A practical design for a robust fault detection and isolation system

DAW-SHANG HWANG [a], SHAO-KUNG CHANG [b] & PAU-LO Hsu [a]

[a] Institute of Control Engineering, National Chiao Tung University, Hsinchu, Taiwan, 300, Republic of China

[b] Mechanical Industry Research Laboratories, Industrial Technology Research Institute, Hsinchu, Taiwan, 300, Republic of China.

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A practical design for a robust fault detection and isolation system

Daw-Shang Hwang†, Shao-Kung Chang‡ and Pau-Lo Hsu†§

Based on a development for the unknown input observer (UIO) design, this paper presents a straightforward method of designing a robust fault detection and isolation (FDI) system for uncertain systems with unknown inputs. By exploiting the properties of a nominal model structure, we form a basic set of residuals which is independent of the unknown input but is dependent on the measurements. Then, with this basic residual set, we append a dynamic weighting matrix to the residuals to increase the design freedom for meeting the required detection and isolation performance. Additionally, a generalized residual generator scheme (GRGS) is proposed to synthesize the parity relations for the isolation of faulty sensors in an uncertain system. The proposed strategy is verified through simulation studies performed on the control of a vertical take-off and landing (VTOL) aircraft in the vertical plane.

1. Introduction

The performance of a model-based fault detection and isolation (FDI) system depends heavily on the accuracy of the mathematical model. An accurate mathematical model of the process, however, is never available because the process is usually nonlinear in nature and its parameters are varying with time. Moreover, the characteristics of disturbances and noise of the system are also unknown so they may not be modelled at all. These difficulties in modelling certainly cannot be well covered in a linear model with constant parameters employed for model-based FDI design. The discrepancy between the mathematical model and the actual process is known as modelling error or model uncertainties, and it may causes false alarms and deteriorate the performance of the model-based FDI system to such an extent that, the FDI system may even become totally useless. Therefore, to design a robust FDI system in the sense that the fuction of FDI can be insensitive to model uncertainties is a key issue in model-based FDI techniques. Recently, robust approaches to fault diagnostic systems have gained much attention, as reported

by Watanabe and Himmelblau (1982), Viswannadham and Srichander (1987), Frank and Wünnenberg (1989), Patton *et al.* (1989), Saif and Guan (1993), Frank (1994).

The 'unknown input decoupling' approach is an important robust technique for FDI. In this approach, according to structured uncertainty concepts, most unknown uncertainties such as modelling error and the disturbances to a system are modelled as unknown input, and their structural characteristics are summarized in the unknown input distribution matrix. Based on this assumption, one desires to make the residual indpendent of all unknown inputs to obtain a robust FDI. Unknown input decoupling designs can be achieved by using the unknown input observer (UIO) (Watanabe and Himmelblau 1982, Viswannadham and Srichander 1987, Frank and Wünnenberg 1989, Saif and Guan 1993, Frank 1994, Chang and Hsu 1995) or eigenstructure assignment (Patton and Kangethe 1989, Patton and Chen 1993, 1994). Such designs generally involve considerable computational complexity. In this paper, by considering a linear system with unknown inputs, we first derive an equivalent system free of unknown inputs for residual generator design. Then, based on the equivalent system description, one can form a basic residual generator in which unknown inputs are perfectly decoupled. With this basic residual generator, we append a dynamic weighting matrix to increase the design freedom and to meet the required detection and isolation performance. The proposed design is more straightforward and simpler than methods based on UIO or eigenstructure assign-

ment. In addition to fault detection, fault isolation is also crucial in a diagnostic system when faults occur in different sectors of the system, e.g. in different sensors, actuators, or components. It is desirable that the residuals generated by the diagnostic system have a structured or fixed-directional form to achieve fault isolation (Gertler 1993, Patton and Chen 1994). The associated isolation problem is also treated in this paper. Here, we adopt the proposed technique in accordance with the concept of a fault identification filter (FIDF) (Chang *et al.* 1995) to design an actuator fault detection filter for a vertical take-off and landing (VTOL) aircraft, in which simultaneous isolation for multiple actuator failures is achieved. In addition, a generalized residual generator scheme (GRGS) to synthesis the parity relations for the isolation of a faulty sensor for that aircraft is also used to demonstrate the validity of the proposed strategies.

The following notation will be used in this paper: $a := b$ means a denotes b. $\mathbf{R} :=$ the field of real numbers; $\mathbf{C} :=$ the field of complex numbers. $\mathbf{R}[s] :=$ the ring of polynomials in s with real coefficients. $\mathbf{C}_+ := \{s \in \mathbf{C} \mid \text{Re}(s) \geq 0\}$; $\mathbf{C}_- := \{s \in \mathbf{C} \mid \text{Re}(s) < 0\}$; $Z[P] :=$ the zeros of the polynomial P. $f = \text{LCM}\{g, h\} := f(s)$ is the monic least common multiplier of $g(s)$ and $h(s)$; i.e. $f(s)$ has its leading coefficient equal to 1, $g \mid f$ and $h \mid f$.

2. Robust residual generation

2.1. Basic concepts of robust residual generation

We consider the state-space model of a dynamic system with input vector u ($u \in \mathbf{R}^p$) and output vector y ($y \in \mathbf{R}^m$) as

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t) + Kf(t) \quad (1\,a)$$

$$y(t) = Cx(t) + Gf(t), \quad (1\,b)$$

where $x \in \mathbf{R}^n$ is the state variable, $f \in \mathbf{R}^q$ represents the effect of actuator faults, component faults, and sensor (instrument) faults, $Kf(t)$ and $Gf(t)$ model the actuator, component, and sensor faults with the distribution matrix K and G whose elements depend on how the actuators, or the components, or the sensors are mounted in the system. $d \in \mathbf{R}^r$ characterizes the uncertainties implicit in the system. The term $Ed(t)$ models the system disturbance either in structured form or unstructured form (Watanabe and Himmelblau 1982, Frank and Wünnenberg 1989). Recently, an identification procedure has been considered to identify the constant E matrix and use the $Ed(t)$ to summarize the effect of unstructured modelling errors. Patton and Chen (1993) proposed a useful method for computing the unknown input distribution matrix E for practical large-scale nonlinear plants by re-identifying plant parameters from different operating points to achieve the E matrix.

