國 立 交 通 大 學

電信工程學系

碩 士 論 文

非線性與立體聲回音消除之收斂性分析

Convergence Analysis of Nonlinear and Stereophonic Acoustic Echo Cancellation

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摘要

由於現代通訊系統的多樣化應用，愈來愈多的聲訊回音消除(AEC)設計必須考量非線性特性(喇叭)及立體聲環境(高相關輸入)。已經有許多針對非線性適應濾波器所提出的演算法，但卻缺少相對應的演算法收斂分析。根據無記憶多項式適濾波器與 NLMS 演算法，我們提出了非線性聲訊回音消除的收斂分析式；另一方面，為了尋找非線性聲訊回音消除較佳的收斂方式，我們也提出並比較幾種不同的係數更新方法。在立體聲回音消除方面，我們利用 Doğancay 在分析單聲道選擇式係數更新收斂性時所採用的方法，提出應用於立體聲回音消除部分係數更新之收斂分析式。另外，結合非線性喇叭與立體聲回音消除的實驗驗證了相關性削減與非線性適應濾波器的功效。最後，電腦模擬用來驗證支持之前的分析。

Convergence Analysis of Nonlinear and Stereophonic Acoustic Echo Cancellation

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Abstract

Because of diversity applications of modern communication system, more and more acoustic echo cancellation (AEC) designs need to consider the nonlinearity characteristic (loudspeaker) and stereophonic environment (high correlated inputs). There are many proposed algorithms for nonlinear adaptive filter but lack of convergence analysis for these algorithms. We propose the convergence analyses for nonlinear AEC based on memoryless polynomial nonlinear filter and NLMS algorithm. Several adaptation strategies for nonlinear AEC are also provided and compared for finding better convergence behavior. In stereophonic AEC, we analyze the convergence of partial update stereophonic AEC by taking use of the Doğancay's method used in selective partial update monophonic AEC study. The combinations of nonlinear loudspeaker and stereophonic AEC are demonstrated to verify the coherence reduction and nonlinear adaptive filter effect. Finally, computer simulations are presented to support the analysis.

Acknowledgement

I would like to express my deepest gratitude to my advisor, Dr. S. F. Hsieh, for his enthusiastic guidance and great patience, especially the autonomy in research. I also appreciate all my lab-mates for their inspiration and help. Finally, I would like to show my thanks to my family, my parents, my sisters and my girl friend, I. H. Lee, for their unceasing encouragement and love.

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Chapter 1

Introduction

Acoustic echo cancellers (AEC) are used to reduce echoes in a wide range of applications, such as hands-free telephones and teleconferencing etc. Generally speaking, the transmitted-back signals not produced by far end talker are considered as "noises" in a hands-free telephony system. These noises include echoes, near end local noise and other audio signal. We will concentrate our studies on echoes cancellation in some specific conditions and environments in this thesis.

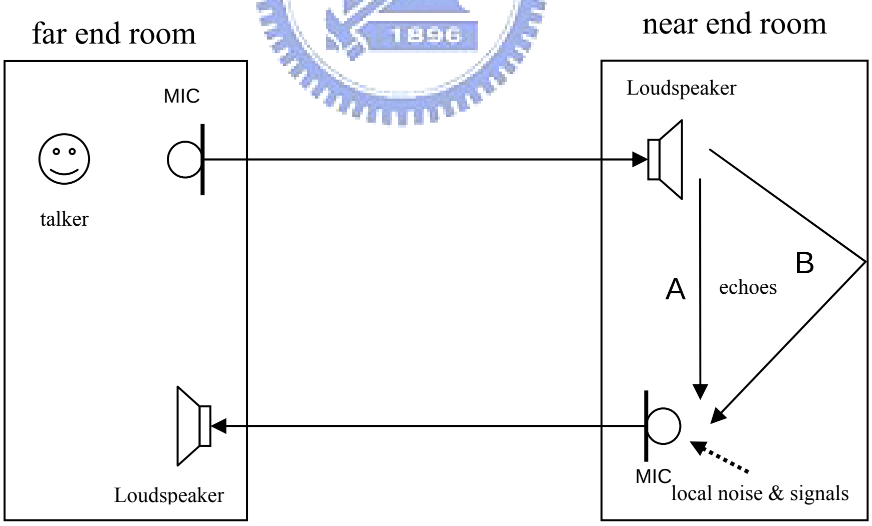

Figure 1.1 Echo signals

In Figure 1.1, near end signals A and B radiated by the loudspeaker are echoes to far end room talker. That is, the far end talker will be annoyed by listening to his or her own speech delayed by the round-trip time of system. The main idea of AEC design is try to reduce the "echo signal" efficiently.

An application system with AEC is shown in Figure 1.2 to overcome acoustic echo and provide satisfactory speech quality. The estimated filter $\hat{\underline{h}}(n)$ approximates to the room equivelent impulse response $\underline{h}(n)$ through AEC system dealing with far end signal $x(n)$ and residual error signal $e(n)$ iteratively. The residual error signal $e(n)$ is obtained by subtracting estimated output $\hat{y}(n)$ from the near end microphone output signal $d(n)$ which includes the real echo signal $y(n)$, local noise $v(n)$ and local signal $t(n)$.

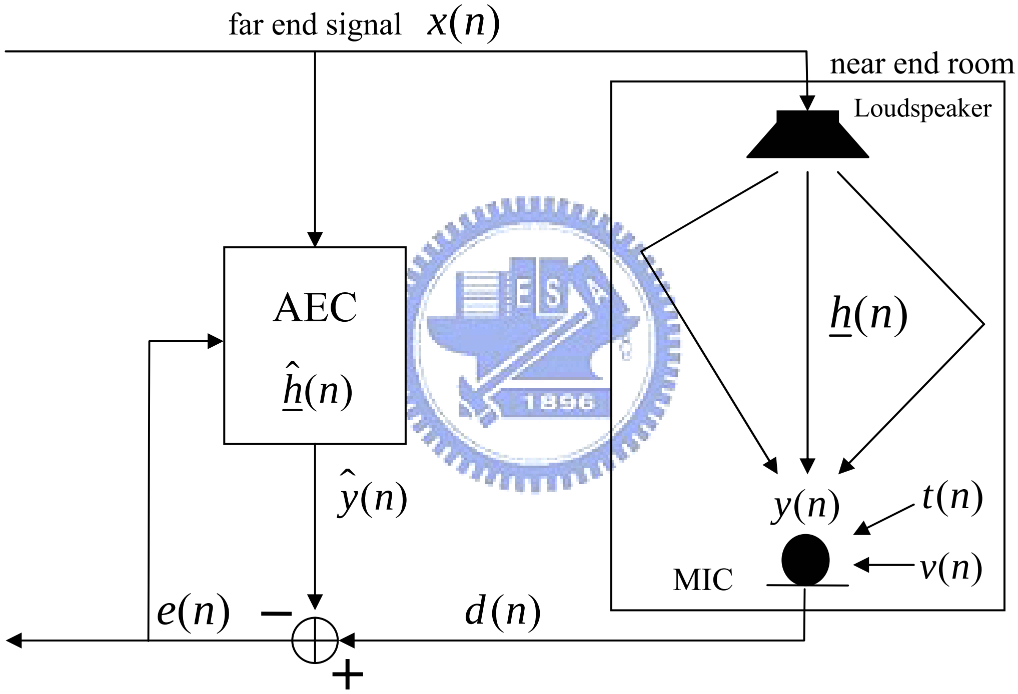

Figure 1.2 Application system with AEC

Many researches of AEC are focus on designing algorithms or system structures to maximize “Convergence Rate / Value” ratio or to minimize complexity both in applications and theory fields [2] [3]. Recently, there are more and more interesting AEC topics could be studied because of the diversity application of modern communication system. There are two main topics we studied about AEC application in this paper: one is nonlinear AEC system and the other is stereophonic AEC system.

AEC in today's speakerphones or video conferencing systems most rely on the assumption of a linear echo path. However, the customer comfort and security are improved, and through this, audio equipments or portable communication systems should be compatible with the miniaturization trend and cannot be very expensive, all of these things result in nonlinearities. The small but inaudible nonlinearities have a dramatic influence on classical linear AEC and nonlinear AEC is the first topic we will discuss in this thesis; we develop the approximate convergence analyses in different conditions and compare several adaptation strategies for specific nonlinear AEC system.

By another way, a stereo teleconferencing system provides a more realistic presence than a monophonic system, because listeners can use spatial information to help distinguish who is speaking. This is especially important for video teleconferencing involving many different talkers. However, in stereophonic AEC the acoustic echo cancellation problem is more difficult to solve because of the necessity to uniquely identify two acoustic paths. This has led to several approaches to the problem that involve techniques to de-correlate the two input signals using nonlinearities preprocessing or partial adaptation scheme. For nonlinear de-correlation preprocessing, it has the disadvantages of larger computation complexity and possible audible sound distortion. Due to that partial adaptation scheme of stereophonic AEC provides an alternative to solve the nonuniqueness problem without the disadvantages of nonlinear de-correlation preprocessing. We also derive the stereophonic AEC partial update scheme convergence analysis in an approximated form in this paper.

This thesis is organized as follows. There will be more details about nonlinear and stereophonic AEC partial update schemes and prepared knowledge in Chapter 2. And we propose convergence analyses in different conditions and compare several adaptation strategies for nonlinear AEC in Chapter 3. Chapter 4 shows the approximated convergence analysis of stereophonic AEC partial update scheme. The demonstration of combining nonlinear loudspeaker and stereophonic AEC is also given in this Chapter. In Chapter 5, computer simulations verify the results of our derivations in Chapter 3 and Chapter 4. Finally, in Chapter 6, we give a conclusion for our study.

The main efforts in this thesis are:

(1) Individual convergence analyses of the linear and nonlinear coefficient error.

(2) Joint coefficient error convergence analysis when the other coefficient set is estimated adaptively through simulation.

(3) Discuss and compare several kinds of adaptation strategies for nonlinear AEC.

(4) Convergence analysis of stereophonic AEC partial update scheme.

(5) Demonstrate the behavior of nonlinear loudspeaker effect in stereophonic AEC and apply the nonlinear adaptive filter in nonlinear-stereophonic AEC.

