma = \sum_{i=1}^{N_t}|h_i|^2 + \sum_{i,j\neq i}^{N_t} h_i^* h_j e^{j(\theta_j-\theta_i)} \leq \sum_{i=1}^{N_t}|h_i|^2 + \sum_{i=1,j\neq i}^{N_t}|h_i^* h_j|. \quad (3)$$

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Let the phase of h_i be ϕ_i. The equality of (3) holds when $(\theta_j - \theta_i) = -(\phi_j - \phi_i)$. It is intuitive to choose

$$\theta_i = -\phi_i, \quad 1 \leq i \leq N_t. \tag{4}$$

Using the solution in (4), all the transmit antennas require to multiply individual phases. However, we notice that multiplying an extra arbitrary phase rotation in both the transmitter and the receiver sides simultaneously does not change γ. In this case, the equality also holds when

$$\theta_i = 0 \text{ if } i = 1; \quad \theta_i = -(\phi_i - \phi_1) \text{ otherwise.} \tag{5}$$

From (5), we do not need to perform precoding for the first transmit antenna and thus the feedback information can be reduced. The above results were also shown in [3] and [19].

From (2) and (3), for a specified σ_x^2/σ_n^2, the average SNR can be upper bounded by

$$\mathbb{E}_{\mathbf{h}}\left\{\rho_e\right\} \leq \frac{\sigma_x^2}{\sigma_n^2}\left(\mathbb{E}_{\mathbf{h}}\left\{|h_i|^2\right\} + (N_t - 1)\mathbb{E}_{\mathbf{h}}\left\{|h_i^* h_j|\right\}\right). \tag{6}$$

Without losing the generality, let us assume that both the real part and the imaginary part of h_i have unit variance. In this case, $\mathbb{E}_{\mathbf{h}}\left\{|h_i|^2\right\} = \sigma_h^2 = 2$. Next, let us see how to obtain $\mathbb{E}_{\mathbf{h}}\left\{|h_i^* h_j|\right\}$.

Lemma 1: Assume that h_i is complex Gaussian with zero mean and unit variance in both the real part and the imaginary part. The probability density function (PDF) of $|h_i^* h_j|$ is given by

$$f_{\mathbf{h}}(x) = \frac{x}{2}\left(K_o(x) + K_2(x)\right) - K_1(x) \tag{7}$$

where $K_\nu(x)$ is the modified Bessel function of the second kind.

Proof: Following the conditions for h_i, the cumulative distribution function (CDF) of $|h_i^* h_j|$ is $F_{\mathbf{h}}(x) = 1 - xK_1(x)$ (see [16]). From [17], we have $\partial K_\nu(x)/\partial x = -1/2\left(K_{\nu-1}(x) + K_{\nu+1}(x)\right)$. Thus, using the fact that $f_{\mathbf{h}}(x) = \partial F_{\mathbf{h}}(x)/\partial x$, we can obtain (7). ■

Lemma 2: Assume that h_i is complex Gaussian with zero mean and unit variance in both the real part and the imaginary part. The mean of the random variable $|h_i^* h_j|$ can be calculated as

$$\mathbb{E}_{\mathbf{h}}\left\{|h_i^* h_j|\right\} = 1.5708. \tag{8}$$

Proof: From Lemma 1, we have

$$\begin{aligned}\mathbb{E}_{\mathbf{h}}\left\{|h_i^* h_j|\right\} &= \int_{-\infty}^{\infty} x f_{\mathbf{h}}(x)dx \\ &= \int_0^{\infty} \frac{x^2}{2}K_0(x)dx + \int_0^{\infty} \frac{x^2}{2}K_2(x)dx \\ &\quad - \int_0^{\infty} xK_1(x)dx \end{aligned} \tag{9}$$

where we have used the fact that $K_\nu(x) = 0$, for $x < 0$. According to [18], we have $\int_0^\infty x^{\alpha-1}K_\nu(x)dx = 2^{\alpha-2}\Gamma\left(\alpha - \nu/2\right)\Gamma\left(\alpha + \nu/2\right)$, where $\Gamma(x)$ is the gamma function. Hence, from (9), we can have (8). ■

