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2-D analytical model for characterizing flexural damping responses of composite laminates

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ABSTRACT

The flexural damping behaviors of composite laminates were characterized analytically in this study. A 2-D analytical model was developed based on the extension of Ni–Adams model [Ni RG, Adams RD. The damping and dynamic moduli of symmetric laminated composite beams—theoretical and experimental results. J Compos Mater 1984;18(2):104–21] accounting for the energy dissipation contributed by the laminar stresses of σ_{xy} and σ_{yy}. The specific damping capacity (SDC) of the composite was determined in accordance with the energy dissipation concept, which was defined as the ratio of the dissipated energy to the stored energy for per circle of vibration. The 2-D analytical model was validated by comparing the SDC of $[0/-60/60]_s$ and $[0/90/45/-45]_s$ laminates with the experimental data and the finite element (FEM) results. In addition, the effects of interlaminar stress on the flexural damping responses of laminated plates were also characterized in the 3-D FEM analysis. Results indicated that the interlaminar stress effect may not be so significant that the current 2-D model is adequate for the evaluation of the damping responses of the composite laminates. Furthermore, the present predictions, as compared to the Ni–Adams [Ni RG, Adams RD. The damping and dynamic moduli of symmetric laminated composite beams—theoretical and experimental results. J Compos Mater 1984;18(2):104–21] and Adams–Maheri [Adams RD, Maheri MR. Dynamic flexural properties of anisotropic fibrous composite beams. Compos Sci Technol 1994;50(4):497–514] models, generally demonstrate good agreements with the experimental data and the FEM results.

1. Introduction

Polymeric composites possessing the high damping properties of polymer matrices and the extraordinary characteristics of material heterogeneity demonstrate the distinctive capability of dissipating energy during flexural vibrations. The extent of the flexural damping capacity of the composite laminates is generally dependent on the material properties, the ply orientation, and the stacking sequences. In past decades, there were several analytical models proposed for characterizing the damping responses of composite laminates with various lay-up configurations [1–11]. A comprehensive review on the models was provided by Chandra et al. [12,13]. Among them, the models developed by Ni and Adams [1], Adams and Maheri [2], Adams and Bacon [3], and Saravanos and Chamis [4] with the attribute of straightforward physical concept and simplified mathematical form were frequently referenced in the literature. Although the mathematical formulations of the models are different, the main concepts employed in the derivation are quite similar.

Based on the Ritz method, Berthelot and Sefrani [5] presented an evaluation to consider the beam width effect on the damping behaviors of unidirectional composites. The analysis was extended into the damping responses of composite laminates [6]. Ohta et al. [7] employed the 3-D theory of elasticity to evaluate the maximum strain and kinetic energies of cross-ply laminates for the damping analysis. The analytical formulation and governing equation were interpreted by using Ritz's method. Berthelot et al. [8] developed a synthesis of damping analysis of laminate materials, laminates with interleaved viscoelastic layers and sandwich materials using finite element analysis. The approach was established in accordance with the first-order laminate theory including the transverse shear effects. The transverse shear effect on the damping responses of 0° and 90° unidirectional laminates was also investigated by Yim and Gillespie [9]. Wei et al. [10] derived the energy dissipation formulation by constructing a damping matrix together with the visco-elasticity theory to predict the damping behavior of fiber composites. Recently, Billups and Cavalli [11] revisited the analytical models and tried to exemplify the differences among them in terms of the numerical results. Unfortunately, the exclusive distinctions in the fundamental mechanics were not pointed out in their study, and only the numerical results obtained from the models were compared with each other as well as with the

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experimental data. As a result, it is required to provide a classification on these analytical models from the mechanics point of view and validate their applicability to model the flexural damping behaviors of composite laminates.

In this study, a 2-D analytical model, accounting for all in-plane stress and strain quantities into the evaluation of energy dissipation, was developed. During the derivation, the distinct differences between other analytical models were indicated lucidly. The model predictions were then compared with the results obtained from the FEM analysis and experimental data. Furthermore, the interlaminar stress effect on the flexural vibration of composite laminates was also examined through the FEM analysis. The applicability of the 2-D analytical models to the characterization of flexural vibration of composite laminates was discussed.