Chapter2

Adaptive Schemes of Nonlinear and Stereophonic AEC

In this Chapter, we will introduce two adaptation schemes for different AEC applications by the normalized least mean square (NLMS) algorithm, which belongs to the category of stochastic gradient algorithms. First, acoustic echo cancellers in today's speakerphones or video conferencing systems rely on the assumption of a linear echo path. Low-cost loudspeakers or audio equipment introduce considerable nonlinearities, which limit the echo return loss enhancement achieved by linear adaptation schemes. This means that an annoying nonlinearly distorted echo will be transmitted back to the far end subscriber. While nonlinearities with memory are of concern usually with high-quality audio equipment, the memoryless nonlinearities are addressed for fast converging and simplicity in our study. According to gradient-type adaptation (i.e., NLMS), joint NLMS adaptation of nonlinear polynomial adaptive filter (preprocessor) and FIR filter are further provided in [5], which will be introduced in Section 2.2 and adapted for derivation in Chapter 3.

Stereophonic AEC is another topic discussed in our thesis, which is the scheme for controlling echo signals in a stereo teleconferencing system. There are two acoustic paths to identify in a stereophonic system, which raises the stereophonic AEC fundament problem: nonuniqueness, this problem will be introduced in Section 2.3 and it can be solved by reduction of inter-channel coherence. Recently, coefficient partial update approaches [14] [15] are adapted to overcome the nonuniqueness problem; it has advantages including not only less computation complexity in coefficient adaptation, but also avoid applying nonlinearities between two channel input signals. Some basic partial update scheme of stereophonic AEC also will be introduced in final Section.

2.1 Configuration and adaptation algorithm

2.1.1 Configuration of an acoustic echo canceller

Figure 2.1 shows the configuration of an acoustic echo canceller. The echo canceller identifies the impulse response $\underline{h}(n)$ between the loudspeaker and the microphone. An adaptive filter $\hat{\underline{h}}(n)$ is used to identify $\underline{h}(n)$; if $\hat{\underline{h}}(n)$ is identical to $\underline{h}(n)$, the echo estimated output $\hat{y}(n)$ will be equal to $y(n)$ and the echo signal can be cancelled perfectly ($e(n)=0$) under noise free ($v(n)=0$) and no double talk

($t(n)=0$) situation. For easy implementation and stability consideration, $\underline{\hat{h}}(n)$ is usually implemented by an FIR filter.

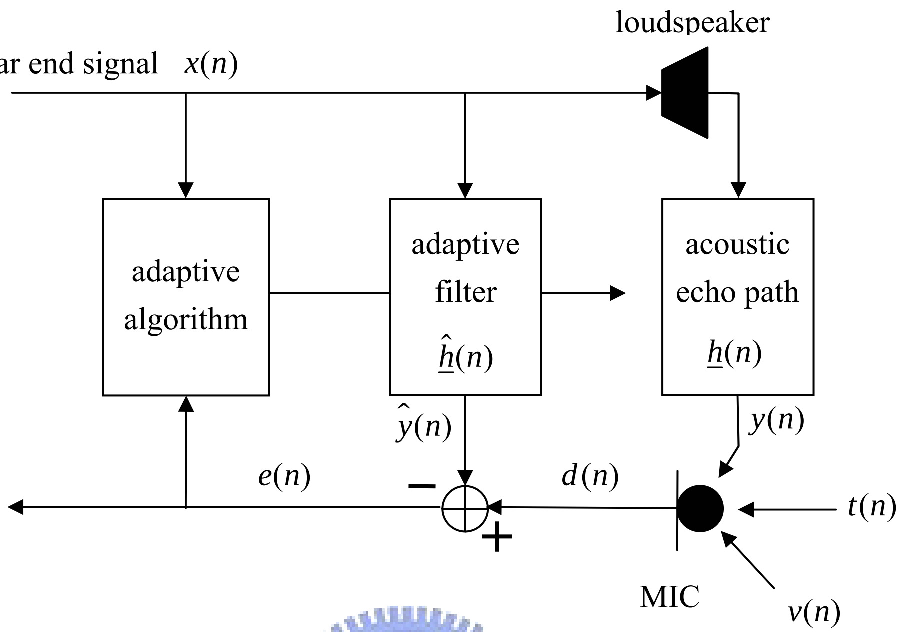

Figure 2.1 Configuration of an acoustic echo canceller

In order to obtain a good replica of the echo, adaptive algorithm is necessary. Since the real echo path $\underline{h}(n)$ is usually unknown and time variant. The adaptive algorithm should satisfy the following implementation requirements:

1. Real-time operation.
2. Fast convergence speed.
3. High echo return loss enhancement (ERLE) or low residual error power

where ERLE is defined as the ratio of the real echo signal power to the residual error signal power:

$$\mathrm{ERLE(dB)} = 10 \cdot \log_{10} \frac{E[y^2(n)]}{E[(y(n)-\hat{y}(n))^2]} \tag{2.1.1}$$

2.1.2 Normalized least mean square algorithm

The least mean square (LMS) algorithm is an iterative algorithm used to estimate the impulse response so as to minimize the mean square error. For LMS algorithm, the coefficients adaptive equation and parameters are described as follows [1]:

$$\underline{\hat{h}}(n+1) = \underline{\hat{h}}(n) + \mu \underline{x}(n)e(n) \tag{2.1.2}$$

$$e(n) = d(n) - \hat{y}(n) \tag{2.1.3}$$

μ : step size

$e(n)$: residual error signal

$d(n) = \underline{h}^T(n)\underline{x}(n) + t(n) + v(n)$: the signal received by microphone

$\hat{y}(n) = \underline{\hat{h}}^T(n)\underline{x}(n)$: echo signal

$\underline{x}(n) = [x(n), \cdots, x(n-L+1)]^T$: far end signal with adaptive filter length L

$\underline{h}(n) = [h_0(n), \ldots, h_{L-1}(n)]^T$: true echo path response

$\underline{\hat{h}}(n) = [\hat{h}_0(n), \ldots, \hat{h}_{L-1}(n)]^T$: estimated echo path response

In order to make the LMS algorithm insensitive to the change of input signal power level, the step size is normalized, resulting in the NLMS coefficients adaptive equation described as:

$$\underline{\hat{h}}(n+1) = \underline{\hat{h}}(n) + \frac{\mu}{\|x(n)\|_2^2}\underline{x}(n)e(n) \tag{2.1.4}$$

Convergence of the mean squared error (MSE) is guaranteed [1] when $0 < \mu < 2$. The NLMS algorithm has been the main algorithm of most studies due to its simplicity for implementation in many applications. We also adapt the NLMS algorithm for the further studies of two AEC applications: nonlinear and stereophonic AEC schemes.

2.2 Nonlinear AEC

Nonlinearity happens in loudspeaker due to loudspeaker over driven. There are two main considerations for nonlinear AEC study, one is finding suitable nonlinear system model and the other is how to combine nonlinear and linear system in one structure. The Volterra series expansion can model a large class of nonlinear systems [18] and it's attractive in adaptive filtering applications because the expansion is a linear combination of nonlinear functions of the input signals. Because of large complexity, the Volterra series always only been taken as the first step to find simpler nonlinear system [6] [7] [12].

There are many kinds of method to build a nonlinear channel before linear response. Most studies take the real loudspeaker as the nonlinear device [4] [5]. Another study models the nonlinear channel by saturation followed by a linear propagation [6]. For simplicity, we set up our nonlinear AEC system consisting of a cascade of memoryless nonlinear system and linear systems liked the following description.

2.2.1 Cascade nonlinear AEC scheme

A nonlinear AEC with a cascade of a memoryless nonlinear preprocessor and an FIR filter [5] is introduced in this Section; and an LMS-type adaptation is derived for a general nonlinear preprocessor. The cascaded nonlinear AEC scheme is shown in Figure 2.2, the adaptation of both nonlinear and linear stages has to rely on the joint residual error signal $e(n)$.

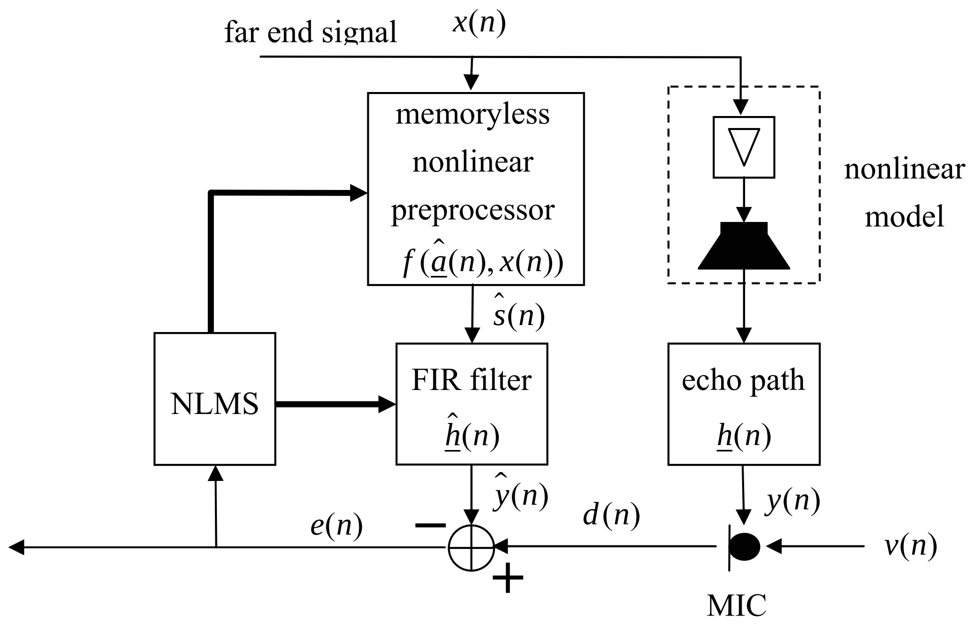

Figure 2.2 Nonlinear AEC cascade scheme

A general joint adaptation scheme for FIR filter $\hat{\underline{h}}(n)$ and nonlinear filter $\hat{\underline{a}}(n)$ will be introduced in following description [5].