Please note that without using the PDF of $|h_i^* h_j|$, we can still derive $\mathbb{E}_{\mathbf{h}}\left\{|h_i^* h_j|\right\}$ as follows: Since h_i and h_j are independent, we have $\mathbb{E}_{\mathbf{h}}\left\{|h_i^* h_j|\right\} = \mathbb{E}_{\mathbf{h}}\left\{|h_i|\right\}\mathbb{E}_{\mathbf{h}}\left\{|h_j|\right\} = \left(\mathbb{E}_{\mathbf{h}}\left\{|h_i|\right\}\right)^2 = \pi/2$, where we have used the property that the expectation of the Rayleigh random variable is $\sqrt{\pi/2}$[14, pp. 279–280].

Theorem 1: Let the channel coefficient h_i be complex Gaussian with zero mean. Without quantization, the SNR gap of the optimal precoder and the equal gain precoder in a MISO channel is given by

$$\frac{\mathbb{E}_{\mathbf{h}}\left\{\rho_o\right\}}{\mathbb{E}_{\mathbf{h}}\left\{\rho_e\right\}} = \frac{N_t}{0.7854N_t + 0.2416}. \tag{10}$$

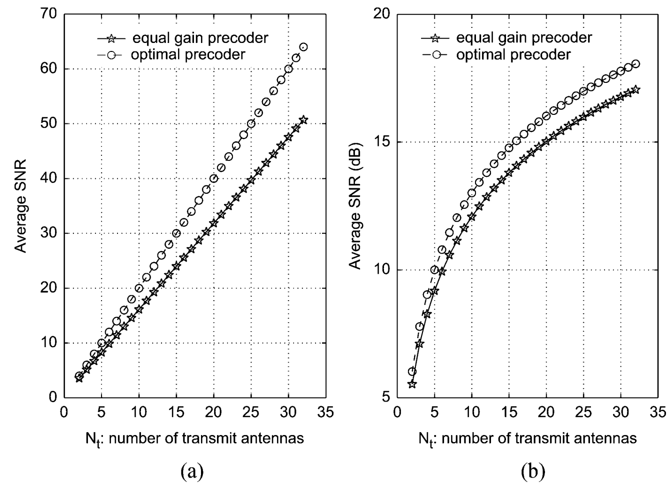

Fig. 1. Average SNR of the optimal and the equal gain precoders, where (a) is without taking dB and (b) is taking dB.

Proof: In the MISO case, the optimal precoder is actually $\mathbf{h}^\dagger$. Its average SNR can be shown to be

$$\mathbb{E}_{\mathbf{h}}\left\{\rho_o\right\} = \frac{\sigma_x^2}{\sigma_n^2}\mathbb{E}_{\mathbf{h}}\left\{\sum_{i=1}^{N_t}|h_i|^2\right\} = N_t\frac{\sigma_x^2}{\sigma_n^2}\mathbb{E}_{\mathbf{h}}\left\{|h_i|^2\right\}. \tag{11}$$

To have a fair comparison, we also assume that the channel coefficients in the optimal precoder have unit variance in both the real and the imaginary parts. Thus, $\mathbb{E}_{\mathbf{h}}\left\{|h_i|^2\right\} = 2$. From (11), we have $\mathbb{E}_{\mathbf{h}}\left\{\rho_o\right\} = \sigma_x^2/\sigma_n^2 2N_t$. From (6) and (8), for the equal gain precoder without quantization, we have

$$\mathbb{E}_{\mathbf{h}}\left\{\rho_e\right\} = \frac{\sigma_x^2}{\sigma_n^2}(2 + (N_t - 1)1.5708). \tag{12}$$

Hence, we can obtain the result in (10). It is worth to emphasize that since this is a fair comparison for the two precoders, there is no need to constrain the variance of h_i in Theorem 1. ■

Please note that $\mathbb{E}_{\mathbf{h}}\left\{\rho_e\right\}$ can also be directly derived from [19, Eq. (29)], without obtaining the PDF of $|h_i^* h_j|$. From Theorem 1, when $N_t \gg 1$, the ratio approximates $1/0.7854 = 1.049$ (dB), which is a constant performance gap. This is an interesting result because it means that the performance loss due to the use of phase alone is at most around 1 dB, despite the increase of the transmit antennas.

