An Analysis on Downlink Capacity of Multi-Cell OFDMA Systems under Randomized Inter-cell/sector Interference

I-Kang Fu and Wern-Ho Sheen
Department of Communication Engineering
National Chiao Tung University , Hsinchu, 300 Taiwan, R.O.C.
e-mail: apatch.cm91g@nctu.edu.tw, whsheen@cm.nctu.edu.tw

Abstract-**An analysis on downlink capacity of multi-cell OFDMA (orthogonal frequency division multiple access) systems is presented in this paper under a model of randomized multiple access interference, which is a result of applying random sub-carrier allocation among different cells/sectors. The analysis shows that the system capacity is either in the bandwidth-limited or power-limited region and that optimum tradeoff between capacity and coverage is achievable in a sense that given a required capacity the maximum coverage can be determined. The capacity is evaluated for different system setups, which are characterized by cell radius, reuse factor and types of antenna system with and without spatial division multiple access (SDMA). Three types of antenna are considered including omni-directional antenna, 3-sector antenna and switched-beam smart antenna. Numerical results show that the use of 3-sector and switched-beam smart antenna can enlarge the bandwidth-limited region and achieves a better capacity and coverage tradeoff.**

Keywords- OFDMA, capacity, interference randomization

Ⅰ. INTRODUCTION

Next-generation mobile communication is envisaged to support multimedia services over a wide variety of operating environments [1]. One key design issue of such a system is to overcome the severe inter-symbol interference (ISI) incurred by high-data-rate transmission and OFDM (orthogonal frequency division multiplexing) is an effective modulation /multiplexing scheme to combat this effect. By using parallel orthogonal sub-carriers along with cyclic-prefix, ISI can be removed completely as long as the cyclic-prefix is lager than the maximum delay spread.

OFDMA (orthogonal frequency division multiple access), a form of combining OFDM and FDMA, has been considered as one of the most promising multiple access schemes for the next generation systems [2,3]. In OFDMA systems, users in the same cell can be maintained as orthogonal to each other, and thus there is no intra-cell multiple access interference (MAI). However, interference from other cells will be very severe, like in the traditional narrow-band systems, if the same sub-channel of a user is reused in the adjacent cells. To overcome this difficulty, allocating sub-carriers into a sub-channel is often *permuted* differently in the adjacent cells/sectors so as to randomize the inter-cell/sector MAI. It eliminates the most severe interfering scenarios and makes the worst case network planning not necessary. An OFDMA system employing sub-carrier permutation has already been adopted as part of the IEEE 802.16 standard [4].

The capacity of OFDMA systems has recently been investigated in some literatures. In [5], the uplink capacity of multi-cell OFDMA systems with universal frequency reuse was investigated through Monte Carlo simulations under various resource allocation methods. The same simulation technique was used in [6] to investigate the downlink capacity of single-cell OFDMA systems. In addition, the downlink capacity of multi-cell OFDMA was compared with CDMA through simulations [7].

Instead of resorting to simulations, this paper aims to provide a mathematical analysis on the downlink capacity of multi-cell OFDMA systems. Based on a model of randomized MAI, the analysis is able to incorporate with the effects of propagation model, cell radius, frequency reuse factor and antenna systems with and without spatial division multiple access (SDMA). Numerical results are given to illustrate the system capacity with various system setups that lead to best design of the system in terms of capacity and coverage trade-off.

Ⅱ. SYSTEM MODEL

A. Sub-carrier Permutation and Interference Modeling

In the IEEE 802.16 downlink OFDMA system, sub-carrier

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allocation is achieved by using a pseudo-random sub-carrier mapping process, called sub-carrier permutation [4]. Allocation of sub-carriers into sub-channels is often *permuted* differently for the co-channel cells/sectors in order to randomize the inter-cell/sector interference. In practice, the sub-carrier permutation of a cell/sector is fixed and planned in advance, usually determined by the cell/sector ID, as in the IEEE 802.16 OFDMA system. In this study, however, the sub-carrier allocations will be considered random and independent from one cell to another for easy analysis. With this arrangement, we say that the co-channel inter-cell/sector MAI has been randomized.