Based on (1) the input–output description of the system is

$$y(s) = G_u(s)u(s) + G_d(s)d(s) + G_f(s)f(s), \quad (2)$$

where

$$G_u(s) = C(sI - A)^{-1}B$$

$$G_d(s) = C(sI - A)^{-1}E$$

$$G_f(s) = C(sI - A)^{-1}K + G.$$

A residual generator is a dynamic system which can be written in the following general form

$$r(s) = H_u(s)u(s) + H_y(s)y(s), \quad (3)$$

where $H_u(s)$ and $H_y(s)$ are transfer matrices that should be realizable using stable linear systems. Note that (3) is called the computation form of the residual generator, since it contains the measurables. By substituting $y(s)$ of (2) into (3), we have the following residual equation

$$r(s) = [H_y(s)G_u(s) + H_u(s)]u(s) + H_y(s)G_d(s)d(s)$$
$$+ H_y(s)G_f(s)f(s). \quad (4)$$

Ideally, the residual is designed to satisfy

(D1) $r(t) = 0$ if $f(t) = 0$

(D2) $r(t) \neq 0$ if $f(t) \neq 0$

for all $u(t)$ and $d(t)$. To fulfil these requirements, $H_u(s)$ and $H_y(s)$ must satisfy

$$H_u(s) = -H_y(s)G_u(s) \quad (5\,a)$$

$$H_y(s)G_d(s) = 0 \quad (5\,b)$$

$$[H_y(s)G_f(s)]_i := [H_f(s)]_i \neq 0, i = 1, 2, \ldots, q, \quad (5\,c)$$

where $[H_y(s)G_f(s)]_i$ denotes the ith column of $H_y(s)G_f(s)$ and $H_f(s)$ is equal to $H_y(s)G_f(s)$. Equation (5 a) ensures that the residual generator is independent of the input signal $u(t)$. Equation (5 b) implies that the unknown input is totally decoupled from the residual. If a residual generator satisfies (5 b), we call it an 'unknown input insensitive residual generator'. Equation (5 c) guarantees that the residual is controllable by the fault vector $f(t)$; in other words, the fault is detectable by monitoring the residual. Unknown input decoupling designs can be achieved by using the unknown input observer or eigenstructure assignment techniques. Alternatively, we present a direct way in the following to synthesize an unknown input insensitive residual generator using the state-space relations.

2.2. Robust parity relations from the state-space model

In the unknown input observer design, a necessary condition for unknown input decoupling is that

rank $(CE) = $ rank (E) (Yang and Wilde 1988). This means that to implement an observer that is insensitive to unknown input, specific state variables must be measured or at least appear as a part of the output. Therefore, in the following we assume that rank $(CE) = $ rank (E). We also assume that the number of sensor output equations m is greater than the number of unknown inputs r in order to be sure *a priori* that the system has at least one redundancy equation.

Consider the state-space model of (1). $\dot{y}(t)$ is obtained by substitution as

$$\dot{y}(t) = CAx(t) + CBu(t) + CEd(t) + CKf(t) + Gf(t). \tag{6}$$

Since CE has full column rank, the Moore–Penrose generalized inverse matrix $(CE)^+$ can be obtained by (Campbell and Meyer 1979)

$$(CE)^+ = [(CE)^T(CE)]^{-1}(CE)^T. \tag{7}$$

Thus, by (6), we can estimate the unknown input effect $Ed(t)$ by the following equation:

$$Ed(t) = E(CE)^+ \{\dot{y}(t) - CAx(t) - CBu(t) - CKf(t) - Gf(t)\}. \tag{8}$$

Substituting (8) into (1 *a*), we arrive at the following state equation independent of the unknown input

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{B}u(t) + \tilde{K}f(t) + E(CE)^+\dot{y}(t) - E(CE)^+Gf(t), \tag{9}$$

where

$$\tilde{A} = A - E(CE)^+CA, \quad \tilde{B} = B - E(CE)^+CB,$$
$$\tilde{K} = K - E(CE)^+CK. \tag{10}$$

Defining a new state vector as

$$z(t) = x(t) - E(CE)^+y(t) - E(CE)^+Gf(t),$$

which leads to the following state-space equations free of unknown inputs for the monitoring system

$$\dot{z}(t) = \tilde{A}z(t) + \tilde{B}u(t) + [\tilde{K} - \tilde{A}E(CE)^+G]f(t) + \tilde{A}E(CE)^+y(t) \tag{11 a}$$

$$y(t) = Cz(t) + CE(CE)^+y(t) + (I_m - CE(CE)^+)Gf(t). \tag{11 b}$$

The input–output equation corresponding to (11) can be further obtained as

$$L_y(s)y(s) = L_u(s)u(s) + L_f(s)f(s), \tag{12}$$

where

$$L_y(s) = (I_m - CE(CE)^+) - C(sI_n - \tilde{A})^{-1}\tilde{A}E(CE)^+ \tag{13 a}$$

$$L_u(s) = C(sI_n - \tilde{A})^{-1}\tilde{B} \tag{13 b}$$

$$L_f(s) = (I_m - CE(CE)^+)G + C(sI_n - \tilde{A})^{-1} \times (\tilde{K} - \tilde{A}E(CE)^+G). \tag{13 c}$$