1) Example 2: Constant Performance Gap Between the Optimal and the Equal Gain Precoders: Let h_i be complex Gaussian with unit variance in both the real and the imaginary parts. The average SNR of both the optimal and the equal gain precoders as a function of the number of transmit antennas are shown in Fig. 1. We observe that when $N_t = 4$, the two precoders have performance gap around 0.75 dB and approximate to 1.049 dB when $N_t > 8$. These results show that there may be no need to use complicated precoding schemes such as the optimal precoder if we are capable to sacrifice around 1-dB performance. Take IEEE 802.11n and IEEE 802.16e-2005 for instance, the antenna number (with RF) is at most four in these standards. Under such situations, the performance loss is at most 0.75 dB.

III. Scalar Quantization

Let us consider the scalar quantization effect for the equal gain precoder. Since we assume the channel coefficients are complex Gaussian, the phase is uniformly distributed in $[-\pi \ \pi]$. In this case, let us use equal space quantization and quantize the phase to the closet available value. For instance, if the bit number, b, to represent the phase per transmit antenna is 2, the available values can be 0, $\pi/2$, π or

$-\pi/2$. Please note that with scalar quantization, the total required bits is $b(N_t-1)$. For vector quantization, it does not have such limitation and it considers the optimal solution for a given bit budget, e.g., [5] and [7].

Now consider how the quantization effect will deteriorate the average SNR. Let $\hat{\theta}_i$ be the quantized phase. From (3), the average SNR due to scalar quantization is given by

$$\mathbb{E}_{\mathbf{h}}\left\{\widetilde{\rho}_e\right\}=\frac{\sigma_x^2}{\sigma_n^2}\left(\mathbb{E}_{\mathbf{h}}\left\{|h_i|^2\right\}+c_1\mathbb{E}_{\mathbf{h}}\left\{\Re\left\{h_1^*h_je^{j\hat{\theta}_j}\right\}\right\}\right.$$
$$\left.+c_2\mathbb{E}_{\mathbf{h}}\left\{\Re\left\{h_i^*h_je^{j(\hat{\theta}_j-\hat{\theta}_i)}\right\}\right\}\right) \quad (13)$$

where $c_1=2(N_t-1)/N_t$ and $c_2=(N_t-1)(N_t-2)/N_t$. In addition, the reason that we separate the terms for $i=1$ and $i\neq 1$ is because $\hat{\theta}_1=0$ from (5). Define the quantization error of the phase as $\epsilon_i=\hat{\theta}_i-\theta_i$. Since $\theta_i=-(\phi_i-\phi_1)$ according to (5), we have $\hat{\theta}_i=-(\phi_i-\phi_1)+\epsilon_i$. Assuming that the quantization error of the phase is independent of channel (see [9]) and using the fact that $h_i^*h_je^{-j(\phi_j-\phi_i)}$ is real, we can rewritten (13) as

$$\mathbb{E}_{\mathbf{h}}\left\{\widetilde{\rho}_e\right\}=\frac{\sigma_x^2}{\sigma_n^2}\left(\mathbb{E}_{\mathbf{h}}\left\{|h_i|^2\right\}+c_1\mathbb{E}_{\mathbf{h}}\left\{|h_1^*h_j|\right\}\mathbb{E}_{\mathbf{h}}\left\{\cos(\epsilon_j)\right\}\right.$$
$$\left.+c_2\mathbb{E}_{\mathbf{h}}\left\{|h_i^*h_j|\right\}\mathbb{E}_{\mathbf{h}}\left\{\cos(\epsilon_j-\epsilon_i)\right\}\right). \quad (14)$$