The out put of the memoryless nonlinear function $f(\hat{\underline{a}}(n), x(n))$ is given by

$$\hat{\underline{s}}(n) = f(\hat{\underline{a}}(n), x(n)) \tag{2.2.1}$$

With the nonlinear coefficient vector

$$\hat{\underline{a}}(n) = \left[\hat{a}_1(n), \hat{a}_2(n), \cdots, \hat{a}_Q(n)\right]^T \tag{2.2.2}$$

Where Q represents the nonlinear order

By another way, the estimated echo signal reads

$$\hat{y}(n) = \hat{\underline{h}}^T(n)\hat{\underline{s}}(n) = \hat{\underline{h}}^T(n)\underline{f}(\hat{\underline{a}}(n), \underline{x}(n)) \tag{2.2.3}$$

Where the estimated impulse response

$$\hat{\underline{h}}(n) = \left[\hat{h}_0(n), \hat{h}_1(n), \cdots, \hat{h}_{L-1}(n)\right]^T \tag{2.2.4}$$

and the recent L 's outputs of memoryless nonlinear function $f(\hat{\underline{a}}(n), x(n))$ is obtained by

$$\hat{\underline{s}}(n) = \left[\hat{s}(n), \hat{s}(n-1), \cdots, \hat{s}(n-L+1)\right]^T$$
$$= \underline{f}(\hat{\underline{a}}(n), \underline{x}(n))$$
$$= \left[f(\hat{\underline{a}}(n), x(n)), f(\hat{\underline{a}}(n), x(n-1)), \cdots, f(\hat{\underline{a}}(n), x(n-L+1))\right]^T \quad (2.2.5)$$

with the $L \times 1$ input vectors

$$x(n) = \left[x(n), x(n-1), \cdots, x(n-L+1)\right]^T \quad (2.2.6)$$

The gradient of residual error power $e^2(n)$, as derived for linear transversal filters in [1] can be calculated according to:

$$\nabla_h(n) = \frac{\partial e^2(n)}{\partial \hat{\underline{h}}(n)} = -2e(n)\hat{\underline{s}}(n) \quad (2.2.7)$$

$$\nabla_a(n) = \frac{\partial e^2(n)}{\partial \hat{\underline{a}}(n)} = -2e(n)\left[f'(\hat{\underline{a}}^T(n), \underline{x}(n))\right]^T \hat{\underline{h}}(n) \quad (2.2.8)$$

with the residual error signal

$$e(n) = d(n) - \hat{\underline{h}}^T(n)\underline{f}(\hat{\underline{a}}(n), \underline{x}(n)) \quad (2.2.9)$$

and the $L \times Q$ matrix

$$f'(\hat{\underline{a}}^T(n), \underline{x}(n)) = \frac{\partial \underline{f}(\hat{\underline{a}}^T(n), \underline{x}(n))}{\partial \hat{\underline{a}}^T(n)}$$
$$= \left[\frac{\partial f(\hat{\underline{a}}(n), x(n))}{\partial \hat{\underline{a}}(n)}, \cdots, \frac{\partial f(\hat{\underline{a}}(n), x(n-L+1))}{\partial \hat{\underline{a}}(n)}\right]^T \quad (2.2.10)$$

If linear coefficient vector is updated with normalized step size $\mu_h(n)$, an NLMS-type adaptation results:

$$\hat{\underline{h}}(n+1) = \hat{\underline{h}}(n) - \frac{\mu_h(n)}{2}\nabla_h(n) = \hat{\underline{h}}(n) + \mu_h(n)\hat{\underline{s}}(n)e(n) \quad (2.2.11)$$

where the normalized step size reads

$$\mu_h(n) = \frac{\mu_h}{\left\|\hat{\underline{s}}(n)\right\|_2^2} \quad (2.2.12)$$

Similarly, the nonlinear coefficient vector is updated with step size μ_a and the adaptation equation results:

$$\underline{\hat{a}}(n+1) = \underline{\hat{a}}(n) - \frac{\mu_a}{2}\nabla_a(n) = \underline{\hat{a}}(n) + \mu_a \left[f'(\underline{\hat{a}}^T(n), \underline{x}(n)) \right]^T \underline{\hat{h}}(n)e(n) \quad (2.2.13)$$

Eq.(2.2.11) and Eq.(2.2.13) demonstrate the general form of a joint NLMS adaptation of preprocessor and FIR filter. As the normalization of the preprocessor depends on the specific realization of f, we distinguish the polynomial case in Subsection 2.2.2.

2.2.2 Polynomial adaptive filter

The preprocessor NLMS adaptation discussed above has been specialized to a polynomial adaptive filter in [5] and [10]. The realization, linear filter with polynomial preprocessor, was derived in [5] assuming that nonlinearities appear only in amplifier and loudspeaker, the nonlinearities are memoryless and room response is linear. Because we use the same nonlinear scheme for our convergence analysis in Chapter 3, the derivations are introduced again briefly in following description.

Describing the nonlinear preprocessor by a Q-th order polynomial and the preprocessor output can be simplified by a de-couple form of nonlinear coefficients $\underline{\hat{a}}(n)$ and input $x(n)$. It reads:

$$f(\underline{\hat{a}}(n), x(n)) = \sum_{q=1}^{Q} \hat{a}_q(n)x^q(n) \quad (2.2.14)$$

Where the n-th preprocessor output reads:

$$\hat{s}(n) = \underline{x}_Q^T(n)\underline{\hat{a}}(n) \quad (2.2.15)$$

and $$e(n) = d(n) - \underline{\hat{a}}^T(n)\underline{\hat{u}}(n) \quad (2.2.16)$$

where $$\underline{x}_Q(n) = \left[x(n), x^2(n), \cdots, x^Q(n) \right]^T \tag{2.2.17}$$

$$\underline{\hat{u}}(n) = X_Q^T(n)\underline{\hat{h}}(n) \tag{2.2.18}$$

$$X_Q(n) = \left[\underline{x}_Q(n), \underline{x}_Q(n-1), \cdots, \underline{x}_Q(n-L+1) \right]^T \tag{2.2.19}$$

Similar to $\mu_h(n)$ in Subsection 2.2.1, the polynomial adaptation is normalized with respect to $\left\| \underline{\hat{u}}(n) \right\|_2^2$. It becomes very small, when a single input sample $x(n)$ is close to zero. To avoid a large adaptation step towards to an unknown direction, adding a small constant δ_u to the normalized term, so that the nonlinear coefficient adaptation equation Eq.(2.2.13) can be written as:

$$\underline{\hat{a}}(n+1) = \underline{\hat{a}}(n) + \frac{\mu_a}{\left\| \underline{\hat{u}}(n) \right\|_2^2 + \delta_u} \underline{\hat{u}}(n) e(n) \tag{2.2.20}$$

In contrast to $\underline{\hat{h}}(0)$, $\underline{\hat{a}}(0)$ must never be zero, as then no adaptation can take place. A good choice is linear initialization, i.e., $\underline{\hat{a}}(0) = [1, 0, \cdots, 0]^T$. This ensures that even if $\underline{\hat{a}}$ is not adapted, the system performs at least as well as a linear AEC. According to the assumption of memoryless polynomial filter, a less or equivalent nonlinear order polynomial coupled with a linear system is our nonlinear AEC configuration.

2.3 Stereophonic AEC

2.3.1 Stereophonic AEC scheme

Stereophonic acoustic echo cancellation can be viewed as a straightforward generalization of the single channel acoustic echo cancellation principle, as illustrated in Figure 2.3.

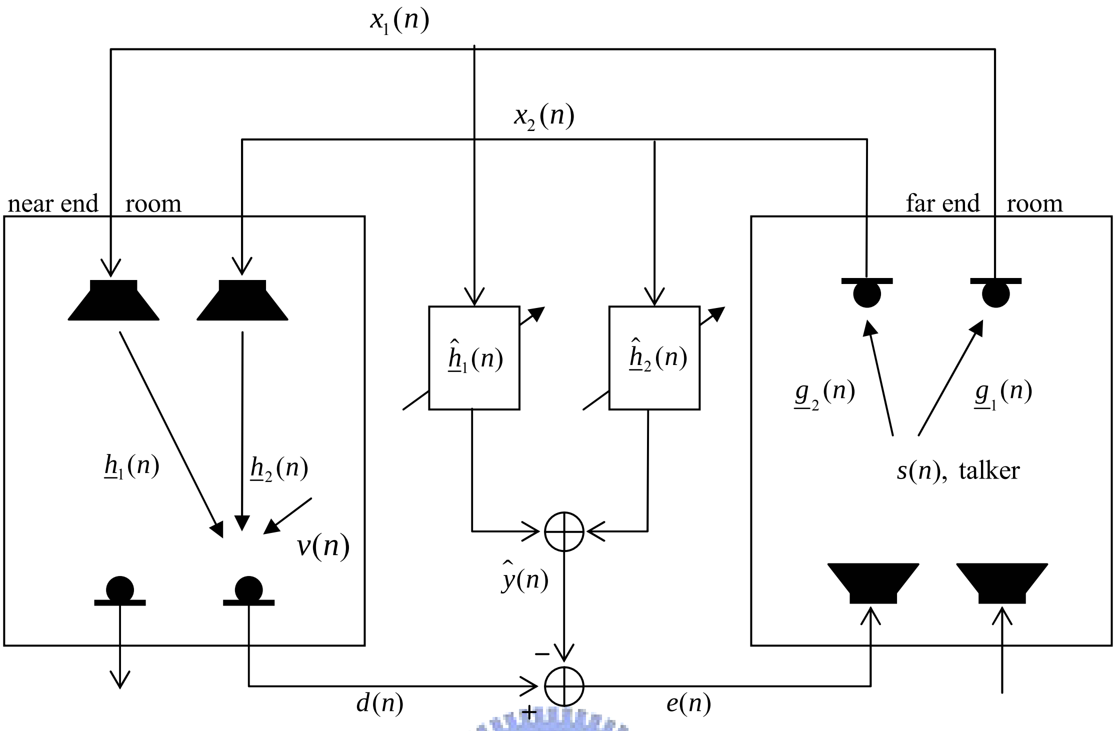

Figure 2.3 Scheme of stereophonic AEC

Figure 2.3 shows the schematic diagram of stereophonic AEC. For simplicity, the acoustic paths to only one microphone are shown in the near end room; it is understood that similar analysis will apply to the other microphone signal. The far end signals $x_1(n)$ and $x_2(n)$ at time index n are generated from the talker speech $s(n)$ by passing far end room impulse response $\underline{g}_1(n)$ and $\underline{g}_2(n)$ separately from the talker to the microphones. Then, $x_1(n)$ ($x_2(n)$) passes the echo path $\underline{h}_1(n)$ ($\underline{h}_2(n)$) which is the equivalent impulse response from the speaker to microphone. The summation of $\underline{x}_1(n) * \underline{h}_1(n)$, $\underline{x}_2(n) * \underline{h}_2(n)$ and local noise $v(n)$ becomes the microphone output signal $d(n)$ .Similarly, adaptive filters ($\hat{\underline{h}}_1(n), \hat{\underline{h}}_2(n)$) generate an echo replica signal $\hat{y}(n)$ to reduce the residual error signal $e(n)$. The pair of adaptive filters ($\hat{\underline{h}}_1(n), \hat{\underline{h}}_2(n)$) are recursively updated with the NLMS algorithm by the residual error signal $e(n)$. The same model applies to the other near end room microphone with the acoustic paths replaced by the ones appropriate to that microphone.

