國 立 交 通 大 學

電 機 與 控 制 工 程 學 系

碩 士 論 文

三層類神經網路的動態最佳學習

Dynamic Optimal Training of A Three Layer Neural Network with Sigmoid Function

研究生：紀右益

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摘要

本篇論文是針對三層類神經網路提出一個動態最佳訓練法則，其中網路的隱藏層和輸出層都有經過一個 S 型激發函數。這種三層的網路可以被運用於處理分類的問題，像是蝴蝶花的品種分類。我們將對這種三層神經網路的動態最佳訓練方法提出一個完整的証明，用來說明這種動態最佳訓練方法保證神經網路能在最短的迭代次數下達到收斂的輸出結果。這種最佳的動態訓練方法不是使用單一固定的學習速率，而是在每一次的迭代過程中不斷的更新，來取得下一次迭代過程所需要的最佳學習速率，以保證最佳的收斂的訓練結果。我們可以由 XOR 和蝴蝶花的測試例子得到良好的結論。

Dynamic Optimal Training of A Three Layer Neural Network with Sigmoid Function

Student: Yu-Yi Chi **Advisor: Chi-Hsu Wang**

Department of Electrical and Control Engineering
National Chiao Tung University

ABSTRACT

This thesis proposes a dynamical optimal training algorithm for a three layer neural network with sigmoid activation functions in the hidden and output layers. This three layer neural network can be used for classification problems, such as the classification of Iris data. Rigorous proof has been presented for the dynamical optimal training process for this three layer neural network, which guarantees the convergence of the training in a minimum number of epochs. This dynamical optimal training does not use fixed learning rate for training. Instead, the learning rates are updated for next iteration to guarantee the optimal convergence of the training result. Excellent results have been obtained for XOR and Iris data set.

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I feel external gratitude to my advisor, Chi-Hsu Wang for teaching me many things. He taught me how to do to the research and the most important is that he taught me how to get along with people.

And I am grateful to everyone in ECL. I am very happy to get along with you.

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CHAPTER 1

Introduction

Artificial neural network (**ANN**) is the science of investigating and analyzing the algorithms of the human brain, and using the similar algorithm to build up a powerful computational system to do the tasks like pattern recognition [1], [2], identification [3], [4] and control of dynamical systems [5], [6], system modeling [7], [8] and nonlinear prediction of time series [9]. The artificial neural network owns the capability, to organize its structural constituents, the same as the human brain. So the most attractive character of artificial neural network is that it can be taught to achieve the complex tasks we just experienced before by using some learning algorithms and training examples. The learning algorithms here can be roughly divided into two parts: one is the *supervised learning* and the other is the unsupervised learning. One most popular algorithm of artificial neural network for classification is the *Error Back-Propagation Algorithm* [10], [11], which is the supervised learning. The well-known error back-propagation algorithm, or simply the back-propagation algorithm, for training multi-layer perceptrons was proposed by Rumelhart in 1986 [12]. The back-propagation algorithm is a generalized form of the *delta learning rule,* which is actually based on the *least-mean square algorithm*. The topic of the training process is to minimize the standard *mean-square error*. Although the way to adjust the weights of network, i.e., the method of steepest descent, is easy to understand, there are several flaws in the back-propagation algorithm. One of them is that we don't have a suitable way to find the stable and optimal *learning rate*. For smaller learning rate, we may have a convergent result. But the speed of the output convergence is very slow and need more number of epochs to train the network. For larger learning rate, the speed of training can be accelerated, but it will cause the training result to fluctuate and even leads to divergent result. Actually the dynamical optimal training was proposed in [13] for a simple two layer neural network (without hidden layers) without any activation functions in the output layer. The basic theme in [13] is to find a stable and optimal learning for the next iteration in back propagation algorithm. Moreover a more complicated three layer neural network (with one hidden layer) with sigmoid activation functions in the hidden and output layers is very useful in performing the classification problems, such as the XOR [14] and Iris data [15], [16]. However its learning process has been very slow in terms of the classical back propagation algorithm. In other words, the dynamical optimal learning algorithm has never been proposed for this type of neural network. Therefore the major purpose of this thesis is to find a proper way to achieve the dynamical optimal training of the three layer neural network with sigmoid activation functions in hidden and output layers. Rigorous proof will be proposed and the popular XOR and Iris data classification benchmarks will be fully illustrated. Excellent results have been obtained by comparing our optimal training results with previous results using fixed small learning rates.

CHAPTER 2

The Perceptron as A Neural Network

In this chapter, the multi-layer feed-forward perceptrons will be introduced. First the single layer perceptron will be explained and it will lead to the multi-layer perceptrons in later sections. Also the back propagation algorithm for multi-layer feed forward perceptrons will be explained in section 2.4.

2.1. Single Layer Perceptron

The first model of the feed-forward network, ***perceptron***, was proposed by F. Rosenblatt in 1958 [17]. He hoped to find a suitable model to simulate the animal's brain and the visual system so that he proposed the "perceptron" model, which is a *supervised learning* model. The supervised learning is also referred to as learning with a teacher, and the teacher here means the input-output data sets for training. It also means that the perceptron can be trained by the given input-output data. The perceptron is a neuronal model consists of two parts. In the first part, the neural model combines all the input signals apply to its corresponding weights. And in second part, there comes the linear combiner followed by a hard limiter. The structure of the perceptron is depicted in Figure 2-1-1 (or Figure 2-1-2). The hard limiter is shown in Figure 2-2.

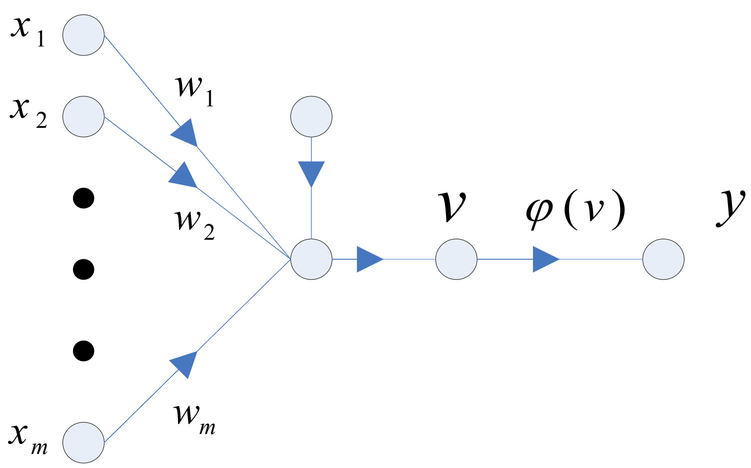


Figure 2-1-1. Single layer perceptron

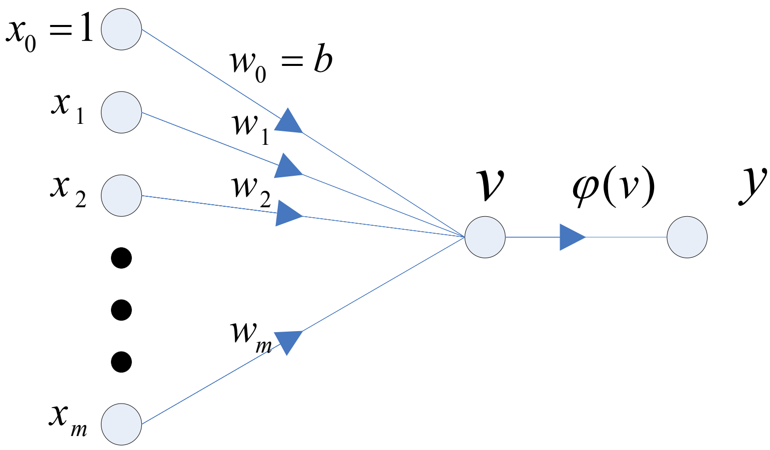


Figure 2-1-2. Another model of single layer perceptron

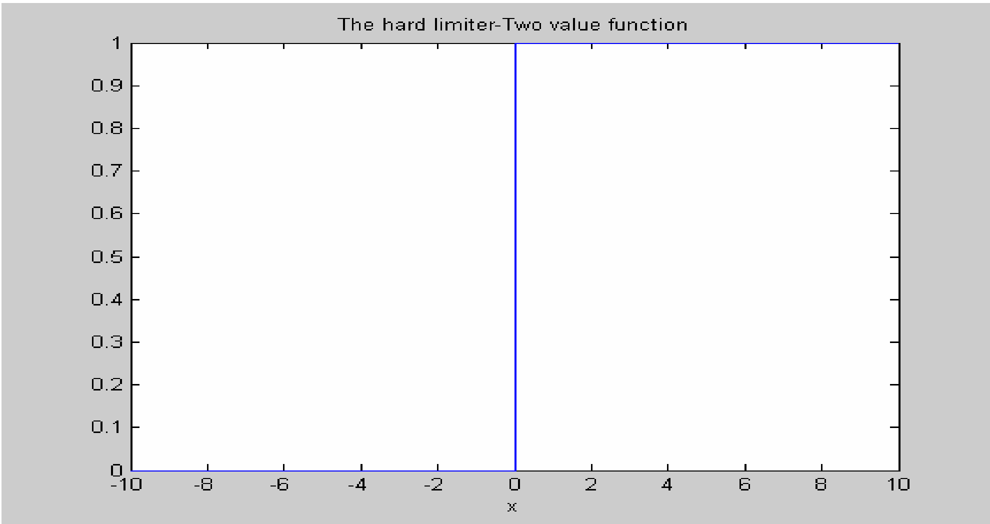


Figure 2-2. The hard limiter for perceptron

In Figure 2-1-1, the input data set of the perceptron is denoted by $\{x_1, x_2, \ldots, x_m\}$ and the corresponding synaptic weights of the perceptron are denoted by $\{w_1, w_2, \ldots, w_m\}$. The external bias is denoted by b. The first part of the percpetron computes the linear combination of the products of input data and synaptic weight with an externally applied bias. So the result of the first part of the perceptron, v, can be expressed as

$$v = b + \sum_{i=1}^{m} w_i x_i \tag{2.1}$$

Then in the second part, the resulted sum v is applied to a hard limiter. Therefore, the output of the perceptron equals +1 or 0. The output of perceptron equals to +1 if the resulted sum v is positive or zero; and the output of perceptron equals to 0 if v is negative. This can be simply expressed by (2.2).

$$y = \begin{cases} +1 \text{ , if } & b + \sum_{i=1}^{m} x_i w_i \geq 0 \\ 0 \text{ , if } & b + \sum_{i=1}^{m} x_i w_i < 0 \end{cases} \tag{2.2}$$

The goal of the perceptron is to classify the input data point represents by the set $\{x_1, x_2, \ldots, x_m\}$ into one of two classes, C_1 and C_2. If the output of the perceptron equals to +1, the input data point represented by the set $\{x_1, x_2, \ldots, x_m\}$will be assigned to class C_1. Otherwise, the input data point will be assigned to class C_2. Since the classification depends on the output of the perceptron, y, and the output y is decided by the resulted sum v. If we only consider the simplest form of the perceptron, the m-dimensional space can only be divided into two decision regions by the hyper-plane, which is defined as

$$b + \sum_{i=1}^{m} x_i w_i = 0 \tag{2.3}$$

One simple example is shown in Figure 2-3. In this case, there are only two input variables x_1 and x_2 for single layer perceptron, for which the decision hyper-plane is a straight line. The hyper-plane can be shown as:

$$b + x_1 w_1 + x_2 w_2 = 0 \tag{2.4}$$

Note that the shift distance of the decision line away from the origin is decided by the parameter b in (2.4) (and (2.3)).

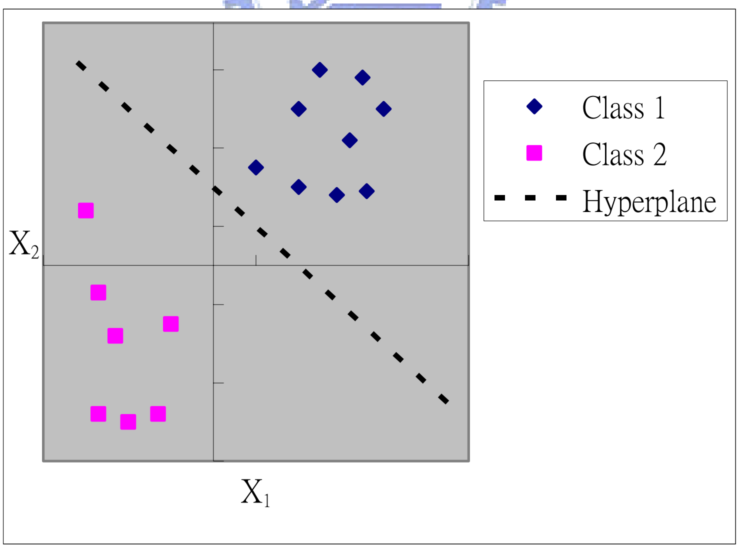


Figure 2-3. The decision boundary for 2-demensional plane

For convenience, we can consider the architecture of single layer perceptron in another form as depicted in Figure 2-1-2 and (2.1) can be rewritten as

$$v = \sum_{i=0}^{m} w_i x_i \tag{2.5}$$

In which, we substitute w_0 for the term b in (2.3) and $x_0 = 1$. The other results are all the same [18].

2.2. The Simple Examples

<u>Example</u> 1. The AND Problem

The **AND** logic problem, which can be solved by the single layer perceptron easily. The truth table for **AND** is shown in Table 2.1.

Table 2.1 The truth table for **AND**

A	B	A **AND** B
0	0	0 (Class 0)
0	1	0 (Class 0)
1	0	0 (Class 0)
1	1	1 (Class 1)

The input data set of the **AND** logic is shown in Figure 2-4, which is obviously a linearly separable problem. For solving this problem, we can build the single layer perceptron by assigning the suitable synaptic weights with external bias for. The architecture of the network is illustrated in Figure 2-5, in which the synaptic weights are assigned as $w_1 = w_2 = 1$ and the external bias $b = -1.5$.

Besides the **AND** logic, the **OR** and **NOT** logic problems are also linear separable. So these three logic problems can be solved by the same way easily. The only difference is the choice of the synaptic weights w_1, w_2 and external bias b.

