European Journal of Operational Research 196 (2009) 968–978

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

Sample size determination for production yield estimation with multiple independent process characteristics

Ya-Chen Hsu [a,*], W.L. Pearn [b], Ya-Fei Chuang [b]

[a] Department of Business Administration, Yuanpei University, Taiwan, ROC
[b] Department of Industrial Engineering and Management, National Chiao Tung University, Taiwan, ROC

ARTICLE INFO

Article history:
Received 20 July 2007
Accepted 24 April 2008
Available online 4 May 2008

Keywords:
Bootstrap resampling
Lower confidence bound
Multiple characteristics
Process capability indices
Estimation accuracy
Sample size determination

ABSTRACT

Capability measure for processes yield with single characteristic has been investigated extensively, but is still comparatively neglected for processes with multiple characteristics. Wu and Pearn [Wu, C.W., Pearn, W.L., 2005. Measuring manufacturing capability for couplers and wavelength division multiplexers (WDM). International Journal of Advanced Manufacturing Technology 25(5/6), 533–541] proposed a capability index for multiple characteristics called C_{PU}^{T}, which provides an exact measure on process yield for multiple characteristics with each characteristic normally distributed. However, the exact sampling distribution of C_{PU}^{T} (multiple characteristics) is analytically intractable. In this paper, we apply the bootstrap method for calculating the lower confidence bounds of the index C_{PU}^{T}, and determine the sample size for a specified estimation accuracy. In order to obtain a desired estimation quality assurance, the sample size determination is essential as it provides the accuracy of the lower bound obtained from the bootstrap method. For convenience of applications, we tabulate the sample size required for various designated accuracy for the engineers/practitioners to use. A real-world example from manufacturing process with multiple characteristics is investigated to illustrate the applicability of the proposed approach.

1. Introduction

Process capability indices (PCIS) are effective tools for quality assurance and process improvement. Numerous capability indices quantifying process potential and process performance are essential to any successful quality improvement activities and quality program implementation. Several basic capability indices have been widely used in manufacturing industry as follows:

$$C_p = \frac{USL - LSL}{6\sigma}, \quad (1)$$

$$C_{PU} = \frac{USL - \mu}{3\sigma}, \quad (2)$$

$$C_{PL} = \frac{\mu - LSL}{3\sigma}, \quad (3)$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \quad (4)$$

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\}, \quad (5)$$

where USL and LSL are the upper and the lower specification limits, μ is the process mean, σ is the process standard deviation, and T is the target value.

In order to calculate the estimator, data must be collected. A great degree of uncertainty may be introduced into the capability assessments due to sampling errors. As the sampling errors have been ignored, the approach, simply by the calculated values of the estimated indices and then making a conclusion on whether the given process is capable, is highly unreliable. A reliable approach for estimating the true value of process capability is to determine the sample size for desired estimation accuracy. The sample size is directly related to the estimation accuracy and the cost of the data collection plan. The capability measurements for processes with single characteristic have been investigated extensively (see Kane, 1986; Pearn et al., 1992; Chen, 1998; Chen and Hsu, 2004; Cheng et al., 2006; Flaig, 2006; Vännman, 2006; Vännman and Albing, 2007). However, the lacks of these studies associated with analyzing the quality and efficiency of a process, are, so far, limited by discussing one single quality specification. In this paper, we consider the process capability with multiple characteristics to determine the sample size for desired estimation accuracy.

For process with multiple characteristics, several approaches have been suggested (see e.g. Bothe, 1992; Chen et al., 2003a,b; Castagliola and Castellanos, 2005; Huang et al., 2005; Wu and Pearn, 2005). For example, Bothe (1992) considered a simple

* Corresponding author. Tel.: +886 968027645.
E-mail address: ychsu.iem93g@nctu.edu.tw (Y.-C. Hsu).

0377-2217/$ - see front matter © 2008 Published by Elsevier B.V.
doi:10.1016/j.ejor.2008.04.029

measurement by taking the minimum measure of each single characteristic. For example, considering a v-characteristics product with v-yield measures $P_1, P_2, \ldots, P_v$, the overall process yield is measured as $P = \min\{P_1, P_2, \ldots, P_v\}$. Furthermore, Chen et al. (2003a) provided the process capability index with multi-characteristics as

$$S_{pk}^T = \frac{1}{3}\Phi^{-1}\left\{\left[\prod_{j=1}^{v}(2\Phi(3S_{pkj}) - 1) + 1\right] \Big/ 2\right\}, \quad (6)$$

where $\Phi(\cdot)$ is the cumulative distribution of the standard normal distribution $N(0,1)$, Φ^{-1} is the inverse function of $\Phi(\cdot)$, S_{pkj} denotes the S_{pk} value of the jth characteristic for $j = 1,2,\ldots,v$, and v is the number of characteristics. This index, which provides an exact measurement on the process yield, establishes the relationship between the manufacturing specification and the actual process performance (Pearn and Cheng, 2007). Wu and Pearn (2005) discussed the process with multi-characteristics for one-sided specification and proposed a capability index as

$$C_{PU}^T = \frac{1}{3}\Phi^{-1}\left\{\prod_{j=1}^{v}\Phi(3C_{PUj})\right\}, \quad (7)$$

where C_{PUj} denotes the C_{PU} value of the jth characteristic for $j = 1,2,\ldots,v$, and v is the number of characteristics. A one-to-one correspondence relationship between the index C_{PU}^T and the overall process yield P can be established as

$$P = \prod_{j=1}^{v} P_j = \prod_{j=1}^{v}\Phi(3C_{PUj}) = \Phi(3C_{PU}^T). \quad (8)$$

Bootstrap approach seems to be a reasonable method for tackling the problem that the sampling distribution of C_{PU}^T (multiple characteristics) is analytically intractable. Since lower confidence bound estimates the minimum process capability conveying critical information regarding product quality, Wu and Pearn (2005) estimated the confidence bound by percentile bootstrap (PB) method. However, there are four types of bootstrap methods to estimate confidence bound, including the standard bootstrap confidence interval (SB), the percentile bootstrap confidence interval (PB), the biased-corrected percentile bootstrap confidence interval (BCPB), and the bootstrap-t (BT) method. And the engineers/practitioners would want to know which one is recommended. In this paper, we compare the performance of confidence bound for the one-sided index C_{PU}^T with multiple characteristics by using these four bootstrap methods. The modified index C_{PU}^T proposed by Wu and Pearn (2005) are calculated. Furthermore, we find that the BCPB method would be the recommended method to estimate confidence bound in the general cases. We also provide the tables about the sample sizes required for various designated estimation accuracy for the engineers/practitioners to use in their factory applications. A real-world example from manufacturing process with multiple characteristics is investigated to illustrate the applicability of the proposed approach.

Table 1
Various C_{PU}^T values and the corresponding process yield

C_{PU}^T	Process yield
1.00	0.9986501020
1.25	0.9999115827
1.33	0.9999669634
1.45	0.9999931931
1.50	0.9999966023
1.60	0.9999992067
1.67	0.9999997278
2.00	0.9999999990

Table 2
Minimal requirement for each single characteristic of various capability levels for multiple characteristics

c'	c_L	
	1.000	1.33
1	1.000	1.330
2	1.068	1.383
3	1.107	1.414
4	1.133	1.436
5	1.153	1.452

