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Applying Particle Swarm Optimization to Parameter Estimation of the Nonlinear Muskingum Model

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Abstract: The Muskingum model is the most widely used method for flood routing in hydrologic engineering. However, the application of the model still suffers from a lack of an efficient method for parameter estimation. Particle swarm optimization (PSO) is applied to the parameter estimation for the nonlinear Muskingum model. PSO does not need any initial guess of each parameter and thus avoids the subjective estimation usually found in traditional estimation methods and reduces the likelihood of finding a local optimum of the parameter values. Simulation results indicate that the proposed scheme can improve the accuracy of the Muskingum model for flood routing. A case study is presented to demonstrate that the proposed scheme is an alternative way to estimate the parameters of the Muskingum model.

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Introduction

Among the many models used for flood routing, the Muskingum method is the most widely used owing to its simplicity. The Muskingum flood routing model was developed by the U.S. Army Corps of Engineers for the Muskingum Conservancy District Flood-Control Project over six decades ago. The following continuity and nonlinear storage equations are commonly used in the Muskingum model (Gill 1978; Tung 1985; Yoon and Padmanabhan 1993; Mohan 1997; Kim et al. 2001; Geem 2006; Al-Humoud and Esen 2006)

$$\frac{dS_t}{dt} = I_t - O_t \quad (1)$$

$$S_t = K[XI_t + (1-X)O_t]^m \quad (2)$$

where S_t, I_t, and O_t denote the instantaneous amounts of storage, inflow, and outflow, respectively, at time t; K=storage-time constant for the river reach, which has a value reasonably close to the flow travel time through the river reach; X=a weighting factor usually varying between 0 and 0.5 for reservoir storage, and between 0 and 0.3 for stream channels; and m=an exponent for considering the effects of nonlinearity. However, the calibration for finding the optimal value of the three parameters K, X, and m can be complicated.

Over the last two decades, many optimization techniques, including Broyden-Fletcher-Goldfarb-Shanno method, genetic algorithm (GA), and harmony search (HS), etc., have been applied to identify the three parameters (Gill 1978; Tung 1985; Yoon and Padmanabhan 1993; Mohan 1997; Kim et al. 2001; Geem 2006). Gill (1978) used a least-squares method (LSM) to find the values of the three parameters in the nonlinear Muskingum model. Tung (1985) proposed parameter estimation using the Hook-Jeeves (HJ) pattern search in conjunction with linear regression (LR), the conjugate gradient (CG), and Davidon-Fletcher-Powell (DFP) techniques. The performance of the methods was compared with Gill's procedure and (HJ+CG) and (HJ+DFP) were found to yield better solutions. Yoon and Padmanabhan (1993) discussed several methods for estimating the parameters. The linear model may be inappropriate when the nonlinear relationship between the storage and discharge exists in most actual river systems. The suggested method for the nonlinear routing model is an iterative procedure and involves the nonlinear least-squares regression (NONLR). Mohan (1997) pointed out all of the foregoing methods do not guarantee the global optimal, and they may be trapped at a local optimum. He used GA to estimate the parameters in the model. The results showed that the estimation by GA was better than by the previous methods and did not require the initial guess to be close to the optimum. Kim et al. (2001) applied the HS to the same problem. Their results showed HS estimation was better than GA and also did not require that the initial guess were close to the optimum.

In this study, the parameter estimation for the nonlinear Muskingum model is performed using the particle swarm optimization (PSO) technique. The results are then compared to those obtained using the previous described techniques.

Routing Procedure of the Nonlinear Muskingum Model

By rearranging Eq. (2), the rate of the outflow can be expressed as

$$O_t = \left(\frac{1}{1-X}\right)\left(\frac{S_t}{K}\right)^{1/m} - \left(\frac{X}{1-X}\right)I_t \quad (3)$$

Combining Eq. (3) and the continuity Eq. (1), the state equation can be obtained as

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$$\frac{\Delta S_t}{\Delta t} = -\left(\frac{1}{1-X}\right)\left(\frac{S_t}{K}\right)^{1/m} + \left(\frac{1}{1-X}\right)I_t \tag{4}$$

$$S_{t+1} = S_t + \Delta S_t \tag{5}$$

$$O_{t+1} = \left(\frac{1}{1-X}\right)\left(\frac{S_{t+1}}{K}\right)^{1/m} - \left(\frac{X}{1-X}\right)\bar{I}_{t+1},$$
$$\text{where } \bar{I}_{t+1} = (I_{t+1} + I_t)/2 \tag{6}$$

The routing procedure involves the following steps (Geem 2006):

Step 1. Assume values for the three parameters, K, X, and m.

Step 2. Calculate storage (S_t) using Eq. (2), where initial outflow is same as initial inflow.

Step 3. Calculate the time rate of change of storage volume using Eq. (4).

Step 4. Estimate the next accumulated storage using Eq. (5).

Step 5. Calculate the next outflow using Eq. (6). $\bar{I}_{t+1}$ is expressed as average inflow $(I_{t+1}+I_t)/2$. I_t replaces $(I_{t+1}+I_t)/2$ when the ratio of storage t and $t+1$ is over 2.