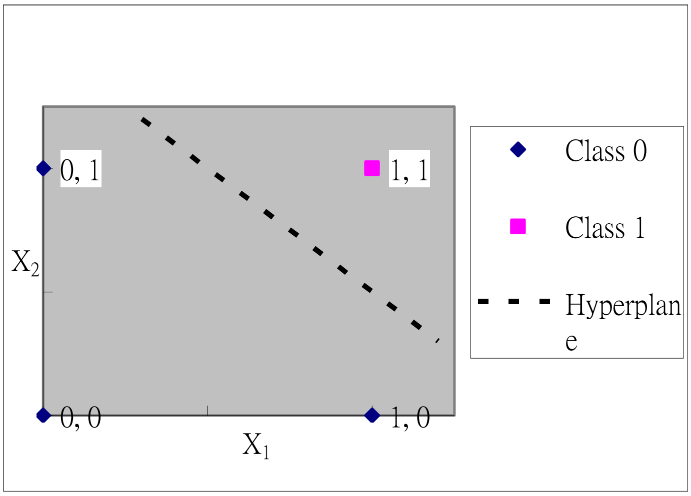


Figure 2-4. The input data of **AND**

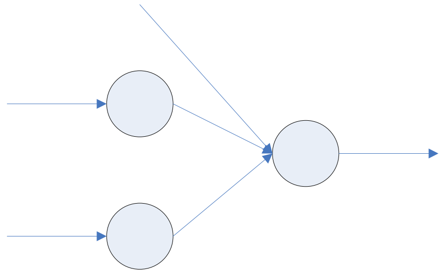


Figure 2-5. The architecture of the network for solving **AND** logic problem

<u>Example</u> 2. The XOR Problem

The Exclusive OR (**XOR**) logic problem can't be solved by single layer perceptron because the **XOR** logic is with nonlinear separable input data set. The truth table for **XOR** is shown in Table 2.2.

Table 2.2 The truth table for **XOR**

A	B	A **XOR** B
0	0	0 (Class 0)
0	1	1 (Class 1)
1	0	1 (Class 1)
1	1	0 (Class 0)

From Figure 2-6, we can see that the input data is not linear separable. So, the single layer perceptron can not work for this problem. We can not find the suitable parameters w_1, w_2 and b to classify the output y in Figure 2-6.

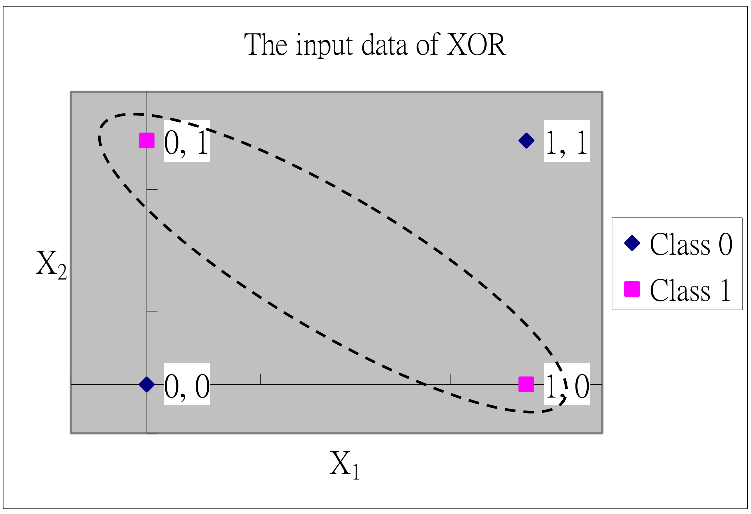


Figure 2-6. The input data of **XOR**

From the statements of section 2.1 and 2.2, we know that the single layer perceptron can only solve the problems which are linear separable. Besides this, another serious defect is that we have no proper learning algorithm to adjust the synaptic weights for the single layer perceptron, and all the parameters can only be assigned by try-and-error method. Until 1985, this problem was solved when the *Back-Propagation algorithm* was proposed. So the single layer perceptron was gradually replaced by Back-Propagation algorithm in the present day.

2.3. Multi-layer Feed-Forward Perceptrons

In this section we will introduce the multi-layer feed-forward network, an important class of neural network. The difference between single layer perceptron and multi-layer feed-forward perceptrons is the "*hidden layer*". The multi-layer network consists of a set of input nodes (input layer), one or more hidden layers, and a set of output nodes that constitute the output layer. The input signals will propagate through the network in the forward direction. A multi-layer feed-forward fully-connected network is shown in Figure 2-7 with only one hidden layer.

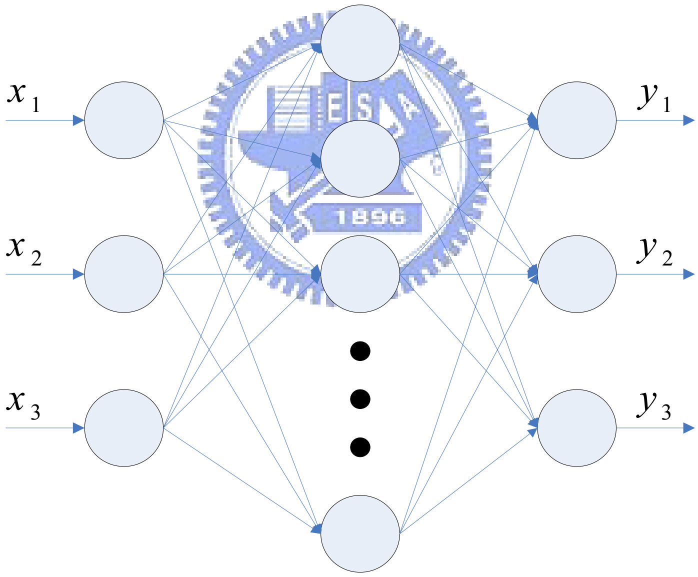


Figure 2-7. A three layer feed-forward network

The major characters of the multi-layer network are as follow:

(1) Besides the neurons of input layer, every neuron of the network has a *nonlinear activation function*. Note that the activation function used in the multi-layer network is *smooth*, means the activation function is differentiable everywhere, which is different from the activation function for single layer perceptron, the hard limiter shown in Fig. 2-2. The most popularly activation function is the *sigmoid function*, whose graph is s-shaped, as shown in Figure 2-8 with

$$y = \frac{1}{1 + \exp(-ax)} \tag{2.6}$$

where y is the output signal of the neuron and v is the input signal. And the parameter a is the slop parameter of the sigmoid function. We can get different sigmoid functions by varying the slop parameter a, but we usually choose the sigmoid function with a = 1. Equation (2.6) is defined as a strictly increasing function that exhibits a graceful balance between linear and nonlinear behaviors.

(2) The multi-layer network contains at least one hidden layer which is between input layer and output layer. The hidden layers enable the multi-layer networks to deal with more complex problems which can not be solved by single layer perceptron. The number of hidden layers for different problems is also an open question. We choose only one hidden layer with sigmoid activation functions in the hidden and output layers to solve the classification problem in this paper. This three layer perceptrons is also the most popular neural network adopted for many engineering applications.

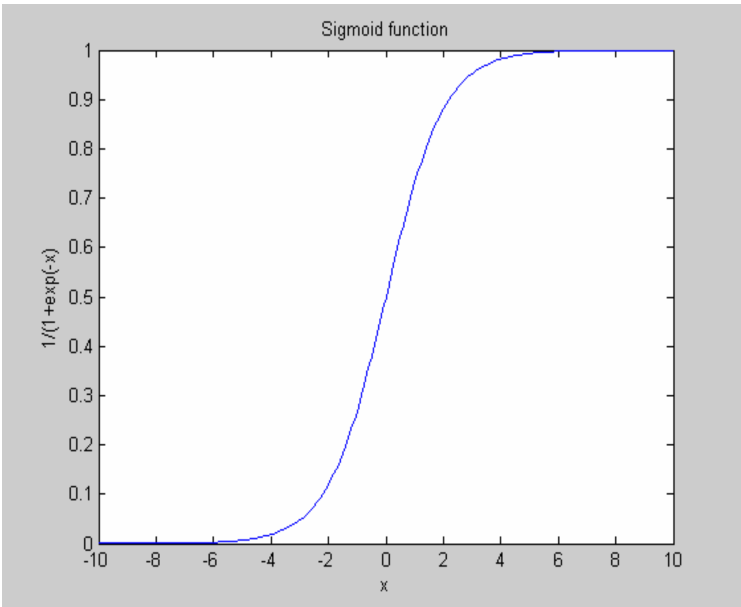


Figure 2-8. The sigmoid function

2.4. The Back-Propagation Algorithm (BPA)

The most serious problem of the single layer perceptron is that we don't have any proper learning algorithm to adjust the synaptic weights of the perceptron. But the multi-layer feed-forward perceptrons don't have this problem. The synaptic weights of the multi-layer perceptrons can be trained by using the highly popular algorithm known as the *error back-propagation algorithm (**EBP** algorithm)*. We can view the error back-propagation algorithm as the general form of the *least-mean square* *algorithm* (**LMS**). The error back-propagation algorithm (or the back-propagation algorithm) consists of two parts, one is the forward pass and the other is the backward pass. In the forward pass, the effect of the input data propagates through the network layer by layer. During this process, the synaptic weights of the networks are all *fixed*. On the other hand, the synaptic weights of the multi-layer perceptrons are all *adjusted* in accordance with the error-correction rule during the backward pass. Before using error-correction rule, we have to define the *error signal* of the learning process. Because the goal of the learning algorithm is to find one set of weights which makes the actual outputs of the network equal to the desired outputs, so we can define the error signal as the difference between the actual output and the desired output. Specifically, the desired response is subtracted from the actual response of the network to produce the error signal. In the backward pass, the error signal propagates backward through the network from output layer. During the backward pass, the synaptic weights of the multi-layer perceptrons are adjusted to make the actual outputs of the network move closer to the desired outputs. This is why we call this algorithm as "error back-propagation algorithm". Now, we will consider the learning process of the back-propagation algorithm. First, the error signal at the output node j at the t^{th} iteration is defined by

$$e_j(t) = y_j^m(t) - d_j(t) \tag{2.7}$$

where d_j is the desired output of output node j and $y^m{}_j$ is the actual output of output node j. The $y^m{}_j$ can also be represented as:

$$y_j^m = \varphi(net_j^m) = \frac{1}{1+\exp(-net_j^m)} \tag{2.8}$$

Where the term $\phi(\cdot)$ is the activation function, which is the sigmoid function, and $net^m{}_j$ is the output of node j in output layer. We can rewrite (2.8) as a more general form:

$$y_j^k = \varphi\left(net_j^k\right) = \frac{1}{1+\exp(-net_j^k)} \tag{2.9}$$

where $y^k{}_j$ is the output of node j in the k^{th} layer and $net^k{}_j$ is the linear combination output of node j in the k^{th} layer, which can be expressed as

$$net_j^k = \sum_i w_{ji}^k y_i^{k-1} - b_j^k \tag{2.10}$$

In (2.10), the term $w^k{}_{ji}$ is the synaptic weight between the j^{th} neuron in the k^{th} layer and the i^{th} neuron in the $(k\text{-}1)^{\text{th}}$ layer, and $b^k{}_j$ is the external bias of the j^{th} neuron in the k^{th} layer. Then we can define the square error of neuron j in output layer by:

$$\zeta_j = \frac{1}{2} e_j^2(t) \tag{2.11}$$

By the same way, we can define the total square error J of the network as:

$$J = \sum_j \zeta_j = \frac{1}{2}\sum_j e_j^2 \tag{2.12}$$

The goal of the back-propagation algorithm is to find one set of weights so that the actual outputs can be as close as possible to the desired outputs. In other words, the purpose of the back-propagation algorithm is to reduce the total square error J, as described in (2.12). In the method of steepest descent, the successive adjustments applied to the weight matrix $\boldsymbol{W}$ are in the direction opposite to the gradient matrix $\partial J / \partial W$. The adjustment can be expressed as:

$$\Delta W(t) = -\eta \left.\frac{\partial J}{\partial W}\right|_t \tag{2.13}$$

where η is the *learning rate parameter* of the back-propagation algorithm. It decides the step-size of correction of the method of steepest descent [18]. Note that the learning rate is a positive constant. By using the chain rule, the element of matrix $\partial J/\partial W$, i.e. $\partial J/\partial w_{ji}^k$, can be represented as

$$\frac{\partial J}{\partial w_{ji}^k} = \frac{\partial J}{\partial y_j^k}\frac{\partial y_j^k}{\partial net_j^k}\frac{\partial net_j^k}{\partial w_{ji}^k} \tag{2.14}$$

We can rewrite $\partial y_j^k / \partial net_j^k$ by using (2.9) to have

$$\frac{\partial y_j^k}{\partial net_j^k} = \varphi'\left(net_j^k\right) = \frac{\partial}{\partial net_j^k}\left(\frac{1}{1+\exp(-net_j^k)}\right) = y_j^k\left(1-y_j^k\right) \tag{2.15}$$

We can also rewrite $\partial net_j^k / \partial w_{ji}^k$ by using (2.10) to have

$$\frac{\partial net_j^k}{\partial w_{ji}^k} = \frac{\partial}{\partial w_{ji}^k}\left(\sum_i w_{ji}^k y_i^{k-1} - b_j\right) = y_i^{k-1} \tag{2.16}$$

The item $\partial J / \partial y_j^k$ in (2.14) can be discussed separately for two cases:

1. If k^{th} layer is the output layer, then $k = m$ and $y^k_j = y^m_j$. So we can substitute (2.12) into $\partial J / \partial y_j^k$ to have

$$\frac{\partial J}{\partial y_j^k} = \frac{\partial}{\partial y_j^k}\left[\frac{1}{2}\sum_j \left(y_j^m - d_j\right)^2\right] = \left(y_j^m - d_j\right) \tag{2.17}$$

2. If the k^{th} layer is not output layer which means the k^{th} layer is one of the hidden layers. So, by using the chain rule we can rewrite $\partial J / \partial y_j^k$ as

$$\frac{\partial J}{\partial y_j^k} = \sum_l \left(\frac{\partial J}{\partial net_l^{k+1}} \frac{\partial net_l^{k+1}}{\partial y_j^k} \right) \tag{2.18}$$

By using (2.10), the term $\partial net_l^{k+1} \big/ \partial y_j^k$

$$\frac{\partial net_l^{k+1}}{\partial y_j^k} = \frac{\partial}{\partial y_j^k} \left(\sum_i w_{li}^{k+1} y_i^k - b_l^{k+1} \right) = w_{lj}^{k+1} \tag{2.19}$$

where $w^{k+1}{}_{lj}$ is the synaptic weight between the j^{th} neuron in the $(k+1)^{\text{th}}$ layer and the l^{th} neuron in the k^{th} layer. For simplicity, we assume the item $\partial J / \partial net_l^{k+1}$ in (2.18) to have the following form:

$$\frac{\partial J}{\partial net_l^{k+1}} = \delta_l^{k+1} \tag{2.20}$$

Substitute (2.19) and (2.20) into (2.18), we have

$$\frac{\partial J}{\partial y_j^k} = \sum_l \left(\delta_l^{k+1} w_{lj}^{k+1} \right) \tag{2.21}$$

From (2.14) to (2.21), we have the following observations:

1. If the synaptic weights are between the hidden layer and output layer, then

$$\frac{\partial J}{\partial w_{ji}^k} = \left(y_j^m - d_j \right) y_j^k \left(1 - y_j^k \right) y_i^{k-1} \tag{2.22}$$

$$\Rightarrow \quad \Delta w_{ji}^k = -\eta \left(y_j^m - d_j \right) y_j^k \left(1 - y_j^k \right) y_i^{k-1} \tag{2.23}$$

2. If the synaptic weights are between the hidden layers or between the input layer and the hidden layer, then

$$\frac{\partial J}{\partial w_{ji}^k} = \left(\sum_l \left(\delta_l^{k+1} w_{lj}^{k+1} \right) \right) y_j^k \left(1 - y_j^k \right) y_i^{k-1} \tag{2.24}$$

$$\Rightarrow \quad \Delta w_{ji}^k = -\eta \left(\sum_l \left(\delta_l^{k+1} w_{lj}^{k+1} \right) \right) y_j^k \left(1 - y_j^k \right) y_i^{k-1} \tag{2.25}$$

Equation (2.23) and (2.25) are the most important formulas of the back-propagation algorithm. The synaptic weights can be adjusted by substituting (2.23) and (2.25) into the following (2.26):

$$w_{ji}^{k}\left(t+1\right)=w_{ji}^{k}\left(t\right)+\Delta w_{ji}^{k}\left(t\right) \tag{2.26}$$

The learning process of Back-Propagation learning algorithm can be expressed by the following steps:

Step1: Decide the structure of the network for the problem.