2. Analytical model

In order to describe the flexural damping responses of composite laminates, a 2-D analytical model was proposed based on the laminated plate theory together with the energy dissipation concept. Without loss of generality, the laminates' flexural vibration was simulated by applying a cyclic bending moment M_x in the x-direction. It is noted that all fiber orientations of the laminates in the following expressions are all referenced to the x-axis. For a symmetric laminated composite subjected to pure bending moment M_x, the corresponding curvature can be derived from the laminated plate theory as [14]

$$\kappa_x = \frac{C^*_{11}}{h^*}M_x \quad \kappa_y = \frac{C^*_{12}}{h^*}M_x \quad \kappa_{xy} = \frac{C^*_{16}}{h^*}M_x \tag{1}$$

where $h^* = \frac{h^3}{12}$, where h is the thickness of the composite laminate and C^*_{ij} is the normalized flexural compliances. The detail derivation of Eq. (1) is not presented at this point, but it can be found elsewhere in the literature [1]. It is noted that in Eq. (1), the bending moment as well as the curvatures are described with regard to the x–y coordinate system, and the information associated with the stacking sequences and the material properties are all included in the compliance matrix C^*_{ij}. From the thin plate assumption that the strain in a material point is linearly proportional to the distance measured from the mid-plane of the laminate in terms of the curvature, the strain components can be expressed explicitly as

$$\varepsilon_x = z\kappa_x \quad \varepsilon_y = z\kappa_y \quad \gamma_{xy} = z\kappa_{xy} \tag{2}$$

where z is the distance referenced to the mid-plane of laminates.

Combining Eqs. (1) and (2) leads to the relationship of the strain components to the bending moment M_x as

$$\varepsilon_x = z\frac{C^*_{11}}{h^*}M_x \quad \varepsilon_y = z\frac{C^*_{12}}{h^*}M_x \quad \gamma_{xy} = z\frac{C^*_{16}}{h^*}M_x \tag{3}$$

Subsequently, in accordance with the energy dissipation concept [15], the SDC of materials can be defined as the ratio of the dissipated energy to the stored energy for per circle of vibration, i.e.,

$$\psi = \frac{\Delta U}{U} \tag{4}$$

where ΔU is the energy dissipation per cyclic vibration, and U is the strain energy stored in the material systems when the deformation is maximum. For the composite laminates under cyclic vibration, the energy dissipation can be evaluated from the summation of those calculated individually from the fiber direction, transverse direction, and in-plane shear direction as

$$\Delta U = \Delta U_1 + \Delta U_2 + \Delta U_{12} \tag{5}$$

where ΔU_1, ΔU_2, and ΔU_{12} denote the energy dissipation in the fiber, transverse, and in-plane shear directions, respectively. The quantities can be further expressed as

$$\Delta U_1 = \psi_L U_1 \tag{6}$$
$$\Delta U_2 = \psi_T U_2 \tag{7}$$
$$\Delta U_{12} = \psi_{LT} U_{12} \tag{8}$$

where ψ_L, ψ_T, and ψ_{LT} indicate the SDC of a unidirectional composite in the fiber, transverse, and in-plane shear directions, respectively; U_1, U_2, and U_{12} denote the corresponding strain energy in the fiber, transverse, and in-plane shear directions as well. Without loss of generality, we take ΔU_1 as an example in the following derivation for the calculation of energy dissipation in the fiber direction. From Eq. (6), ΔU_1 can be expressed in terms of strain energy in the fiber direction as

$$\Delta U_1 = \psi_L U_1 = \psi_L \frac{1}{2}\int_v \varepsilon_1\sigma_1 \mathrm{d}v \tag{9}$$