Step 6. Repeat Steps 2–5.

PSO

PSO is a stochastic optimization technique developed by Kennedy and Eberhart (1995), inspired by social behavior of bird flocking or fish schooling (Clerc and Kennedy 2002). PSO provides a population-based search procedure in which individuals called particles change their position with time. In the past several years, PSO has been successfully applied including hydrological modeling. For example, Chau (2004) used a PSO model adopted to train perceptrons. The perceptron is a type of artificial neural network which the inputs are fed directly to the outputs via the weighted connections. The optimal weightings are determined by PSO in training process. The approach is demonstrated to be feasible and effective by predicting real-time water levels in the Shing Mun River of Hong Kong with different lead times on the basis of the upstream gauging stations or stage/time history at the specific station. Chau (2005) also presented the application of a split-step PSO model for training perceptrons to forecast real-time algal bloom dynamics in Tolo Harbour of Hong Kong. In this study, parameter estimation for the nonlinear Muskingum model using the PSO is developed.

In a PSO system, particles fly around in a multidimensional search space. During flight, each particle adjusts its position according to its own experience, and according to the experience of a neighboring particle, making use of the best position encountered by itself and its neighbor. PSO shares many similarities with evolutionary computation techniques such as GA (Clerc and Kennedy 2002; Boeringer and Werner 2004; Robinson and Rahmat-Samii 2004). The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation. The PSO algorithm can be expressed as follows: At kth iteration, a current ith particle position in the multidimensional search space which represents (Y_i^k). A current velocity (V_i^k) controls its fly speed and direction. The velocity each particle updates along each dimension toward local and global best positions in Eq. (7), and the position update in Eq. (8) that are given by

$$V_i^{k+1} = V_i^k + c_1 r_1 (P_i^k - Y_i^k) + c_2 r_2 (P_g^k - Y_i^k) \tag{7}$$

$$Y_i^{k+1} = Y_i^k + V_i^{k+1} \tag{8}$$

where P_i=best previous position of particle (also known as p_{best}); P_g=global best position among all the particles (also known as g_{best}); c_1 and c_2=constants known as acceleration coefficients which control how far a particle will move in a single iteration; and r_1 and r_2=elements from two uniform random numbers in the range (0, 1).

Eqs. (9) and (10) update the local bests (P_i) and the global best (P_g)

$$P_i = \begin{cases} P_i : f(Y_i) \geq f(P_i) \\ Y_i : f(Y_i) < f(P_i) \end{cases} \tag{9}$$

$$P_g = \min[f(P_i)], \quad i = 1, 2, \ldots, M \tag{10}$$

where f=the objective function and M=the total number of particles.

Application

Outflow hydrographs along with routed flows can be obtained using the proposed nonlinear Muskingum model. To investigate the performance of PSO, a typical problem is used as an example. PSO for the estimating of the parameters in the nonlinear Muskingum model was applied to an example which was first proposed by Wilson (1974). The ranges of three parameters in the nonlinear Muskingum model are K=0.01–0.20, X=0.2–0.3, and m=1.5–2.5. The objective function of PSO is to minimize the sum of the squared deviations (SSQs) between the computed and observed outflows

$$\text{Minimize } \text{SSQ} = \sum_{t=1}^{n} (O_t - \hat{O}_t)^2 \tag{11}$$

where $\hat{O}_t$ denotes the computed outflow at time t. In this PSO algorithm, a population of 100 individuals is used. The maximum number of iterations in the program is 100. The values of the acceleration constants (c_1 and c_2) are both set to 1.0.

Fig. 1 shows a comparison of the performances of the different parameter estimation procedures. Columns 1–3 of Table 1 are the actual data (Wilson 1974); columns 4–9 show the routed flow data obtained by using the Muskingum model. Column 4 uses the LSM (Gill 1978); Column 5 uses the HJ+DFP (Tung 1985); Column 6 uses NONLR (Yoon and Padmanabhan 1993); Column 7 uses GA (Mohan 1997); Column 8 uses HS (Kim et al. 2001); Column 9 uses PSO programs to estimate the parameters which are then used to determine the routed flows.

Finally, the new method (PSO) and the five conventional methods are compared using the SSQ and sum of absolute deviations (SADs) between the computed and observed outflows presented in Table 2. The results show that the SSQ is attained using PSO. It has been demonstrated that PSO gets better results than other methods except HS. Fig. 2 shows the sensitivity of SSQ and SAD to the PSO parameters (c_1 and c_2).These parameters range from 0.2 to 2. The finding implies clearly that the value of SSQ is