Step2: Choosing a suitable value between 0 and 1 for the learning rate η.

Step3: Picking the initial synaptic weights from a uniform distribution whose value is usually small, like between -1 and 1.

Step4: Calculate the output signal of the network by using (2.9).

Step5: Calculate the error energy function J by using (2.12).

Step6: Using (2.26) to update the synaptic weights.

Step7: Back to step4 and repeat step4 to step6 until the error energy function J is small enough.

Although the network with BPA can deal with more complex problems which can not be solved by single layer perceptron, the BPA still has the following problems:

1. Number of hidden layers

According to the result of theoretical researches, the number of hidden layer does not need over two layers. Although nearly all problems can be solved by two hidden layers or even one hidden layer, we really have no idea to choose the number of hidden layers. Even the number of neurons in the hidden layer is an open question.

2. Initial synaptic weights

One defect of the method of steepest descent is the "*local minimum*" problem. This problem relates to the initial synaptic weights. How to choose the best initial weights is still a topic of neural network.

3. The most suitable learning rate

The learning rate decides the step-size of learning process. Although smaller learning rate can have a better chance to have convergent results, the speed of convergence is very slow and need more number of epochs. For larger learning rate, it can speed up the learning process, but it will create more unpredictable results. So how to find a suitable learning rate is also an important problem of neural network.

CHAPTER 3

Dynamic Optimal Training of A Three-Layer Neural Network with Sigmoid Function

In this chapter, we will try to solve the problem of finding the suitable learning rate for the training of a three layer neural network. For solving this problem, we will find the dynamic optimal learning rate of every iteration. The neural network with one hidden layer is enough to solve the most problems in classification. So the dynamical optimal training algorithm will be proposed in this Chapter for a three layer neural network with sigmoid activation functions in the hidden and output layers in this chapter.

3.1. The Architecture of A Three-Layer Network

Consider the following three layer neural network, as shown in the Figure 3-1, which has only one hidden layer. The architecture of the neural network is *M-H-N* network. There are *M* neurons in the input layer and *H* neurons in the hidden layer and *N* neurons in the output layer.

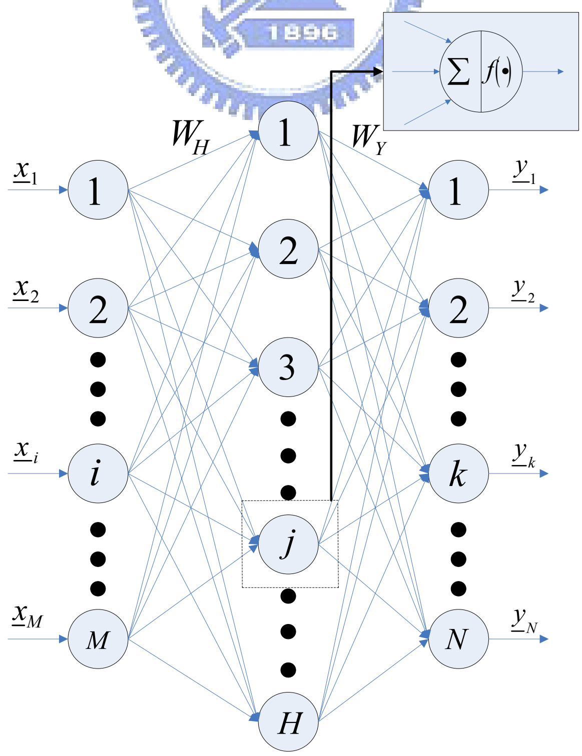


Figure 3-1. The three layer neural network

There is no activation function in the neurons of the input layer. And the activation function for the neurons in the hidden and output layers is defined by

$$f(x)=\frac{1}{1+\exp(-x)} \tag{3.1}$$

Note that we don't consider the bias in this chapter.

3.2. The Dynamic Optimal Learning Rate

Suppose we are given the following training matrix and the desired output matrix:

$$X_{M\times P}=\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1P} \\ x_{21} & x_{22} & \cdots & x_{2P} \\ \vdots & \vdots & \ddots & \vdots \\ x_{M1} & x_{M2} & \cdots & x_{MP} \end{bmatrix}_{M\times P} \tag{3.2}$$

$$D=\begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1P} \\ d_{21} & d_{22} & \cdots & d_{2P} \\ \vdots & \vdots & \ddots & \vdots \\ d_{N1} & d_{N2} & \cdots & d_{NP} \end{bmatrix}_{N\times P} \tag{3.3}$$

There are P columns in (3.2) and (3.3) which implies that there are P sets of training examples. And the weighting matrix between the input layer and the hidden layer is expressed as:

$$W_H=\begin{bmatrix} w_{11}^{hidden} & w_{12}^{hidden} & \cdots & w_{1M}^{hidden} \\ w_{21}^{hidden} & w_{22}^{hidden} & \cdots & w_{2M}^{hidden} \\ \vdots & \vdots & \ddots & \vdots \\ w_{H1}^{hidden} & w_{H2}^{hidden} & \cdots & w_{HM}^{hidden} \end{bmatrix}_{H\times M} \tag{3.4}$$

The weighting matrix between the hidden layer and the output layer is expressed as:

$$W_Y=\begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1H} \\ w_{21} & w_{22} & \cdots & w_{2H} \\ \vdots & \vdots & \ddots & \vdots \\ w_{N1} & w_{N2} & \cdots & w_{NH} \end{bmatrix}_{N\times H} \tag{3.5}$$

Then, we can get the linear combiner output of the hidden layer due to (3.2) and (3.4) which is expressed as:

$$S_H=W_H\cdot X=\begin{bmatrix} s_{11}^{hidden} & s_{12}^{hidden} & \cdots & s_{1P}^{hidden} \\ s_{21}^{hidden} & s_{22}^{hidden} & \cdots & s_{2P}^{hidden} \\ \vdots & \vdots & \ddots & \vdots \\ s_{H1}^{hidden} & s_{H2}^{hidden} & \cdots & s_{HP}^{hidden} \end{bmatrix}_{H\times P} \tag{3.6}$$

And by using (3.1) and (3.6), the output matrix of the hidden layer is expressed as below.

$$V=\begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1P} \\ v_{21} & v_{22} & \cdots & v_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ v_{H1} & v_{H2} & \cdots & v_{HP} \end{bmatrix}_{H\times P} = \begin{bmatrix} f\left(s_{11}^{hidden}\right) & f\left(s_{12}^{hidden}\right) & \cdots & f\left(s_{1P}^{hidden}\right) \\ f\left(s_{21}^{hidden}\right) & f\left(s_{22}^{hidden}\right) & \cdots & f\left(s_{2P}^{hidden}\right) \\ \vdots & \vdots & \ddots & \vdots \\ f\left(s_{H1}^{hidden}\right) & f\left(s_{H2}^{hidden}\right) & \cdots & f\left(s_{HP}^{hidden}\right) \end{bmatrix}_{H\times P}$$

$$=\begin{bmatrix} \frac{1}{1+e^{-s_{11}^{hidden}}} & \frac{1}{1+e^{-s_{12}^{hidden}}} & \cdots & \frac{1}{1+e^{-s_{1P}^{hidden}}} \\ \frac{1}{1+e^{-s_{21}^{hidden}}} & \frac{1}{1+e^{-s_{22}^{hidden}}} & \cdots & \frac{1}{1+e^{-s_{2P}^{hidden}}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{1+e^{-s_{H1}^{hidden}}} & \frac{1}{1+e^{-s_{H2}^{hidden}}} & \cdots & \frac{1}{1+e^{-s_{HP}^{hidden}}} \end{bmatrix}_{H\times P} \tag{3.7}$$

The linear combiner output of the output layer due to (3.5) and (3.7) is expressed as:

$$S_Y = W_Y \cdot V = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1P} \\ s_{21} & s_{22} & \cdots & s_{2P} \\ \vdots & \vdots & \ddots & \vdots \\ s_{N1} & s_{N2} & \cdots & s_{NP} \end{bmatrix}_{N\times P} \tag{3.8}$$

And the output of the output layer, which is also the actual output of the network:

$$Y=\begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1P} \\ y_{21} & y_{22} & \cdots & y_{2P} \\ \vdots & \vdots & \ddots & \vdots \\ y_{N1} & y_{N2} & \cdots & y_{NP} \end{bmatrix}_{N\times P} = \begin{bmatrix} f(s_{11}) & f(s_{12}) & \cdots & f(s_{1P}) \\ f(s_{21}) & f(s_{22}) & \cdots & f(s_{2P}) \\ \vdots & \vdots & \ddots & \vdots \\ f(s_{N1}) & f(s_{N2}) & \cdots & f(s_{NP}) \end{bmatrix}_{N\times P}$$

$$=\begin{bmatrix} \frac{1}{1+e^{-s_{11}}} & \frac{1}{1+e^{-s_{12}}} & \cdots & \frac{1}{1+e^{-s_{1P}}} \\ \frac{1}{1+e^{-s_{21}}} & \frac{1}{1+e^{-s_{22}}} & \cdots & \frac{1}{1+e^{-s_{2P}}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{1+e^{-s_{N1}}} & \frac{1}{1+e^{-s_{N2}}} & \cdots & \frac{1}{1+e^{-s_{NP}}} \end{bmatrix}_{N\times P} \tag{3.9}$$

Substituting (3.8) into (3.1) then we can get (3.9).

The most important parameter of the BP algorithm is the error signal. And the error signal is the difference between the actual output of the network and the desire output. So, we define the error function E as:

$$E = Y - D = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1P} \\ e_{21} & e_{22} & \cdots & e_{2P} \\ \vdots & \vdots & \ddots & \vdots \\ e_{N1} & e_{N2} & \cdots & e_{NP} \end{bmatrix}_{N\times P}$$

$$= \begin{bmatrix} y_{11}-d_{11} & y_{12}-d_{12} & \cdots & y_{1P}-d_{1P} \\ y_{21}-d_{21} & y_{22}-d_{22} & \cdots & y_{2P}-d_{2P} \\ \vdots & \vdots & \ddots & \vdots \\ y_{N1}-d_{N1} & y_{N2}-d_{N2} & \cdots & y_{NP}-d_{NP} \end{bmatrix}_{N\times P} \tag{3.10}$$

Finial, we define the total squared error, the energy function of the network, J as follows:

$$J = \frac{1}{2}\sum_{p=1}^{P}\sum_{k=1}^{N}\left(y_{kp} - d_{kp}\right)^2 = \frac{1}{2}Tr\left(E^T \bullet E\right) \tag{3.11}$$

The topic of the learning process is to minimize the total square error J. According to Back-propagation algorithm, we use the method of steepest descent to adjust the synaptic weights. And we apply formula (3.12) and (3.14) to update the weighting matrixes W_H and W_Y. Equation (3.12) and (3.14) are expressed as follow.

$$W_H\left(t+1\right) = W_H\left(t\right) - \beta_H \left.\frac{\partial J}{\partial W_H}\right|_t \tag{3.12}$$

or $$w_{ji}^{hidden}\left(t+1\right) = w_{ji}^{hidden}\left(t\right) - \beta_Y \cdot \left.\frac{\partial J}{\partial w_{ji}^{hidden}}\right|_t \ , \quad \left(j = 1,2,\cdots,H \ ; i = 1,2,\cdots,M\right)$$

(3.13)

$$W_Y\left(t+1\right) = W_Y\left(t\right) - \beta_Y \left.\frac{\partial J}{\partial W_Y}\right|_t \tag{3.14}$$

or $$w_{kj}\left(t+1\right) = w_{kj}\left(t\right) - \beta_Y \cdot \left.\frac{\partial J}{\partial w_{kj}}\right|_t \ , \quad \left(k = 1,2,\cdots,N \ ; j = 1,2,\cdots,H\right)$$

(3.15)

Where t denotes the t^{th} iteration and β_H and β_Y are the learning rate of W_H and W_Y respectively.