The characteristics of MAI in a coded OFDM-based system were investigated for Rayleigh fading channels in [8]. Simulation and analytic results showed that MAI of sub-carriers can be well modeled as independent additive white Gaussian noise (AWGN) for both the synchronous and asynchronous systems. Using the AWGN modeling of MAI and the assumption of random permutation that implies that each sub-carrier sees the same amount of average interference, we are able to model the coded OFDMA system as a system of frequency hopping spread spectrum (FHSS) corrupted by partial band noise jamming, as did in [5]. Therefore, we have the following equality

$$\frac{S}{I+N}=\frac{1}{PG}\cdot\frac{E_b}{I_0+N_0}, \qquad (1)$$

where S is the received signal power for the desired sub-channel, I and N are the received interference and thermal noise power for the total number of sub-carriers, respectively, PG is processing gain, E_b is the received signal energy per bit, and N_0 and I_0 are the thermal noise and interference density, respectively. $PG = N_{sub-channel}/(m\cdot c)$, where m is the modulation order, and c the coding rate.

B. Antenna Systems

Three types of antenna systems will be considered, including omni-directional antenna, 3-sector antenna and switched-beam smart antenna. For omni-directional antenna, the antenna pattern $G(\theta)$ is equal to $0\ dBi$, where θ is the azimuth angle, and for 3-sector antenna, the antenna pattern of each sector is the one specified in [9]

$$G_j(\theta)=14-\min\left[12\left(\frac{18\cdot(\theta-\varphi_j)^2}{7\pi}\right),20\right]\ \ dBi, \qquad (2)$$

where $-\pi<\theta-\varphi_j\le\pi$, φ_j is the steering direction of the antenna for *j*-th sector, which are 0, $2\pi/3$ and $4\pi/3$ for $j=1,2,3$, respectively.

Switched-beam smart antenna forms a number of fixed beams to cover different area in the cell, and let subscriber station served by the beam with the highest gain. Consider a linear equally spaced (LES) antenna array with each antenna element as an omni-directional one. Its antenna pattern can be given by [10]

$$G_j(\theta)=\left|\frac{\sin\left(\beta Ud/2\,(\cos\varphi_j-\cos\theta)\right)}{\sin\left(\beta d/2\,(\cos\varphi_j-\cos\theta)\right)}\right|^2, \qquad (3)$$

where U is the number of antenna elements, d is the distance between antenna elements, $\beta=2\pi/\lambda$ is the phase propagation factor, λ is the wavelength, and φ_j is the steering direction of *j*-th beam.

Note that SDMA can be applied in the systems with 3-sector or switched-beam smart antenna systems. By SDMA we mean that all sub-channels are reused in each sector (beam), and there will be intra-cell interference when SDMA is employed.

III. DOWNLINK CAPACITY

The downlink capacity of the considered multi-cell OFDMA system is analyzed in this section. Consider the cellular structure in Figure 1, with the reference base station (BS) denoted as BS_0 and Q interfering ones as BS_q, $q=1,2,\ldots,Q$. Each cell has the same radius R, and the locations of interfering base station can be determined by knowing the frequency reuse factor K. The distance between adjacent interfering base stations is $D=\sqrt{3K}\cdot R$.

Three assumptions are made in the following analysis. First, each subscriber station (SS) is ideally power controlled by its serving base station. Second, for 3-sector or switched-beam smart antenna, subscriber stations are served by the sector (beam) with highest antenna gain, which is achieved by intra-cell handoff. Third, the subscriber stations are uniformly distributed over the cell.

Consider a subscriber station SS_i served by *j*-th sector (beam) of BS_0. By ideal power control, the received $E_b/(I_0+N_0)$ will equal to target value ρ.

$$\rho=PG\cdot\frac{p_{j,i}\cdot G_{0,j}(\theta_{0,i})\cdot L_{0,i}}{I_i+N}, \qquad (4)$$

where $p_{j,i}$ is the downlink transmit power for SS_i in *j*-th sector (beam), $G_{0,j}\left(\theta_{0,i}\right)$ is the antenna gain toward the direction $\theta_{0,i}$, and $L_{0,i}$ is the propagation loss between BS_0 and SS_i, which is equal to

$$L_{0,i} = A \cdot r_{0,i}^{-l} \cdot \chi_{0,i}, \tag{5}$$

where $r_{0,i}$ is the distance from BS_0 to SS_i, and l is the path-loss exponent, A is a constant, which is determined by path-loss model, and $\chi_{0,i}$ is a log-normal random variable with zero mean and variance σ_{dB}^2 in dB. The effect of small-scale multi-path fading is included in the calculation of the link performance and thus is not treated explicitly here. From (4), the required transmit power is given by

$$p_{j,i} = \frac{\rho \cdot \left(I_i + N\right)}{PG \cdot A \cdot r_{0,i}^{-l} \cdot \chi_{0,i} \cdot G_{0,j}\left(\theta_{0,i}\right)}. \tag{6}$$