Table 3
The total rank of the four bootstrap methods as $C_{PU}^T = 1, 1.33$ and $v = 2(1)5$

n	$C_{PU}^T = 1$				$C_{PU}^T = 1.33$			
	SB	PB	BCPB	PT	SB	PB	BCPB	PT
$v = 2$								
30	3	2	1.006	3.994	2.996	1.996	1.008	4.000
40	2.996	2.004	1.004	3.996	2.982	2.008	1.034	3.974
50	2.992	2.002	1.024	3.982	2.99	2.012	1.030	3.968
60	2.982	2.014	1.042	3.962	2.984	2.010	1.056	3.950
70	2.994	2.004	1.018	3.984	2.984	2.020	1.030	3.966
80	2.988	2.012	1.026	3.974	2.972	2.028	1.072	3.928
90	2.994	2.016	1.022	3.968	2.970	2.026	1.084	3.920
100	2.980	2.020	1.026	3.974	2.992	2.002	1.072	3.934
125	3.000	2.010	1.018	3.972	2.968	2.040	1.104	3.888
150	2.988	2.012	1.040	3.960	2.974	2.056	1.094	3.876
200	3.004	2.018	1.036	3.942	2.972	2.018	1.148	3.858
$v = 3$								
30	2.966	2.028	1.1	3.906	2.942	2.048	1.112	3.896
40	2.95	2.04	1.106	3.904	2.954	2.046	1.132	3.868
50	2.97	2.034	1.1	3.896	2.946	2.044	1.15	3.86
60	2.942	2.07	1.124	3.864	2.922	2.094	1.216	3.768
70	2.954	2.052	1.12	3.874	2.916	2.112	1.29	3.678
80	2.944	2.062	1.172	3.818	2.904	2.108	1.332	3.656
90	2.94	2.078	1.166	3.816	2.836	2.168	1.428	3.568
100	2.934	2.072	1.162	3.828	2.894	2.136	1.352	3.61
125	2.964	2.06	1.124	3.852	2.832	2.172	1.59	3.406
150	2.97	2.044	1.136	3.848	2.804	2.176	1.62	3.398
200	2.918	2.114	1.2	3.768	2.784	2.23	1.636	3.344
$v = 4$								
30	2.97	2.052	1.116	3.856	2.94	2.076	1.17	3.814
40	2.912	2.084	1.208	3.796	2.882	2.11	1.3	3.708
50	2.922	2.094	1.224	3.76	2.826	2.186	1.508	3.478
60	2.91	2.096	1.222	3.772	2.798	2.188	1.53	3.478
70	2.882	2.126	1.254	3.736	2.822	2.186	1.664	3.328
80	2.916	2.108	1.26	3.716	2.79	2.274	1.666	3.268
90	2.892	2.12	1.318	3.668	2.742	2.272	1.784	3.196
100	2.886	2.124	1.28	3.706	2.73	2.308	1.85	3.11
125	2.922	2.126	1.308	3.644	2.634	2.4	2.054	2.912
150	2.92	2.1	1.328	3.648	2.684	2.412	2.034	2.868
200	2.88	2.152	1.394	3.57	2.632	2.434	2.322	2.606
$v = 5$								
30	2.938	2.084	1.222	3.756	2.86	2.15	1.398	3.59
40	2.878	2.118	1.314	3.688	2.786	2.216	1.652	3.344
50	2.892	2.148	1.326	3.63	2.792	2.24	1.648	3.316
60	2.85	2.16	1.408	3.582	2.758	2.254	1.828	3.156
70	2.846	2.158	1.45	3.542	2.686	2.336	2.026	2.942
80	2.902	2.18	1.382	3.528	2.654	2.4	2.11	2.83
90	2.876	2.172	1.5	3.452	2.572	2.432	2.288	2.702
100	2.83	2.192	1.56	3.406	2.582	2.478	2.384	2.55
125	2.904	2.156	1.514	3.42	2.56	2.482	2.6	2.354
150	2.856	2.228	1.572	3.342	2.466	2.524	2.648	2.346
200	2.834	2.244	1.578	3.34	2.426	2.614	2.878	2.074

2. Capability measures for multiple characteristics

Capability measure for processes with single characteristic has been investigated extensively. For normally distributed processes with a one-sided specification limit, USL or LSL, the process yield

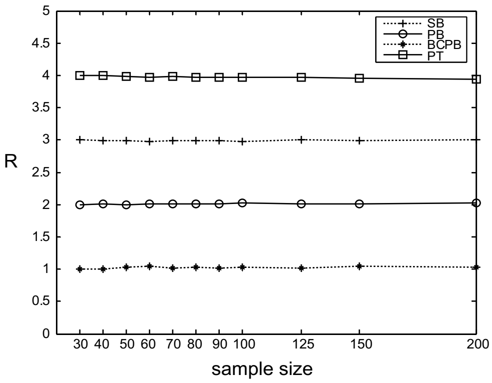

Fig. 1a. The total rank of the four bootstrap methods as $C_{PU}^{T}=1$, $v = 2$.

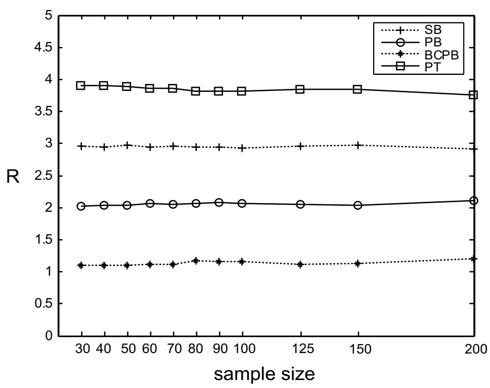

Fig. 1b. The total rank of the four bootstrap methods as $C_{PU}^{T}=1$, $v = 3$.

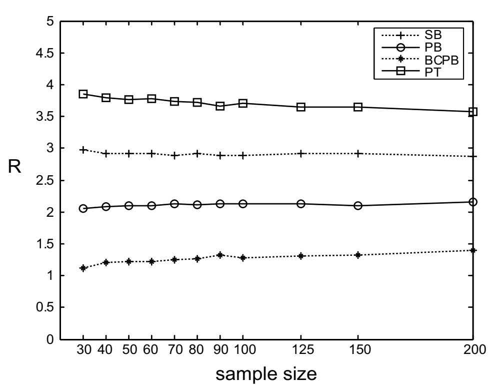

Fig. 1c. The total rank of the four bootstrap methods as $C_{PU}^{T}=1$, $v = 4$.

is listed in the following, where Z follows the standard normal distribution $N(0,1)$

$$P(X < USL) = P\left(\frac{X-\mu}{\sigma} < \frac{USL-\mu}{\sigma}\right) = P(Z < 3C_{PU}) = \Phi(3C_{PU}), \quad (9)$$

$$P(X > LSL) = P\left(\frac{X-\mu}{\sigma} > \frac{LSL-\mu}{\sigma}\right) = P(-Z < 3C_{PL}) = \Phi(3C_{PL}). \quad (10)$$

For easier presentation, we denote C_I as either C_{PU} or C_{PL}. Thus, process capability index C_I provides an exact measure of the potential process yield for processes with a one-sided manufacturing specification. The corresponding process yield for a well controlled normally distributed process is easily calculated as Φ $(3C_I)$.

Considering processes with v-characteristics (assuming characteristics are mutually independent) and v yield measures

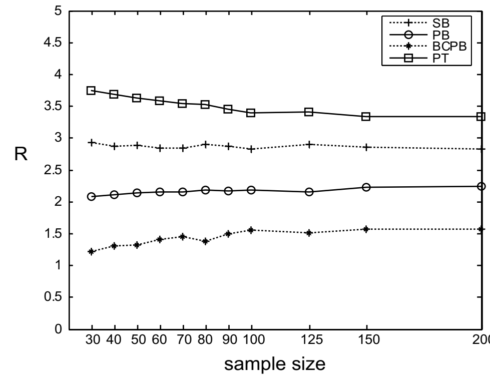

Fig. 1d. The total rank of the four bootstrap methods as $C_{PU}^{T}=1$, $v = 5$.

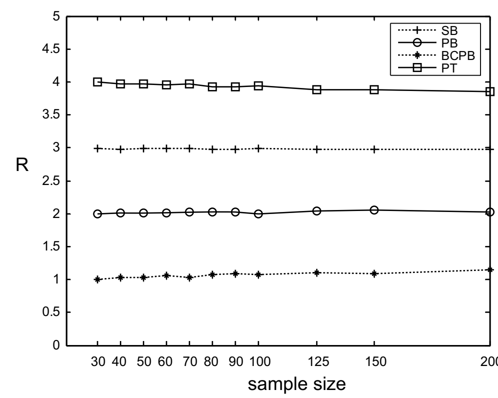

Fig. 2a. The total rank of the four bootstrap methods as $C_{PU}^{T}=1.33$, $v = 2$.

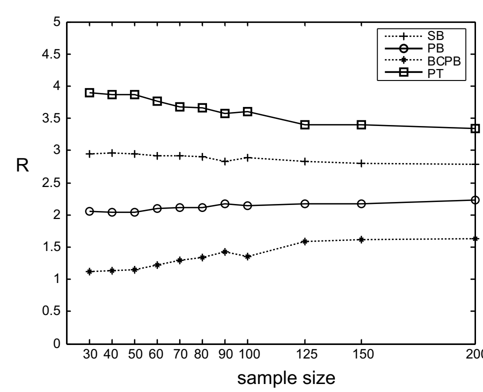

Fig. 2b. The total rank of the four bootstrap methods as $C_{PU}^{T}=1.33$, $v = 3$.