Now, let us consider the derivative of (3.15) only,

$$\frac{\partial J}{\partial w_{kj}} = \frac{\partial J}{\partial y_{k1}}\cdot\frac{\partial y_{k1}}{\partial s_{k1}}\cdot\frac{\partial s_{k1}}{\partial w_{kj}} + \frac{\partial J}{\partial y_{k2}}\cdot\frac{\partial y_{k2}}{\partial s_{k2}}\cdot\frac{\partial s_{k2}}{\partial w_{kj}} + \cdots + \frac{\partial J}{\partial y_{kP}}\cdot\frac{\partial y_{kP}}{\partial s_{kP}}\cdot\frac{\partial s_{kP}}{\partial w_{kj}}$$

$$= \sum_{p=1}^{P} \frac{\partial J}{\partial y_{kp}} \cdot \frac{\partial y_{kp}}{\partial s_{kp}} \cdot \frac{\partial s_{kp}}{\partial w_{kj}} \tag{3.16}$$

In which,

$$\frac{\partial J}{\partial y_{kp}} = \frac{\partial}{\partial y_{kp}} \left[\frac{1}{2} \sum_{p=1}^{P} \sum_{k=1}^{N} \left(y_{kp} - d_{kp} \right)^2 \right] = \left(y_{kp} - d_{kp} \right) \tag{3.17}$$

$$\frac{\partial y_{kp}}{\partial s_{kp}} = \frac{\partial}{\partial s_{kp}} \left(1 + e^{-s_{kp}} \right)^{-1} = \left(1 + e^{-s_{kp}} \right)^{-2} \cdot e^{-s_{kp}} = y_{kp} \left(1 - y_{kp} \right) \tag{3.18}$$

$$\frac{\partial s_{kp}}{\partial w_{kj}} = \frac{\partial}{\partial w_{kj}} \left(w_{k1} v_{1p} + w_{k2} v_{2p} + \cdots + w_{kH} v_{Hp} \right) = v_{jp} \tag{3.19}$$

Accordingly, the use of (3.17)、(3.18) and (3.19) in (3.16) yields

$$\frac{\partial J}{\partial w_{kj}} = \sum_{p=1}^{P} \left(y_{kp} - d_{kp} \right) \cdot y_{kp} \left(1 - y_{kp} \right) \cdot v_{jp} \tag{3.20}$$

So, we can rewrite (3.15) as

$$w_{kj}\left(t+1 \right) = w_{kj}\left(t \right) - \beta_Y \cdot \sum_{p=1}^{P} \left(y_{kp} - d_{kp} \right) \cdot y_{kp} \left(1 - y_{kp} \right) \cdot v_{jp} \Big|_t$$

$$\left(k = 1, 2, \cdots, N \ ; j = 1, 2, \cdots, H \right) \tag{3.21}$$

Equation (3.21) is the formula that we use to adjust the synaptic weights W_Y. Furthermore, let us consider the weighting matrix W_H between the input layer and the hidden layer, as expressed in (3.13):

$$w_{ji}^{hidden}\left(t+1 \right) = w_{ji}^{hidden}\left(t \right) - \beta_Y \cdot \frac{\partial J}{\partial w_{ji}^{hidden}} \Big|_t \tag{3.13}$$

For the convenience, we just discuss the derivative of (3.13) in the beginning.

$$\frac{\partial J}{\partial w_{ji}^{hidden}} = \frac{\partial J}{\partial v_{j1}} \frac{\partial v_{j1}}{\partial w_{ji}^{hidden}} + \frac{\partial J}{\partial v_{j2}} \frac{\partial v_{j2}}{\partial w_{ji}^{hidden}} + \cdots + \frac{\partial J}{\partial v_{jP}} \frac{\partial v_{jP}}{\partial w_{ji}^{hidden}}$$

$$= \left(\frac{\partial J}{\partial y_{11}} \frac{\partial y_{11}}{\partial v_{j1}} + \frac{\partial J}{\partial y_{21}} \frac{\partial y_{21}}{\partial v_{j1}} + \cdots + \frac{\partial J}{\partial y_{N1}} \frac{\partial y_{N1}}{\partial v_{j1}} \right) \cdot \frac{\partial v_{j1}}{\partial w_{ji}^{hidden}} +$$

$$\left(\frac{\partial J}{\partial y_{12}} \frac{\partial y_{12}}{\partial v_{j2}} + \frac{\partial J}{\partial y_{22}} \frac{\partial y_{22}}{\partial v_{j2}} + \cdots + \frac{\partial J}{\partial y_{N2}} \frac{\partial y_{N2}}{\partial v_{j2}} \right) \cdot \frac{\partial v_{j2}}{\partial w_{ji}^{hidden}} +$$

$$\cdots\cdots\cdots +$$

$$\left(\frac{\partial J}{\partial y_{1P}}\frac{\partial y_{1P}}{\partial v_{jP}}+\frac{\partial J}{\partial y_{2P}}\frac{\partial y_{2P}}{\partial v_{jP}}+\cdots+\frac{\partial J}{\partial y_{NP}}\frac{\partial y_{NP}}{\partial v_{jP}}\right)\cdot\frac{\partial v_{jP}}{\partial w_{ji}^{hidden}}$$

$$=\sum_{p=1}^{P}\frac{\partial v_{jp}}{\partial w_{ji}^{hidden}}\sum_{k=1}^{N}\left(\frac{\partial J}{\partial y_{kp}}\frac{\partial y_{kp}}{\partial v_{jp}}\right)$$

$$=\sum_{p=1}^{P}\frac{\partial v_{jp}}{\partial s_{jp}^{hidden}}\frac{\partial s_{jp}^{hidden}}{\partial w_{ji}^{hidden}}\sum_{k=1}^{N}\left(\frac{\partial J}{\partial y_{kp}}\frac{\partial y_{kp}}{\partial v_{jp}}\right) \tag{3.22}$$

In which,

$$\frac{\partial y_{kp}}{\partial v_{jp}}=\frac{\partial y_{kp}}{\partial s_{kp}}\frac{\partial s_{kp}}{\partial v_{jp}}=y_{kp}\left(1-y_{kp}\right)\cdot w_{kj} \tag{3.23}$$

In which we substitute (3.18) into (3.23).

$$\frac{\partial v_{jp}}{\partial s_{jp}^{hidden}}=\frac{\partial}{\partial s_{jp}^{hidden}}\left(1+e^{-s_{jp}^{hidden}}\right)^{-1}=\left(1+e^{-s_{jp}^{hidden}}\right)^{-2}e^{-s_{jp}^{hidden}}=v_{jp}\left(1-v_{jp}\right) \tag{3.24}$$

$$\frac{\partial s_{jp}^{hidden}}{\partial w_{ji}^{hidden}}=\frac{\partial}{\partial w_{ji}^{hidden}}\left(w_{j1}^{hidden}x_{1p}+w_{j2}^{hidden}x_{2p}+\cdots+w_{jM}^{hidden}x_{Mp}\right)=x_{jp} \tag{3.25}$$

Accordingly, the use of (3.17)、(3.23)、(3.24) and (3.25) in (3.22) yields

$$\frac{\partial J}{\partial w_{ji}^{hidden}}=\sum_{p=1}^{P}\left[v_{jp}\left(1-v_{jp}\right)\cdot x_{ip}\cdot\sum_{k=1}^{N}\left(\left(y_{kp}-d_{kp}\right)\cdot y_{kp}\left(1-y_{kp}\right)\cdot w_{kj}\right)\right] \tag{3.26}$$

So, we can rewrite (3.13) as

$$w_{ji}^{hidden}\left(t+1\right)=w_{ji}^{hidden}\left(t\right)-\beta_H\cdot\left\{\sum_{p=1}^{P}\left[v_{jp}\left(1-v_{jp}\right)\cdot x_{ip}\cdot\sum_{k=1}^{N}\left(\left(y_{kp}-d_{kp}\right)\cdot y_{kp}\left(1-y_{kp}\right)\cdot w_{kj}\right)\right]\right\}$$

$$,\qquad \left(j=1,2,\cdots,H\ ;i=1,2,\cdots,M\right) \tag{3.27}$$

Equation (3.27) is the formula that we use to adjust the synaptic weights W_H. By using (3.21) and (3.27), we can adjust the synaptic weights of the network.

3.3. Dynamical Optimal Training via Lyapunov's Method

In control system, we know that we can use the *Lyapunov function* to consider the stability of the system. The basic philosophy of Lyapunov's direct method is the mathematical extension of a fundamental physical observation: if the total energy of a mechanical (or electrical) system is continuously dissipated, then the system, whether linear or nonlinear must eventually settle down to an equilibrium point [19]. So by the same meaning, we define the Lyapunov function as

$$V = J \tag{3.28}$$

Where the item J is total square error, defined in (3.11). And Equation (3.28) is positive definite, which means that $V = J > 0$. The difference of the Lyapunov function is

$$\Delta V = J_{t+1} - J_t \tag{3.29}$$

Where J_{t+1} expresses the total square error of the $(t+1)^{\text{th}}$ iteration. If Equation (3.29) is negative, the system is guaranteed to be stable. Then, for $\triangle V < 0$ we have

$$J_{t+1} - J_t < 0 \tag{3.30}$$

So, by using (3.11) we can get

$$\begin{aligned} J_{t+1} - J_t &= \frac{1}{2}\sum_{p=1}^{P}\sum_{k=1}^{N}\left(y_{kp}(t+1) - d_{kp}\right)^2 - \frac{1}{2}\sum_{p=1}^{P}\sum_{k=1}^{N}\left(y_{kp}(t) - d_{kp}\right)^2 \\ &= \frac{1}{2}\sum_{p=1}^{P}\sum_{k=1}^{N}\left[\left(y_{kp}(t+1) - d_{kp}\right)^2 - \left(y_{kp}(t) - d_{kp}\right)^2\right] \\ &= G\left(\beta_H(t), \beta_Y(t)\right) \end{aligned} \tag{3.31}$$

In Equation (3.31), the item J_t is already known and the J_{t+1} is the function of $\beta_H(t)$ and $\beta_Y(t)$, so J_{t+1}-J_t can be expressed as a new function G of $\beta_H(t)$ and $\beta_Y(t)$. For simplification, we can assume that $\beta_H(t) = \beta_Y(t) = \beta(t)$, then the function G only has one parameter $G(\beta(t))$. The Equation (3.31) can be rewritten as

$$J_{t+1} - J_t = G\left(\beta(t)\right) \tag{3.32}$$

If the parameter $\beta(t)$ satisfies $J_{t+1} - J_t = G(\beta(t)) < 0$, then the set of $\beta(t)$ is the stable range of the learning rate of the system at the t^{th} iteration. In this stable range, if the $\beta_{opt}(t)$ satisfies that $J_{t+1} - J_t$ is at its minimum, we call $\beta_{opt}(t)$ the optimal learning rate at the t^{th} iteration. The optimal learning rate $\beta_{opt}(t)$ will not only guarantee the stability of the training process, but also has the fastest speed of convergence.

In order to find the optimal learning rate $\beta_{opt}(t)$ from the function $J_{t+1} - J_t$ analytically, we need to have an explicit form of $J_{t+1} - J_t$, like the simplest form of $J_{t+1} - J_t$ (for a simple two layer neural network) in [13]. But the function $J_{t+1} - J_t$ here is a very complicated nonlinear algebraic equation, it is nearly impossible to have a simple explicit form. However we have also defined the function $J_{t+1} - J_t$ in (3.31) by progressively evolving the equations from the beginning of this Chapter. Therefore we can indeed defined (3.31) implicitly in Matlab coding. In this case, we can apply the Matlab routine ***fminbnd*** to find the optimal learning rate $\beta_{opt}(t)$ from $J_{t+1} - J_t$. The calling sequence of ***fminbnd*** is:

FMINBND *Scalar bounded nonlinear function minimization.*
X = FMINBND(FUN,x1,x2) starts at X0 and finds a local minimizer X of the function FUN in the interval x1 <= X <= x2. FUN accepts scalar input X and returns a scalar function value F evaluated at X.

The capability of the Matlab routine "***fminbnd***" is to find a local minimizer β_{opt} of the function $G(\beta)$, which has only one independent variable β, in a given interval. So we have to define an interval when we use this routine. For the following two examples in Chapter 4, we set the interval as [0.01, 100] to set the allowable learning rates between 0.01 and 100. Note that for simplicity, we assume that $\beta_H(t) = \beta_Y(t) = \beta(t)$ in (3.31), therefore there is only one variable in (3.31). So we use the routine "***fminbnd***" to find the optimal learning rate. However, we can also find the learning rate $\beta_H(t)$ and $\beta_Y(t)$ respectively by using another Matlab routine "***fminunc***". But this routine "***fminunc***" can only find the minimizer $\beta_H(t)$ and $\beta_Y(t)$ around one specific point, not around an interval. Therefore it is very limited in application and is not appropriate for this case.

Algorithm 1: Dynamic optimal training algorithm for a three-layer neural network

Step 0: The following $W_H(t)$, $W_Y(t)$, $G(\beta(t))$, $\beta_{opt}(t)$ and $Y(t)$ denote their respective values at iteration t.

Step 1: Given the initial weighting matrix $W_H(1)$, $W_Y(1)$, the training input matrix X and the desired output matrix D then we can find the actual output matrix of the network $Y(1)$ and the nonlinear function $G(\beta(1))$.

Step 2: Using Matlab routine "***fminbnd***" with the interval [0.01, 100] to solve the nonlinear function $G(\beta(1))$ and find the optimal learning rate $\beta_{opt}(1)$.

Step 3: Iteration count t=1. Start the back propagation training process.

Step 4: Find if the desired output matrix D and the actual output matrix of the network $Y(1)$ are close enough or not? If Yes, GOTO Step 9.

Step 5: Update the synaptic weights matrix to yield $W_H(t+1)$ and $W_Y(t+1)$ by using (3.27) and (3.21) respectively.

Step 6: Find the actual output matrix of the network $Y(t+1)$ and the nonlinear function $G(\beta(t+1))$.

Step 7: Use Matlab routine "***fminbnd***" to find the optimal learning rate $\beta_{opt}(t+1)$ for the next iteration.

Step 8: $t=t+1$ and GOTO Step 4.

Step 9: End.

CHAPTER 4

Experimental Results

In this chapter, the classification problems of XOR and Iris data will be solved via our new dynamical optimal training algorithm in Chapter 3. The training results will be compared with the conventional BP training using fixed learning rate.

4.1. Example 1: The XOR Problem

The task is to train the network to produce the Boolean “Exclusive OR” (**XOR**) function of two variables. The **XOR** operator yields true if exactly one (but not both) of two conditions is true, otherwise the **XOR** operator yields false. We need only consider four input data (0,0), (0,1), (1,1), and (1,0) in this problem. The first and third input patterns are in class 0, which means the **XOR** operator yields “**False**” when the input data is (0,0) or (1,1). The distribution of the input data is shown in Figure 4-1. Because there are two variables of **XOR** function, we choose the input layer with two nodes and the output layer with one node. Then we use one hidden layer with two neurons to solve **XOR** problem [14], as shown in Figure 4-2. The architecture of the neural network is 2-2-1 network.