Define M_j be the maximum number of users in *j*-th sector (beam) (one channel per user). By applying the weak law of large number, the total transmission power $\sum_{i=1}^{M_j} p_{j,i}$ of the sector (beam) can be approximated as $M_j \cdot E\left[p_j\right]$ for a sufficiently large M_j, where p_j is a random variable that characterizes the *BS* transmit power for a user uniformly located over *j*-th sector (beam). Thus, M_j can be obtained as

$$M_j \approx \begin{cases} \left\lfloor P_{\max,j} \big/ E\left[p_j\right] \right\rfloor & if \left\lfloor P_{\max,j} \big/ E\left[p_j\right] \right\rfloor \le N_{sub-channel,j} \\ N_{sub-channel,j} & if\ N_{sub-channel,j} < \left\lfloor P_{\max,j} \big/ E\left[p_j\right] \right\rfloor \end{cases}, \tag{7}$$

where $P_{\max,j}$ and $N_{sub-channel,j}$ are the maximum transmit power and the number of sub-channel for *j*-th sector (beam), respectively, and $\lfloor x \rfloor$ is the largest integer less than ***x***. $E\left[p_j\right]$ is the averaged transmit power at BS for an SS uniformly located over *j*-th sector (beam), which can be numerically derived and calculated [11].

When $\left\lfloor P_{\max,j} / E\left[p_j\right] \right\rfloor \le N_{sub-channel,j}$ the system capacity is in the power-limited region, otherwise it is in the bandwidth-limited region, i.e. $N_{sub-channel,j} < \left\lfloor P_{\max,j} / E\left[p_j\right] \right\rfloor$. Power-limited means that although there still are some sub-channels (bandwidth) available, *BS* already uses up all its power. This happens when BS has a too large coverage (cell radius). On the other hand, bandwidth-limited means that BS runs out of sub-channels even though it has not yet reached its maximum transmit power, and that implies the coverage can be extended. With (7), the cell capacity can be approximated as

$$C = \sum_{j=1}^{J} M_j \cdot \frac{m \cdot c \cdot N_{sub-carrier}}{T_{OFDM}} \quad \left(\text{bits/sec/cell}\right), \tag{8}$$

where T_{OFDM} is the OFDM symbol duration, and J is the number of sectors (beams) in a cell.

IV. NUMERICAL RESULTS

In this section, numerical results are given to illustrate the downlink capacity of the considered OFDMA systems in various system setups. When switched-beam smart antenna is considered, twelve antenna elements are used to generate the same number of beams in the following results.

Table 1 gives the OFDMA parameters used in the numerical results. For a reuse factor K, the total system bandwidth is $6 \cdot K$ MHz. In the case of SDMA, all sub-channels are allocated to each sector (beam) so that $N_{sub-channel,j} = 420$, and independent sub-carrier allocation is applied to each sector (beam). In the no SDMA case, on the other hand, sub-channels are divided equally into each sector (beam), i.e., $N_{sub-channel,j} = 140\,(35)$ and no intra-cell interference.

The required E_b/N_0 (ρ) is set to 7dB for QPSK and $1/2$ coding rate in Rayleigh fading channel. Therefore, the processing gain (*PG*) is equal to 420. Maximum transmit power of base station is assumed to be 50Watts, which is equally divided to each sector (beam). Furthermore, $A = 6.01 \times 10^{-13}$ and $l = 4.375$, the same as those used in the IEEE 802.16 type-B path-loss model with 2GHz carrier frequency and $\sigma_{dB}^2 = 8$ dB [9].

In Figure 2, the capacity versus cell radius is presented for the single cell OFDMA systems. Clearly, we observe that the system capacity is either bandwidth-limited or power-limited, according to the cell radius. For example, in the case of omni-directional antenna, the system capacity is in the bandwidth-limited region when $R \le 2\ km$, where the capacity is upper-limited to 5 Mbits/sec/cell. The results in Figure 2 provide an optimum tradeoff between capacity and coverage in a sense that given a required capacity the maximum cell radius is determined. It is also observed that the system capacity may drop significantly after the cell radius is larger than certain point. This says that a good tradeoff between capacity and coverage may exist only within a range of cell radius.