$P_1, P_2, \ldots, P_V$, Wu and Pearn (2005) suggested that the overall process yield should be calculated as $P = P_1 \times P_2 \times \ldots \times P_V$ which is significantly less than the calculated one. From the definition of one-sided yield index in (9), the process yield index with single characteristic can be rewritten as

$$C_{PU} = \frac{1}{3}\Phi^{-1}\left\{\Phi\left(\frac{USL-\mu}{\sigma}\right)\right\}, \quad (11)$$

where $\Phi(\cdot)$ is the cumulative distribution of the standard normal distribution $N(0,1)$, and Φ^{-1} is the inverse function of $\Phi(\cdot)$. For the process with multiple quality characteristics, a simple measure by taking the minimum of the measure of each single characteristic

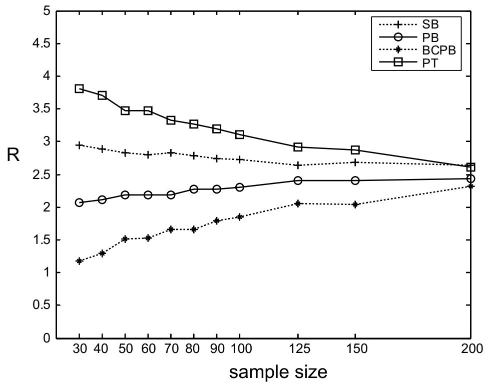

Fig. 2c. The total rank of the four bootstrap methods as $C_{PU}^{T} = 1.33$, $v = 4$.

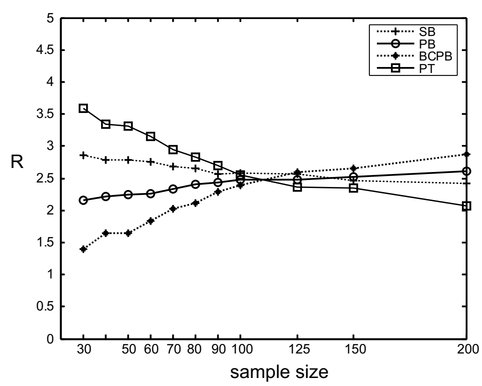

Fig. 2d. The total rank of the four bootstrap methods as $C_{PU}^{T} = 1.33$, $v = 5$.

Table 4
Sample size n required for $R_\gamma \geqslant R_{PU}$, with quality characteristics $v = 3$, $R_{PU} = 0.75(0.01)0.95$, $\gamma = 0.9, 0.95, 0.975, 0.99$, and $C_{PU}^{T} = 1$

R_{PU}	$\gamma = 0.90$		$\gamma = 0.95$		$\gamma = 0.975$		$\gamma = 0.99$	
	n	R_γ	n	R_γ	n	R_γ	n	R_γ
0.75	–	–	–	–	–	–	16	0.7518
0.76	–	–	–	–	6	–	21	0.7609
0.77	–	–	–	–	7	0.7776	24	0.7731
0.78	–	–	–	–	14	0.7832	28	0.7807
0.79	–	–	–	–	18	0.7904	31	0.7901
0.80	–	–	6	–	22	0.8005	36	0.8012
0.81	–	–	12	0.8119	26	0.8107	40	0.8103
0.82	–	–	17	0.8222	32	0.8226	49	0.8201
0.83	–	–	23	0.8316	38	0.8304	56	0.8305
0.84	–	–	28	0.8414	44	0.8403	65	0.8414
0.85	6	–	35	0.8536	52	0.8502	75	0.8502
0.86	18	0.8608	41	0.8600	63	0.8613	88	0.8609
0.87	26	0.8708	51	0.8710	73	0.8700	105	0.8710
0.88	33	0.8805	60	0.8802	88	0.8800	124	0.8812
0.89	44	0.8909	76	0.8912	105	0.8908	146	0.8610
0.90	54	0.9005	93	0.9004	128	0.9000	176	0.9005
0.91	71	0.9107	115	0.9102	158	0.9103	213	0.9101
0.92	92	0.9205	146	0.9201	197	0.9204	268	0.9202
0.93	121	0.9306	188	0.9303	253	0.9300	339	0.9302
0.94	164	0.9400	251	0.9402	337	0.9400	451	0.9400
0.95	231	0.9500	350	0.9502	473	0.9505	634	0.9500

has been considered. Wu and Pearn (2005) proposed the following overall capability index is referred to as:

$$C_{PU}^{T} = \frac{1}{3}\Phi^{-1}\left\{\prod_{j=1}^{v}\Phi(3C_{PUj})\right\}, \tag{12}$$

Table 5
Sample size n required for $R_\gamma \geqslant R_{PU}$, with quality characteristics $v = 3$, $R_{PU} = 0.75(0.01)0.95$, $\gamma = 0.9, 0.95, 0.975, 0.99$, and $C_{PU}^{T} = 1.33$

R_{PU}	$\gamma = 0.90$		$\gamma = 0.95$		$\gamma = 0.975$		$\gamma = 0.99$	
	n	R_γ	n	R_γ	n	R_γ	n	R_γ
0.75	–	–	–	–	6	–	19	0.7533
0.76	–	–	–	–	8	0.7608	22	0.7600
0.77	–	–	–	–	14	0.7721	24	0.7719
0.78	–	–	–	–	17	0.7820	28	0.7813
0.79	–	–	–	–	21	0.7923	33	0.7914
0.80	–	–	6	0.8074	24	0.8008	38	0.8019
0.81	–	–	16	0.8119	28	0.8109	41	0.8101
0.82	–	–	19	0.8200	33	0.8209	50	0.8219
0.83	–	–	24	0.8310	40	0.8334	57	0.8320
0.84	6	–	31	0.8421	44	0.8406	65	0.8400
0.85	12	0.8518	35	0.8519	54	0.8506	77	0.8524
0.86	21	0.8615	42	0.8613	62	0.8602	88	0.8603
0.87	26	0.8700	51	0.8705	73	0.8704	102	0.8705
0.88	34	0.8807	62	0.8817	89	0.8814	122	0.8814
0.89	43	0.8910	74	0.8902	104	0.8904	145	0.8906
0.90	55	0.9010	91	0.9009	129	0.9017	174	0.9010
0.91	72	0.9118	113	0.9101	157	0.9109	213	0.9101
0.92	89	0.9202	144	0.9211	196	0.9201	269	0.9204
0.93	118	0.9303	184	0.9301	254	0.9301	348	0.9301
0.94	159	0.9400	249	0.9400	340	0.9400	501	0.9421
0.95	226	0.9501	353	0.9501	481	0.9500	658	0.9500

where C_{PUj} denotes the C_{PU} value of the jth characteristic for $j = 1, 2, \ldots, v$, and v is the number of characteristics. The index, C_{PU}^{T}, can be a generalization of the single characteristic yield index. Let $C_{PU}^{T} = c$, we have

$$\left\{\prod_{j=1}^{v}\Phi(3C_{PUj})\right\} = \Phi(3c). \tag{13}$$

In fact, Wu and Pearn (2005) showed that the one-to-one correspondence relationship between the index C_{PU}^{T} and the overall process yields P can be established as follows:

$$P = \prod_{j=1}^{v}P_j = \prod_{j=1}^{v}\Phi(3C_{PUj}) = \Phi(3C_{PU}^{T}). \tag{14}$$

Hence, the new index C_{PU}^{T} provides an exact measure on the overall process yield when the characteristics are mutually independent. For example, if $C_{PU}^{T} = 1.00$, the entire process yield would be exactly 99.865%, and each single characteristic yield is no less than $(0.9986501)^{1/5} = 0.9997299$ (equivalent to 270 NCPPM). Table 1 displays various commonly used capability requirement and the corresponding overall process yield.

Wu and Pearn (2005) also showed that for process with v characteristics, if the requirement for the overall process capability is $C_{PU}^{T} \geqslant c_0$, a sufficient condition (which is minimal) for the requirement to each single characteristic can be obtained by the following. Let c' be the minimum C_{PUj} required for each single characteristic, then

$$\frac{1}{3}\Phi^{-1}\left\{\prod_{j=1}^{v}\Phi(3C_{PUj})\right\} \geqslant \frac{1}{3}\Phi^{-1}\left\{\prod_{j=1}^{v}\Phi(3c')\right\} \geqslant c_0. \tag{15}$$

We can obtain the lower bound of each single characteristic to be

$$c_L = \frac{1}{3}\Phi^{-1}\left(\sqrt[v]{\Phi(3c_0)}\right). \tag{16}$$

Table 2 displays the minimum c_L of C_{PUj} for the required overall process capability C_{PU}^{T} are 1.00 and 1.33 for $v = 1(1)5$ characteristics. For example, if the overall capability requirement $C_{PU}^{T} \geqslant 1.00$ would be satisfied, it means each single characteristic yield is no less than $(0.9986501)^{1/5} =$ 0.9997299 (equivalent to 270 NCPPM), and the capability for all the five characteristics is the following, for $j = 1, 2, \ldots, 5$.