Evidently, (12) must hold when it is evaluated using the actual measurement data $u(t)$ and $y(t)$. A basic unknown input insensitive residual generator is then simply formed as

$$e(s) = L_y(s)y(s) - L_u(s)u(s) \tag{14 a}$$

$$= L_f(s)f(s). \tag{14 b}$$

Note that (14 *a*) is the computational form (containing the measurables), while (14 *b*) is the internal form (containing the faults). By (14), the derived parity equations are now insensitive to the unknown input. However, a drawback to the use of (14 *a*) is that it will result in an unstable residual generator when the monitored system is unstable. To increase the design freedom and to meet the required isolation and robust performance, we use a dynamic weighting matrix $W(s)$ to weight (or transform) the previous residual. That is, further equations (residuals) can be generated from (14) by applying a linear transformation as

$$r(s) = W(s)e(s) \tag{15 a}$$

$$= W(s)\{L_y(s)y(s) - L_u(s)u(s)\} \tag{15 b}$$

$$= W(s)L_f(s)f(s). \tag{15 c}$$

Now, the residual generator design amounts to selecting the transformation matrix $W(s)$ so that the residual generator is stable and the residual $r(t)$ can possess desirable properties for diagnostic performance. Rewrite (15) in a scalar residual form

$$r_i(s) = W_i^T(s)\{L_y(s)y(s) - L_u(s)u(s)\} \tag{16 a}$$

$$= W_i^T(s)L_f(s)f(s), \tag{16 b}$$

where r_i is the ith element of the residual vector $r(s)$, and $W_i^T(s) \in R^{1 \times m}(s)$ is the ith row of $W(s)$. We decompose the denominators of the entry (i, j) of $L_y(s)$, $L_u(s)$ and $L_f(s)$ into the following form

$$L_y = \left[\frac{N_{yij}(s)}{D_{yij-}(s)D_{yij+}(s)} \right], \quad L_u = \left[\frac{N_{uij}(s)}{D_{uij-}(s)D_{uij+}(s)} \right]$$

and

$$L_f = \left[\frac{N_{fij}(s)}{D_{fij-}(s)D_{fij+}(s)} \right], \tag{17}$$

where N_{yij}, D_{yij-}, $D_{yij+} \in \mathbf{R}[s]$ are mutually coprime, D_{yij-} and D_{yij+} are monic with $Z[D_{yij-}] \in \mathbf{C}_-$ and $Z[D_{yij+}] \in \mathbf{C}_+$, and N_{uij}, D_{uij-}, D_{uij+}, N_{fij}, D_{fij-} and D_{fij+} are defined in the similar way. By computing

$$D_{i+}(s) = \text{LCM}\{D_{yij+}(s), D_{uik+}(s), D_{fil+}(s),$$

$$i = 1, \ldots, m, k = 1, \ldots, p, l = 1, \ldots, q\} \quad (18)$$

and denoting

$$W_i^{\mathsf{T}}(s) = \left[\frac{\tilde{\beta}_{i1}(s)}{\alpha_{i1}(s)} \quad \frac{\tilde{\beta}_{i2}(s)}{\alpha_{i2}(s)} \quad \cdots \quad \frac{\tilde{\beta}_{im}(s)}{\alpha_{im}(s)} \right] \quad (19)$$

we conclude that if

$$\tilde{\beta}_{ij}(s) = \beta_{ij}(s)D_{j+}(s) \quad (20)$$

and $\alpha_{ij}(s)$ is Hurwitz, for $j = 1, 2, \ldots, m$, then

$$[H_y(s)]^i = W_i^{\mathsf{T}}(s)L_y(s) \quad \text{and} \quad [H_u(s)]^i = -W_i^{\mathsf{T}}(s)L_u(s)$$

$$(21\,a)$$

$$[H_f(s)]^i = W_i^{\mathsf{T}}(s)L_f(s) \quad (21\,b)$$

are stable, where $[H_y(s)]^i$, $[H_u(s)]^i$ and $[H_f(s)]^i$ denote the ith row of $H_y(s)$, $H_u(s)$ and $H_f(s)$, respectively. Moreover, $\alpha_{ij}(s)$ and $\beta_{ij}(s)$ can be designed to satisfy requirements of realizability (filter matrices are required to be proper), robustness issue and fast detection. The above derivations for finding an unknown input insenstive residual generator are summarized as in the following algorithm.

Data   Given system matrices A, B, C, E, K and G.

Step 1. Comput $(CE)^+$, $\tilde{A}$, $\tilde{B}$ and $\tilde{K}$ (Equation (10)).

Step 2. Compute $L_y(s)$, $L_u(s)$ and $L_f(s)$ (Equation (13)) and decompose $L_y(s)$, $L_u(s)$ and $L_f(s)$ in the form as (17).

Step 3. Find $D_{i+}(s)$ defined in (18).

Step 4. For $j = 1, 2, \ldots, m$:

(a) choose $\beta_{ij}(s)$ and set $\tilde{\beta}_{ij}(s)$ as in (20);

(b) choose an adequate Hurwitz polynomial $\alpha_{ij}(s)$ that satisfies proper requirements.

Step 5. Compute $[H_y(s)]^i$ and $[H_u(s)]^i$ according to (21 a).

The ith residual is then generated by

$$r_i(s) = [H_y(s)]^i y(s) + [H_u(s)]^i u(s)$$

and the fault/residual map is given by

$$r_i(s) = [H_f(s)]^i f(s) = W_i^{\mathsf{T}}(s)L_f(s)f(s).$$

Clearly, if the jth element of $[H_f(s)]^i$ is non-zero then the jth fault can be detected by the ith residual. Using the unknown input insensitive residual generator as proposed above makes the FDI scheme robust for up to $m - 1$ unknown inputs. That is, under certain conditions, it is possible to achieve perfect decoupling with respect to $m - 1$ unknown inputs.

2.3. Techniques for fault isolation

By the above algorithm, an unknown input insensitive residual generator has been designed for fault detection. However, in general, faults may occur in different sectors of the system, e.g. in different sensors, actuators or components. To localize each individual fault uniquely, residual sets are usually enhanced in one of the following ways (Gertler 1993, Patton and Chen 1994).

(A) *Structured residuals.* In response to a single fault, only a fault-specific subset of the residuals becomes non-zero.

(B) *Fixed-direction residuals.* In response to a single fault, the residual vector is confined to a fault specific direction.