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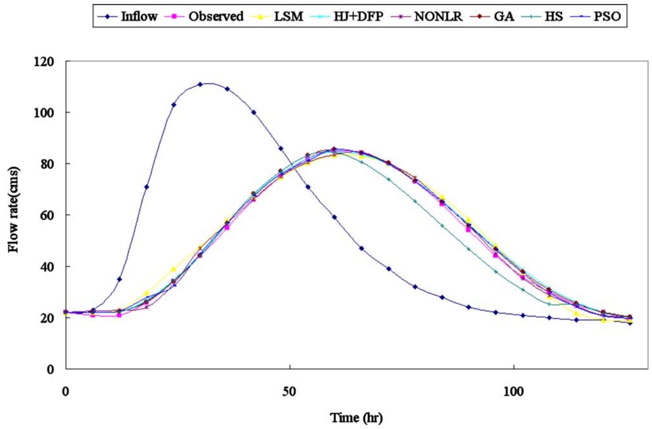

Fig. 1. Inflow and outflow hydrographs for the example problem computed with parameters obtained from selected estimation methods

the minimum for the example studied when $c_1=1$ and $c_2=1$. Thus, the PSO yielded good SSQ and SAD over a wide range of c_1 and c_2 values.

Conclusions

The newly developed heuristic algorithm, PSO, is applied to the parameter estimation problem of the nonlinear Muskingum model. While possessing similar capabilities to the GA, the particle swarm model is much simpler to implement. The proposed approach is compared with the other optimization methods for an example case from the literature. PSO has the advantage that it does not require assumption of the initial values of the model parameters. The results demonstrate that PSO can achieve a high degree of accuracy to estimate the three parameters and this results in accurate predictions of outflow. Consequently, the model also shows robustness in forecasting outflow. With the PSO method, no derivative is required, and the semibiological evolution will approach the nearly global optimum solution. PSO appears to offer good applicability in the hydrology field and further applications should be explored.

Table 1. Inflow and Outflow Hydrographs for the Example and the Results of Various Parameter Estimation Methods

Time (h)	Inflow (cms.)	Observed outflow (cms.)	Computed outflow (cms.)					
			LSM	HJ+DFP	NONLR	GA	HS	PSO
0	22	22	22.0	22.0	22.0	22.0	22.0	22.0
6	23	21	22.0	22.0	22.6	22.0	22.0	22.0
12	35	21	22.8	22.4	23.0	22.4	22.4	22.6
18	71	26	29.6	26.7	24.2	26.3	26.6	28.1
24	103	34	39.1	34.8	33.2	34.2	34.4	32.2
30	111	44	47.6	44.7	47.1	44.2	44.1	45.0
36	109	55	58.0	56.9	56.8	56.9	56.8	57.0
42	100	66	67.1	67.7	66.2	68.2	68.1	67.5
48	86	75	74.8	76.3	75.0	77.1	77.1	75.9
54	71	82	80.4	82.2	80.7	83.2	83.3	81.2
60	59	85	83.2	84.7	83.5	85.7	85.9	85.6
66	47	84	82.8	83.5	84.3	84.2	84.5	84.2
72	39	80	80.1	79.8	79.9	80.2	80.6	79.6
78	32	73	74.5	73.3	74.3	73.3	73.7	73.3
84	28	64	67.2	65.5	65.3	65.0	65.4	65.0
90	24	54	58.1	56.5	55.9	55.8	56.0	56.2
96	22	44	48.1	47.5	45.1	46.7	46.7	46.5
102	21	36	37.6	38.7	35.4	38.0	37.8	37.3
108	20	30	28.2	31.4	28.7	30.9	30.9	29.7
114	19	25	21.9	25.9	24.3	25.7	25.3	24.3
120	19	22	19.1	22.1	20.9	22.1	21.8	20.6
126	18	19	19.0	20.2	20.4	20.2	20.0	19.6

Table 2. Estimated Parameters and Fit Quality Indicators Obtained by All Methods

	K	X	m	SSQ	SAD
LSM	0.0100	0.2500	2.3470	143.60	46.4
HJ+DFP	0.0669	0.2685	1.9291	49.64	25.2
NONLR	0.0600	0.2700	2.3600	43.26	25.2
GA	0.1033	0.2873	1.8282	38.23	23.0
HS	0.0883	0.2873	1.8630	36.78	23.4
PSO	0.1824	0.3330	2.1458	36.89	24.1

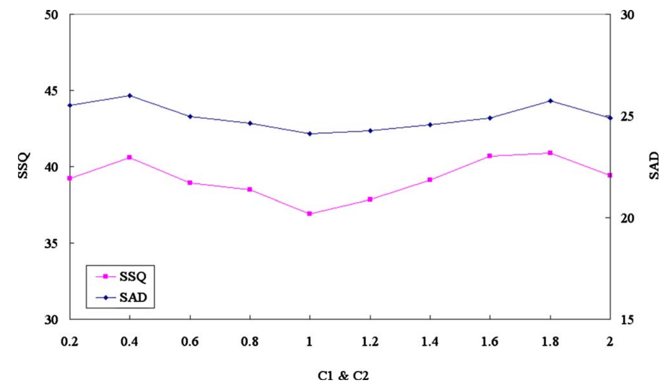

Fig. 2. Sensitivity of SSQ and SAD to selected values of c_1 and c_2

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