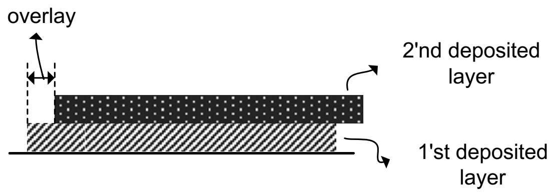

Fig. 3. Deposited layers on TFT-LCD.

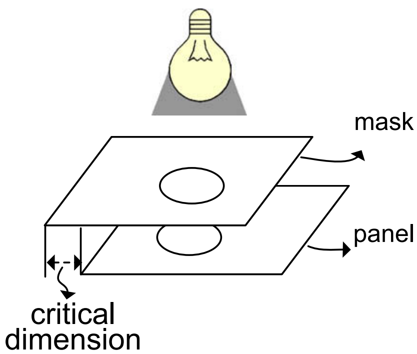

Fig. 4. Exposure process on panel window.

Table 6
Specifications for thin film transistor liquid crystal display

Parameter	Specifications
Overlay	⩽0.1 μm
Critical dimension	⩽0.3 μm
Uniformality	⩽0.03

$$C_{PUj} = \frac{1}{3}\Phi^{-1}\left(\sqrt[5]{\Phi(3)}\right) = 1.153 \quad \text{for } j = 1, 2, \ldots, 5. \tag{17}$$

3. Bootstrap methods for calculating the lower bounds of C_{PU}^T

3.1. Lower confidence bounds on C_{PU}^T

For each single characteristic, the C_{PU_j} values can be estimated by their natural estimators $\widehat{C}_{PU_j} = (USL_j - \bar{x}_j)/s_j, j = 1, 2, \ldots, v$, where $\bar{x}_j$ and s_j are the sample mean and the sample standard deviation of the jth characteristic, respectively. Thus, the estimator of $\widehat{C}_{PU}^T$ are defined as

$$\widehat{C}_{PU}^T = \frac{1}{3}\Phi^{-1}\left\{\prod_{j=1}^{v}\Phi(3\widehat{C}_{PUj})\right\}. \tag{18}$$

In order to calculate the estimator of C_{PU}^T, however, sample data must be collected. Therefore, due to sampling errors, a great degree of uncertainty may be introduced into capability assessments. It is highly unreliable simply by the calculated values of the estimated indices and then making a conclusion on whether the given process is capable. Since the sampling errors have been ignored, a reliable approach for estimating the true value of process index is to construct the lower confidence bound.

Determination of the lower confidence bound on the actual process capability is essential for quality assurance. The lower confidence bound can not only be essential to production yield assurance, but also be used in capability testing for decision making. Since the sample size provides the accuracy of the lower bound, for the given desired estimation accuracy R_{PU} ($R_{PU} = C_{PU}^{T(LB)}/\widehat{C}_{PU}^T$, where $C_{PU}^{T(LB)}$ is the lower confidence bound on C_{PU}^T) and the confidence level γ (ensures that the risk of making incorrect decisions will be no larger than the preset Type I error $1-\gamma$), the approximate sample size must be obtained. Before estimating the sample size, it is necessary to determine a desired lower confidence bound for C_{PU}^T, depending on the ratio of $R_{PU} = C_{PU}^{T(LB)}/\widehat{C}_{PU}^T$. Hence, we need to compute the lower confidence bound to determine sample sizes required for specified estimation accuracy of the C_{PU}^T.

While the sampling distribution of the estimator $\widehat{C}_{PU}^T$ for multiple samples is unknown, we use the nonparametric bootstrap method and the following to estimate the lower confidence bound $C_{PU}^{T(LB)}$. Efron (1981) introduced a nonparametric, computational intensive but effective estimation method, called the "Bootstrap", which is a data-based simulation technique for statistical inference. The merit of the nonparametric bootstrap approach is that it does not rely on any assumptions regarding the underlying distribution. The bootstrap sampling is equivalent to sampling (with replacement) from the empirical probability distribution function. The essence of bootstrapping is that, without any knowledge about a population, the distribution found in a random sample of size n from the population is the best guide to the distribution in the population. By resampling observations from the observed data, the population that consists of the n observed sample values is used to model the unknown real population.

In the bootstrap, B new samples, each of the same size as the observed data n, are drawn with replacement from the population. Efron and Tibshirani (1986) developed four types of bootstrap confidence interval, including the standard bootstrap confidence interval (SB), the percentile bootstrap confidence interval (PB), the biased-corrected percentile bootstrap confidence interval (BCPB), and the bootstrap-t (BT) method. Franklin and Wasserman (1992) investigated the lower confidence bounds for the capability indices, C_p, C_{pk} and C_{pm} using these bootstrap methods. Some simulations results indicate that for normal processes the bootstrap confidence limits perform equally well (see Chou et al., 1990 and Bissell, 1990). In the following, we give an overview of four Bootstrap confidence intervals. These are employed to determine the lower confidence bounds of the index.

3.2. Bootstrap methods

3.2.1. Standard bootstrap (SB)

From the B bootstrap estimator $\widehat{C}_{PU}^{T*}$, the sample average and the sample standard deviation are calculated as follows:

$$\overline{\widehat{C}_{PU}^{T*}} = \frac{1}{B}\sum_{i=1}^{B}\widehat{C}_{PU}^{T*}(i), \tag{19}$$

$$S^*_{\widehat{C}_{PU}^T} = \sqrt{\frac{1}{B-1}\sum_{I=1}^{B}[\widehat{C}_{PU}^{T*}(i) - \overline{\widehat{C}_{PU}^{T*}}]^2}, \tag{20}$$

Table 7
Calculations for process capability of the overlay, critical dimension and uniformality

Characteristics	USL	$\bar{x}$	s	$\widehat{C}_{PUj}$	L_C
Overlay	0.1	0.0795	0.0065	1.0499	0.9394
Critical dimension	0.1	0.2693	0.0083	1.2298	1.1016
Uniformality	0.03	0.0267	0.00097	1.1404	1.0215

Table 8
Calculations for overall yield index

Characteristic	$\widehat{C}_{PU}^T$	NCPPM	$C_{PU}^{T(LB)}$	NCPPM
LCD	1.0085	1241	0.9277	2692