Subsequently, through the coordinate transformation relation, the stress and strain components in the fiber direction can be correlated to the corresponding components in the x–y coordinate; therefore, the energy dissipation becomes

$$\Delta U_1 = \frac{1}{2}\int_v \psi_L(c^2\varepsilon_x + s^2\varepsilon_y + cs\gamma_{xy})(c^2\sigma_x + s^2\sigma_y + 2cs\sigma_{xy})\mathrm{d}v \tag{10}$$

where "s" denotes $\sin(\theta)$, "c" indicates $\cos(\theta)$, and θ is the fiber orientation with respect to the x–y coordinate. In conjunction with the constitutive relation and Eq. (3), the energy dissipation in Eq. (10) is written in terms of the bending moment M_x as

$$\begin{aligned}\Delta U_1 = \frac{1}{2}\int_v &\psi_L(c^2C^*_{11} + s^2C^*_{12} + csC^*_{16}) \\ &\times \left[c^2(\overline{Q}_{11}C^*_{11} + \overline{Q}_{12}C^*_{12} + \overline{Q}_{16}C^*_{16}) + s^2(\overline{Q}_{12}C^*_{11} + \overline{Q}_{22}C^*_{12}\right. \\ &\left. + \overline{Q}_{26}C^*_{16}) + 2cs(\overline{Q}_{16}C^*_{11} + \overline{Q}_{26}C^*_{12} + \overline{Q}_{66}C^*_{16})\right]\left(M_x\frac{z}{h^*}\right)^2\mathrm{d}v\end{aligned} \tag{11}$$

where $[\overline{Q}]$ is the compliance matrix for the off-axis composites.

When the dimensions of the laminated plate are assumed to be L in length, h in thickness, and "1" in width, the above integration on the energy dissipation can be further deduced as

$$\begin{aligned}\Delta U_1 = 2\int_0^{L/2} M_x^2\mathrm{d}x \int_0^{h/2} &\psi_L(c^2C^*_{11} + s^2C^*_{12} + csC^*_{16}) \\ &\times \left[c^2(\overline{Q}_{11}C^*_{11} + \overline{Q}_{12}C^*_{12} + \overline{Q}_{16}C^*_{16}) + s^2(\overline{Q}_{12}C^*_{11} + \overline{Q}_{22}C^*_{12}\right. \\ &\left. + \overline{Q}_{26}C^*_{16}) + 2cs(\overline{Q}_{16}C^*_{11} + \overline{Q}_{26}C^*_{12} + \overline{Q}_{66}C^*_{16})\right]\left(\frac{z}{h^*}\right)^2\mathrm{d}z\end{aligned} \tag{12}$$

It is noted that Eq. (12) indicates the total energy dissipation in the fiber direction obtained by calculating the energy dissipation in each ply and then adding them together by means of the integration operation. In addition, for a laminated plate subjected to a bending moment M_x, the total strain energy is written as

$$U = \frac{1}{2}\int_0^L M_x\kappa_x\mathrm{d}x = \frac{C^*_{11}}{h^*}\int_0^{L/2} M_x^2\mathrm{d}x \tag{13}$$

Thus, from the energy dissipation concept provided in Eq. (4), the SDC of composite laminate in the fiber direction ψ_1 can be deduced as

$$\psi_1 = \frac{\Delta U_1}{U} = \frac{2}{h^* C_{11}^*} \int_0^{h/2} \psi_L (c^2 C_{11}^* + s^2 C_{12}^* + csC_{16}^*) \times \Big[c^2(\overline{Q}_{11}C_{11}^* + \overline{Q}_{12}C_{12}^* + \overline{Q}_{16}C_{16}^*) + s^2(\overline{Q}_{12}C_{11}^* + \overline{Q}_{22}C_{12}^* + \overline{Q}_{26}C_{16}^*) + 2cs(\overline{Q}_{16}C_{11}^* + \overline{Q}_{26}C_{12}^* + \overline{Q}_{66}C_{16}^*)\Big] z^2\,dz \tag{14}$$