If $L_f(s)$ in (13 c) has full column rank, then residuals can be generated in the fixed-direction form so that mutual isolation of multiple faults becomes possible. In this case, the method for designing a fault identification filter (FIDF) in Chang *et al.* (1995) is adopted. Since $L_f(s)$ has full column rank q, one can easily find a matrix $W_0 \in \mathbf{R}^{q \times m}$ so that

$$[W_0 L_f(s)] \in \mathbf{R}^{q \times q}$$

is square and of full rank, i.e. $[W_0 L_f(s)]$ is invertible. Therefore, if we design the weighting matrix

$$W(s) = H_f(s)(W_0 L_f(s))^{-1} W_0, \quad (22)$$

where $H_f(s)$ is in a diagonal form, mutual fault isolation can be achieved in this proposed design. Detailed procedures are outlined in Chang *et al.* (1995).

If $L_f(s)$ in (13 c) is not of full column rank, then mutual isolation of multiple faults is not achievable. Under such a circumstance, one way of isolating different faults is to generate a residual vector $r(t)$ in structured form. A structured set implies that each residual is completely unaffected by other different subsets of faults. The number of different residual generators to be designed is then equal to the number of the fault subset. Each residual generator is designed so that the generated residual signal is insensitive only to one subset of faults. Typically, observer schemes that employ a bank of observers are adopted for fault isolation (e.g. Frank 1987, Frank and Wünnenberg 1989, Ge and Fang, 1989). Based upon a similar concept of the generalized observer scheme
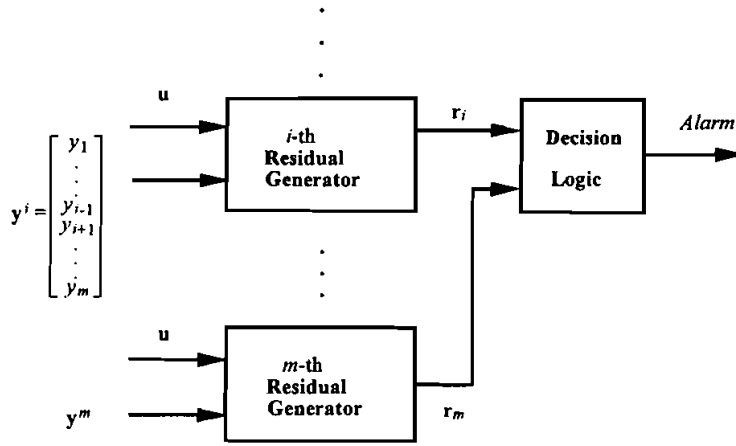
Figure 1. Generalized residual generator scheme (GRGS).

(GOS—Frank 1987, Frank and Wünnenberg 1989) with the observer replaced by the residual generator, in the following we use a generalized residual generator scheme (GRGS) to generate structured residuals for sensor fault isolation.

As illustrated in Fig. 1, the ith residual generator is driven by all but the ith sensor, so that a fault in the ith sensor affects all but the ith residual. Moreover, a decision logic is necessary to complete the task of fault isolation. Here, we define two additional quantities used in the decision logic for fault detection and isolation:

$$\psi(t) = \left[\frac{1}{m}\sum_{i=1}^{m} r_i^2(t)\right]^{1/2} \qquad (23\,a)$$

and

$$\theta_i(t) = \cos^{-1}\left(\frac{\langle r(t), H_{fi}\rangle}{|r(t)|\,|H_{fi}|}\right) \quad \text{for } i = 1, 2, \ldots, m. \qquad (23\,b)$$

$\psi(t)$ is the magnitude of $r(t)$ and can be further formed as a flag function indicating whether any failure occurs. When $\psi(t)$ becomes large, it indicates the presence of certain failures. The $\theta_i(t)$ is the angle between the residual vector and ith failure direction H_{fi} where H_{fi} is the ith column of steady gain of $H_f(0)$. If $\theta_i(t)$ is close to zero, the ith sensor is then declared to be faulty, thus fault isolation is achieved. However, as multiple faults occur, its fault specific direction is changing with fault size

$$r_{ss} = \sum_{i=1}^{m} H_{fi} f_i,$$

where r_{ss} is the steady-state value of residual vector r. This fault specific direction r_{ss} is no longer suitable for fault isolation. Therefore, in this scheme, only a single fault is permitted at a time for fault isolation. We remark that all the derivations in this paper can also deal with discrete-time cases by replacing the Laplace transform

by the z-transform and the region on the left half s-plane by the region inside the unit circle.

3. Numerical examples

A linearized dynamic model of the VTOL aircraft in the vertical plane is considered here (Narendra and Tripathi 1973). For typical loading and flight conditions at an airspeed of 135 Kt, the aircraft has the following nominal system matrices

$$A = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.7070 & 1.4200 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.5200 & 4.4900 \\ 0.0000 & 0.0000 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

where the state x and control input u are

$$x = \begin{bmatrix} \text{horizontal velocity (Kt)} \\ \text{vertical velocity (Kt)} \\ \text{pitch rate} \quad (\deg\,s^{-1}) \\ \text{pitch angle} \quad (\deg) \end{bmatrix},$$

$$u = \begin{bmatrix} \text{collective pitch control} \\ \text{longitudal cyclic pitch control} \end{bmatrix}$$

Consider that the variations in the plant parameters due to the change of airspeed are provided as

$$\Delta A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 2.0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

where ΔA is the modelling error in the system matrix A. The unknown input term Ed can be thus represented as $E = [0\ 0\ 1\ 0]^{T}$ and $d = [0, 0.5, 0, 2]x$.

Note that the eigenvalues of this unstable system are $\{-2.0729, -0.2325, 0.2757 \pm 0.2577j\}$. The unstable plant is first stabilized by using a state feedback type controller and the system matrix of the stabilized system becomes

$$A_{c} = \begin{bmatrix} -5.0817 & 0.0835 & 0.6464 & 1.9753 \\ 10.2660 & -2.7417 & -1.6492 & -6.02388 \\ 25.4477 & 1.2858 & -3.5767 & -12.9620 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The proposed strategy for detection of actuator fault and sensor fault will then be discussed separately.