Table 9
The 150 sample observations for three quality characteristics

Overlay (μm): USL = 0.1 μm									
0.0779	0.0697	0.0764	0.0763	0.0834	0.0860	0.0778	0.0849	0.0846	0.0649
0.0853	0.0801	0.0711	0.0847	0.0817	0.0747	0.0886	0.0777	0.0889	0.0716
0.0802	0.0776	0.0800	0.0811	0.0873	0.0804	0.0810	0.0729	0.0782	0.0794
0.0711	0.0712	0.0724	0.0839	0.0831	0.0846	0.0803	0.0851	0.0701	0.0741
0.0706	0.0826	0.0665	0.0843	0.0862	0.0824	0.0810	0.0804	0.0838	0.0693
0.0757	0.0842	0.0765	0.0742	0.0838	0.0832	0.0837	0.0745	0.0820	0.0911
0.0786	0.0751	0.0738	0.0801	0.0853	0.0667	0.0778	0.0888	0.0890	0.0638
0.0796	0.0859	0.0718	0.0799	0.0637	0.0789	0.0878	0.0926	0.0674	0.0745
0.0859	0.0913	0.0863	0.0695	0.0878	0.0753	0.0790	0.0798	0.0801	0.0736
0.0746	0.0885	0.0788	0.0746	0.0862	0.0787	0.0753	0.0793	0.0776	0.0945
0.0833	0.0709	0.0804	0.0780	0.0888	0.0842	0.0794	0.0793	0.0771	0.0835
0.0691	0.0806	0.0805	0.0735	0.0843	0.0837	0.0727	0.0834	0.0752	0.0877
0.0771	0.0850	0.0755	0.0826	0.0776	0.0833	0.0669	0.0740	0.0839	0.0743
0.0781	0.0754	0.0840	0.0840	0.0962	0.0780	0.0801	0.0742	0.0781	0.0908
0.0911	0.0849	0.0764	0.0932	0.0783	0.0732	0.0722	0.0775	0.0787	0.0715
Critical dimension (μm): USL = 0.1 μm									
0.2559	0.2627	0.2717	0.2656	0.2756	0.2747	0.2645	0.2671	0.2588	0.2703
0.2689	0.2633	0.2694	0.2573	0.2691	0.2776	0.2550	0.2632	0.2624	0.2605
0.2783	0.2623	0.2691	0.2571	0.2616	0.2759	0.2670	0.2688	0.2598	0.2620
0.2788	0.2507	0.2661	0.2726	0.2807	0.2735	0.2673	0.2478	0.2831	0.2653
0.2691	0.2792	0.2718	0.2791	0.2770	0.2581	0.2731	0.2660	0.2612	0.2718
0.2657	0.2711	0.2579	0.2649	0.2760	0.2707	0.2769	0.2605	0.2648	0.2723
0.2657	0.2650	0.2764	0.2827	0.2734	0.2676	0.2757	0.2662	0.2758	0.2753
0.2514	0.2654	0.2754	0.2842	0.2524	0.2734	0.2687	0.2743	0.2631	0.2719
0.2726	0.2828	0.2750	0.2721	0.2633	0.2608	0.2877	0.2628	0.2894	0.2638
0.2700	0.2654	0.2819	0.2728	0.2713	0.2670	0.2580	0.2730	0.2652	0.2794
0.2656	0.2850	0.2735	0.2774	0.2730	0.2757	0.2640	0.2707	0.2564	0.2634
0.2638	0.2727	0.2681	0.2647	0.2720	0.2687	0.2627	0.2828	0.2838	0.2700
0.2638	0.2640	0.2797	0.2708	0.2704	0.2475	0.2713	0.2710	0.2870	0.2610
0.2651	0.2729	0.2698	0.2702	0.2694	0.2586	0.2619	0.2790	0.2723	0.2833
0.2709	0.2592	0.2740	0.2598	0.2557	0.2790	0.2714	0.2874	0.2656	0.2789
Uniformality: USL = 0.03									
0.0272	0.0264	0.0255	0.0267	0.0248	0.0272	0.0270	0.0267	0.0257	0.0265
0.0264	0.0265	0.0252	0.0278	0.0263	0.0272	0.0252	0.0264	0.0264	0.0247
0.0271	0.0276	0.0268	0.0293	0.0283	0.0265	0.0269	0.0275	0.0277	0.0257
0.0255	0.0269	0.0259	0.0271	0.0273	0.0256	0.0278	0.0283	0.0267	0.0277
0.0254	0.0265	0.0280	0.0283	0.0262	0.0269	0.0267	0.0266	0.0263	0.0261
0.0269	0.0270	0.0262	0.0279	0.0252	0.0255	0.0277	0.0254	0.0262	0.0279
0.0265	0.0271	0.0286	0.0252	0.0261	0.0266	0.0278	0.0270	0.0255	0.0274
0.0244	0.0272	0.0279	0.0259	0.0266	0.0265	0.0256	0.0274	0.0266	0.0282
0.0268	0.0260	0.0256	0.0253	0.0268	0.0287	0.0270	0.0294	0.0265	0.0258
0.0275	0.0265	0.0282	0.0270	0.0266	0.0267	0.0254	0.0270	0.0277	0.0257
0.0278	0.0255	0.0274	0.0260	0.0273	0.0269	0.0256	0.0293	0.0256	0.0274
0.0249	0.0265	0.0269	0.0269	0.0268	0.0267	0.0262	0.0266	0.0271	0.0269
0.0252	0.0257	0.0286	0.0267	0.0265	0.0270	0.0270	0.0261	0.0264	0.0263
0.0280	0.0271	0.0267	0.0274	0.0266	0.0277	0.0252	0.0268	0.0267	0.0257
0.0264	0.0273	0.0244	0.0263	0.0264	0.0258	0.0268	0.0260	0.0276	0.0256

where $\widehat{C}^{T*}_{PU}(i)$ is the ith bootstrap estimate. The quantity $S^*_{\widehat{C}^T_{PU}}$ is actually an estimator of the standard deviation of $\widehat{C}^T_{PU}$, and if $\widehat{C}^T_{PU}$ is approximately normal distribution, the $(1-2\alpha)$ 100% SB confidence interval can be obtained as

$$[\widehat{C}^T_{PU} - Z_\alpha S^*_{\widehat{C}^T_{PU}}], \tag{21}$$

where Z_α is the upper α quantile of the standard normal distribution.

3.2.2. The percentile bootstrap (PB)

From the ordered collection of $\widehat{C}^{T*}_{PU}(i)$, select the α percent and the $(1-\alpha)$ percent points as the end points, and the PB confidence interval is

$$[\widehat{C}^{T*}_{PU}(\alpha B)]. \tag{22}$$

3.2.3. Biased-corrected percentile bootstrap (BCPB)

The bootstrap distribution may be biased while the percentile confidence interval is possible due to sampling errors. In other words, that bootstrap distributions obtained using only a sample of the complete bootstrap distribution may be shifted higher or lower than expected. Thus, a three steps procedure has been developed to correct for this potential bias Efron (1982). First, using the ordered distribution of $\widehat{C}^{T*}_{PU}$, calculate the probability of

$$P_0 = P[\widehat{C}^{T*}_{PU} \leqslant \hat{c}^T_{PU}]. \tag{23}$$

Second, calculate

$$Z_0 = \Phi^{-1}(P_0), \tag{24}$$
$$P_L = \Phi(2Z_0 - Z_\alpha), \tag{25}$$
$$P_U = \Phi(2Z_0 + Z_\alpha), \tag{26}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. Finally, the BCPB confidence is obtained as

$$[\widehat{C}^{T*}_{PU}(P_L B)]. \tag{27}$$

3.2.4. Bootstrap-t (BT)

While the distribution of the statistic is skewed, the percentile bootstrap confidence interval is probably lower. Thus, the

Table 10
The average rank of the four bootstrap methods as C_{PU} = 1,1.33 and v = 2

n		$C_{PU}^{T}=1$					$C_{PU}^{T}=1.33$				
		N_1	N_2	N_3	N_4	R	N_1	N_2	N_3	N_4	R
30	SB	0	1	498	1	3	1	0	499	0	2.996
	PB	2	497	0	1	2	2	498	0	0	1.996
	BCPB	498	1	1	0	1.006	497	2	1	0	1.008
	PT	0	1	1	498	3.994	0	0	0	500	4
40	SB	1	0	499	0	2.996	2	6	491	1	2.982
	PB	0	499	0	1	2.004	3	491	5	1	2.008
	BCPB	499	0	1	0	1.004	493	1	2	4	1.034
	PT	0	1	0	499	3.996	3	1	2	494	3.974
50	SB	1	3	495	1	2.992	2	3	493	2	2.99
	PB	2	495	3	0	2.002	2	493	2	3	2.012
	BCPB	495	1	1	3	1.024	492	4	1	3	1.03
	PT	2	1	1	496	3.982	4	0	4	492	3.968
60	SB	1	7	492	0	2.982	3	6	487	4	2.984
	PB	1	492	6	1	2.014	4	488	7	1	2.01
	BCPB	492	1	1	6	1.042	487	5	1	7	1.056
	PT	6	0	1	493	3.962	6	1	5	488	3.95
70	SB	1	2	496	1	2.994	1	8	489	2	2.984
	PB	0	498	2	0	2.004	2	488	8	2	2.02
	BCPB	497	0	0	3	1.018	493	2	2	3	1.03
	PT	2	0	2	496	3.984	4	2	1	493	3.966
80	SB	0	7	492	1	2.988	4	9	484	3	2.972
	PB	1	492	7	0	2.012	3	482	13	2	2.028
	BCPB	495	1	0	4	1.026	485	4	1	10	1.072
	PT	4	0	1	495	3.974	8	5	2	485	3.928
90	SB	1	4	492	3	2.994	5	11	478	6	2.97
	PB	0	494	4	2	2.016	4	482	11	3	2.026
	BCPB	495	1	2	2	1.022	483	3	3	11	1.084
	PT	4	1	2	493	3.968	8	4	8	480	3.92
100	SB	1	10	487	2	2.98	3	6	483	8	2.992
	PB	2	488	8	2	2.02	8	484	7	1	2.002
	BCPB	494	2	1	3	1.026	484	5	2	9	1.072
	PT	3	0	4	493	3.974	5	5	8	482	3.934
125	SB	1	2	493	4	3	5	14	473	8	2.968
	PB	1	494	4	1	2.01	3	478	15	4	2.04
	BCPB	495	3	0	2	1.018	480	2	4	14	1.104
	PT	3	1	3	493	3.972	12	6	8	474	3.888
150	SB	1	8	487	4	2.988	4	22	457	17	2.974
	PB	2	490	8	0	2.012	6	465	24	5	2.056
	BCPB	492	1	2	5	1.04	480	5	3	12	1.094
	PT	5	1	3	491	3.96	10	8	16	466	3.876
200	SB	0	8	482	10	3.004	7	15	463	15	2.972
	PB	3	487	8	2	2.018	12	469	17	2	2.018
	BCPB	492	3	0	5	1.036	466	11	6	17	1.148
	PT	5	2	10	483	3.942	16	5	13	466	3.858