It is noted that the specific damping capacity in Eq. (14) is not relied on the amplitude of vibration but the material properties and lay-up configuration. In addition, ψ_1 in Eq. (14) indicates the SDC of composite laminates in the fiber direction which is different from ψ_L defined in Eq. (6). If the kth ply is occupying the region measured from $z = z^{k-1}$ to $z = z^k$ in the coordinate system, the integral operation in the Eq. (14) can be replaced by a summation operator as

$$\psi_1 = \frac{8\psi_T}{C_{11}^* N^3} \sum_{k=1}^{N/2} (c^2 C_{11}^* + s^2 C_{12}^* + csC_{16}^*) \times \Big[c^2(\overline{Q}_{11}^k C_{11}^* + \overline{Q}_{12}^k C_{12}^* + \overline{Q}_{16}^k C_{16}^*) + s^2(\overline{Q}_{12}^k C_{11}^* + \overline{Q}_{22}^k C_{12}^* + \overline{Q}_{26}^k C_{16}^*) + 2cs(\overline{Q}_{16}^k C_{11}^* + \overline{Q}_{26}^k C_{12}^* + \overline{Q}_{66}^k C_{16}^*)\Big] W_k \tag{15}$$

where $W_k = (k^3 - (k-1)^3)$ is the weighting factor in the kth layer.

In the same manner, the damping capacity in the transverse and in-plane shear direction can also be calculated, respectively, as

$$\psi_2 = \frac{\Delta U_2}{U} = \frac{8\psi_T}{C_{11}^* N^3} \sum_{k=1}^{N/2} (s^2 C_{11}^* + c^2 C_{12}^* - csC_{16}^*) \times \Big[s^2(\overline{Q}_{11}^k C_{11}^* + \overline{Q}_{12}^k C_{12}^* + \overline{Q}_{16}^k C_{16}^*) + c^2(\overline{Q}_{12}^k C_{11}^* + \overline{Q}_{22}^k C_{12}^* + \overline{Q}_{26}^k C_{16}^*) - 2cs(\overline{Q}_{16}^k C_{11}^* + \overline{Q}_{26}^k C_{12}^* + \overline{Q}_{66}^k C_{16}^*)\Big] W_k \tag{16}$$

$$\psi_{12} = \frac{\Delta U_{12}}{U} = \frac{8\psi_{LT}}{C_{11}^* N^3} \sum_{k=1}^{N/2} [2csC_{11}^* - 2csC_{12}^* - (c^2 - s^2)C_{16}^*] \times \Big[cs(\overline{Q}_{11}^k C_{11}^* + \overline{Q}_{12}^k C_{12}^* + \overline{Q}_{16}^k C_{16}^*) - cs(\overline{Q}_{12}^k C_{11}^* + \overline{Q}_{22}^k C_{12}^* + \overline{Q}_{26}^k C_{16}^*) - (c^2 - s^2)(\overline{Q}_{16}^k C_{11}^* + \overline{Q}_{26}^k C_{12}^* + \overline{Q}_{66}^k C_{16}^*)\Big] W_k \tag{17}$$

As a result, from the energy dissipation concept defined in Eqs. (4) and (5), the SDC of the composite laminate under flexural vibration would be given as

$$\psi = \psi_1 + \psi_2 + \psi_{12} \tag{18}$$

Presently, it is worth mentioning that in the aforementioned derivation, all in-plane quantities, i.e., ε_x, ε_y, γ_{xy}, σ_x, σ_y and σ_{xy}, were taken into account for the calculation of strain energy and energy dissipation as well, which should be the same as the Saravanos–Chamis model [4]. In fact, in the Saravanos–Chamis model, only the matrix form was presented, and the explicit expression for the SDC of laminates was not clearly addressed in the literature. For the sake of comparison, the explicit formulation in the Saravanos–Chamis model was reproduced by the authors although the process is quite tedious. Results confirmed that the mathematical forms obtained from the two models coincide with each other. However, the present analytical model initiated from the fundamental mechanics point of view provides a definite and straightforward evaluation on the SDC of composite laminates, which can be easily employed to compare with other analytical models.