According to NLMS algorithm and system scheme, the adaptation equation to one near end microphone reads:

$$\underline{\hat{h}}_1(n+1) = \underline{\hat{h}}_1(n) + \mu(n)\underline{x}_1(n)e(n) \tag{2.3.1}$$

$$\underline{\hat{h}}_2(n+1) = \underline{\hat{h}}_2(n) + \mu(n)\underline{x}_2(n)e(n) \tag{2.3.2}$$

Where $$\mu(n) = \frac{\mu}{\left\|\underline{x}_i(n)\right\|_2^2} \tag{2.3.3}$$

$$e(n) = d(n) - \hat{y}(n) \tag{2.3.4}$$

$$d(n) = \underline{h}_1^T(n)\underline{x}_1(n) + \underline{h}_2^T(n)\underline{x}_2(n) + v(n) \tag{2.3.5}$$

$$\hat{y}(n) = \underline{\hat{h}}_1^T(n)\underline{x}_1(n) + \underline{\hat{h}}_2^T(n)\underline{x}_2(n) \tag{2.3.6}$$

2.3.2 The nonuniqueness problem

We will show that the solution of normal equation is not as obvious as in the single-channel case in this Subsection. Since the two input signals are obtained by filtering from a common source, a problem of nonuniqueness is expected [11] in stereophonic AEC.

For simplicity, the near end signal and local noise are assumed to 0 in following description. In monophonic AEC system, the residual error signal $e(n)$ is represented as:

$$e(n) = \underline{h}^T(n)\underline{x}(n) - \underline{\hat{h}}^T(n)\underline{x}(n) = \underline{\tilde{h}}^T(n)\underline{x}(n) \tag{2.3.7}$$

When $e(n) \to 0$, it also indicates $\underline{\tilde{h}}(n) \to \underline{0}$, i.e., The smaller residual error signal the smaller coefficient misalignment we have in monophonic AEC.

In stereophonic AEC, the residual error signal $e(n)$ reads:

$$\begin{aligned} e(n) &= \left[\underline{h}_1^T(n)\underline{x}_1(n) + \underline{h}_2^T(n)\underline{x}_2(n) \right] - \left[\hat{\underline{h}}_1^T(n)\underline{x}_1(n) + \hat{\underline{h}}_2^T(n)\underline{x}_2(n) \right] \\ &= \left[\tilde{\underline{h}}_1^T(n)G_1 + \tilde{\underline{h}}_2^T(n)G_2 \right]\underline{s}(n) \end{aligned} \tag{2.3.8}$$

G_i is the far end channel matrix which is the combination of far end room impulse response g_i and its shift. In Eq.(2.3.8), the residual error $e(n) \to 0$ only indicates $\left[\tilde{\underline{h}}_1^T(n)G_1 + \tilde{\underline{h}}_2^T(n)G_2 \right] \to 0$. Actually, we hope the coefficient misalignment can become small when residual error is small, i.e., $\tilde{\underline{h}}_1(n) \to 0$ and $\tilde{\underline{h}}_2(n) \to 0$ when $e(n) \to 0$. To achieve this requirement, Eq.(2.3.8) can be rewritten as another form:

$$e(n) = \tilde{\underline{h}}_1^T(n)\underline{k} + \tilde{\underline{h}}_2^T(n)\underline{r} \tag{2.3.9}$$

and $\underline{k}$ is independent of $\underline{r}$

According to that, $e(n) \to 0$ also indicates $\tilde{\underline{h}}_1(n) \to 0$ and $\tilde{\underline{h}}_2(n) \to 0$. That's why the nonlinearities or partial update scheme are applied in stereophonic AEC to reduce the coherence between two far end input signals. i.e., reducing the coherence between $x_1(n) = g_1 * \underline{s}(n)$ and $x_2(n) = g_2 * \underline{s}(n)$.

2.3.3 Coefficients partial update

Many partial update schemes for monophonic AEC had been proposed before, for example: the MMax-NLMS algorithm [17]. Furthermore, the coefficient error convergence behavior of monophonic partial update AEC has also been analyzed [16]. In [16], the coefficient block is updated only when the corresponding input block has the maximum magnitude, a general representation of the partial adaptation equation in monophonic AEC can be represented as follows:

$$\underline{\hat{h}}(n+1) = \underline{\hat{h}}(n) + \mu(n) A_i \underline{x}(n) e(n) \qquad i = \arg\max_{1 \le j \le B} \left\| \underline{x}_j(n) \right\|_2^2 \tag{2.3.10}$$

where B : partial update block numbers

$$\mu(n) = \frac{\mu}{\left\| \underline{x}_i(n) \right\|_2^2}$$

and A_i is a $L \times L$ diagonal matrix defined by

$$A_i = diag(0, \ \cdots \ ,0, \underbrace{1, \ \cdots \ ,1}_{i'th \text{ block}}, 0, \ \cdots \ ,0) \tag{2.3.11}$$

The introduction of A_i combines the effect of selective partial update with original NLMS adaptation equation. According to that, Eq.(2.3.10) can be applied in other analysis, e.g., stereophonic AEC partial update.

By the other way, there exists a dual motivation to develop algorithms which have improved convergence performance due to reduction of inter-channel coherence whilst maintaining computation complexity to be as low as possible for practice reasons. For stereophonic partial update AEC, there are also some schemes had been demonstrated recently [14] [15]. In [15], the estimated coefficient vector is divided into two parts: front and back half. Which part coefficients can be update is decided by a certain criterion proposed in [15]. We modify the stereophonic partial update scheme in [15] by an equal probability block update assumption. According to this assumption and applying the adaptation equation Eq.(2.3.10), we try to give a convergence analysis for partial update stereophonic AEC in Chapter 4.

Chapter 3

Nonlinear Adaptive Filter

Some methods have been studied for nonlinear adaptive filter. Volterra series based filters [7]: can model a large class of nonlinear systems and are attractive in adaptive filtering applications because the expansion is a linear combination of nonlinear functions of the input signals. But it has disadvantage of a high computation complexity. The second method is nonlinear state space structure; this is an IIR filter model suitable for long memories requirement with lower computation complexity than the FIR filter but it has the unstable problem [8]. Neural network is the third method; this cascade structure offers a new perspective but need a pre-identification procedure [9].

More recently, cascade filter structures have been proposed [4], [5] and [10]. In [4], the nonlinear adaptive filter is composed of two distinct modules organized in a cascade structure: a nonlinear module based on polynomial Volterra filters models the loudspeaker, and a second module of standard linear filtering identifies the impulse response of the acoustic path. In [5], Stenger introduced a nonlinear AEC by cascading a memoryless nonlinear preprocessor with a FIR filter, and derived an

LMS-type adaptive algorithm for a general nonlinear processor. Two special cases and RLS adaptation for speeding up are also discussed in his study. The most important thing in this paper is: it provides a general parametric nonlinear function to represent a nonlinear preprocessor in mathematical form and gives the general form of a joint NLMS-type adaptation of both nonlinear and linear stages. In [10], Niemistö proposed another cascade filter structure by swapping the order of the nonlinear processor and the linear filter provided in [5], i.e., post-processor. Niemistö compared the adaptive polynomial preprocessor and post-processor with traditional Volterra filter through simulations on hands-free equipments and showed the ERLE improvement with the extra operations of the proposed structure. In [6], Costa introduced five general nonlinear structures and made a comparison on complexity and ERLEs.

Although, Stenger [5] and Niemistö [10] both introduced some study results on nonlinear cascade structure with adaptive polynomial filter, especially on updating algorithm and system structure. The convergent analysis for the adaptive polynomial filter remains unclear. We will perform the convergent analysis on the nonlinear AEC filter, including the linear FIR and the nonlinear polynomial filter.

We will introduce the preprocessor of nonlinear adaptive filter in Section 3.1, including diagrams, notations and recursively updated equations of both nonlinear and linear coefficients. In Section 3.2 and 3.3, we derive the general iteration formula for the linear and nonlinear coefficient error variance independently. In real situation, linear and nonlinear coefficient estimation errors are mutually related; they are dependent. We try to give an approximate equation to predict the error behavior of two filter's coefficients to match the real situation in Section 3.4. Finally, some modified adaptation strategies will be introduced in Section 3.5.

3.1 Preprocessor of nonlinear adaptive filter

Stenger has proposed a preprocess structure with cascade of a memoryless polynomial filter and a FIR filter [5]. The system diagram is in Figure 3.1. The echo signal $\underline{d}(n)$ at time index n is generated by passing the far end signal $x(n)$ with nonlinear loudspeaker response $\underline{a}$ and echo path $\underline{h}$ in a cascade order. For adaptive filters, far end signal $x(n)$ is powered first then the nonlinear adaptive filter output $\hat{s}(n)$ is the linear combination of the powered input $x^{i}(n)$ and estimated nonlinear coefficients. Finally, the FIR filter's output $\hat{y}(n)$ is given by $\hat{\underline{s}}(n) * \hat{\underline{h}}(n)$. Nonlinear and linear coefficients are adapted by joint NLMS adaptation (see also Sec. 2.2) with the same residual echo $e(n)$ but different powered inputs ($\hat{\underline{u}}(n)$, $\hat{\underline{s}}(n)$) at each iteration..