bootstrap-*t* is developed and that the generated distribution will mimic the distribution of *T*. First, approximate the distribution of a statistic of $T=(\widehat{C}_{PU}^{T}-C_{PU}^{T})/S_{C_{PU}^{T}}$ by using bootstrap. By taking bootstrap samples from the original data values the bootstrap approximation in this case can be obtained, calculate the corresponding estimates $\widehat{C}_{PU}^{T*}(i)$ and their standard error, and then find the *T*-values $T=(\widehat{C}_{PU}^{T*}-\widehat{C}_{PU}^{T})/S_{C_{PU}^{T}}^{*}$. The $(1-2\alpha)$ 100% BT confidence interval can be obtained as

$$[\widehat{C}_{PU}^{T}-t_{\alpha}^{*}S_{C_{PU}^{T}}^{*}], \tag{28}$$

where t_{α}^{*} and $t_{1-\alpha}^{*}$ are the upper α and $1-\alpha$ quantile of the bootstrap *T*-distribution respectively.

4. Performance comparisons of bootstrap methods

We use bootstrap methods to calculate the lower confidence bound of C_{PU}^{T}, used to demonstrate the estimation accuracy $R_{PU}=C_{PU}^{T(LB)}/\widehat{C}_{PU}^{T}$. Then we can determine the required sample sizes for specified estimation accuracy on C_{PU}^{T}. We also rank the four bootstrap methods according to R_{PU} for ascertaining their performance.

Considering the data generated by MATLAB program with multiple independent characteristics from normal distribution, the assumption of normality for each single characteristic is required for the process yield calculation. But the bootstrap approach, which does not require any assumption, is a general nonparametric method. Let the capability C_{PU_j} of each single characteristic satisfy the minimal value (see Table 2) required for overall process capability C_{PU}^{T}. For example, if a process has a capability requirement $C_{PU}^{T}\geqslant 1.00$ with v = 5, i.e., the capability for all the five characteristics is the following $C_{PUj}\geqslant 1.153$ for $j=1,2,\ldots,5$. We repeated 500 simulations and then obtained each rank of the four bootstrap methods. The calculation of the total weighted average rank *R* of each bootstrap method is as follows:

$$R=\frac{1}{500}\sum_{i=1}^{4}N_i\times i, \tag{29}$$

Table 11
The average rank of four bootstrap methods as $C_{PU} = 1, 1.33$ and $v = 3$

n		$C^T_{PU} = 1$					$C^T_{PU} = 1.33$				
		N_1	N_2	N_3	N_4	R	N_1	N_2	N_3	N_4	R
30	SB	5	11	480	4	2.966	5	19	476	0	2.942
	PB	3	482	13	2	2.028	3	473	21	3	2.048
	BCPB	480	4	2	14	1.1	479	3	1	17	1.112
	PT	12	3	5	480	3.906	14	4	2	480	3.896
40	SB	7	12	480	1	2.95	4	20	471	5	2.954
	PB	3	477	17	3	2.04	5	470	22	3	2.046
	BCPB	479	4	2	15	1.106	473	6	3	18	1.132
	PT	11	7	1	481	3.904	18	4	4	474	3.868
50	SB	2	15	479	4	2.97	2	25	471	2	2.946
	PB	3	478	18	1	2.034	5	469	25	1	2.044
	BCPB	481	3	1	15	1.1	472	4	1	23	1.15
	PT	14	4	2	480	3.896	21	2	3	474	3.86
60	SB	9	16	470	5	2.942	8	34	447	11	2.922
	PB	3	468	20	9	2.07	6	451	33	10	2.094
	BCPB	473	6	7	14	1.124	456	9	6	29	1.216
	PT	15	10	3	472	3.864	30	6	14	450	3.768
70	SB	5	19	470	6	2.954	5	47	433	15	2.916
	PB	2	473	22	3	2.052	7	439	45	9	2.112
	BCPB	475	6	3	16	1.12	444	9	5	42	1.29
	PT	18	2	5	475	3.874	45	5	16	434	3.678
80	SB	8	24	456	12	2.944	12	42	428	18	2.904
	PB	7	462	24	7	2.062	9	438	43	10	2.108
	BCPB	463	8	9	20	1.172	436	8	10	46	1.332
	PT	23	5	12	460	3.818	43	12	19	426	3.656
90	SB	8	24	458	10	2.94	15	69	399	17	2.836
	PB	3	465	22	10	2.078	12	407	66	15	2.168
	BCPB	466	6	7	21	1.166	415	9	23	53	1.428
	PT	23	5	13	459	3.816	58	15	12	415	3.568
100	SB	9	23	460	8	2.934	15	49	410	26	2.894
	PB	4	463	26	7	2.072	12	423	50	15	2.136
	BCPB	465	7	10	18	1.162	428	13	14	45	1.352
	PT	22	7	6	465	3.828	46	16	25	413	3.61
125	SB	6	18	464	12	2.964	19	79	369	33	2.832
	PB	5	467	21	7	2.06	18	386	88	8	2.172
	BCPB	471	11	3	15	1.124	386	19	9	86	1.59
	PT	18	4	12	466	3.852	77	16	34	373	3.406
150	SB	3	22	462	13	2.97	25	85	353	37	2.804
	PB	9	467	17	7	2.044	25	377	83	15	2.176
	BCPB	470	7	8	15	1.136	378	18	20	84	1.62
	PT	19	3	13	465	3.848	72	20	45	363	3.398
200	SB	15	42	412	31	2.918	26	97	336	41	2.784
	PB	8	436	47	9	2.114	30	350	95	25	2.23
	BCPB	456	10	12	22	1.2	364	34	22	80	1.636
	PT	21	12	29	438	3.768	82	18	46	354	3.344

where N_i is the total number of rank i ($i = 1,2,\ldots,5$) during the simulations. For example, if a process has a capability requirement $C^T_{PU} \geqslant 1.00$ with $v = 2$ and sample size $n = 30$, we can obtained the weighted average rank R of the SB method to be

$$R = (0 \times 1 + 1 \times 2 + 498 \times 3 + 1 \times 4)/500 = 3. \tag{30}$$

In Table 3, the weighted average rank R of four bootstrap method is illustrated with various sample size $n = 30(10)100$, 125, 150, 200, and $v = 2(1)5$ as $C^T_{PU} = 1$ and 1.33. For example, if the sample size n is 60, $v = 2$ and the C^T_{PU} is 1.33, the weighted average rank R of the four bootstrap methods are 2.984, 2.010, 1.056, 3.950, respectively (see Table 3).

From Table 3, the BCPB (biased-corrected percentile bootstrap) method performs better than other ones when $C^T_{PU} = 1$. For the fixed values of C^T_{PU} and n, the weighted average rank of each method gets closer to each other as v increases. This fact indicates that the performances of four methods are not much different when v is large.

It is shown in Figs. 1a–1d, 2a–2d that the BCPB method is distinctly better when sample size $n < 125$. However, as the sample size is greater than 125, the performances of four methods are not much different. Furthermore, as $C^T_{PU} = 1.33$ and the quality characteristic $v = 5$, the weighted average rank R of BCPB method is larger than others. This indicates that BCPB method performs worse than the other ones when $n > 125$, $v = 5$ and $C^T_{PU} = 1.33$ (see Fig. 2d). Actually, the estimation of the four methods is similar in this situation. As a result of this fact, we recommend the BCPB method is the best one to calculate $\widehat{C}^T_{PU}$ when the sample size $n < 125$. In the following section, we use BCPB method to evaluate the estimator $\widehat{C}^T_{PU}$.