In the forgoing derivation, if the stress components σ_y and σ_{xy} were neglected in the energy dissipation formulation given by Eq. (10), the corresponding damping capacity in the fiber direction would become

$$\psi_1 = \frac{8\psi_L}{C_{11}^* N^3} \sum_{k=1}^{N/2} (c^2 C_{11}^* + s^2 C_{12}^* + csC_{16}^*)[c^2(\overline{Q}_{11}^k C_{11}^* + \overline{Q}_{12}^k C_{12}^* + \overline{Q}_{16}^k C_{16}^*)] W_k \tag{19}$$

Similarly, the SDC in other directions would be deduced as

$$\psi_2 = \frac{8\psi_T}{C_{11}^* N^3} \sum_{k=1}^{N/2} (s^2 C_{11}^* + c^2 C_{12}^* - csC_{16}^*)[s^2(\overline{Q}_{11}^k C_{11}^* + \overline{Q}_{12}^k C_{12}^* + \overline{Q}_{16}^k C_{16}^*)] W_k \tag{20}$$

and

$$\psi_{12} = \frac{8\psi_{LT}}{C_{11}^* N^3} \sum_{k=1}^{N/2} [2csC_{11}^* - 2csC_{12}^* - (c^2 - s^2)C_{16}^*][cs(\overline{Q}_{11}^k C_{11}^* + \overline{Q}_{12}^k C_{12}^* + \overline{Q}_{16}^k C_{16}^*)] W_k \tag{21}$$

Basically, Eqs. (19)–(21) are the SDC provided in the Adams–Maheri model. Furthermore, if $\sigma_y = \sigma_{xy} = 0$ and $\varepsilon_y = 0$ were assumed in the energy dissipation function in Eq. (10), the SDC was deduced as

$$\psi_1 = \frac{8\psi_L}{C_{11}^* N^3} \sum_{k=1}^{N/2} (c^2 C_{11}^* + csC_{16}^*)[c^2(\overline{Q}_{11}^k C_{11}^* + \overline{Q}_{12}^k C_{12}^* + \overline{Q}_{16}^k C_{16}^*)] W_k \tag{22}$$

$$\psi_2 = \frac{8\psi_T}{C_{11}^* N^3} \sum_{k=1}^{N/2} (s^2 C_{11}^* - csC_{16}^*)[s^2(\overline{Q}_{11}^k C_{11}^* + \overline{Q}_{12}^k C_{12}^* + \overline{Q}_{16}^k C_{16}^*)] W_k \tag{23}$$

$$\psi_{12} = \frac{8\psi_{LT}}{C_{11}^* N^3} \sum_{k=1}^{N/2} [2csC_{11}^* - (c^2 - s^2)C_{16}^*][cs(\overline{Q}_{11}^k C_{11}^* + \overline{Q}_{12}^k C_{12}^* + \overline{Q}_{16}^k C_{16}^*)] W_k \tag{24}$$

which are the expressions produced in the Ni–Adams model.

In view of the forgoing, it was found that there are some physical quantities neglected in the Ni–Adams and Adams–Maheri models; therefore, the numerical results obtained from the models would be different from the current 2-D analytical model. The validations on the analytical models were conducted in the following section by comparing the results with the finite element analysis and the experimental data.

3. Finite element analysis

In order to verify the analytical model presented in the previous section, a 3-D finite element analysis was conducted to characterize the SDC of a laminated plate subjected to pure bending. Because in the analytical models the calculations of SDC are based on the 2-D stress states, for the comparison purpose, the in-plane stress and strain components obtained from finite element analysis were also employed in the evaluation of energy dissipation and the SDC as well although the 3-D finite element analysis was performed in reality. On the other hand, in order to understand the interlaminar stress effect on the SDC of the composite laminates, the 3-D results obtained from the FEM analysis were compared with those calculated in the 2-D models.