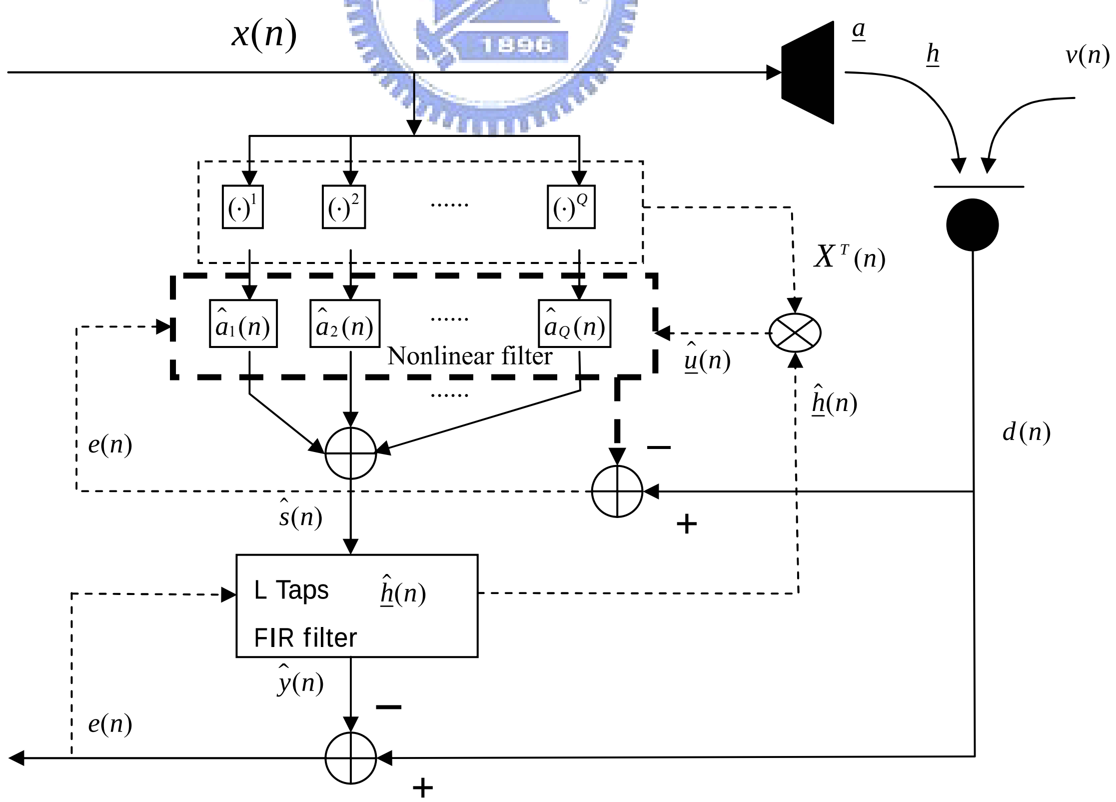

Figure 3.1 Preprocessor scheme of nonlinear adaptive filter

We take this cascade filter structure as our reference model in following derivations. It makes sense that nonlinear adaptive filter cascaded in the order of linear filter after nonlinear filter corresponding to nonlinear loudspeaker following by linear echo path response. This cascaded order is also the Hammerstein model described in [6].

Although the general nonlinear AEC notations have been mentioned before [see also Section 2.2], we still organized the necessary notations for the derivations convenient in following:

$$Q = \textit{memoryless polynomial filter order}$$

$$\sigma_v^2 = E[v(n)^2]\text{, } \textit{noise power}$$

$$\sigma_x^2 = E[x(n)^2]\text{, } \textit{far-end input signal power}$$

$$\underline{\tilde{h}}(n) = \underline{h} - \underline{\hat{h}}(n) = [\tilde{h}_0(n), \tilde{h}_1(n), \cdots, \tilde{h}_{L-1}(n)]^T\text{, } \textit{linear coefficient error vector}$$

$$\underline{\tilde{a}}(n) = \underline{a} - \underline{\hat{a}}(n) = [\tilde{a}_1(n), \tilde{a}_2(n), \cdots, \tilde{a}_Q(n)]^T\text{, } \textit{nonlinear coefficient error vector}$$

$$X(n) = \begin{bmatrix} x(n) & x^2(n) & \cdots & x^Q(n) \\ x(n-1) & x^2(n-1) & \cdots & x^Q(n-1) \\ \vdots & \vdots & \ddots & \vdots \\ x(n-L+1) & x^2(n-L+1) & \cdots & x^Q(n-L+1) \end{bmatrix}_{L\times Q}\text{, } \textit{powered input matrix}$$

$$\underline{\hat{s}}(n) = X(n)\underline{\hat{a}}(n) = [\hat{s}_0(n), \hat{s}_1(n), \cdots, \hat{s}_{L-1}(n)]^T\text{, } \textit{nonlinear powered input vector}$$

$$\underline{\hat{u}}(n) = X^T(n)\underline{\hat{h}}(n) = [\hat{a}_1(n), \hat{a}_2(n), \cdots, \hat{a}_Q(n)]^T\text{, } \textit{linear powered input vector}$$

$$\mu_h(n) = \frac{\mu_h}{\left\|\underline{\hat{s}}(n)\right\|_2^2}\text{, } \textit{normalized stepsize of linear coefficient}$$

$$\mu_a(n) = \frac{\mu_a}{\left\|\underline{\hat{u}}(n)\right\|_2^2 + \delta_u}\text{, } \textit{normalized stepsize of nonlinear coefficient}$$

$$R_{\underline{x}}(n) = \sigma_x^2 \cdot I$$

$$R_{\underline{\tilde{h}}}(n) = E[\underline{\tilde{h}}(n)\underline{\tilde{h}}^T(n)]\text{, } \textit{correlation matrix of linear coefficient error}$$

$$R_{\underline{\tilde{a}}}(n) = E[\underline{\tilde{a}}(n)\underline{\tilde{a}}^T(n)]\text{, } \textit{correlation matrix of nonlinear coefficient error}$$

Assuming the input signal $x(n)$ is continuous-valued White Gaussian noise (WGN) in the following Sections. For simplicity, nonlinear loudspeaker and linear echo path responses are assumed to be time invariant. (i.e., $\underline{a}(n)=\underline{a}$ and $\underline{h}(n)=\underline{h}$) and the near-end speech $t(n)=0$ is also assumed.

According to the NLMS algorithm and memoryless polynomial filter [5], [10] [see also Sec. 2.2], we write the nonlinear and linear coefficients recursive updating equation as follows:

Nonlinear coefficient updating equation:

$$\hat{\underline{a}}(n+1)=\hat{\underline{a}}(n)+\frac{\mu_a}{\left\|\hat{\underline{u}}(n)\right\|_2^2+\delta_u}\hat{\underline{u}}(n)e(n) \tag{3.1.1}$$

where $\quad e(n)=d(n)-\hat{\underline{a}}^T(n)\hat{\underline{u}}(n)$

Linear coefficient updating equation:

$$\hat{\underline{h}}(n+1)=\hat{\underline{h}}(n)+\frac{\mu_h}{\left\|\hat{\underline{s}}(n)\right\|_2^2}\hat{\underline{s}}(n)e(n) \tag{3.1.2}$$

where $\quad e(n)=d(n)-\hat{\underline{h}}^T(n)\hat{\underline{s}}(n)$

The echo signal $e(n)$ is the same in one iteration for both linear and nonlinear coefficient updating equations, it reads as:

$$\hat{\underline{h}}^T(n)\hat{\underline{s}}(n)=\hat{\underline{h}}^T(n)X(n)\hat{\underline{a}}(n)=\hat{\underline{a}}^T(n)\hat{\underline{u}}(n) \tag{3.1.3}$$

In next Section, we derive an equation to predict the variance of linear coefficients error first with the assumption: nonlinear coefficients are perfect, i.e., $\tilde{\underline{a}}(n)=\underline{0}$.

3.2 Linear coefficient error convergence analysis

In 3.2, we will derive the variance of linear coefficient error under the assumption of perfect nonlinear coefficients. First, express the linear and nonlinear coefficient error vector as

$$\tilde{\underline{h}}(n) = \underline{h} - \hat{\underline{h}}(n) \tag{3.2.1}$$

$$\tilde{\underline{a}}(n) = \underline{a} - \hat{\underline{a}}(n) \tag{3.2.2}$$

Rearranging Eq.(3.1.2), we can have:

$$\begin{aligned}
\tilde{\underline{h}}(n+1) &= \tilde{\underline{h}}(n) - \mu_h(n)\hat{\underline{s}}(n)\left[d(n) - \hat{\underline{h}}^T(n)\hat{\underline{s}}(n)\right] \\
&= \tilde{\underline{h}}(n) - \mu_h(n)\hat{\underline{s}}(n)\left[(\hat{\underline{a}}(n) + \tilde{\underline{a}}(n))^T X^T(n)\underline{h} + v(n) - \hat{\underline{s}}^T(n)\hat{\underline{h}}(n)\right] \\
&= \tilde{\underline{h}}(n) - \mu_h(n)\hat{\underline{s}}(n)\left[\hat{\underline{s}}^T(n)\tilde{\underline{h}}(n) + \underline{h}^T X(n)\tilde{\underline{a}}(n) + v(n)\right] \\
&= \left[I - \mu_h(n)\hat{\underline{s}}(n)\hat{\underline{s}}^T(n)\right]\tilde{\underline{h}}(n) - \mu_h(n)\hat{\underline{s}}(n)\underline{h}^T X(n)\tilde{\underline{a}}(n) - \mu_h(n)\hat{\underline{s}}(n)v(n) \qquad (3.2.3)
\end{aligned}$$

For simplicity, the nonlinear coefficient is assumed perfect. i.e., $\hat{\underline{a}}(n) = \underline{a}$ and $\hat{\underline{s}}(n) = X(n)\hat{\underline{a}}(n) = X(n)\underline{a} = \underline{s}(n)$. Eq.(3.2.3) can be rewritten as follows:

$$\tilde{\underline{h}}(n+1) = \left[I - \mu_h(n)\underline{s}(n)\underline{s}^T(n)\right]\tilde{\underline{h}}(n) - \mu_h(n)\underline{s}(n)v(n) \tag{3.2.4}$$

Secondly, we define the correlation matrix of linear coefficient error by $R_{\tilde{\underline{h}}}(n+1)$ then we have:

$$\begin{aligned}
R_{\tilde{\underline{h}}}(n+1) &= E\left[\tilde{\underline{h}}(n+1)\tilde{\underline{h}}^T(n+1)\right] \\
&= E\left[\tilde{\underline{h}}(n)\tilde{\underline{h}}^T(n)\right] - E\left[\tilde{\underline{h}}(n)\tilde{\underline{h}}^T(n)\mu_h(n)\underline{s}(n)\underline{s}^T(n)\right] - E\left[\mu_h(n)\underline{s}(n)\underline{s}^T(n)\tilde{\underline{h}}(n)\tilde{\underline{h}}^T(n)\right] \\
&\quad + E\left[\mu_h^2(n)\underline{s}(n)\underline{s}^T(n)\tilde{\underline{h}}(n)\tilde{\underline{h}}^T(n)\underline{s}(n)\underline{s}^T(n)\right] + E\left[\mu_h^2(n)v^2(n)\underline{s}(n)\underline{s}^T(n)\right] \qquad (3.2.5)
\end{aligned}$$

The cross-product terms of $\left[I - \mu_h(n)\underline{s}(n)\underline{s}^T(n)\right]\tilde{\underline{h}}(n)$ and $\mu_h(n)\underline{s}(n)v(n)$ disappear because of the independence and zero mean assumptions of the near-end noise $v(n)$.