In the case of SDMA, all sub-channels are reused in each sector (beam). Therefore, the limitation on the sub-channel number in Equation (7) is released mostly, and accordingly the capacity is shown to be substantially improved in the bandwidth-limited region. Especially, the capacity can be up to 28.214 Mbits/sec/cell when switched-beam smart antenna is considered. In the power-limited region, nevertheless, the capacity gain is diminishing even becomes negative due to the additional intra-cell interference incurred by applying SDMA.

In Figure 3, the downlink capacity with omni-directional antenna is given for the multi-cell environment. The results show that the system capacity is substantially degraded by inter-cell interference, as compared to the single-cell case in Figure 2. The degradation can be recovered mostly by using $K = 3$, 4 or 7 in the bandwidth limited region at the expense of larger bandwidth. In the power-limited region, however, using $K > 1$ provides only very little improvement because the limiting factor in this region is the excessive path loss due to a large cell radius; there is no way to significantly increase SINR by reducing interference because of the presence of additive white Gaussian noise. In addition, using $K = 4$ or 7 only provides a marginal capacity improvement over $K = 3$ in the bandwidth-limited region because the system capacity is mainly limited by the number of available sub-channels rather than inter-cell interference on which $K = 4$ or 7 can provide improvement.

Figure 4 presents the downlink capacity of multi-cell OFDMA system with 3-sector antenna. For the cases of no SDMA, using 3-sector antenna can increase the system capacity and/or enlarge the bandwidth-limited region because of the antenna gain and reduced inter-cell interference, as compared to the case of omni-directional antenna. In particular, for $K = 1$ the power-limited capacity is increased, and for $K > 1$ in addition to the capacity improvement in the power-limited region, the bandwidth-limited region is enlarged, and that says that the maximum capacity can be maintained for a larger coverage. Also shown in Figure 4, SDMA can significantly increase the system capacity in the bandwidth-limited region for $K > 1$. For $K = 1$, however, since the system capacity is basically power-limited, there is no advantage of using SDMA; SDMA introduce intra-cell interference between sectors that results in capacity reduction.

Figure 5 shows the downlink capacity of multi-cell OFDMA systems with switched-beam smart antenna. Compared to Figure 4, the capacity is improved and the bandwidth-limited region is enlarged further, respectively through higher antenna gain and narrower width of each beam.

V. CONCLUSIONS

The downlink capacity of multi-cell OFDMA systems is analyzed in this paper under a model of randomized multiple access interference. The analysis provides a convenient way to achieve the best tradeoff between capacity and coverage. The system capacity was evaluated for different system setups, which are characterized by cell radius, reuse factor and type of antenna with or without use of SDMA. Three types of antenna are considered including omni-directional antenna, 3-sector and switched-beam smart antenna. Numerical results show that 3-sector and switched-beam smart antenna can be employed to significantly increase the system capacity if the cell radius and reuse factor can be properly selected.

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Table 1 OFDMA parameters used in numerical results

Parameter	Value
Bandwidth per cell	6 MHz
FFT size	2048
OFDM symbol duration, T_{OFDM}	336 μs
Cyclic-prefix	37.333 μs
Sub-carrier frequency spacing	3.348 kHz
Number of sub-channels, $N_{sub\text{-}channel}$	420
Number of sub-carriers in a sub-channel, $N_{sub\text{-}carrier}$	4
Number of data sub-carriers	1680
Number of sub-carriers for guard band	368

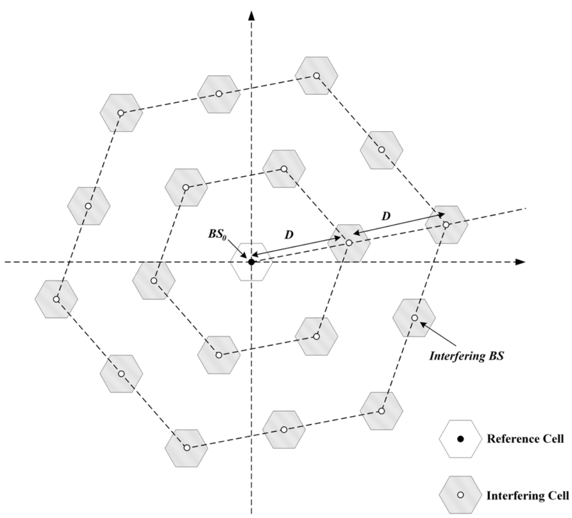

Figure 1 Geometry of interfering cells (Q = 18)