3.1. Actuator FDI

In the present actuator fault detection filter design, the fault distribution matrices K and G are set as

$$K = B \quad \text{and} \quad G = 0_{4*2}.$$

Following (13)–(14) the transfer matrices of $L_{f}(s)$, $L_{y}(s)$ and $L_{u}(s)$ of this residual generator can be determined. In this example, the transfer matrix $L_{f}(s)$ is of full rank, implying that mutual isolation of multiple actuator faults

is achievable. The FIDF design strategy can be used to construct a decoupled map from fault to residual. In the present design, a weighting matrix W_{0} is simply selected as

$$W_{0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

In addition, the desired map from fault to residue is selected as

$$H_{f}(s) = \begin{bmatrix} \dfrac{2}{(s+2)} & 0 \\ 0 & \dfrac{2}{(s+2)} \end{bmatrix}.$$

By (22), the weighting matrix $W(s)$ can be thus determined as

$$W(s) = H_{f}(s)(W_{0}L_{f}(s))^{-1}W_{0}.$$

Now, we have the residual generator insensitive to the parameter variation in the following form:

$$r(s) = H_{y}(s)y(s) + H_{u}(s)u(s) = H_{f}(s)f(s),$$

where

$$H_{y}(s) = W(s)L_{y}(s) =$$

$$\begin{bmatrix} \dfrac{3.8138(s+0.0355)}{(s+2)} & \dfrac{0.0885(s+3.8451)(s-3.0970)}{(s+2)(s+0.5)} \\ \dfrac{1.7805(s+0.0426)}{(s+2)} & \dfrac{-0.2221(s+1.3309)(s-0.3222)}{(s+2)(s+0.5)} \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{-0.0360(s-29.1032)}{(s+2)(s+0.5)} & \dfrac{-0.0360(s-29.1032)}{(s+2)(s+0.5)} \\ \dfrac{-0.0165(s+2.4920)}{(s+2)(s+0.5)} & \dfrac{-0.0165(s+2.4920)}{(s+2)(s+0.5)} \end{bmatrix}$$
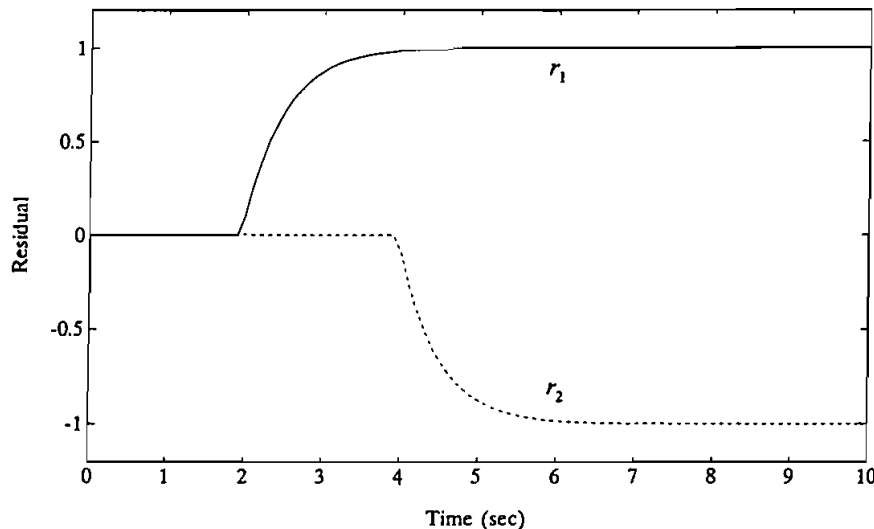

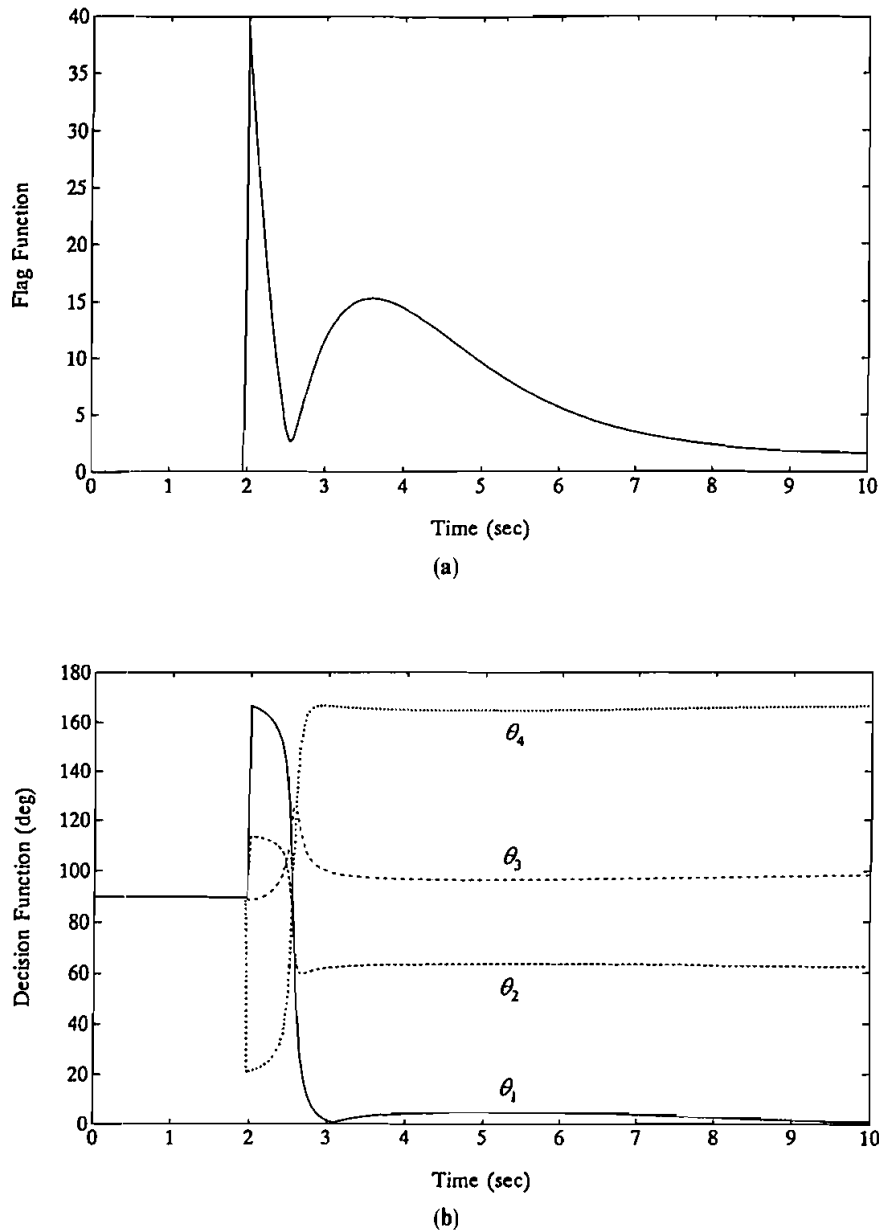Figure 2. Residual responses for actuator fault detection.