5. Sample size required for designated estimation accuracy

The sample size determination is important, as it directly relates to the cost of data collection plan. We develop a MATLAB program to compute the required sample size n. The BCPB method is the recommended one to calculate the estimator $\widehat{C}^T_{PU}$. In Section 5, we use the simulation data which is randomly generated from normal distribution to determine the required sample size n.

Table 12
The average rank of four bootstrap methods as C_{PU} = 1,1.33 and v = 4

n		$C_{PU}^{T}=1$					$C_{PU}^{T}=1.33$				
		N_1	N_2	N_3	N_4	R	N_1	N_2	N_3	N_4	R
30	SB	2	21	467	10	2.97	4	30	458	8	2.94
	PB	3	470	25	2	2.052	2	465	26	7	2.076
	BCPB	478	3	2	17	1.116	469	1	6	24	1.17
	PT	18	5	8	469	3.856	25	4	10	461	3.814
40	SB	6	35	456	3	2.912	8	49	437	6	2.882
	PB	3	457	35	5	2.084	7	438	48	7	2.11
	BCPB	461	4	5	30	1.208	442	7	10	41	1.3
	PT	30	4	4	462	3.796	43	6	5	446	3.708
50	SB	7	31	456	6	2.922	18	73	387	22	2.826
	PB	1	458	34	7	2.094	13	395	78	14	2.186
	BCPB	458	4	6	32	1.224	400	16	14	70	1.508
	PT	34	7	4	455	3.76	69	16	22	393	3.478
60	SB	11	30	452	7	2.91	17	80	390	13	2.798
	PB	6	450	34	10	2.096	15	391	79	15	2.188
	BCPB	457	6	6	31	1.222	399	13	12	76	1.53
	PT	26	14	8	452	3.772	70	16	19	395	3.478
70	SB	16	38	435	11	2.882	16	91	359	34	2.822
	PB	4	444	37	15	2.126	21	371	102	6	2.186
	BCPB	447	9	14	30	1.254	372	20	12	96	1.664
	PT	33	10	13	444	3.736	91	18	27	364	3.328
80	SB	8	39	440	13	2.916	12	112	345	31	2.79
	PB	8	438	46	8	2.108	18	351	107	24	2.274
	BCPB	450	7	6	37	1.26	370	19	19	92	1.666
	PT	34	16	8	442	3.716	101	17	29	353	3.268
90	SB	12	45	428	15	2.892	23	114	332	31	2.742
	PB	12	428	48	12	2.12	26	330	126	18	2.272
	BCPB	435	12	12	41	1.318	343	32	15	110	1.784
	PT	41	15	13	431	3.668	110	23	26	341	3.196
100	SB	14	46	423	17	2.886	25	122	316	37	2.73
	PB	11	427	51	11	2.124	27	317	131	25	2.308
	BCPB	440	16	8	36	1.28	332	28	23	117	1.85
	PT	37	9	18	436	3.706	116	34	29	321	3.11
125	SB	9	47	418	26	2.922	34	152	277	37	2.634
	PB	18	416	51	15	2.126	28	281	154	37	2.4
	BCPB	434	19	6	41	1.308	292	31	35	142	2.054
	PT	39	18	25	418	3.644	146	36	34	284	2.912
150	SB	7	55	409	29	2.92	30	155	258	57	2.684
	PB	29	409	45	17	2.1	28	275	160	37	2.412
	BCPB	424	26	12	38	1.328	295	33	32	140	2.034
	PT	42	8	34	416	3.648	147	37	51	265	2.868
200	SB	21	55	387	37	2.88	35	187	205	73	2.632
	PB	24	397	58	21	2.152	45	232	184	39	2.434
	BCPB	412	24	19	45	1.394	240	37	45	178	2.322
	PT	44	23	37	396	3.57	182	42	67	209	2.606

Based on the procedure above, a Matlab algorithm for calculating the required sample size is developed as follows:

Algorithm for the required sample size

Step 1. Input the value of characteristics v, the designated estimation accuracy R_{PU} and the initial sample size values of Lo and Hi. Compute $R_{PU}(Hi)$ and $R_{PU}(Lo)$ to ensure that $R_{PU}(Hi) > R_{PU}$ and $R_{PU}(Lo) < R_{PU}$.

Step 2. Let $n = (Lo + Hi)/2$. Compute $R_{PU}(n)$. If $-Hi - Lo- \leqslant 1$, stop and choose $n = \{x|Min|R_{PU}(x) - R_{PU}|, x \in (Hi, Lo)\}$, and then return n (always rounding up if n is not an integer) as the required sample size.

Step 3. If $R_{PU}(n) > R_{PU}$, $Hi \leftarrow n$; otherwise, $Lo \leftarrow n$. Go back to Step 2.We implement the algorithm and develop a MATLAB program to compute the required sample size. Tables 4, 5 tabulate the required sample size for R_{PU} = 0.75(0.01) 0.95 and γ = 0.9, 0.95, 0.975, 0.99.

Let the desired estimation accuracy be R_{PU} and the confidence level be γ, and then the minimum sample size n(always rounding up if n is not an integer) can be calculated. Tables 4, 5 display the sample size n required for $R_{\gamma} \geqslant R_{PU}$ with quality characteristic v = 3, R_{PU} = 0.75(0.01)0.95 and γ = 0.9, 0.95, 0.975, and 0.99 when $C_{PU}^{T} = 1$ and 1.33. For example, if R_{PU} is set to 0.89, $C_{PU}^{T} = 1$, and γ = 0.95, the sample size needed is n = 76. We conclude that a minimum sample size of n = 76 is required to be 95% so that the true C_{PU} is no less than R_{γ} = 89.12% of the sample estimate $\widehat{C}_{PU}$. Thus, if the sample estimate $\widehat{C}_{PU}^{T} = 1.2$, the true value of C_{PU}^{T} is no less than $1.2 \times 89.12\%$ = 1.069, with 95% confidence.

From Tables 4 and 5, we can find that as R_{PU} and γ increase, the required sample size n increases. However, some values of sample size can not be obtained (see the sign "–" in the column of the sample size n) when the values of R_{PU} and confidence level γ are small. This is due to the problem of the bootstrap resampling procedure.

Table 13
The average rank of four bootstrap methods as C_{PU} = 1,1.33 and ν = 5

n		$C_{PU}^T = 1$					$C_{PU}^T = 1.33$				
		N_1	N_2	N_3	N_4	R	N_1	N_2	N_3	N_4	R
30	SB	4	33	453	10	2.938	8	64	418	10	2.86
	PB	4	455	36	5	2.084	5	422	66	7	2.15
	BCPB	458	5	5	32	1.222	425	10	6	59	1.398
	PT	34	7	6	453	3.756	62	4	11	423	3.59
40	SB	7	50	440	3	2.878	19	87	376	18	2.786
	PB	7	438	44	11	2.118	16	372	100	12	2.216
	BCPB	441	5	10	44	1.314	376	19	8	97	1.652
	PT	45	7	7	441	3.688	90	21	16	373	3.344
50	SB	11	51	419	19	2.892	14	101	360	25	2.792
	PB	4	429	56	11	2.148	13	372	97	18	2.24
	BCPB	436	10	9	45	1.326	376	15	18	91	1.648
	PT	50	10	15	425	3.63	97	13	25	365	3.316
60	SB	12	65	409	14	2.85	17	117	336	30	2.758
	PB	4	421	66	9	2.16	20	347	119	14	2.254
	BCPB	425	6	9	60	1.408	344	20	14	122	1.828
	PT	59	8	16	417	3.582	121	14	31	334	3.156
70	SB	9	75	400	16	2.846	14	156	303	27	2.686
	PB	12	407	71	10	2.158	30	297	148	25	2.336
	BCPB	415	9	12	64	1.45	302	31	19	148	2.026
	PT	65	9	16	410	3.542	158	12	31	299	2.942
80	SB	12	59	395	34	2.902	27	159	274	40	2.654
	PB	6	414	64	16	2.18	17	289	171	23	2.4
	BCPB	427	8	12	53	1.382	290	26	23	161	2.11
	PT	57	18	29	396	3.528	168	24	33	275	2.83
90	SB	8	82	374	36	2.876	28	195	240	37	2.572
	PB	25	381	77	17	2.172	33	249	187	31	2.432
	BCPB	396	25	12	67	1.5	259	28	23	190	2.288
	PT	71	12	37	380	3.452	182	27	49	242	2.702
100	SB	19	83	362	36	2.83	24	217	203	56	2.582
	PB	25	368	93	14	2.192	37	219	212	32	2.478
	BCPB	384	25	18	73	1.56	235	31	41	193	2.384
	PT	76	20	29	375	3.406	206	31	45	218	2.55
125	SB	11	73	369	47	2.904	25	225	195	55	2.56
	PB	30	378	76	16	2.156	42	207	219	32	2.482
	BCPB	388	30	19	63	1.514	197	38	33	232	2.6
	PT	72	18	38	372	3.42	236	30	55	179	2.354
150	SB	24	84	332	60	2.856	45	233	166	56	2.466
	PB	22	364	92	22	2.228	52	170	242	36	2.524
	BCPB	382	25	18	75	1.572	186	43	32	239	2.648
	PT	72	27	59	342	3.342	222	50	61	167	2.346
200	SB	26	88	329	57	2.834	40	252	163	45	2.426
	PB	28	350	94	28	2.244	49	146	254	51	2.614
	BCPB	372	37	21	70	1.578	137	54	42	267	2.878
	PT	74	27	54	345	3.34	278	44	41	137	2.074