Lemma 3: Assume that the phase is uniformly distributed in $[-\pi\ \pi]$. Let the number of bits to represent the phase per transmit antenna (excluding the first antenna) be b. Using equal space quantization, we have the following result

$$\mathbb{E}_{\mathbf{h}}\left\{\cos(\epsilon_j-\epsilon_i)\right\}=\frac{2^{2b}}{2\pi^2}\left(1-\cos\left(\frac{2\pi}{2^b}\right)\right). \quad (15)$$

Proof: Let us use ϵ_i to denote the phase error of the ith transmit antenna. According to [12], if the phase is uniformly distributed in $[-\pi\ \pi]$, the phase error ϵ_i is uniformly distributed in $\left[-\pi/2^b\ \pi/2^b\right]$. Now let us find the distribution of $\epsilon_i-\epsilon_j$. From [10], the distribution of $-\epsilon_j$ is still uniformly distributed in $\left[-\pi/2^b\ \pi/2^b\right]$. It is known that the PDF of the random variable $z=x+y$, where x and y are independent, can be obtained by convoluting $f_{\mathbf{x}}(x)$ with $f_{\mathbf{y}}(y)$[10]. Thus, the distribution of $\epsilon_i-\epsilon_j=\epsilon_i+(-\epsilon_j)$ is the linear convolution of the two identical PDFs with uniformly distribution in $\left[-\pi/2^b\ \pi/2^b\right]$. Hence, letting $z=\epsilon_i-\epsilon_j$ the PDF of z can be shown to be

$$f_{\mathbf{z}}(z)=\begin{cases}\frac{2^{2b}}{4\pi^2}z+\frac{2^b}{2\pi}, & \frac{-2\pi}{2^b}\le z\le 0\\ -\frac{2^{2b}}{4\pi^2}z+\frac{2^b}{2\pi}, & 0<z\le\frac{2\pi}{2^b}\\ 0, & \text{otherwise.}\end{cases} \quad (16)$$

From (16), we know that $f_{\mathbf{z}}(z)\cos(z)$ is an even function. According to [10], we have the property: $\mathbb{E}\left\{g(x)\right\}=\int_{-\infty}^{\infty}g(x)f_{\mathbf{x}}(x)dx$, where $g(x)$ is function of x. Thus, we have

$$\mathbb{E}_{\mathbf{h}}\left\{\cos(\epsilon_j-\epsilon_i)\right\}=\int_{-2^b/2\pi}^{2^b/2\pi}f_{\mathbf{z}}(z)\cos(z)dz$$
$$=2\int_0^{2\pi/2^b}\left(-\frac{2^{2b}}{4\pi^2}z+\frac{2^b}{2\pi}\right)\cos(z)dz. \quad (17)$$

Using integration by part: $\int z\cos(z)dz=z\sin(z)+\cos(z)+C$. Thus, we can rewrite (17) as

$$\mathbb{E}_{\mathbf{h}}\left\{\cos(\epsilon_j-\epsilon_i)\right\}$$
$$=2\left(-\frac{2^{2b}}{4\pi^2}(z\sin(z)+\cos(z))\Big|_0^{2\pi/2^b}+\frac{2^b}{2\pi}\sin(z)\Big|_0^{2\pi/2^b}\right). \quad (18)$$

Manipulating (18), we can obtain the result in (15). ■

Similarly, the expectation value of $\cos(\epsilon_i)$ can be calculated as

$$\mathbb{E}_{\mathbf{h}}\left\{\cos(\epsilon_i)\right\}=\int_{-\pi/2^b}^{\pi/2^b}\cos(\epsilon)\frac{2^b}{2\pi}d\epsilon=\frac{2^b}{\pi}\sin\left(\frac{\pi}{2^b}\right). \quad (19)$$