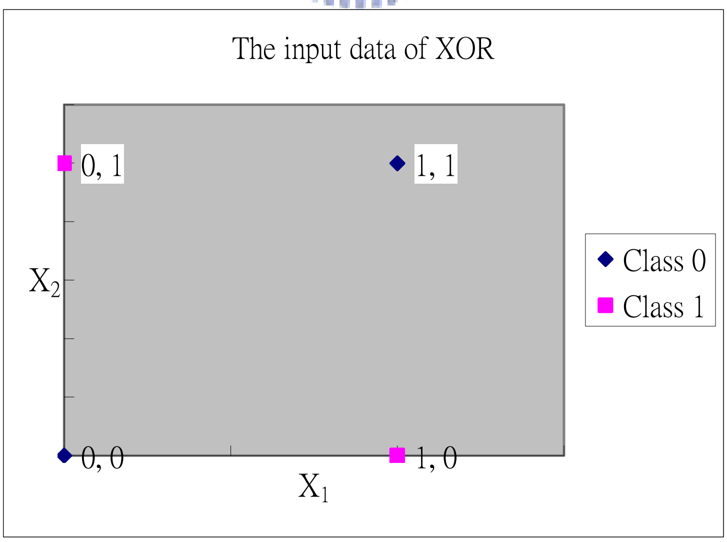


Figure 4-1. The distribution of **XOR** input data sets

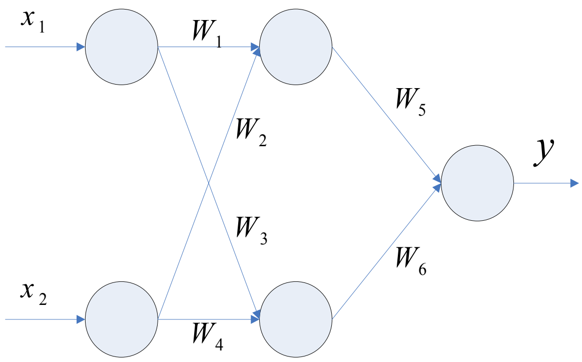
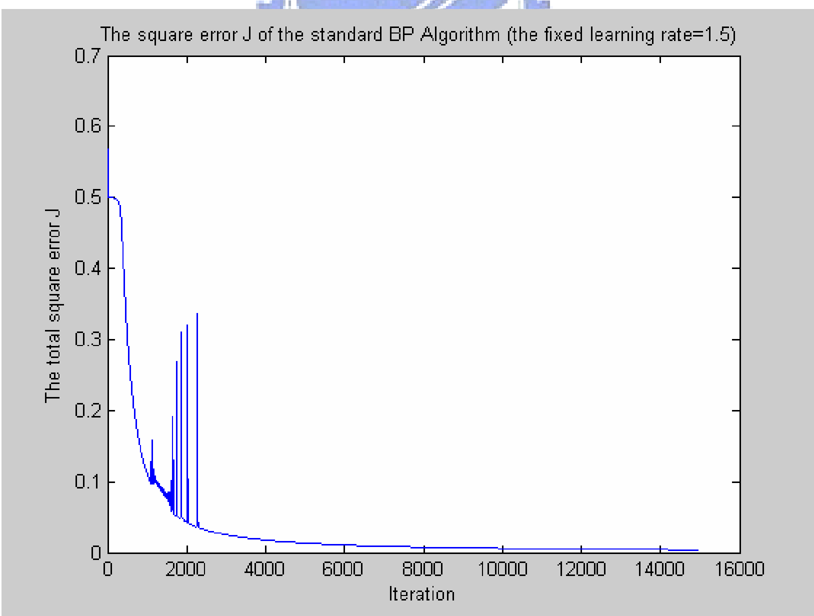


Figure 4-2. The neural network for solving **XOR**

First, we use the standard BP algorithm with fixed learning rates (β = 1.5, 0.9, 0.5 and 0.1) to train the XOR, and the training results are shown in Figure 4-3-1 ~ 4-3-4. The result of using BP algorithm with dynamic optimal learning rates to train the XOR is shown Figure 4-4.



ut layer Hidde

Figure 4-3-1. The square error J of the standard BPA with fixed β = 1.5

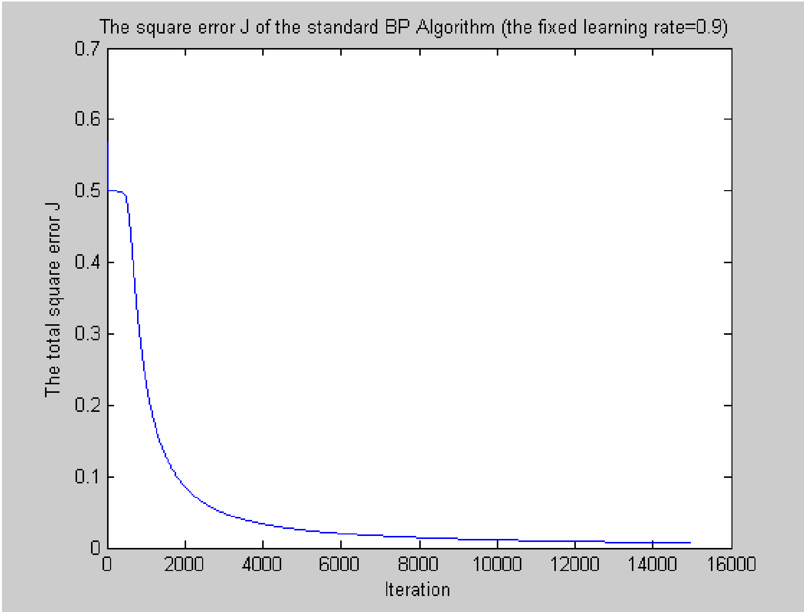


Figure 4-3-2. The square error J of the standard BPA with fixed $\beta = 0.9$

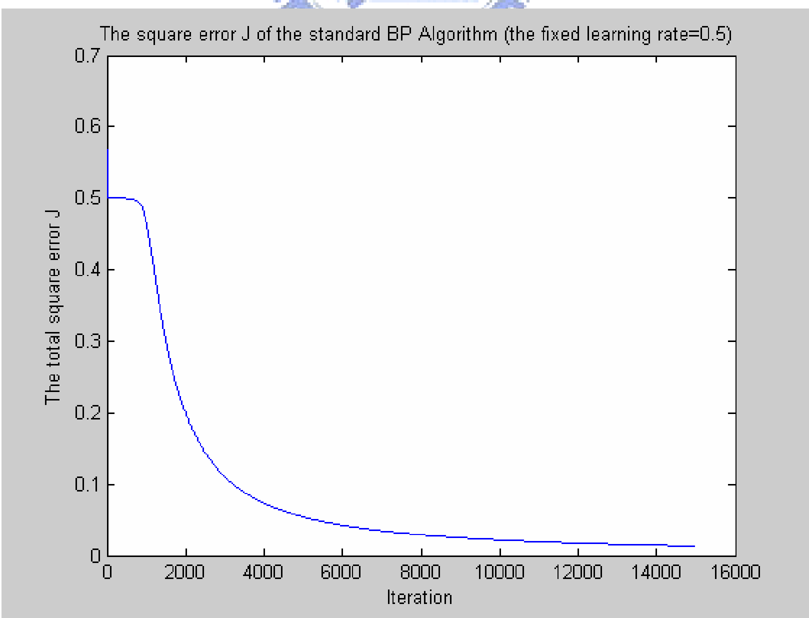


Figure 4-3-3. The square error J of the standard BPA with fixed $\beta = 0.5$

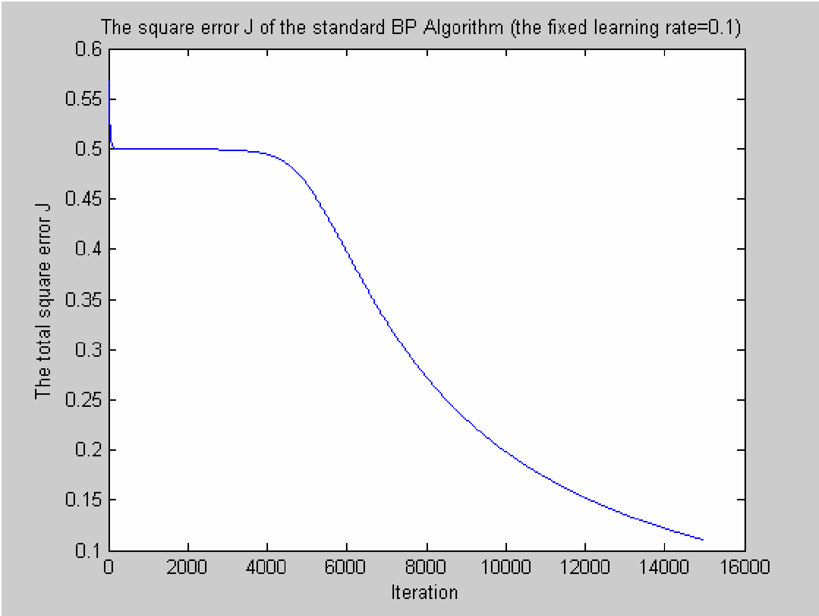


Figure 4-3-4. The square error J of the standard BPA with fixed $\beta = 0.1$

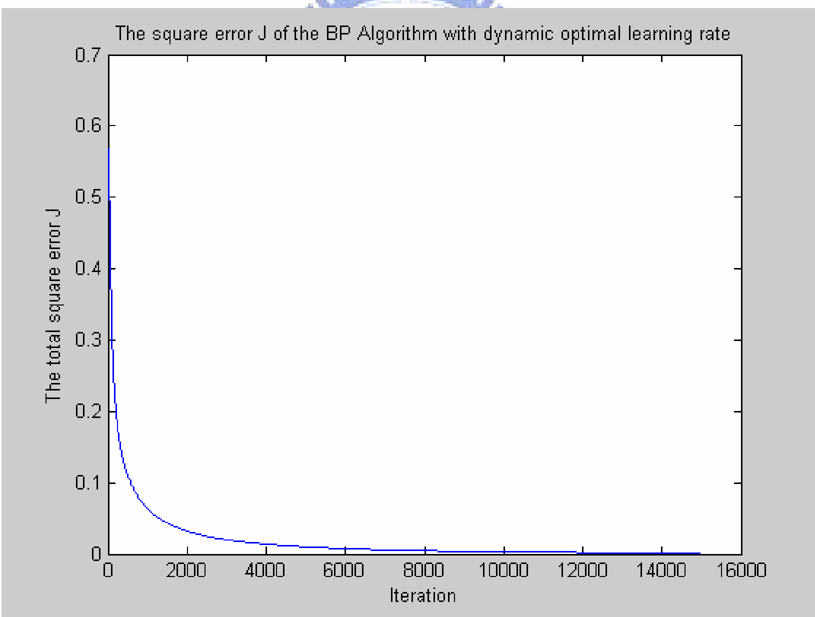


Figure 4-4. The square error J of the BPA with dynamic optimal training

The following Figure 4-5 shows the plot of (3.32) for $-1 < \beta < 100$, which is $G(\beta) = \Delta J(\beta) = J_{t+1} - J_t$, at iteration count $t = 1$. The Matlab routine *fminbnd* will be invoked to find β_{opt} with the constraint that $G(\beta_{\mathrm{opt}}) < 0$ with maximum absolute value. This β_{opt} is the learning rate for iteration count $t = 2$.The β_{opt} is found to be 7.2572 for iteration count 2. The dynamic learning rate of every iteration is shown in Figure 4-6.

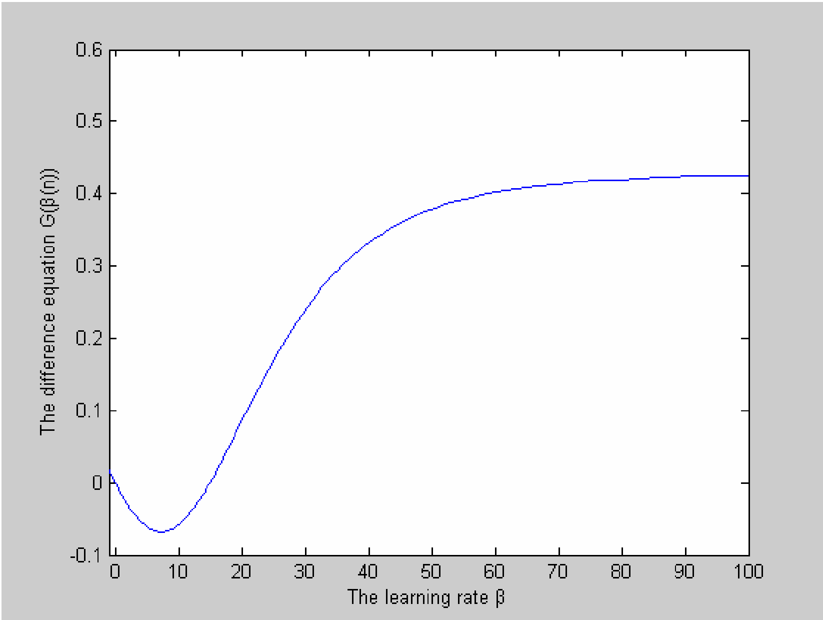


Figure 4-5. The difference equation $G(\beta(n))$ and $\beta_{opt} = 7.2572$

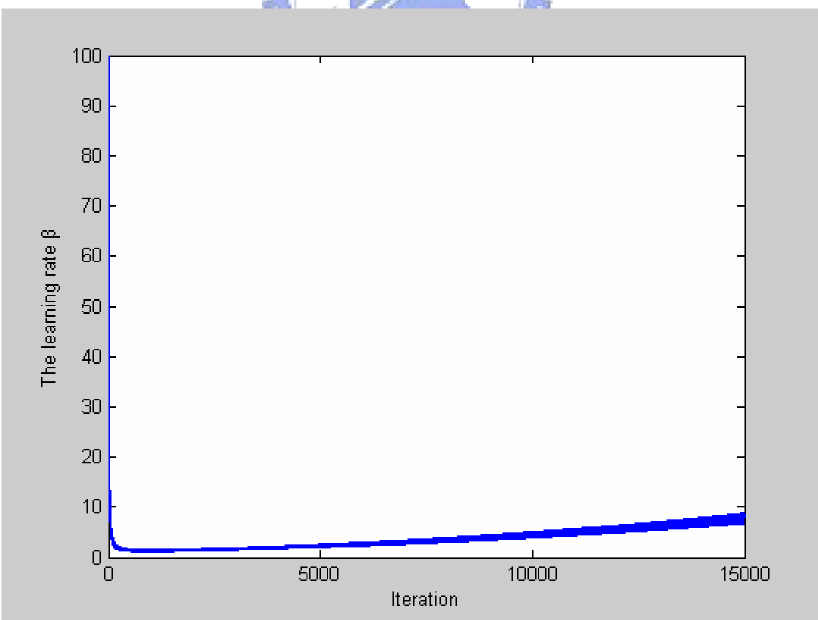


Figure 4-6. The dynamic learning rates of every iteration

The comparison of these cases is shown in Figure 4-7. In Figure 4-7, it is obvious that our dynamical optimal training yields the best training results in minimum epochs.

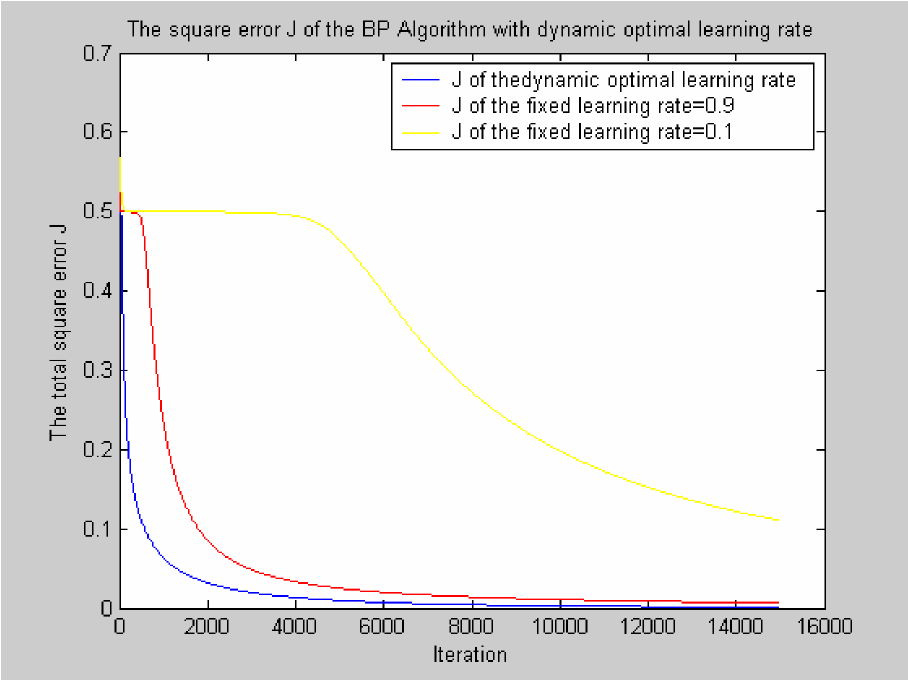


Figure 4-7. Training errors of dynamic optimal learning rates and fixed learning rates

Table 4-1 shows the training result via dynamical optimal training for **XOR** problem.