Figure 3.   Responses of the decision logic to a 1·0 bias fault on the first sensor: (a) flag function $\psi(t)$; (b) decision function.

and

$$H_u(s) = -W(s)L_u(s) = -H_f(s).$$

Figure 2 shows the diagnostic results of the proposed actuator FDI, where bias faults of 1·0 for the first actuator and − 1·0 for the second actuator are issued at $t = 2·0$ s and at $t = 3·0$ s separately. On this diagonal map, the design is immune to the prescribed structured uncertainty, and multiple failures occurring in the actuators can be properly identified.

3.2. Sensor FDI

To achieve sensor fault detection of this VTOL aircraft, the present design results will entail the $L_f(s)$ to be singular. That is, the requirement of simultaneous isolation of multiple faults is not achievable here. The proposed GRGS is adopted here to generate structured residuals. Following the design procedure, we construct four independent schemes for sensor fault isolation. The designed FDI filter matrices for each scheme are the following.
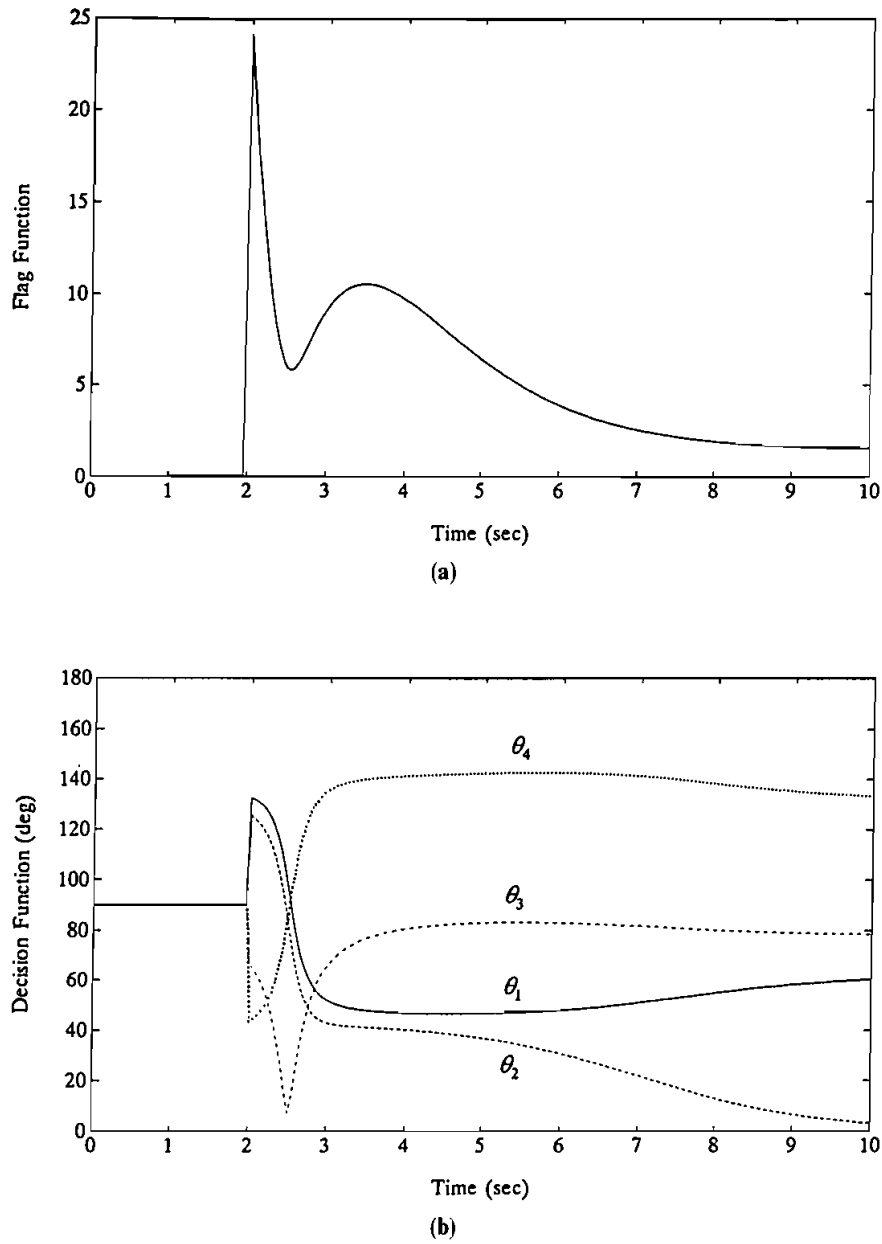
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Figure 4. Responses of the decision logic to a 1·0 bias fault on the second sensor: (a) flag function $\psi(t)$; (b) decision function.