In the finite element analysis, the SDC of composite laminates can be determined by using the following formulation as

$$\psi = \frac{\sum_{k=1}^{m} \Delta U^{(k)}}{U} \tag{25}$$

where U is the total strain energy in the composites, and $\Delta U^{(k)}$ indicates the strain energy dissipation in the kth element. In the 2-D case, the energy dissipation in the kth element could be determined as

$$\Delta U_{2\text{-D}}^{(k)} = \Delta U_1^{(k)} + \Delta U_2^{(k)} + \Delta U_{12}^{(k)} \tag{26}$$

where $\Delta U_1^{(k)}$, $\Delta U_2^{(k)}$, and $\Delta U_{12}^{(k)}$ represent the energy dissipation in the fiber, transverse, and in-plane shear directions, respectively. On the other hand, for the 3-D case, the interlaminar energy dissipation, i.e., ΔU_3, ΔU_{23}, and ΔU_{13}, has to be taken into account. Therefore, the energy dissipation formulation in the kth element would become

$$\Delta U_{3\text{-D}}^{(k)} = \Delta U_1^{(k)} + \Delta U_2^{(k)} + \Delta U_{12}^{(k)} + \Delta U_3^{(k)} + \Delta U_{23}^{(k)} + \Delta U_{13}^{(k)} \tag{27}$$

where $\Delta U_3 = \psi_T U_3$, $\Delta U_{23} = \psi_{LT} U_{23}$, and $\Delta U_{13} = \psi_{LT} U_{13}$. It is noted that since the information for out-of plane (interlaminar) damping properties is lacking, the in-plane properties (ψ_T and ψ_{LT}) were employed instead for the calculation of the out-of-plane energy dissipation [16]. The material properties for the glass/epoxy and graphite/epoxy used in the FEM simulation is summarized, respectively, in Table 1, which was found in the reference [16]. In addition, the total strain energy in the FEM analysis for the 2-D and 3-D cases were calculated, respectively, as follows:

$$U_{3\text{-D}} = \sum_{k=1}^{m} \frac{1}{2}(\sigma_1^k \varepsilon_1^k + \sigma_2^k \varepsilon_2^k + \sigma_3^k \varepsilon_3^k + \sigma_{23}^k \varepsilon_{23}^k + \sigma_{13}^k \varepsilon_{13}^k + \sigma_{12}^k \varepsilon_{12}^k) \tag{28}$$

$$U_{2\text{-D}} = \sum_{k=1}^{m} \frac{1}{2}(\sigma_1^k \varepsilon_1^k + \sigma_2^k \varepsilon_2^k + \sigma_{12}^k \varepsilon_{12}^k) \tag{29}$$

With Eq. (25) together with Eqs. (26)–(29), the corresponding SDC of the laminates can be directly evaluated from FEM analysis. In this study, the FEM analysis was conducted using a commercial code, ANSYS 10.0, where SOLID46 element is selected to construct the laminated composite structures. In addition, to simulate the pure bending, couple forces were applied on both ends of the laminated plate as shown in Fig. 1. For the prevention of rigid body translation and rotation, the following displacement conditions

$$\text{at } (x_1, x_2, x_3) = \left(\frac{L}{2}, 0, 0\right), \quad u_1 = 0, \quad u_2 = 0 \quad \text{and} \quad u_3 = 0 \tag{30}$$

$$\text{at } (x_1, x_2, x_3) = \left(\frac{L}{2}, \frac{-W}{2}, 0\right), \quad u_2 = 0 \tag{31}$$

$$\text{at } (x_1, x_2, x_3) = \left(\frac{-L}{2}, 0, 0\right), \quad u_2 = 0; \quad u_3 = 0 \tag{32}$$

were imposed on the composite laminates. In the above expression, W and L represent the width and length of the laminates, respectively; u_1, u_2, and u_3 denote the displacement in x_1, x_2, and x_3 directions, respectively.

Table 1
Material properties of graphite/epoxy and glass/epoxy composites

Materials	E_L (Gpa)	E_T (Gpa)	G_{LT} (Gpa)	υ_{LT}	ψ_L (%)	ψ_T (%)	ψ_{LT} (%)
Graphite/epoxy composites (HMS/DX-210)	172.7	7.20	3.76	0.291	0.45	4.22	7.05
Glass/epoxy composites (GLASS/DX-210)	37.78	10.90	4.91	0.291	0.87	5.05	6.91

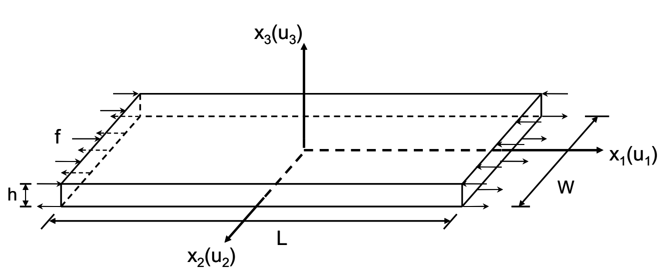

Fig. 1. Schematic of the laminated plates with boundary conditions.