Table 4.1. The training result for **XOR** using dynamical optimal training

Iterations / Training Results	1000	5000	10000	15000
W_1 (after trained)	6.5500	7.7097	8.8191	8.4681
W_2 (after trained)	6.5652	7.7145	8.1921	8.4703
W_3 (after trained)	0.8591	0.9265	0.9473	0.9573
W_4 (after trained)	0.8592	0.9265	0.9473	0.9573
W_5 (after trained)	14.9536	26.2062	33.0393	37.8155
W_6 (after trained)	-19.0670	-32.9550	-41.3692	-47.2513
Actual Output Y for $(x_1, x_2) = (0,0)$	0.1134	0.0331	0.0153	0.0089
Actual Output Y for $(x_1, x_2) = (0,1)$	0.8232	0.9300	0.9616	0.9750
Actual Output Y for $(x_1, x_2) = (1,0)$	0.8232	0.9300	0.9616	0.9750
Actual Output Y for $(x_1, x_2) = (1,1)$	0.2291	0.0925	0.0511	0.0334
J	0.0639	0.0097	0.0029	0.0012

Table 4-2 shows the training result via the standard BP with fixed $\beta = 0.9$ for **XOR** problem.

Table 4.2. The training result for **XOR** using fixed learning rate $\beta = 0.9$

Iterations / Training Results	1000	5000	10000	15000
W_1 (after trained)	4.7659	7.2154	7.6576	7.8631
W_2 (after trained)	4.8474	7.2228	7.6624	7.8670
W_3 (after trained)	0.7199	0.8996	0.9234	0.9331
W_4 (after trained)	0.7228	0.8996	0.9234	0.9331
W_5 (after trained)	6.2435	20.6467	25.5617	28.2288
W_6 (after trained)	-8.1214	-26.1034	-32.1610	-35.4456
Actual Output Y for $(x_1, x_2) = (0,0)$	0.2811	0.0613	0.0356	0.0264
Actual Output Y for $(x_1, x_2) = (0,1)$	0.6742	0.8885	0.9263	0.9415
Actual Output Y for $(x_1, x_2) = (1,0)$	0.6745	0.8885	0.9263	0.9415
Actual Output Y for $(x_1, x_2) = (1,1)$	0.4192	0.1479	0.0982	0.0781
J	0.2334	0.0252	0.0109	0.0068

To compare Table 4.1 with Table 4.2, we can see that the training result via dynamical optimal training is faster with better result than other approaches.

Now, we will use another method, the back-propagation algorithm with momentum, to solve **XOR** problem again. Then we will compare its training errors with that of dynamic optimal training and see if dynamic optimal training is indeed better. The back-propagation algorithm with momentum is to modify the Equation (2.13) by including a momentum term as follows:

$$\Delta W(t) = -\eta \left.\frac{\partial J}{\partial W}\right|_t + \alpha \cdot \Delta W(t-1) \tag{4.1}$$

where α is usually a positive number called the *momentum constant* and usually in the range [0, 1). The training results of the BPA with momentum for **XOR** problem are shown in Figure 4-8-1 ~ 4-8-3. The comparison of these cases is shown in Figure 4-9. In Figure 4-9, we can see that some training results of BPA with momentum are as well as the dynamic training but the most results of BPA with momentum are unpredictable. The training results still depend on the chosen learning rates and momentum.

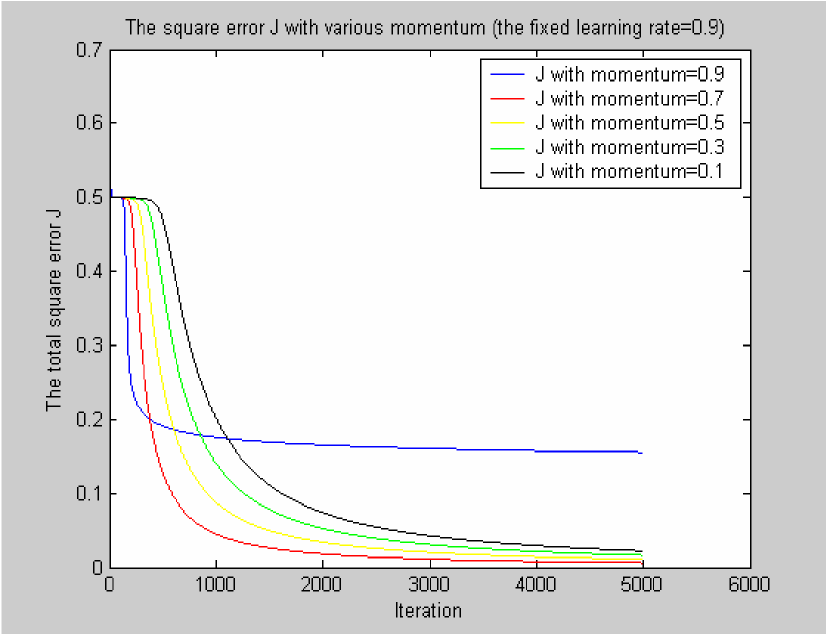


Figure 4-8-1. The square error J of the BPA with variant momentum($\beta = 0.9$)

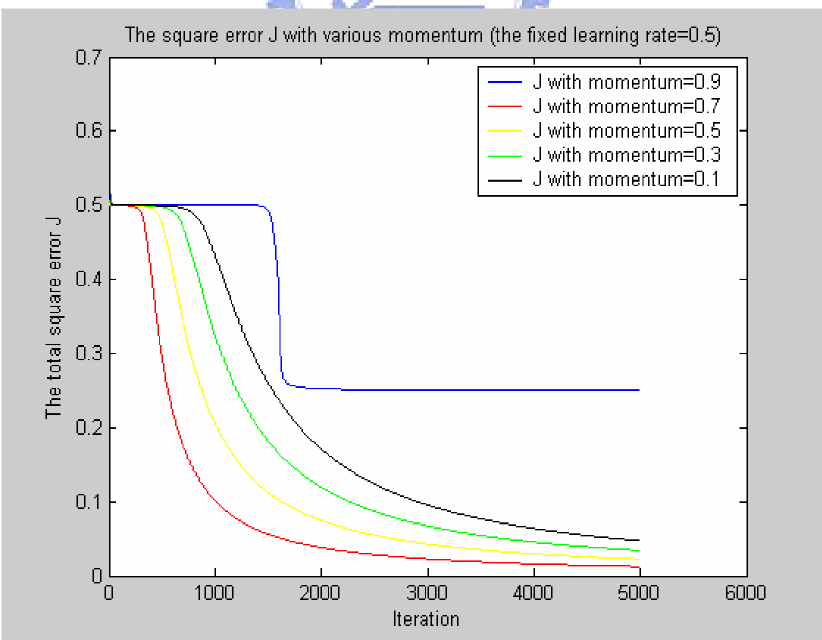


Figure 4-8-2. The square error J of the BPA with variant momentum($\beta = 0.5$)

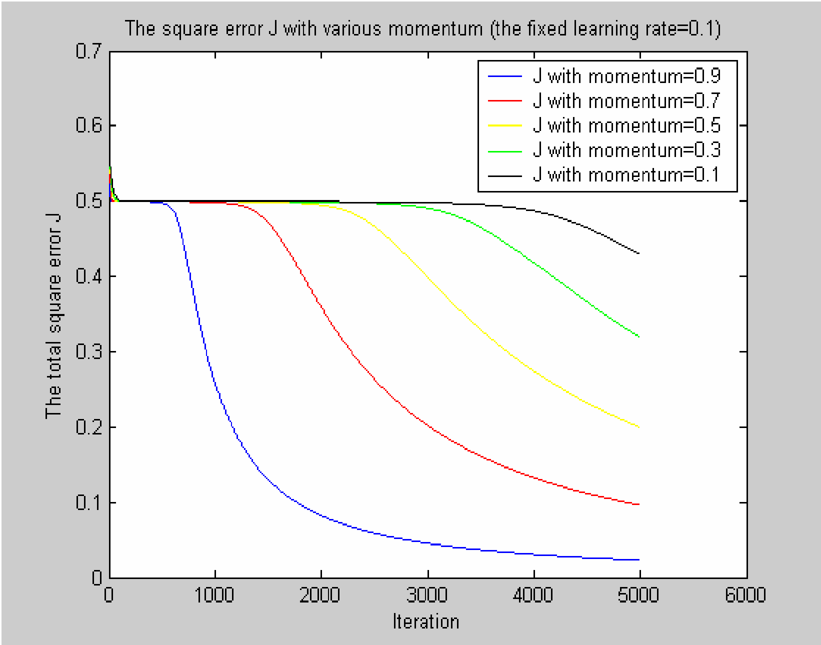


Figure 4-8-3. The square error J of the BPA with variant momentum ($\beta = 0.1$)

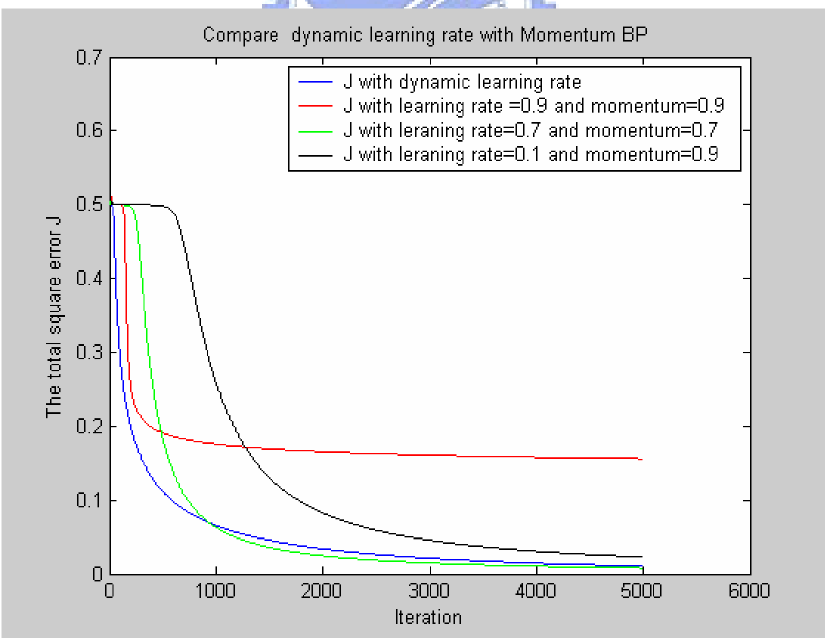


Figure 4-9. Total square errors of dynamic training and the BPA with different learning rates and momentum

4.2. Example 2: Classification of Iris Data Set

In this example, we will use the same neural network as before to classify Iris data sets [15], [16]. Generally, Iris has three kinds of subspecies, and the classification will depend on the length and width of the petal and the length and width of the sepal. The total Iris data are shown in Figure 4-10-1 and 4-10-2. And the training data sets, the first 75 samples of total data, are shown in Figures 4-11-1 and 4-11-2. The Iris data samples are available in [20]. There are 150 samples of three species of the Iris flowers in this data. We choose 75 samples to train the network and using the other 75 samples to test the network. We will have four kinds of input data, so we adopt the network which has four nodes in the input layer and three nodes in the output layer for this problem. Then the architecture of the neural network is a 4-4-3 network as shown in Figure 4-12. In which, we use the network with four hidden nodes in the hidden layer.

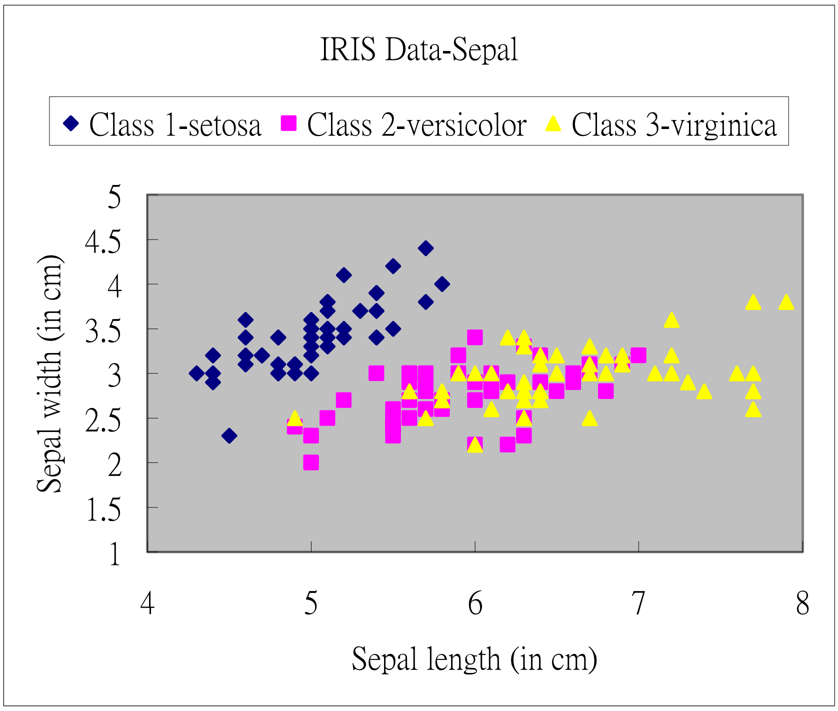


Figure 4-10-1. The total Iris data set (Sepal)

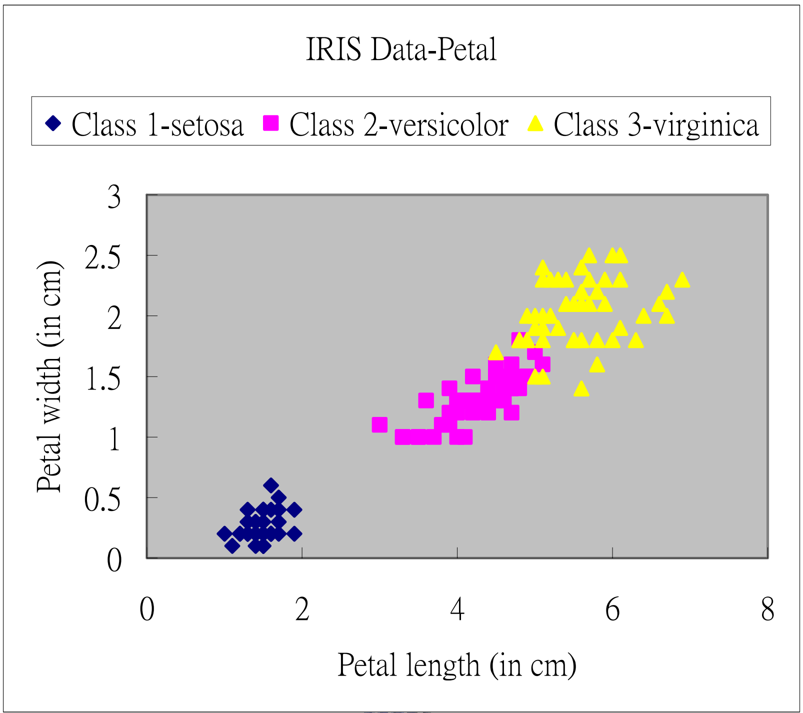


Figure 4-10-2. The total Iris data set (Petal)