Assume h_i is complex Gaussian with zero mean and unit variance in the real and imaginary parts. Using (14), (19), and Lemma 3, we have

$$\mathbb{E}_{\mathbf{h}}\left\{\widetilde{\rho}_e\right\}=\frac{2\sigma_x^2}{\sigma_n^2}\left(1+0.7854\frac{(N_t-1)}{N_t}\frac{2^b}{\pi}\right.$$
$$\left.\left(2\sin\left(\frac{\pi}{2^b}\right)+\frac{2^b}{2\pi}(N_t-2)\left(1-\cos\left(\frac{2\pi}{2^b}\right)\right)\right)\right). \quad (20)$$

Moreover, from Theorem 1, we know $\mathbb{E}_{\mathbf{h}}\left\{\rho_e\right\}$. Hence, we have the following theorem.

Theorem 2: For the equal gain precoder (with solution θ_i given in (5)) in a MISO channel with complex Gaussian distribution, the SNR degradation due to scalar quantization is given by (21), shown at the bottom of the page.

The above result is obtained by setting θ_i according to (5), i.e., $\theta_1=0$ and there is no information feedback for θ_1. It is interesting to ask: When we feed back bits for θ_1, will the performance due to scalar quantization improve? To answer this question, let θ_i be set according to (4). From (3), we have

$$\mathbb{E}_{\mathbf{h}}\left\{\widehat{\rho}_e\right\}=\frac{\sigma_x^2}{\sigma_n^2}\left(\mathbb{E}_{\mathbf{h}}\left\{|h_i|^2\right\}+(N_t-1)\right.$$
$$\left.\times\mathbb{E}_{\mathbf{h}}\left\{\Re\left\{h_i^*h_je^{-j(\phi_j-\phi_i)}\right\}\right\}\mathbb{E}_{\mathbf{h}}\left\{e^{j(\epsilon_j-\epsilon_i)}\right\}\right)$$
$$=\frac{\sigma_x^2}{\sigma_n^2}\left(\mathbb{E}_{\mathbf{h}}\left\{|h_i|^2\right\}+(N_t-1)\right.$$
$$\left.\times\mathbb{E}_{\mathbf{h}}\left\{|h_i^*h_j|\right\}\mathbb{E}_{\mathbf{h}}\left\{\cos(\epsilon_j-\epsilon_i)\right\}\right). \quad (22)$$

Using (14), (22), and Lemma 3, we can calculate $\mathbb{E}\left\{\widehat{\rho}_e\right\}$. Hence, we have the following corollary.

Corollary: For the equal gain precoder (with solution θ_i given in (4)) in a MISO channel with complex Gaussian random distribution, the SNR degradation due to equal scalar quantization is given by

$$\frac{\mathbb{E}\left\{\rho_e\right\}}{\mathbb{E}\left\{\widehat{\rho}_e\right\}}=\frac{2+1.5708(N_t-1)}{2+1.5708(N_t-1)\left(1-\cos(\frac{2\pi}{2^b})\right)\frac{2^{2b}}{2\pi^2}}. \quad (23)$$

1) Example 3: Scalar Quantization for Equal Gain Precoder (Theoretical Result): Let us see the quantization effect for $\theta_1=0$ first. Using (21), we plot the performance loss due to quantization in Fig. 2. We see that for $N_t=2$, 3 and 4, the performance loss for $b=2$ is less than 0.5 dB. Moreover, for $N_t<16$, the performance loss for $b=3$

$$\frac{\mathbb{E}_{\mathbf{h}}\left\{\rho_e\right\}}{\mathbb{E}_{\mathbf{h}}\left\{\widetilde{\rho}_e\right\}}=\frac{0.7854N_t+0.2146}{1+0.7854\frac{(N_t-1)}{N_t}\frac{2^b}{\pi}\left(2\sin(\frac{\pi}{2^b})+\frac{2^b}{2\pi}(N_t-2)\left(1-\cos(\frac{2\pi}{2^b})\right)\right)}. \quad (21)$$