Scheme 1

$$[H_y]^1 = \left[0 \quad \frac{-1\cdot4468(s - 0\cdot6912)}{(s + 1)} \right.$$

$$\left. \frac{0\cdot0017(s - 1675)}{(s + 1)(s + 2\cdot1953)} \quad \frac{0\cdot0017(s - 1675)}{(s + 1)(s + 2\cdot1953)} \right]$$

$$[H_u]^1 = \left[\frac{5\cdot1311(s + 0\cdot5)}{(s + 1)(s + 2\cdot1965)} \quad \frac{-8\cdot3248(s + 0\cdot5)}{(s + 1)(s + 2\cdot1965)} \right].$$

Scheme 2

$$[H_y]^2 = \left[\frac{-32\cdot8958(s - 0\cdot6912)(s + 0\cdot0440)}{(s + 1)^2} \quad 0 \right.$$

$$\frac{0\cdot3092(s - 24\cdot4530)(s + 1\cdot2377)}{(s + 1)(s + 2\cdot1949)}$$

$$\left. \frac{0\cdot3092(s - 24\cdot4530)(s + 1\cdot2377)}{(s + 1)(s + 2\cdot1949)} \right]$$
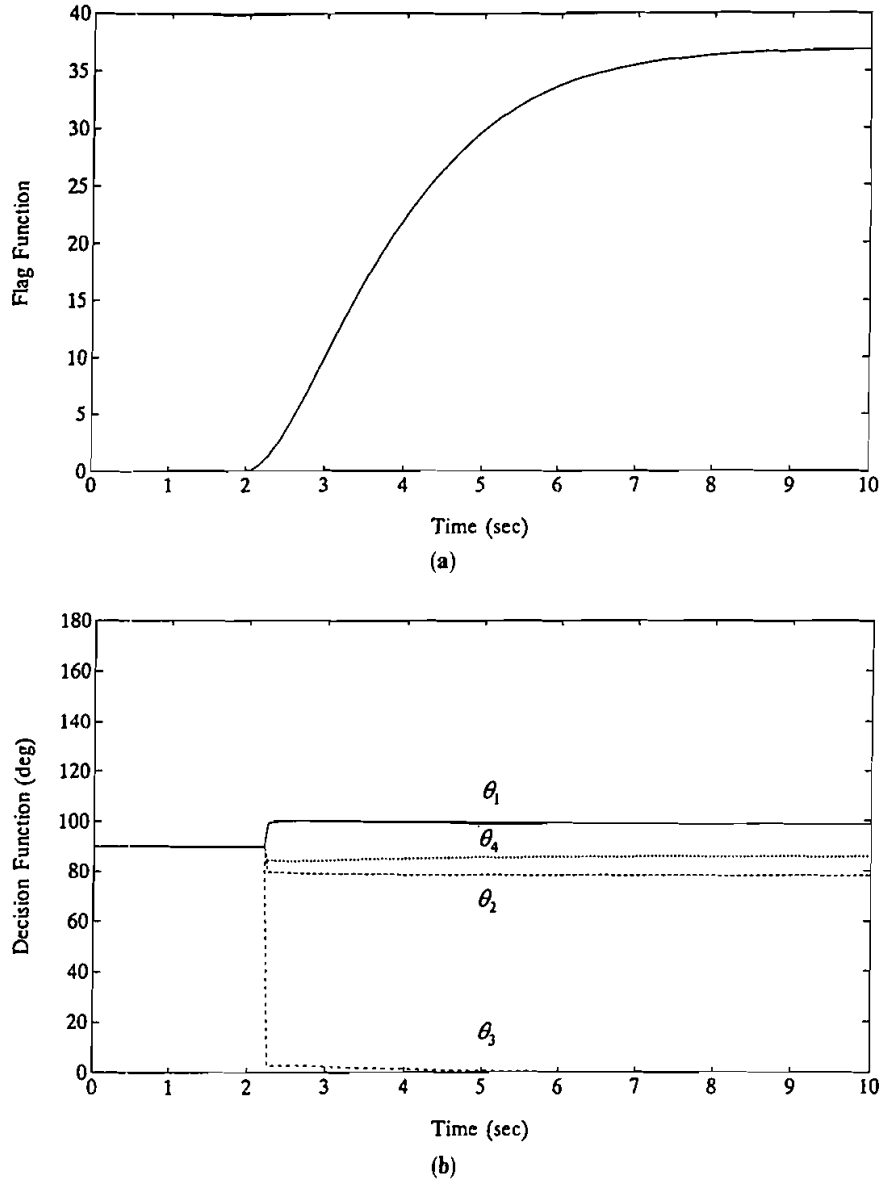
Figure 5. Responses of the decision logic to a 1·0 bias fault on the third sensor: (a) flag function $\psi(t)$; (b) decision function.

$$[H_u]^2 = \left[\frac{8\cdot9918(s + 1\cdot0282)(s + 0\cdot1293)}{(s + 1)^2(s + 3\cdot0101)} \right.$$

$$\left. \frac{14\cdot5399(s + 1\cdot3309)(s + 0\cdot3222)}{(s + 1)^2(s + 2\cdot1965)} \right].$$

Scheme 3

$$[H_y]^3 = \left[\frac{-22\cdot5541(s - 1\cdot0053)(s + 0\cdot0440)}{(s + 1)^2} \right.$$

$$\frac{-22\cdot5541(s - 1\cdot0053)(s + 0\cdot0440)}{(s + 1)^2} \quad 0$$

$$\left. \frac{0\cdot4781(s - 210\cdot3661)(s + 0\cdot1654)}{(s + 1)^2(s + 3\cdot0101)} \right]$$

$$[H_u]^3 = \left[\frac{8\cdot9918(s + 1\cdot0282)(s + 0\cdot1293)}{(s + 1)^2(s + 3\cdot0101)} \right.$$

$$\left. \frac{-167\cdot2638(s + 0\cdot5104 \pm 0\cdot5843j)}{(s + 1)^2(s + 3\cdot0101)} \right].$$