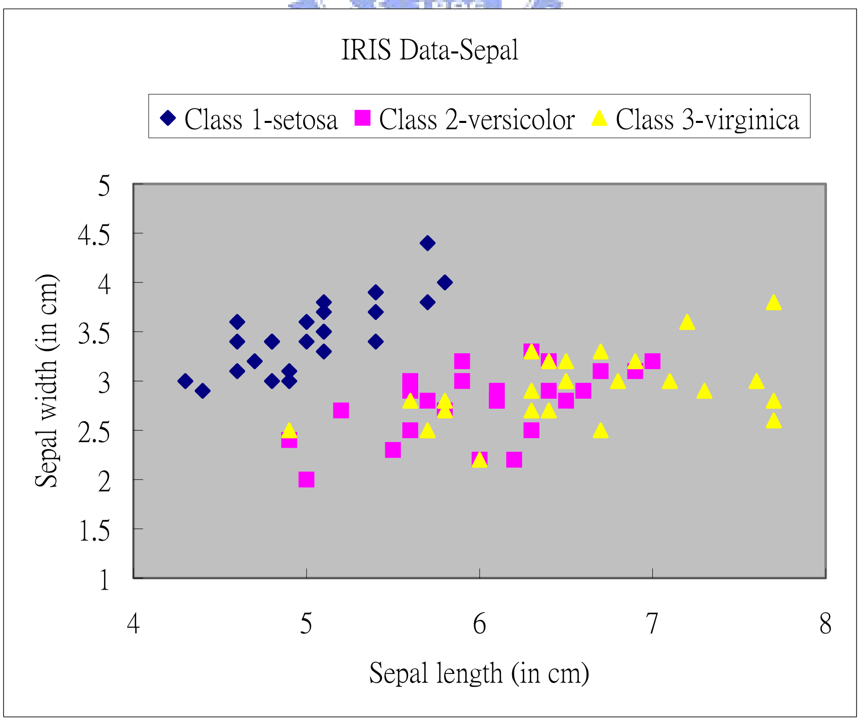


Figure 4-11-1. The training set of Iris data (Sepal)

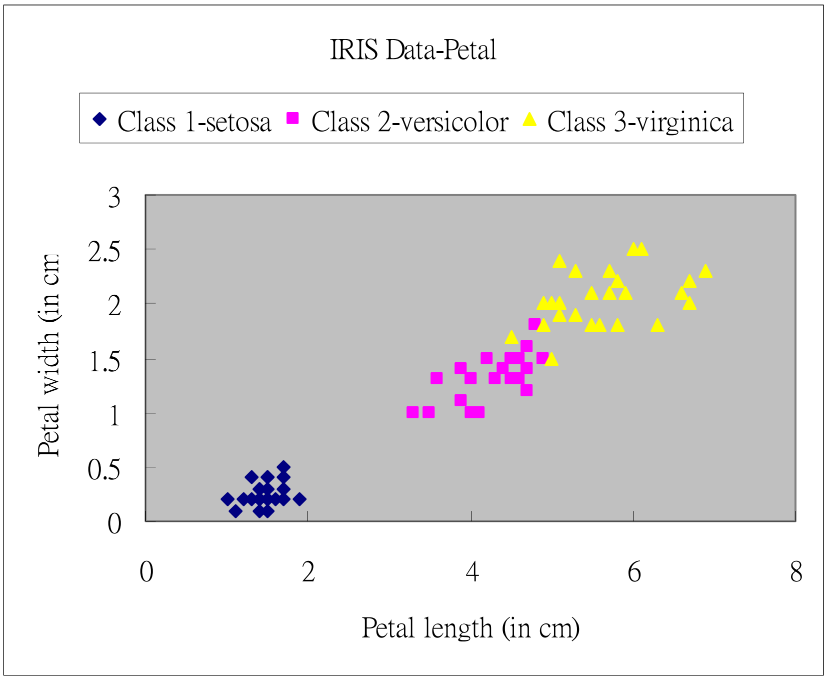


Figure 4-11-2. The training set of Iris data (Petal)

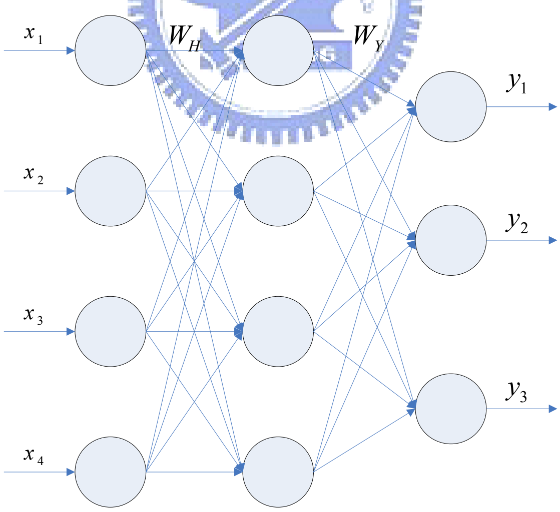


Figure 4-12. The neural network for solving Iris problem

First, we use the standard BPA with fixed learning rates (β = 0.1, 0.01 and 0.001) to solve the classification of Iris data sets, and the training results are shown in Figure 4-13-1 ~ 4-13-3. The result of BPA with dynamic optimal learning rates is shown in Figure 4-14.

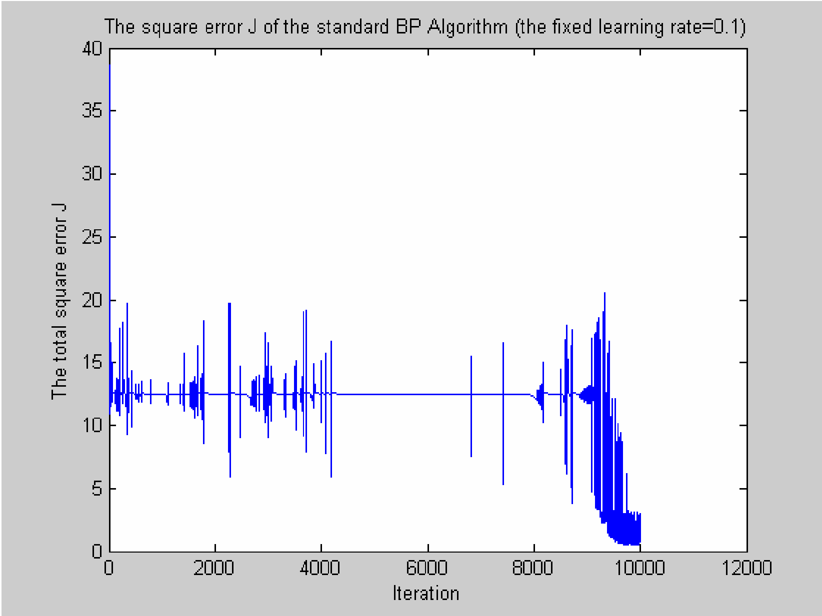


Figure 4-13-1. The square error J of the standard BPA with fixed $\beta = 0.1$

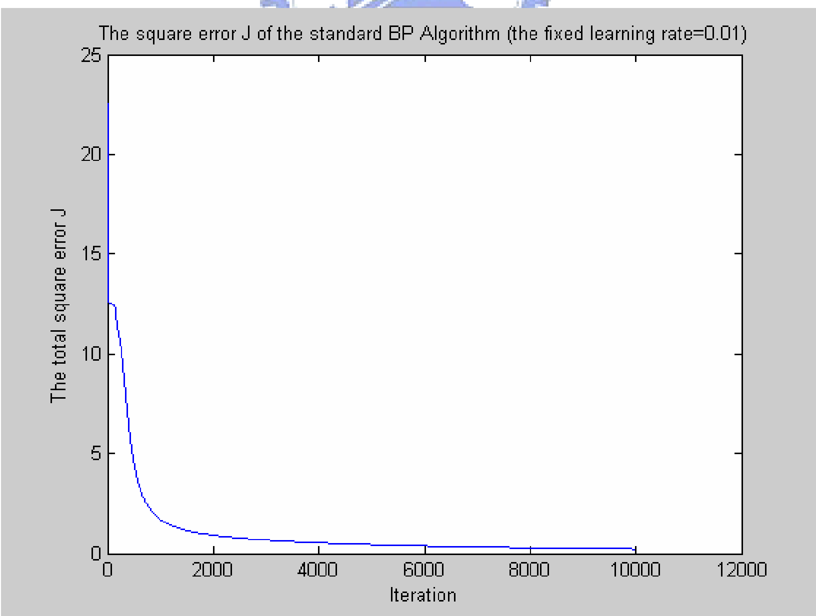


Figure 4-13-2. The square error J of the standard BPA with fixed $\beta = 0.01$

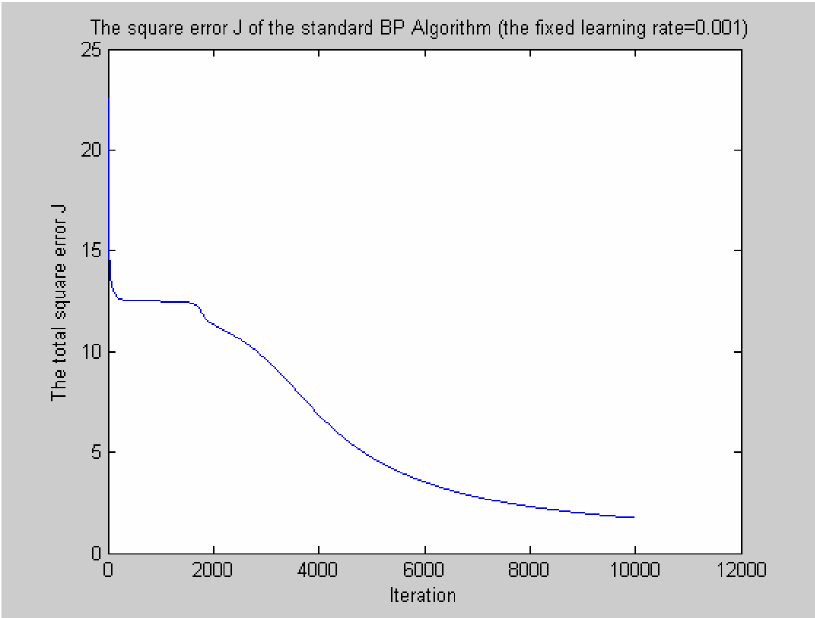


Figure 4-13-3. The square error J of the standard BPA with fixed $\beta = 0.001$

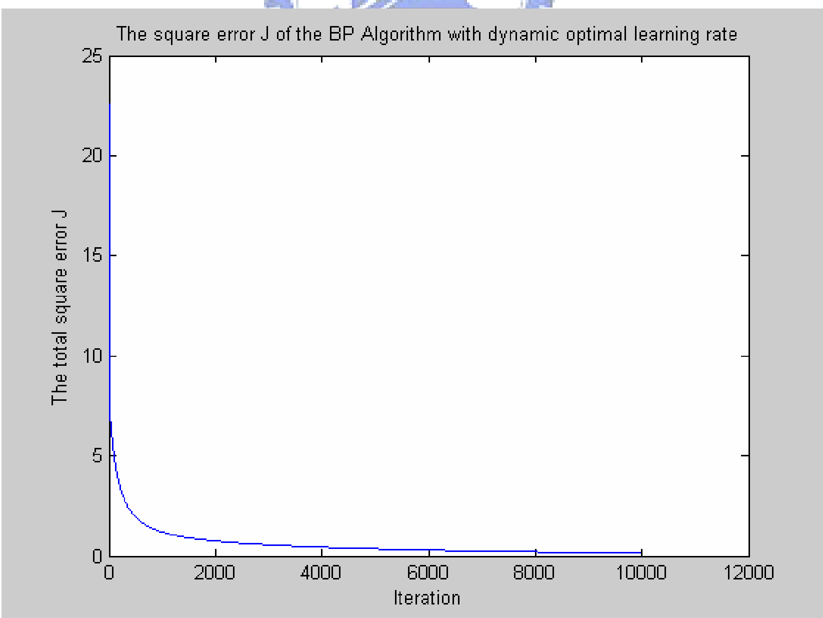


Figure 4-14. The square error J of the BPA with dynamic optimal training

Figure 4-15 shows that the convergence speed of the network with dynamic learning rate is absolutely faster than the network with fixed learning rates. Because the optimal learning rate of every iteration is almost in the range [0.01, 0.02], so the convergence speed of the fixed learning rate β = 0.01 is similar to the convergence speed of the dynamic learning rate. But dynamic learning rate approach still performs better than those of using fixed learning rates.