6. Application

We consider the following case from a manufacturing factory located on the Science-Based Industrial Park in Taiwan, making the thin film transistor liquid crystal display (TFT-LCD). There are three major process groups in TFT-LCD manufacturing process, array process; cell process and module assemble process. The array process is similar to the semiconductor manufacturing process, except that transistors are fabricated on a glass substrate instead of a silicon wafer. Photolithography (one of the array process) is a critical step within LCD manufacturing process since the panel quality depends on the entire pattern formation. Film deposition is done before photolithography. Overlay is a key parameter in deposition process and uniformality is a key parameter in coating and exposure, which are two processes in photolithography. We focus on these key parameters, such as overlay, critical dimension and uniformality.

In Fig. 3, between one deposited layer and another, a distance called overlay may be existed. There are three steps in photolithography process, coating, exposure, and development. It might result deviation as exposure on panel window, called critical dimension (see Fig. 4). In addition, coating photoresist on panel has to be uniform. The specifications of these three key parameters are shown in Table 6. Since the assumption of normality for each single characteristic is required for the process yield calculation, the historical data of each key characteristic indicates the process being pretty approximate to a normal distribution. Thus, we can conclude that each characteristic data collected from the process is in control and normally distributed.

To obtain the sample size required n under the desired estimation accuracy $R_{pm}^{(PS)}$, we can find it in Table 4. If the practitioners set $R_{pm}^{(PS)}$ to be 0.92, and γ = 0.95 the sample size needed is n = 146. We conclude that a minimum sample size of n = 146 is required to be 95% so that the true C_{PU}^T is no less than R_γ = 92.11% of the sample estimator $\widehat{C}_{PU}^T$. Thus, if the sample estimator $\widehat{C}_{PU}^T = 1.3$, the true value of C_{PU}^T is no less than $1.3 \times 92.11\%$ = 1.197, with 95% confidence. Hence, sample data collected from 150 LCD is displayed in Table 9 of the Appendix. And the upper specification limit, the calculated sample mean, location departure, sample standard deviation, the estimated C_{PU_j} and the lower confidence bound L_C for each characteristic are summarized in Table 7.

6.1. Overall process yield analysis

The sample estimators of C_{PU}^T and the BCPB method lower confidence bound of C_{PU}^T for the single characteristic overlay, critical dimension and uniformality coupler can be summarized in Table 8.

Table 8 displays the manufacturing capability and its corresponding NCPPM for the LCD process using the estimated $\widehat{C}_{PU}^T$ values (uncorrected) and the lower confidence bounds $C_{PU}^{T(LB)}$ (corrected). The $C_{PU}^{T(LB)}$ (the LCB of C_{PU}^T) obtained using BCPB method is certainly more reliable than the estimated $\widehat{C}_{PU}^T$ index values (an approach widely used in current industrial applications), since the sampling errors are considered in the LCB approach. In fact, as the sample estimate $\widehat{C}_{PU}^T$ may overestimate the true capability (overall process yield), it conveys unreliable and misleading information, which should be avoided in factory applications. Based on the value of $C_{PU}^{T(LB)}$, we thus can assure that the production yield is 99.7308%, and the number of the nonconformities is less than 2692 PPM.

7. Conclusions

In this paper, we considered the problem of finding the lower confidence bound and sample sizes required for specified estimation accuracy for the C_{PU}^T. Since the sampling distribution of C_{PU}^T is analytically intractable, we applied the bootstrap method to calculate the estimator of C_{PU}^T and compared the estimation accuracy of the four bootstrap methods. The results indicated that the BCPB method has good performance when the sample size is smaller than 125. The lower confidence bounds present a measure on the minimum capability of the process based on the sample data. We also investigated the lower confidence bound values and sample sizes required for specified estimation accuracy using BCPB method. The proposed approach ensures that the risk of making incorrect decisions is no larger than the preset Type I error $1-\gamma$. We also provided tables for the engineers/practitioners to use for their in-plant applications. A real-world example from TFT-LCD manufacturing process is investigated to illustrate the applicability of our approach. In the future, we can consider same approach for indices with two-sided specifications, such as C_{pk}, C_{pm} and C_{pmk}. We also consider multiple characteristics with some correlations.

Appendix

See Tables 9–13.

References

Bissell, A.F., 1990. How reliable is your capability index? Applied Statistics 39, 331–340.

Bothe, D.R., 1992. A capability study for an entire product. ASQC Quality Congress Transactions, 172–178.

Castagliola, P., Castellanos, J.V.G., 2005. Capability indices dedicated to the two quality characteristics case. Quality Technology and Quantitative Management 2 (2), 201–220.

Chen, K.S., 1998. Estimation of the process incapability index. Communications in Statistics – Theory and Methods 27 (5), 1263–1274.

Chen, S.M., Hsu, Y.S., 2004. Uniformly most powerful test for process capability index C_{pk}. Quality Technology and Quantitative Management 1 (2), 257–269.

Chen, K.S., Pearn, W.L., Lin, P.C., 2003a. Capability measures for processes with multiple characteristics. Quality and Reliability Engineering International 19 (2), 101–110.

Chen, T.W., Chen, K.S., Lin, J.Y., 2003b. Fuzzy evaluation of process capability for bigger–the best type products. The International Journal of Advanced Manufacturing Technology 21, 820–826.

Cheng, S.W., Leung, B., Spiring, F.A., 2006. Assessing process capability: A case study. Quality Technology and Quantitative Management 3 (2), 191–206.

Chou, Y.M., Owen, D.B., Borrego, S.A., 1990. Lower confidence limits on process capability indices. Journal of Quality Technology 22, 223–229.

Efron, B., 1981. Nonparametric standard errors and confidence intervals. Canadian Journal of Statistics 9, 139–172.

Efron, B., 1982. The Jackknife, The Bootstrap and Other Resampling Plans, vol. 38. Society for Industrial and Applied Mathematics, Philadelphia, PA.

Efron, B., Tibshirani, R.J., 1986. Bootstrap methods for standard errors, confidence interval, and other measures of statistical accuracy. Statistical Science 1, 54–77.

Flaig, J.J., 2006. Selecting optimal specification limits. Quality Technology and Quantitative Management 3 (2), 207–216.

Franklin, L.A., Wasserman, G.S., 1992. Bootstrap lower confidence limits for capability indices. Journal of Quality Technology 24 (4), 196–210.

Huang, M.L., Chen, K.S., Li, R.K., 2005. Graphical analysis of capability of a process producing a product family. Quality and Quantity 39 (5), 643–657.

Kane, V.E., 1986. Process capability indices. Journal of Quality Technology 18 (1), 41–52.

Pearn, W.L., Cheng, Y.C., 2007. Estimating process yield based on S_{pk} for multiple samples. International Journal of Production Research 45 (1), 49–64.

Pearn, W.L., Kotz, S., Johnson, N.L., 1992. Distributional and inferential properties of process capability indices. Journal of Quality Technology 24 (4), 216–231.

Vännman, K., 2006. Safety regions in process capability plots. Quality Technology and Quantitative Management 3 (2), 227–246.

Vännman, K., Albing, M., 2007. Process capability plots for one-sided specification limits. Quality Technology and Quantitative Management 4 (4), 569–590.

Wu, C.W., Pearn, W.L., 2005. Measuring manufacturing capability for couplers and wavelength division multiplexers (WDM). International Journal of Advanced Manufacturing Technology 25 (5/6), 533–541.