It is reasonable to assume the linear coefficients error $\underline{\tilde{h}}(n)$ and nonlinear powered input vector $\underline{s}(n)$, the linear combination between different powered input signal $x(n)$ and nonlinear coefficients, are all independent. Eq.(3.2.5) can be rearranged as follows:

$$R_{\underline{\tilde{h}}}(n+1) = R_{\underline{\tilde{h}}}(n) - 2\mu_h(n)R_{\underline{s}}(n)R_{\underline{\tilde{h}}}(n) + \mu_h^2(n)E\left[\underline{s}(n)\underline{s}^T(n)\underline{\tilde{h}}(n)\underline{\tilde{h}}^T(n)\underline{s}(n)\underline{s}^T(n)\right] + \mu_h^2(n)\sigma_v^2 R_{\underline{s}}(n) \tag{3.2.6}$$

Since $\underline{\tilde{h}}(n)\underline{\tilde{h}}^T(n)$ is symmetric matrix and $\underline{s}(n)$ is zero mean, approximate WGN, the third terms in Eq.(3.2.6) can be further simplified. By the other way, its input data correlation matrix $R_{\underline{s}}(n)$ is different from the original one $R_{\underline{x}}(n)$. Appendix A. gives the detailed mathematical derivation of approximating $R_{\underline{s}}(n)$ by $\sigma_s^2 I_L$. Then we can obtain

$$R_{\underline{\tilde{h}}}(n+1) = R_{\underline{\tilde{h}}}(n) - 2\frac{\mu_h}{L}R_{\underline{\tilde{h}}}(n) + \frac{\mu_h^2}{L^2}\left[2R_{\underline{\tilde{h}}}(n) + tr\left(R_{\underline{\tilde{h}}}(n)\right)I\right] + \frac{\mu_h^2}{L^2\sigma_s^2}\sigma_v^2 I \tag{3.2.7}$$

Taking trace, Eq.(3.2.10) becomes

$$tr\left[R_{\underline{\tilde{h}}}(n+1)\right] = \left[1 - \frac{2\mu_h}{L} + (2+L)\frac{\mu_h^2}{L^2}\right]tr\left[R_{\underline{\tilde{h}}}(n)\right] + \frac{\mu_h^2\sigma_v^2}{L\sigma_s^2} \tag{3.2.8}$$

By recursion, Eq.(3.2.8) can be represented as follows:

$$T_{\tilde{h}}(n) = C_{\tilde{h}}^n T_{\tilde{h}}(0) + \frac{K_{\tilde{h}} \times (1 - C_{\tilde{h}}^n)}{1 - C_{\tilde{h}}} \approx \frac{K_{\tilde{h}}}{1 - C_{\tilde{h}}} \quad \text{when } n \to \infty \tag{3.2.9}$$

where
$$T_{\tilde{h}}(0) \triangleq \|\underline{h}\|_2^2$$
$$C_{\tilde{h}} \triangleq 1 - \frac{2\mu_h}{L} + (2+L)\frac{\mu_h^2}{L^2}$$
$$K_{\tilde{h}} = \frac{\mu_h^2\sigma_v^2}{L\sigma_s^2}$$

$C_{\tilde{h}}$ is the convergence rate of linear coefficient error variance. It can be determined with the knowledge of step size μ_h and linear adaptive filter length L. Convergence value is determined by $C_{\tilde{h}}$ and $K_{\tilde{h}}$. Besides μ_h and L, power of the local noise (σ_v^2) and the nonlinear input (σ_s^2) are necessary for finding $K_{\tilde{h}}$. According to Eq.(3.2.9), we can easily find the variance of linear coefficient error at the n'th iteration for nonlinear adaptive filter under assumption of perfectly known nonlinear coefficients.

3.3 Nonlinear coefficient error convergence analysis

The nonlinear coefficient error variance will be derived in this section. Similarly, the linear coefficients are assumed to be perfect. The derivation procedure is almost the same as section 3.2, but there are still some differences between linear and nonlinear derivations. Let's examine the nonlinear coefficient updated equation in Eq.(3.1.1).

In contrast to $\left\|\hat{\underline{s}}(n)\right\|_2^2$ in Sec.3.2, $\left\|\hat{\underline{u}}(n)\right\|_2^2$ becomes very small when a single input sample $x(n)$ is close to zero. To avoid a large adaptation step towards a random direction in this situation, a small constant δ_u (usually is 1) is added to the normalized term. Substitute estimated coefficient error vector Eq.(3.2.1) and Eq.(3.2.2) into Eq.(3.1.1) and rearrange it as follows:

$$\tilde{\underline{a}}(n+1)=\tilde{\underline{a}}(n)-\mu_a(n)\hat{\underline{u}}(n)\left[(\hat{\underline{h}}(n)+\tilde{\underline{h}}(n))^T X(n)\underline{a}+v(n)-\hat{\underline{u}}^T(n)\hat{\underline{a}}(n)\right]$$
$$=\tilde{\underline{a}}(n)-\mu_a(n)\hat{\underline{u}}(n)\left[\hat{\underline{u}}^T(n)\tilde{\underline{a}}(n)+\underline{a}^T X^T(n)\tilde{\underline{h}}(n)+v(n)\right]$$
$$=\left[I-\mu_a(n)\hat{\underline{u}}(n)\hat{\underline{u}}^T(n)\right]\tilde{\underline{a}}(n)-\mu_a(n)\hat{\underline{u}}(n)\underline{a}^T X^T(n)\tilde{\underline{h}}(n)-\mu_a(n)\hat{\underline{u}}(n)v(n) \quad (3.3.1)$$

Eq.(3.3.1) is an iterative representation of nonlinear coefficient error vector under the condition that linear coefficients are not perfect. Assume the linear coefficients are perfect first in this section. Because of this assumption, the third term in Eq.(3.3.1) can be eliminated and linear powered input vector $\underline{u}(n)$ is only produced by product of perfect linear coefficients $\underline{h}$ and powered input matrix $X(n)$. We have

$$\tilde{\underline{a}}(n+1)=\left[I-\mu_a(n)\underline{u}(n)\underline{u}^T(n)\right]\tilde{\underline{a}}(n)-\mu_a(n)\underline{u}(n)v(n) \quad (3.3.2)$$

The correlation matrix of nonlinear coefficient error at $n+1$ 'th iteration $R_{\tilde{\underline{a}}}(n+1)$ can be written as follows by substituting Eq.(3.3.2) into its definition.

$$R_{\tilde{\underline{a}}}(n+1)=E\left[\tilde{\underline{a}}(n+1)\tilde{\underline{a}}^T(n+1)\right]$$
$$=E\left[\tilde{\underline{a}}(n)\tilde{\underline{a}}^T(n)\right]-E\left[\tilde{\underline{a}}(n)\tilde{\underline{a}}^T(n)\mu_a(n)\underline{u}(n)\underline{u}^T(n)\right]-E\left[\mu_a(n)\underline{u}(n)\underline{u}^T(n)\tilde{\underline{a}}(n)\tilde{\underline{a}}^T(n)\right]$$
$$+E\left[\mu_a^2(n)\underline{u}(n)\underline{u}^T(n)\tilde{\underline{a}}(n)\tilde{\underline{a}}^T(n)\underline{u}(n)\underline{u}^T(n)\right]+E\left[\mu_a^2(n)v^2(n)\underline{u}(n)\underline{u}^T(n)\right] \quad (3.3.3)$$

Similarly, the cross-product terms of $\left[I-\mu_a(n)\underline{u}(n)\underline{u}^T(n)\right]\tilde{\underline{a}}(n)$ and $\mu_a(n)\underline{u}(n)v(n)$ disappear because of the independence and zero mean assumptions of the near-end noise $v(n)$ .Eq.(3.3.3) can be further simplified by assuming the independence of $\tilde{\underline{a}}(n)$ and $\underline{u}(n)$. It becomes

$$R_{\tilde{\underline{a}}}(n+1)=R_{\tilde{\underline{a}}}(n)-2\mu_a(n)R_{\underline{u}}(n)R_{\tilde{\underline{a}}}(n)+\mu_a^2(n)E\left[\underline{u}(n)\underline{u}^T(n)\tilde{\underline{a}}(n)\tilde{\underline{a}}^T(n)\underline{u}(n)\underline{u}^T(n)\right]$$
$$+\mu_a^2(n)\sigma_v^2R_{\underline{u}}(n) \quad (3.3.4)$$

The third term in Eq.(3.3.4) can not be simplified through the assumption applied in Sec. 3.2. Due to the different statistic characteristic between $\underline{s}(n)$ and $\underline{u}(n)$, we can not approximate $R_{\underline{u}}(n)$ by the product form between a constant value and identity matrix. Taking trace on both side of Eq.(3.3.4) and assuming the correlation matrix $R_{\underline{\tilde{a}}}(n)$ is diagonal, so that we have

$$tr\left[R_{\underline{\tilde{a}}}(n+1)\right] = \left(1 - 2\mu_a(n)\frac{tr\left[R_{\underline{u}}(n)\right]}{Q} + \mu_a^2(n)\frac{E\left\{tr\left[\underline{u}^T(n)\underline{u}(n)\underline{u}(n)\underline{u}^T(n)\right]\right\}}{Q}\right) tr\left[R_{\underline{\tilde{a}}}(n)\right] + \mu_a^2(n)\sigma_v^2 tr\left[R_{\underline{u}}(n)\right] \qquad (3.3.5)$$

Substitute the normalized step-size (see also Sec. 3.1) into Eq.(3.3.5), the equation reads now

$$tr\left[R_{\underline{\tilde{a}}}(n+1)\right] = \left(1 - \frac{E\left[\left\|\underline{u}(n)\right\|_2^2\right]}{E\left[\left\|\underline{u}(n)\right\|_2^2\right] + \delta_u}\frac{2\mu_a}{Q} + \frac{E\left[\left\|\underline{u}(n)\right\|_2^4\right]}{E\left[\left\|\underline{u}(n)\right\|_2^2\right]^2 + 2\delta_u E\left[\left\|\underline{u}(n)\right\|_2^2\right] + \delta_u^2}\frac{\mu_a^2}{Q}\right) tr\left[R_{\underline{\tilde{a}}}(n)\right] + \frac{\mu_a^2\sigma_v^2}{E\left[\left\|\underline{u}(n)\right\|_2^2\right]} \qquad (3.3.6)$$