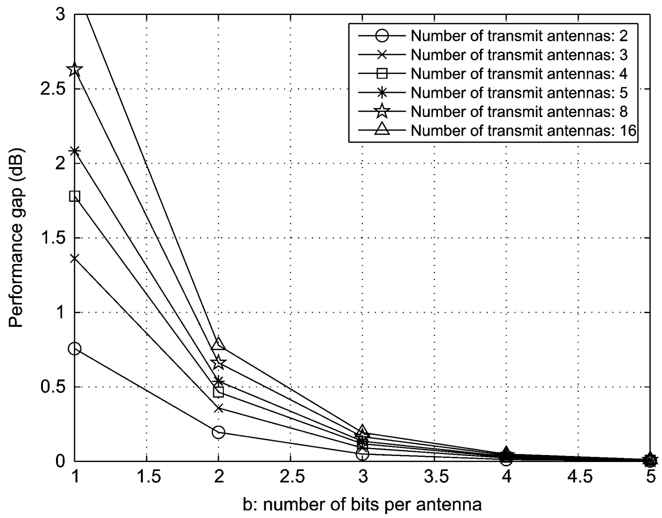

Fig. 2. The gap of average SNR between the equal gain precoders with and without quantization.

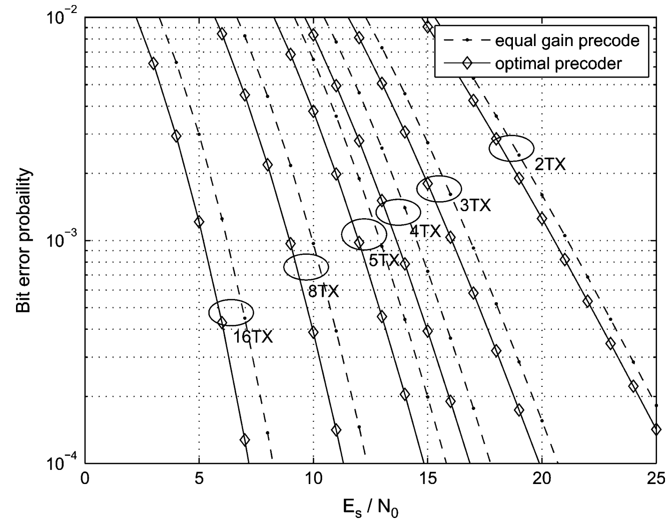

Fig. 4. Performance comparison of the optimal precoder and the equal gain precoder without quantization.

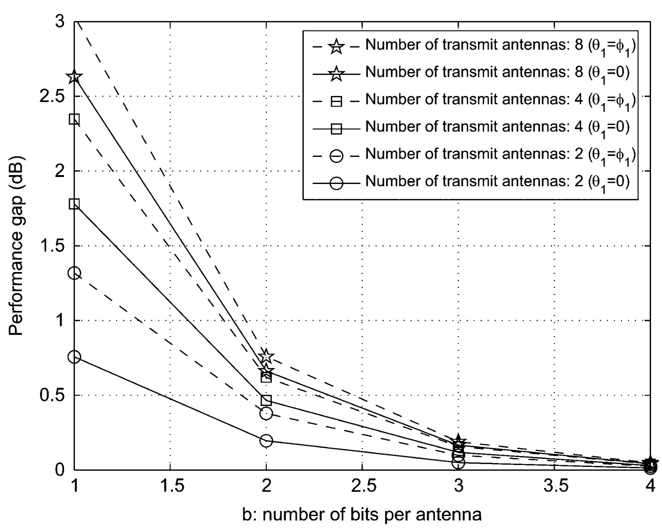

Fig. 3. Comparison of SNR degradation due to quantization for $\theta_1 = 0$ and $\theta_1 = \phi_1$.