4. Results and discussion

The flexural damping responses of the graphite epoxy composite laminates $[0/-60/60]_s$ and $[0/90/45/-45]_s$ obtained from the analytical models are compared with the Ni–Adams model and Adams–Maheri model in Fig. 2 and 3. The FEM results calculated based on the 2-D stress states and 3-D stress states together with the experimental data [1] are also included in the comparison. It was found that the Ni–Adams and Adams–Maheri models are mostly deviated from the FEM results and experimental data except in some fiber orientations. Moreover, the current 2-D analytical model with all in-plane stress and strain quantities considered in the calculation of energy dissipation is close to the 2-D finite element results and experimental results. Nevertheless, there is still little discrepancy between the present model and 2-D FEM analysis, which could be attributed to the edge effect caused by either

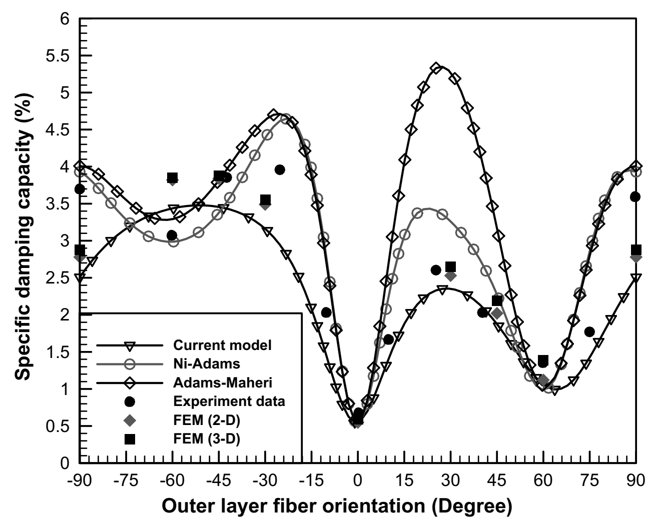

Fig. 2. Flexural damping properties of graphite/epoxy laminates $[0/-60/60]_s$.

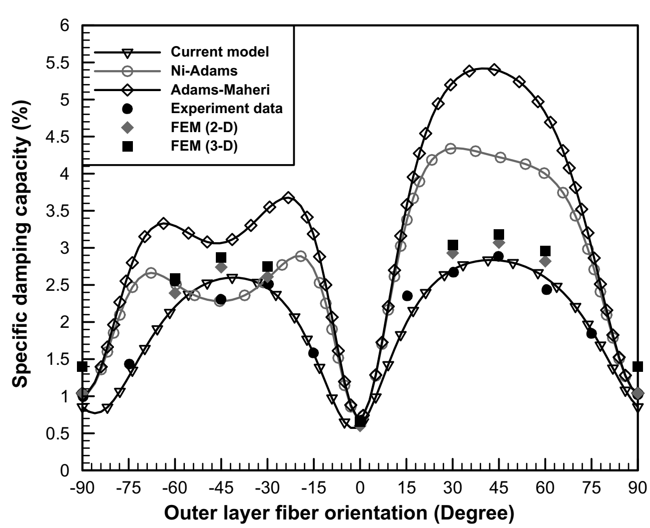

Fig. 3. Flexural damping properties of graphite/epoxy laminates $[0/90/45/-45]_s$.

the free edges or the loading edges of the laminates. In general, the variation of the stresses because of the edge effect was not taken into account in the analytical model, but it was taken into account in the FEM analysis.