δ_u is a small constant, if it satisfies the condition $\delta_u \ll E\left[\left\|\underline{u}(n)\right\|_2^2\right]$ and $E\left[\left\|\underline{u}(n)\right\|_2^2\right] > 1$. Eq.(3.3.6) can be approximated to be

$$tr\left[R_{\underline{\tilde{a}}}(n+1)\right] = \left(1 - \frac{2\mu_a}{Q} + \frac{E\left[\left\|\underline{u}(n)\right\|_2^4\right]}{E\left[\left\|\underline{u}(n)\right\|_2^2\right]^2}\frac{\mu_a^2}{Q}\right) tr\left[R_{\underline{\tilde{a}}}(n)\right] + \frac{\mu_a^2\sigma_v^2}{E\left[\left\|\underline{u}(n)\right\|_2^2\right]} \qquad (3.3.7)$$

where $E\left[\left\|\underline{u}(n)\right\|_2^2\right]$ and $E\left[\left\|\underline{u}(n)\right\|_2^4\right]$ are decided by different order moment value of $x(n)$ and linear coefficient vector $\underline{h}$. In order to keep the main object of this section, the detail mathematical procedure about them has been given in Appendix B. We define another two symbols P_{u2} and P_{u4} to represent $E\left[\left\|\underline{u}(n)\right\|_2^2\right]$ and $E\left[\left\|\underline{u}(n)\right\|_2^4\right]$ respectively. Finally, the variance of nonlinear coefficient error becomes

$$T_{\tilde{a}}(n) = C_{\tilde{a}}^{n} T_{\tilde{a}}(0) + \frac{K_{\tilde{a}} \times (1 - C_{\tilde{a}}^{n})}{1 - C_{\tilde{a}}} \approx \frac{K_{\tilde{a}}}{1 - C_{\tilde{a}}} \quad when\ n \to \infty \tag{3.3.8}$$

where

$$C_{\tilde{a}} = 1 - 2\frac{\mu_a}{Q} + \frac{\mu_a^2}{P_{u2}^2}\frac{P_{u4}}{Q}$$

$$K_{\tilde{a}} = \frac{\mu_a^2 \sigma_v^2}{P_{u2}}$$

$$P_{u2} \triangleq E\left[\left\|\underline{u}(n)\right\|_2^2\right] \quad and \quad P_{u4} \triangleq E\left[\left\|\underline{u}(n)\right\|_2^4\right]$$

In contrast to the variance of linear coefficient error in Sec. 3.2, the variance of nonlinear coefficient error is more complicated. There are two values more needed to know (P_{u2} and P_{u4}) first for getting the values of nonlinear coefficient error at n'th iteration. According to Eq.(3.3.8), we can get the variance of nonlinear coefficient error at the n'th iteration in nonlinear adaptive filter under assumption of perfect linear coefficients.

3.4 Joint coefficient error convergence analysis

In a real nonlinear AEC environment, the behaviors of linear and nonlinear adaptive filters affect each other. We assume the nonlinear and echo responses are time invariant for simplicity. The nonlinear coefficients are updated first then the linear coefficients follow according to last time parameters in one iteration, we call this updating strategy joint.

3.4.1 Imperfect linear coefficient error convergence analysis

Different from Section 3.2, we will derive the linear coefficient error variance by considering the effect of nonlinear coefficient estimation error in this section. Eq.(3.2.3) will be the linear coefficient updating equation by the relationship of

$\underset{\sim}{\tilde{h}}(n) = \underline{h} - \underline{\hat{h}}(n)$:

$$\underline{\tilde{h}}(n+1) = \left[I - \mu_h(n)\underline{\hat{s}}(n)\underline{\hat{s}}^T(n) \right]\underline{\tilde{h}}(n) - \mu_h(n)\underline{\hat{s}}(n)\underline{h}^T X(n)\underline{\tilde{a}}(n) - \mu_h(n)\underline{\hat{s}}(n)v(n) \qquad (3.4.1)$$

There are two different terms worthy to note. The third term in Eq.(3.4.1) is produced by nonlinear coefficient error $\underline{\tilde{a}}(n)$ which is not present in Section 3.2, and the nonlinear powered input $\underline{\hat{s}}(n)$ is also related to the nonlinear coefficient error, i.e.,

$$\underline{\hat{s}}(n) = X(n)\underline{\hat{a}}(n) = X(n)\left[\underline{a} - \underline{\tilde{a}}(n) \right] \qquad (3.4.2)$$

Divide the correlation matrix of linear coefficient error into two parts according to the nonlinear coefficient error effect is direct or not, it can be written as follows:

$$\begin{aligned}
R_{\underline{\tilde{h}}}(n+1) &= E\left\{ \underline{\tilde{h}}(n+1)\underline{\tilde{h}}^T(n+1) \right\} \\
&= E\ \Big\{ \Big[\left(I - \mu_h(n)\underline{\hat{s}}(n)\underline{\hat{s}}^T(n) \right)\underline{\tilde{h}}(n) - \mu_h(n)\underline{\hat{s}}(n)\underline{h}^T X(n)\underline{\tilde{a}}(n) - \mu_h(n)\underline{\hat{s}}(n)v(n) \Big] \\
&\qquad \times \Big[\underline{\tilde{h}}^T(n)\left(I - \mu_h(n)\underline{\hat{s}}(n)\underline{\hat{s}}^T(n) \right) - \mu_h(n)\underline{\tilde{a}}^T(n)X^T(n)\underline{h}\underline{\hat{s}}^T(n) - \mu_h(n)\underline{\hat{s}}^T(n)v(n) \Big] \Big\} \\
&= R_{\underline{\tilde{h}}}(n+1)\big|_{indirect\ \underline{\tilde{a}}(n)\ effect} + R_{\underline{\tilde{h}}}(n+1)\big|_{direct\ \underline{\tilde{a}}(n)\ effect}
\end{aligned} \qquad (3.4.3)$$

The first term in Eq.(3.4.3) is similar to the Eq.(3.2.7) but the nonlinear powered input $\underline{\hat{s}}(n)$ is iteration variant now, the nonlinear coefficients are also estimated imperfectly. It is equal to:

$$\begin{aligned}
&R_{\underline{\tilde{h}}}(n+1)\big|_{indirect\ \underline{\tilde{a}}(n)\ effect} \\
&= R_{\underline{\tilde{h}}}(n) - 2\frac{\mu_h}{L}R_{\underline{\tilde{h}}}(n) + \frac{\mu_h^2}{L^2}\left[2R_{\underline{\tilde{h}}}(n) + tr\left(R_{\underline{\tilde{h}}}(n) \right) I \right] + \frac{\mu_h^2}{L^2\sigma_{\hat{s}}^2(n)}\sigma_v^2 I
\end{aligned} \qquad (3.4.4)$$

where the estimated nonlinear powered input variance $\sigma_{\hat{s}}^2$ can be rewritten as: (see also Appendix A)

$$\sigma_{\hat{s}}^2(n) = a_1^2\overline{X^2} + \left(a_2^2 + 2a_1a_3 \right)\overline{X^4} + a_3^2\overline{X^6} + E\left[\tilde{a}_1^2(n) \right]\overline{X^2} + E\left[\tilde{a}_2^2(n) \right]\overline{X^4} + E\left[\tilde{a}_3^2(n) \right]\overline{X^6} \qquad (3.4.5)$$

It's decided by different moment orders of the powered input, as well as perfect nonlinear coefficients and nonlinear coefficient estimation error. We assume that different nonlinear order coefficient errors have the same error variance which is equal to the average power of nonlinear correlation matrix. Eq.(3.4.5) can be approximated as follows with $Q=3$:

$$\begin{aligned}\sigma_{\hat{s}}^2(n) &\approx a_1^2\overline{X^2}+\left(a_2^2+2a_1a_3\right)\overline{X^4}+a_3^2\overline{X^6}+\frac{tr\left[R_{\underline{\tilde{a}}}(n)\right]}{Q}\left(\overline{X^2}+\overline{X^4}+\overline{X^6}\right)\\ &=a_1^2\sigma_x^2+3\left(a_2^2+2a_1a_3\right)\sigma_x^4+15a_3^2\sigma_x^6+\frac{tr\left[R_{\underline{\tilde{a}}}(n)\right]}{Q}\left(\sigma_x^2+3\sigma_x^4+15\sigma_x^6\right)\end{aligned} \quad (3.4.6)$$

It's clear to know that the behavior of nonlinear coefficient error variance will be introduced in Eq.(3.4.6) to affect the estimation powered input variance ($\sigma_{\hat{s}}^2(n)$) in each iteration. Now, let's look at the second term in Eq.(3.4.3) where the nonlinear coefficient estimation error vector are affected directly by multiplication between the third term and others in Eq.(3.4.1), it reads as :

$$\begin{aligned}&R_{\underline{\tilde{h}}}(n+1)\big|_{direct\ \underline{\tilde{a}}(n)\ effect}=\\ &-\mu_h(n)\left\{E\left[\underline{\tilde{h}}(n)\underline{\tilde{a}}^T(n)X^T(n)\underline{h}\underline{\hat{s}}^T(n)\right]+E\left[\underline{\hat{s}}(n)\underline{h}^TX(n)\underline{\tilde{a}}(n)\underline{\tilde{h}}^T(n)\right]\right\}\\ &+\mu_h^2(n)\left\{E\left[\underline{\hat{s}}(n)\underline{\hat{s}}^T(n)\underline{\tilde{h}}(n)\underline{\tilde{a}}^T(n)X^T(n)\underline{h}\underline{\hat{s}}^T(n)\right]+E\left[\underline{\hat{s}}(n)\underline{h}^TX(n)\underline{\tilde{a}}(n)\underline{\tilde{h}}^T(n)\underline{\hat{s}}(n)\underline{\hat{s}}^T(n)\right]\right.\\ &+E\left[\underline{\hat{s}}(n)\underline{h}^TX(n)\underline{\tilde{a}}(n)\underline{\tilde{a}}^T(n)X^T(n)\underline{h}\underline{\hat{s}}^T(n)\right]+E\left[\underline{\hat{s}}(n)v(n)\underline{\tilde{a}}^T(n)X^T(n)\underline{h}\underline{\hat{s}}^T(n)\right]\\ &\left.+E\left[\underline{\hat{s}}(n)\underline{h}^TX(n)\underline{\tilde{a}}(n)\underline{\hat{s}}^T(n)v(n)\right]\right\}\end{aligned} \quad (3.4.7)$$