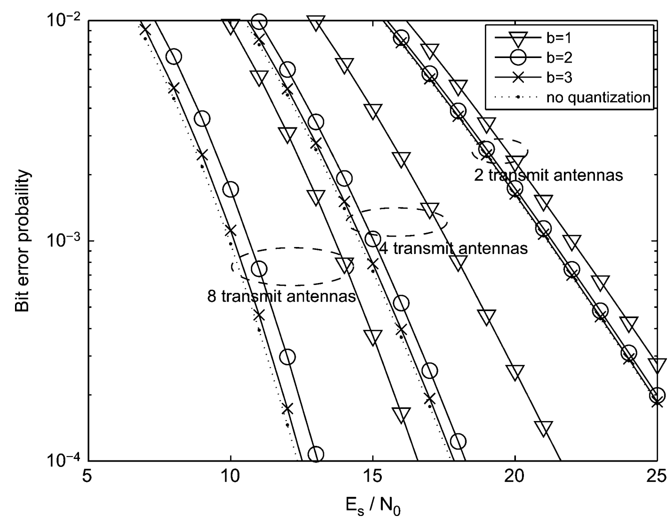

Fig. 5. Scalar quantization effect of the equal gain precoders.

is less than 0.25 dB. These theoretical results can provide useful design references to determine b for the equal gain precoder with scalar quantization.

Next, let us answer the question: will the performance improve by using $\theta_i = \phi_i$, i.e., extra feedback in θ_1. Fig. 3 shows the performance loss due to quantization for $\theta_1 = 0$ and $\theta_1 = \phi_1$ according to (21) and (23), respectively. It is interesting to note that the performance with extra feedback for θ_1 is worse than that without extra feedback. The reason is explained as follows: the two solutions in (4) and (5) are actually equivalent without quantization. However, with quantization, we need to quantize θ_1 as well in (4). This demands extra bits to represent θ_1. If we have infinite bits to represent θ_1, the two solutions would lead to the same performance. However, with limited feedback, solution in (4) is worse than that in (5). Please note that if we consider the optimal bit allocation for scalar quantization to each antenna as in [19], more bit budget will lead to better performance.

IV. Simulation Result

The simulation is conducted using the following parameters: The channel is quasi-stationary and the channel coefficients are i.i.d. complex Gaussian random variables with zero mean and unit variance. The modulation level is 16-QAM. More than 60 000 channel realizations are used. We use the solution in (5). For description convenience, we let b be the number of bits to represent the phase of each transmit antenna (except the first antenna) for the equal gain precoder. We let B be the number of total feedback bits. Thus, we have the relationship that $B = (N_t - 1)b$ for the equal gain precoder.

1) Example 4: Comparison of the Optimal and the Equal Gain Precoders: To see the best performance that the optimal and the equal gain precoders can achieve, we do not quantize the precoding vectors $\mathbf{p}$ in this example. Fig. 4 shows the bit error probability (BEP) performance of these two precoders without quantization. We observe that the optimal precoder outperforms the equal gain precoder around 0.5 dB when $N_t = 2$. When $N_t = 8$, the gap is around 0.9 dB and when $N_t > 16$, the gap is around 1 dB. This result shows that the performance gap between the optimal and the equal gain precoders is around 1 dB, which corroborates the theoretical result in Theorem 1.

2) Example 5: Quantization Effect of the Equal Gain Precoder: Fig. 5 shows the BEP performance of the equal gain precoder due to quantization effect. We observe several interesting results: First, as N_t increases, b needs to be increased to achieve comparable performance without quantization. For instance, when $N_t = 2$, $b = 2$ can achieve comparable performance with its corresponding performance without