Scheme 4

$$[H_y]^4 = \left[\frac{2\cdot8363(s + 0\cdot0353)}{(s + 1)} \quad \frac{9\cdot5439(s + 0\cdot0353)}{(s + 1)} \right.$$

$$\left. \frac{-0\cdot0304(s + 1220\cdot6)}{(s + 1)(s + 1\cdot0113)} \quad 0 \right]$$

$$[H_u]^4 = \left[\frac{32\cdot5752(s - 0\cdot003)}{(s + 1)(s + 1\cdot0113)} \quad \frac{32\cdot5752(s - 0\cdot003)}{(s + 1)(s + 1\cdot0113)} \right].$$
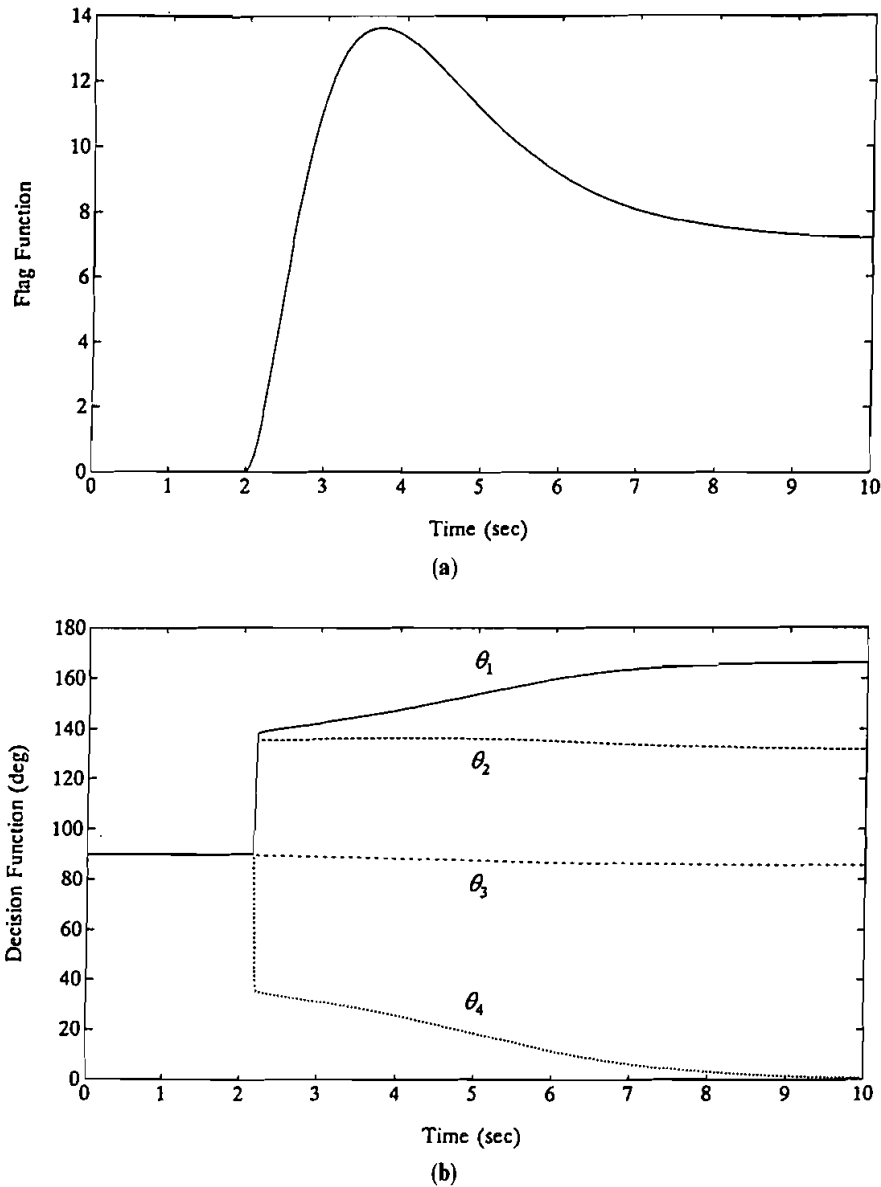
Figure 6.   Responses of the decision logic to a 1·0 bias fault on the fourth sensor: (a) flag function $\psi(t)$; (b) decision function.

The fault/residual map for these schemes is

$$[H_f]^i = [H_y]^i.$$

The norm of the residue is selected as a flag function for fault detection as in (23 a). Moreover, the steady-state gain matrix of this GRGS can be computed as

$$H_f(0) = \begin{bmatrix} 0 & 0 & -1.2955 & -1.2955 \\ 1 & 0 & -4.2636 & -4.2636 \\ 1 & 1 & 0 & -5.5265 \\ 0.1 & -0.3369 & -36.6043 & 0 \end{bmatrix}.$$

Each column of $H_f(0)$ is selected as the estimated failure direction for single fault isolation. Figures 3–6 show the

responses of the flag function and the decision function (fault angles $\theta_i(t)$) of the proposed GRGS FDI scheme for the four sensors, respectively. A bias fault of magnitude 1·0 is issued at $t = 2·0$ s for each sensor separately. As shown in the figures, the flag functions provide good indication of fault occurrence and fault isolation is achieved by monitoring fault angles as they approach zero.

5.   Conclusions

This paper proposes a robust FDI by constructing the diagnostic system with a transfer matrix approach by employing the inverse of the matrix CE. The unknown inputs are thus perfectly decoupled from the fault residual

signals in the present diagnostic system. The present approach can be applied to systems under structured uncertainties to achieve multiple fault isolation for actuators. Furthermore, the syntheses of the obtained parity relations forms the GRGS for isolating the sensor fault. In the illustrated example of a VTOL aircraft, simulation results indicate that the present FDI design is immune to the external disturbance and that it adequately identifies the fault occurrences.

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