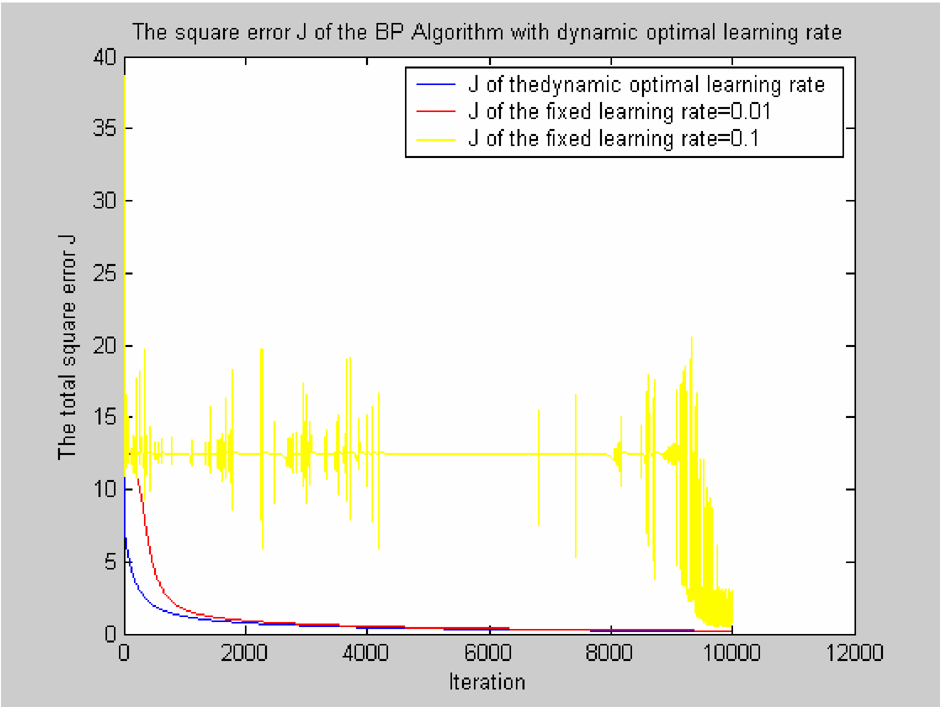


Figure 4-15. Training errors of dynamic optimal learning rates and fixed learning rates

After 10000 training iterations, the resulting weights and total square error J are shown below.

$$W_H = \begin{bmatrix} 1.2337 & -0.5033 & 1.3225 & 1.3074 \\ -0.3751 & 3.4714 & -2.6777 & -1.6052 \\ 3.7235 & 5.1603 & -4.0019 & -10.4289 \\ 1.9876 & -2.7186 & 4.6171 & 4.3400 \end{bmatrix}$$

$$W_Y = \begin{bmatrix} -2.2947 & 7.8444 & 3.4765 & -5.0185 \\ -2.5365 & -8.5464 & 9.3797 & -2.1300 \\ 2.0822 & -3.7674 & -10.4114 & 3.0220 \end{bmatrix}$$

Total square error J = 0.1582

The actual output and desired output of 10000 training iteration are shown in Table 4.3 and the testing output and desired output are shown in Table 4.4. After we substitute the above weighting matrices into the network and perform real testing, we find that there is no classification error by using training set (the first 75 data set). However there are 5 classification errors by using testing set (the later 75 data set), which are index 34, 51, 55, 57, 59 in Table 4-4.

Table 4.3. Actual and desired outputs after 10000 iterations

Index	Actual Output			Desired Output		
	Class 1	**Class 2**	**Class 3**	**Class 1**	**Class 2**	**Class 3**
1	0.9819	0.0216	0.0001	1.0000	0.0000	0.0000
2	0.9807	0.0233	0.0001	1.0000	0.0000	0.0000
3	0.9817	0.0220	0.0001	1.0000	0.0000	0.0000
4	0.9810	0.0229	0.0001	1.0000	0.0000	0.0000
5	0.9821	0.0215	0.0001	1.0000	0.0000	0.0000
6	0.9819	0.0215	0.0001	1.0000	0.0000	0.0000
7	0.9819	0.0218	0.0001	1.0000	0.0000	0.0000
8	0.9817	0.0219	0.0001	1.0000	0.0000	0.0000
9	0.9804	0.0237	0.0001	1.0000	0.0000	0.0000
10	0.9810	0.0228	0.0001	1.0000	0.0000	0.0000
11	0.9820	0.0215	0.0001	1.0000	0.0000	0.0000
12	0.9816	0.0221	0.0001	1.0000	0.0000	0.0000
13	0.9810	0.0229	0.0001	1.0000	0.0000	0.0000
14	0.9821	0.0222	0.0001	1.0000	0.0000	0.0000
15	0.9822	0.0213	0.0001	1.0000	0.0000	0.0000
16	0.9822	0.0213	0.0001	1.0000	0.0000	0.0000
17	0.9821	0.0214	0.0001	1.0000	0.0000	0.0000
18	0.9819	0.0217	0.0001	1.0000	0.0000	0.0000
19	0.9818	0.0216	0.0001	1.0000	0.0000	0.0000
20	0.9821	0.0215	0.0001	1.0000	0.0000	0.0000
21	0.9811	0.0226	0.0001	1.0000	0.0000	0.0000
22	0.9819	0.0216	0.0001	1.0000	0.0000	0.0000
23	0.9829	0.0218	0.0001	1.0000	0.0000	0.0000
24	0.9800	0.0240	0.0001	1.0000	0.0000	0.0000
25	0.9808	0.0231	0.0001	1.0000	0.0000	0.0000
26	0.0214	0.9910	0.0049	0.0000	1.0000	0.0000
27	0.0215	0.9908	0.0049	0.0000	1.0000	0.0000

28	0.0211	0.9909	0.0050	0.0000	1.0000	0.0000
29	0.0174	0.9846	0.0090	0.0000	1.0000	0.0000
30	0.0205	0.9902	0.0055	0.0000	1.0000	0.0000
31	0.0202	0.9895	0.0058	0.0000	1.0000	0.0000
32	0.0210	0.9903	0.0052	0.0000	1.0000	0.0000
33	0.0243	0.9895	0.0046	0.0000	1.0000	0.0000
34	0.0213	0.9910	0.0049	0.0000	1.0000	0.0000
35	0.0189	0.9861	0.0076	0.0000	1.0000	0.0000
36	0.0208	0.9899	0.0055	0.0000	1.0000	0.0000
37	0.0213	0.9902	0.0052	0.0000	1.0000	0.0000
38	0.0212	0.9910	0.0049	0.0000	1.0000	0.0000
39	0.0203	0.9898	0.0057	0.0000	1.0000	0.0000
40	0.0251	0.9892	0.0045	0.0000	1.0000	0.0000
41	0.0216	0.9909	0.0049	0.0000	1.0000	0.0000
42	0.0172	0.9836	0.0095	0.0000	1.0000	0.0000
43	0.0218	0.9907	0.0049	0.0000	1.0000	0.0000
44	0.0072	0.8555	0.1140	0.0000	1.0000	0.0000
45	0.0216	0.9907	0.0049	0.0000	1.0000	0.0000
46	0.0058	0.7622	0.2006	0.0000	1.0000	0.0000
47	0.0218	0.9907	0.0049	0.0000	1.0000	0.0000
48	0.0090	0.9154	0.0618	0.0000	1.0000	0.0000
49	0.0210	0.9908	0.0051	0.0000	1.0000	0.0000
50	0.0215	0.9909	0.0049	0.0000	1.0000	0.0000
51	0.0007	0.0093	0.9940	0.0000	0.0000	1.0000
52	0.0007	0.0101	0.9934	0.0000	0.0000	1.0000
53	0.0007	0.0119	0.9920	0.0000	0.0000	1.0000
54	0.0008	0.0162	0.9888	0.0000	0.0000	1.0000
55	0.0007	0.0095	0.9939	0.0000	0.0000	1.0000
56	0.0007	0.0103	0.9933	0.0000	0.0000	1.0000
57	0.0007	0.0102	0.9933	0.0000	0.0000	1.0000
58	0.0011	0.0330	0.9752	0.0000	0.0000	1.0000
59	0.0007	0.0108	0.9929	0.0000	0.0000	1.0000
60	0.0007	0.0099	0.9936	0.0000	0.0000	1.0000
61	0.0024	0.2245	0.7853	0.0000	0.0000	1.0000
62	0.0008	0.0131	0.9912	0.0000	0.0000	1.0000
63	0.0008	0.0138	0.9906	0.0000	0.0000	1.0000
64	0.0007	0.0094	0.9939	0.0000	0.0000	1.0000

65	0.0007	0.0093	0.9940	0.0000	0.0000	1.0000
66	0.0007	0.0100	0.9935	0.0000	0.0000	1.0000
67	0.0016	0.0894	0.9242	0.0000	0.0000	1.0000
68	0.0013	0.0511	0.9596	0.0000	0.0000	1.0000
69	0.0007	0.0093	0.9940	0.0000	0.0000	1.0000
70	0.0010	0.0291	0.9785	0.0000	0.0000	1.0000
71	0.0007	0.0102	0.9933	0.0000	0.0000	1.0000
72	0.0007	0.0098	0.9936	0.0000	0.0000	1.0000
73	0.0007	0.0103	0.9933	0.0000	0.0000	1.0000
74	0.0017	0.1066	0.9075	0.0000	0.0000	1.0000
75	0.0008	0.0164	0.9887	0.0000	0.0000	1.0000

Table 4.4. Actual and desired outputs in real testings

Index	Actual Output			Desired Output		
	Class 1	Class 2	Class 3	Class 1	Class 2	Class 3
1	0.9794	0.0249	0.0001	1.0000	0.0000	0.0000
2	0.9812	0.0224	0.0001	1.0000	0.0000	0.0000
3	0.9818	0.0218	0.0001	1.0000	0.0000	0.0000
4	0.9818	0.0218	0.0001	1.0000	0.0000	0.0000
5	0.9810	0.0229	0.0001	1.0000	0.0000	0.0000
6	0.9804	0.0236	0.0001	1.0000	0.0000	0.0000
7	0.9813	0.0223	0.0001	1.0000	0.0000	0.0000
8	0.9823	0.0214	0.0001	1.0000	0.0000	0.0000
9	0.9823	0.0213	0.0001	1.0000	0.0000	0.0000
10	0.9810	0.0228	0.0001	1.0000	0.0000	0.0000
11	0.9818	0.0219	0.0001	1.0000	0.0000	0.0000
12	0.9820	0.0216	0.0001	1.0000	0.0000	0.0000
13	0.9810	0.0228	0.0001	1.0000	0.0000	0.0000
14	0.9813	0.0226	0.0001	1.0000	0.0000	0.0000
15	0.9817	0.0219	0.0001	1.0000	0.0000	0.0000
16	0.9820	0.0216	0.0001	1.0000	0.0000	0.0000
17	0.9650	0.0442	0.0002	1.0000	0.0000	0.0000
18	0.9819	0.0220	0.0001	1.0000	0.0000	0.0000
19	0.9813	0.0224	0.0001	1.0000	0.0000	0.0000
20	0.9816	0.0219	0.0001	1.0000	0.0000	0.0000
21	0.9804	0.0236	0.0001	1.0000	0.0000	0.0000

22	0.9820	0.0215	0.0001	1.0000	0.0000	0.0000
23	0.9816	0.0222	0.0001	1.0000	0.0000	0.0000
24	0.9820	0.0215	0.0001	1.0000	0.0000	0.0000
25	0.9817	0.0220	0.0001	1.0000	0.0000	0.0000
26	0.0215	0.9909	0.0049	0.0000	1.0000	0.0000
27	0.0210	0.9909	0.0051	0.0000	1.0000	0.0000
28	0.0170	0.9839	0.0095	0.0000	1.0000	0.0000
29	0.0196	0.9886	0.0064	0.0000	1.0000	0.0000
30	0.0239	0.9898	0.0046	0.0000	1.0000	0.0000
31	0.0215	0.9907	0.0050	0.0000	1.0000	0.0000
32	0.0219	0.9907	0.0049	0.0000	1.0000	0.0000
33	0.0220	0.9906	0.0049	0.0000	1.0000	0.0000
***34**	**0.0019**	**0.1415**	**0.8726**	**0.0000**	**1.0000**	**0.0000**
35	0.0140	0.9710	0.0179	0.0000	1.0000	0.0000
36	0.0217	0.9901	0.0051	0.0000	1.0000	0.0000
37	0.0212	0.9909	0.0050	0.0000	1.0000	0.0000
38	0.0201	0.9897	0.0058	0.0000	1.0000	0.0000
39	0.0224	0.9903	0.0049	0.0000	1.0000	0.0000
40	0.0199	0.9889	0.0061	0.0000	1.0000	0.0000
41	0.0197	0.9889	0.0062	0.0000	1.0000	0.0000
42	0.0210	0.9905	0.0052	0.0000	1.0000	0.0000
43	0.0215	0.9907	0.0050	0.0000	1.0000	0.0000
44	0.0232	0.9900	0.0047	0.0000	1.0000	0.0000
45	0.0206	0.9899	0.0055	0.0000	1.0000	0.0000
46	0.0223	0.9905	0.0048	0.0000	1.0000	0.0000
47	0.0216	0.9905	0.0050	0.0000	1.0000	0.0000
48	0.0215	0.9908	0.0049	0.0000	1.0000	0.0000
49	0.0300	0.9869	0.0042	0.0000	1.0000	0.0000
50	0.0215	0.9905	0.0050	0.0000	1.0000	0.0000
***51**	**0.0060**	**0.7787**	**0.1866**	**0.0000**	**0.0000**	**1.0000**
52	0.0026	0.2674	0.7388	0.0000	0.0000	1.0000
53	0.0032	0.3974	0.5942	0.0000	0.0000	1.0000
54	0.0007	0.0095	0.9938	0.0000	0.0000	1.0000
***55**	**0.0159**	**0.9807**	**0.0117**	**0.0000**	**0.0000**	**1.0000**
56	0.0009	0.0230	0.9835	0.0000	0.0000	1.0000
***57**	**0.0165**	**0.9827**	**0.0104**	**0.0000**	**0.0000**	**1.0000**
58	0.0007	0.0094	0.9939	0.0000	0.0000	1.0000

***59**	**0.0133**	**0.9690**	**0.0199**	**0.0000**	**0.0000**	**1.0000**
60	0.0018	0.1281	0.8862	0.0000	0.0000	1.0000
61	0.0007	0.0106	0.9930	0.0000	0.0000	1.0000
62	0.0007	0.0095	0.9939	0.0000	0.0000	1.0000
63	0.0018	0.1130	0.9011	0.0000	0.0000	1.0000
64	0.0033	0.4128	0.5766	0.0000	0.0000	1.0000
65	0.0011	0.0335	0.9748	0.0000	0.0000	1.0000
66	0.0007	0.0094	0.9939	0.0000	0.0000	1.0000
67	0.0008	0.0166	0.9885	0.0000	0.0000	1.0000
68	0.0007	0.0101	0.9934	0.0000	0.0000	1.0000
69	0.0007	0.0096	0.9938	0.0000	0.0000	1.0000
70	0.0007	0.0094	0.9939	0.0000	0.0000	1.0000
71	0.0007	0.0105	0.9931	0.0000	0.0000	1.0000
72	0.0007	0.0123	0.9917	0.0000	0.0000	1.0000
73	0.0010	0.0284	0.9791	0.0000	0.0000	1.0000
74	0.0007	0.0099	0.9935	0.0000	0.0000	1.0000
75	0.0011	0.0370	0.9717	0.0000	0.0000	1.0000

CHAPTER 5

Conclusions

Although the back propagation algorithm is a useful tool to solve the problems of classification, optimization, prediction etc, it still has many defects. One of those defects is that we don't know how to choose the suitable learning rate to get converged training results. But by using the dynamical training algorithm for three layer neural network that we proposed in the end of Chapter 3, we can find the dynamic optimal learning rate very easily. And the dynamic learning rate guarantees that the total square error J is a decreasing function. This means that actual outputs will be closer to desired outputs for more iterations. The classification problems of XOR and Iris data are proposed in Chapter 4. They are solved by using the dynamical optimal training for a three layer neural network with sigmoid activation functions in hidden and output layers. Excellent results are obtained in the XOR and Iris data problems. Therefore the dynamic training algorithm is actually very powerful for getting better results than the other conventional back propagation algorithm with unknown fixed learning rates. So the goal of removing the defects of the back propagation algorithm with fixed learning rate is achieved by using the dynamical optimal training algorithm.

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