In addition, in the 3-D finite element, because of the inclusion of the interlaminar stress (out of plane stress) effect, the SDC are consistently higher than those obtained from the 2-D FEM analysis and the 2-D analytical model. A comparison of the results indicates that the differences between the 2-D analysis and 3-D FEM results are not so significant that the 2-D analytical model that provides a lower bound solution can be utilized for the evaluation of damping responses of composite laminates. Similar tendencies are also observed in the glass/epoxy composites as shown in Figs. 4 and 5. As a result, the 2-D analytical model, although it does not account for the interlaminar stress effect, demonstrates the same tendency as the FEM results and the experimental data as well. It is suggested that the 2-D analytical model with physical and mathematical simplicity is suitable for characterizing the damping responses of the composite laminates.

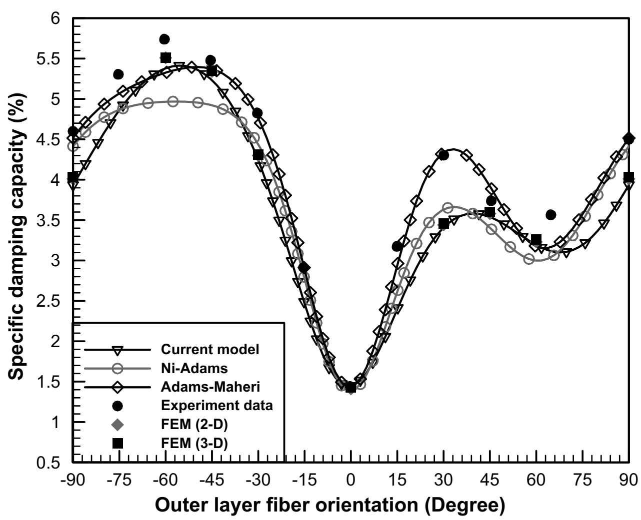

Fig. 4. Flexural damping properties of glass/epoxy laminates $[0/-60/60]_s$.

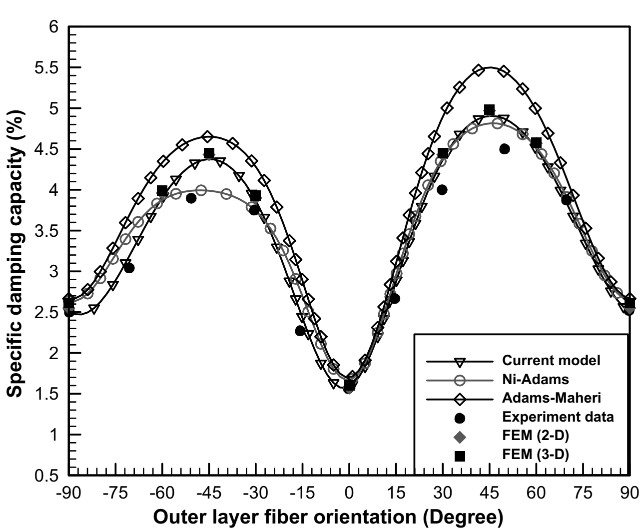

Fig. 5. Flexural damping properties of glass/epoxy laminates $[0/90/45/-45]_s$.

5. Conclusions

The 2-D analytical models for characterizing flexural damping properties of composite laminates were reviewed in the study. It was found that when the in-plane stress and strain quantities that were ignored in the Ni–Adams and Adams–Maheri models were included in the present model for the calculation of energy dissipation, the results were the same as those deduced from the Saravanos–Chamis model. When compared to the Ni–Adams and Adams–Maheri models, the present models demonstrate good agreement with the experimental data. In addition, to investigate the interlaminar stress effect on the SDC of laminates, the FEM analysis based on the 2-D stress and strain components and 3-D stress and strain quantities was performed, respectively. The 3-D results, because of more energy consumption, are considered to exhibit relatively higher SDC values than the 2-D cases. On the other hand, because the discrepancy is not substantial, the interlaminar effect may be neglected in the modeling of the damping responses of composite laminates. Moreover, the 2-D FEM results and the present 2-D analytical model demonstrate good agreements with each other, which indicate that the present 2-D analytical model could provide lower bound solutions in the description of the damping responses of composite laminates.

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