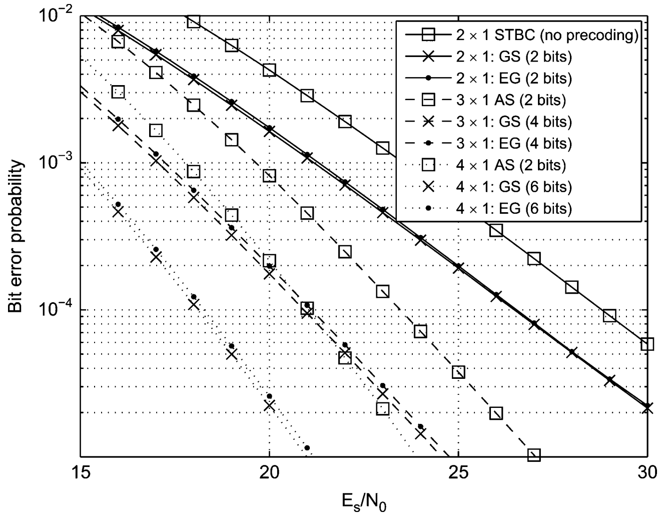

Fig. 6. Performance comparison for STBC, equal gain (EG), Grassmannian (GS) and antenna selection (AS) precoders.

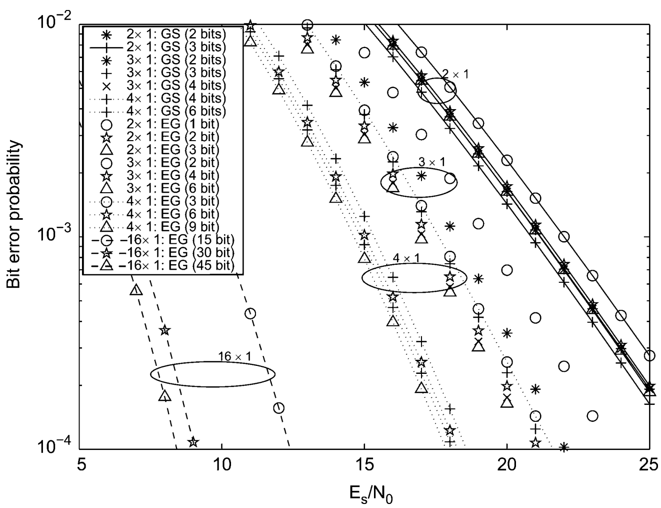

Fig. 7. Performance comparison for the (EG) precoder, and the Grassmannian (GS) precoder.

quantization, but when $N_t = 8$, $b = 3$ is required to achieve comparable performance. Moreover, we found the performance gap with and without quantization in general matches the results in Theorem 2 (also see Fig. 2). The exception is when $N_t = 4$ and 8 with $b = 1$, where the assumption that the quantization error and the precoding coefficients are independent may no longer be valid.

3) Example 6: Comparison of Various Precoders With Quantization Effect: We compare the performance of the equal gain precoder, the Grassmannian precoder [3] and the antenna selection precoder. The performance comparison is shown in Fig. 6. For fair comparison, total required bits are shown for all precoders. To evaluate the performance improvement due to precoding, we also include the 2×1 STBC performance as shown in the solid-square curve. From the figure, we see that the three precoders have the same diversity gain and hence their slopes are the same. However, the Grassmannian and the equal gain precoders can achieve a better performance than the antenna selection precoder. Moreover, we see in this simulation case that with the same required total bits B, the equal gain precoder can achieve comparable performance with the Grassmannian precoder (less than 0.2-dB performance gap).

To have a more detailed comparison between the Grassmannian and the equal gain precoders, let us use all available Grassmannian codewords found in http://cobweb.ecn.purdue.edu/~djlove/grass. The performance curves for these two precoders are shown in Fig. 7. We see that for $b = 1$, i.e., $B = N_t - 1$, the equal gain precoder seems not perform well when $N_t > 2$. For instance, when $N_t = 3$ and $b = 1$, i.e., $B = 2$, the performance gap between these two precoders is up to 2.5 dB (see star and circle curves). However, as $b \geq 2$, the equal gain precoder improves rapidly and can achieve comparable performance with Grassmannian. Although for $b = 1$ the equal gain precoder may not achieve comparable performance with the Grassmannian in some cases, it has several advantages as we mentioned in the introduction. Moreover, the equal gain precoder can be easily extended to arbitrary number of transmit antennas (see curves for $N_t = 16$), where constructing the codebook of the Grassmannian precoder may not be easy in this case [4].

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