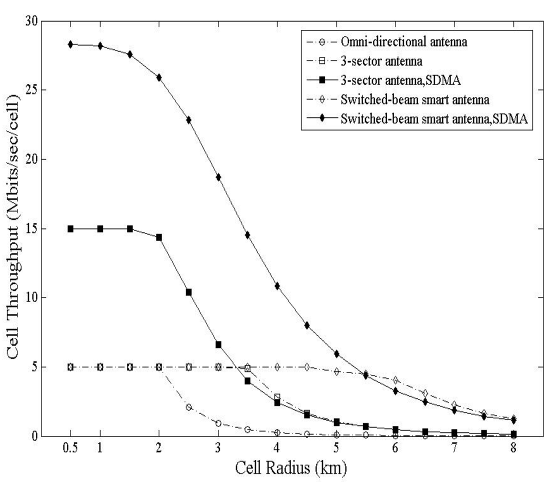

Figure 2 Downlink capacity of single cell OFDMA systems

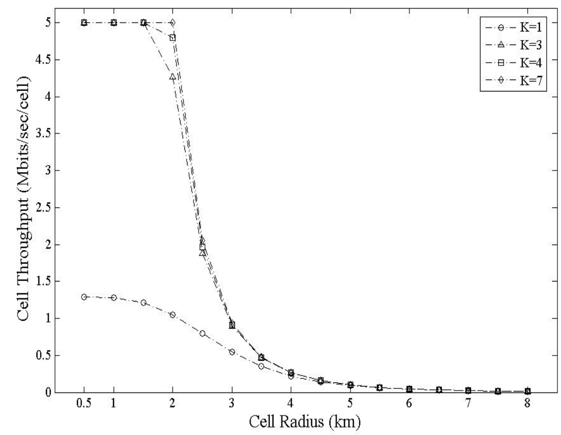

Figure 3 Downlink capacity of multi-cell OFDMA systems with omni-directional antenna

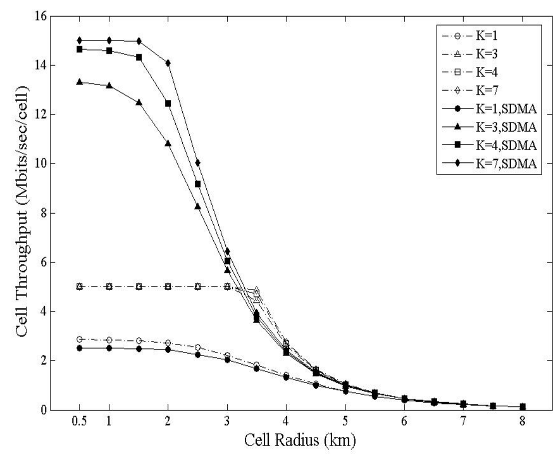

Figure 4 Downlink capacity of multi-cell OFDMA system with 3-sector antenna

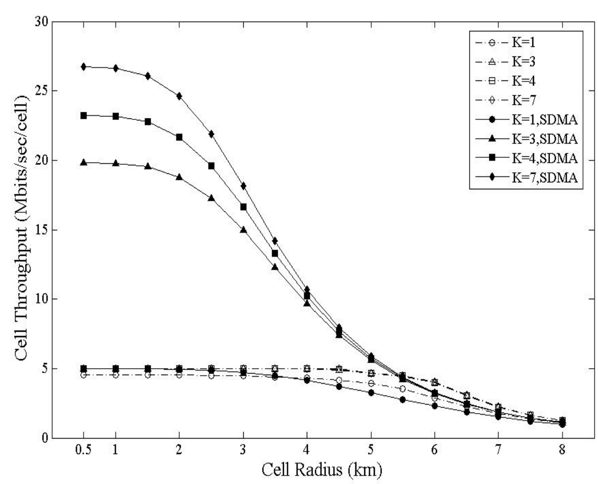

Figure 5 Downlink capacity of multi-cell OFDMA systems with switched-beam smart antenna