There remains only the fifth term in Eq.(3.4.7) according to the assumption : both expectation values of nonlinear and linear coefficient error are zero and local noise mean is zero. By eliminating the other terms, we have:

$$R_{\underline{\tilde{h}}}(n+1)\big|_{direct\ \underline{\tilde{a}}(n)\ effect}=\mu_h^2(n)E\left[\underline{\hat{s}}(n)\underline{h}^TX(n)\underline{\tilde{a}}(n)\underline{\tilde{a}}^T(n)X^T(n)\underline{h}\underline{\hat{s}}^T(n)\right] \quad (3.4.8)$$

It's straight forward that nonlinear coefficient errors need to be highlighted in this equation to represent its effect in the behavior of linear coefficient error variance. Substituting Eq.(3.4.2) and linear powered input into Eq.(3.4.7), we can have:

$$\mu_h^2(n)E\left[\hat{\underline{s}}(n)\underline{u}^T(u)\tilde{\underline{a}}(n)\tilde{\underline{a}}^T(n)\underline{u}(n)\hat{\underline{s}}^T(n)\right]$$
$$=\mu_h^2(n)\Big\{E\left[\underline{s}(n)\underline{u}^T(u)\tilde{\underline{a}}(n)\tilde{\underline{a}}^T(n)\underline{u}(n)\underline{s}^T(n)\right]+E\left[X(n)\tilde{\underline{a}}(n)\underline{u}^T(u)\tilde{\underline{a}}(n)\tilde{\underline{a}}^T(n)\underline{u}(n)\underline{s}^T(n)\right]$$
$$+E\left[\underline{s}(n)\underline{u}^T(n)\tilde{\underline{a}}(n)\tilde{\underline{a}}^T(n)\underline{u}(n)\tilde{\underline{a}}^T(n)X^T(n)\right]+E\left[X(n)\tilde{\underline{a}}(n)\underline{u}^T(u)\tilde{\underline{a}}(n)\tilde{\underline{a}}^T(n)\underline{u}(n)\tilde{\underline{a}}^T(n)X^T(n)\right]\Big\} \quad (3.4.9)$$

Because of $E\left[\tilde{\underline{a}}(n)\right]=\underline{0}$, we make an assumption that its third moment terms will close to zero, i.e., the second and third terms in Eq.(3.4.9) will be zero. Besides, the fourth term in Eq.(3.4.9) is also assumed too small to take account in Eq.(3.4.9). Eq (3.4.9) can be approximated as:

$$R_{\tilde{\underline{h}}}(n+1)\big|_{direct\ \tilde{\underline{a}}(n)\ effect}=\mu_h^2(n)E\left[\underline{s}(n)\underline{u}^T(u)\tilde{\underline{a}}(n)\tilde{\underline{a}}^T(n)\underline{u}(n)\underline{s}^T(n)\right] \quad (3.4.10)$$

Re-arrange Eq.(3.4.3) as follows:

$$R_{\tilde{\underline{h}}}(n+1)=R_{\tilde{\underline{h}}}(n+1)\big|_{indirect\ \tilde{\underline{a}}(n)\ effect}+R_{\tilde{\underline{h}}}(n+1)\big|_{direct\ \tilde{\underline{a}}(n)\ effect}$$
$$=R_{\tilde{\underline{h}}}(n)-2\frac{\mu_h}{L}R_{\tilde{\underline{h}}}(n)+\frac{\mu_h^2}{L^2}\left[2R_{\tilde{\underline{h}}}(n)+tr\left(R_{\tilde{\underline{h}}}(n)\right)I\right]+\frac{\mu_h^2}{L^2\sigma_{\hat{s}}^2(n)}\sigma_v^2 I$$
$$+\mu_h^2(n)E\left[\hat{\underline{s}}(n)\underline{h}^T X(n)\tilde{\underline{a}}(n)\tilde{\underline{a}}^T(n)X^T(n)\underline{h}\hat{\underline{s}}^T(n)\right] \quad (3.4.11)$$

Take trace over both sides of Eq.(3.4.11), it reads as:

$$tr\left[R_{\tilde{\underline{h}}}(n+1)\right]$$
$$=\left[1-2\frac{\mu_h}{L}+\frac{\mu_h^2}{L^2}(2+L)\right]tr\left[R_{\tilde{\underline{h}}}(n)\right]+\frac{\mu_h^2}{L^2\sigma_{\hat{s}}^4}\frac{E\left[\left\|\underline{s}(n)\right\|_2^2\left\|\underline{u}(n)\right\|_2^2\right]}{Q}tr\left[R_{\tilde{\underline{a}}}(n)\right]+\frac{\mu_h^2\sigma_v^2}{L\sigma_{\hat{s}}^2}$$
$$\approx\left[1-2\frac{\mu_h}{L}+\frac{\mu_h^2}{L^2}(2+L)\right]tr\left[R_{\tilde{\underline{h}}}(n)\right]+\frac{\mu_h^2}{L\sigma_{\hat{s}}^4}\frac{\left\|\underline{h}\right\|_2^2P_{nl}}{Q}tr\left[R_{\tilde{\underline{a}}}(n)\right]+\frac{\mu_h^2\sigma_v^2}{L\sigma_{\hat{s}}^2} \quad (3.4.12)$$

When nonlinear order $Q=3$

$$P_{nl}=a_1^2\overline{X^4}+\left(a_1^2+a_2^2+2a_1a_3\right)\overline{X^6}+\left(a_1^2+a_2^2+a_3^2+2a_1a_3\right)\overline{X^8}+\left(a_3^2+a_2^2+2a_1a_3\right)\overline{X^{10}}+a_3^2\overline{X^{12}}$$

The detail derivation of $E\left[\left\|\underline{s}(n)\right\|_2^2\left\|\underline{u}(n)\right\|_2^2\right]$ can be obtained from Appendix Eq.(A.9) and Eq.(B.7). According to Eq.(3.4.12), we can find that the behavior of linear coefficient error variance is determined not only by linear coefficient error variance but also by nonlinear coefficient error (i.e., $tr\left[R_{\tilde{\underline{a}}}(n)\right]$ and $\sigma_{\hat{s}}^2$). In order to know behaviors of both nonlinear and linear coefficient error variance in imperfect estimation, derivation on nonlinear coefficient error variance convergent behavior is also necessary.

3.4.2 Imperfect Nonlinear Coefficient Error Convergence Analysis

Similarly, we start from Eq.(3.3.1) with the consideration of linear coefficient estimation is imperfect. i.e., $\hat{\underline{h}}(n) = \underline{h} - \hat{\underline{h}}(n)$; it reads as:

$$\tilde{\underline{a}}(n+1) = \left[I - \mu_a(n)\hat{\underline{u}}(n)\hat{\underline{u}}^T(n) \right]\tilde{\underline{a}}(n) - \mu_a(n)\hat{\underline{u}}(n)\underline{a}^T X^T(n)\tilde{\underline{h}}(n) - \mu_a(n)\hat{\underline{u}}(n)v(n) \qquad (3.4.13)$$

Linear coefficient error effect is introduced by $\tilde{\underline{h}}(n)$ and $\hat{\underline{u}}(n)$ in Eq.(3.4.13). On account of convenient representation, the correlation matrix of nonlinear coefficient error is still divided into two parts: one part is similar to Eq.(3.3.3) but the linear powered input variance $\left\|\hat{\underline{u}}(n)\right\|_2^2$ is a random variable at each iteration, the other part is collection of the terms including linear coefficient error effect directly. Through that, the correlation matrix of nonlinear coefficient error can be represented as follows:

$$\begin{aligned}
R_{\tilde{\underline{a}}}(n+1) &= E\left\{ \tilde{\underline{a}}(n+1)\tilde{\underline{a}}^T(n+1) \right\} \\
&= E\Big\{ \left[\left(I - \mu_a(n)\hat{\underline{u}}(n)\hat{\underline{u}}^T(n) \right)\tilde{\underline{a}}(n) - \mu_a(n)\hat{\underline{u}}(n)\underline{a}^T X^T(n)\tilde{\underline{h}} - \mu_a(n)\hat{\underline{u}}(n)v(n) \right] \\
&\qquad \times \left[\tilde{\underline{a}}^T(n)\left(I - \mu_a(n)\hat{\underline{u}}(n)\hat{\underline{u}}^T(n) \right) - \mu_a(n)\tilde{\underline{h}}^T X(n)\underline{a}\hat{\underline{u}}^T(n) - \mu_a(n)\hat{\underline{u}}^T(n)v(n) \right] \Big\} \\
&= R_{\tilde{\underline{a}}}(n+1)\big|_{indirect\ \tilde{\underline{h}}(n)\ effect} + R_{\tilde{\underline{a}}}(n+1)\big|_{direct\ \tilde{\underline{h}}(n)\ effect}
\end{aligned} \qquad (3.4.14)$$

Here is the first term in Eq.(3.4.14):

$$\begin{aligned}
&R_{\tilde{\underline{a}}}(n+1)\big|_{indirect\ \tilde{\underline{h}}(n)\ effect} \\
&= R_{\tilde{\underline{a}}}(n) - 2\mu_a(n)R_{\hat{\underline{u}}}(n)R_{\tilde{\underline{a}}}(n) + \mu_a^2(n)E\left[\hat{\underline{u}}(n)\hat{\underline{u}}^T(n)\tilde{\underline{a}}(n)\tilde{\underline{a}}^T(n)\hat{\underline{u}}(n)\hat{\underline{u}}^T(n) \right] + \mu_a^2(n)\sigma_v^2 R_{\hat{\underline{u}}}(n)
\end{aligned} \qquad (3.4.15)$$

The correlation matrix of estimated linear powered input $\hat{\underline{u}}(n)$ has the linear coefficient error vector $\underline{\tilde{h}}(n)$ effect. It also needs to be updated at each iteration.

The second term in Eq.(3.4.14) is the multiplication between the third term and other terms in Eq.(3.4.13). It is represented by:

$$
\begin{aligned}
& R_{\underline{\tilde{a}}}(n+1)\big|_{direct\ \underline{\tilde{h}}(n)\ effect} = \\
& -\mu_a(n)\left\{E\left[\underline{\tilde{a}}(n)\underline{\tilde{h}}^T(n)X(n)\underline{a}\underline{\hat{u}}^T(n)\right]+E\left[\underline{\hat{u}}(n)\underline{a}^T X^T(n)\underline{\tilde{h}}(n)\underline{\tilde{a}}^T(n)\right]\right\} \\
& +\mu_a^2(n)\left\{E\left[\underline{\hat{u}}(n)\underline{\hat{u}}^T(n)\underline{\tilde{a}}(n)\underline{\tilde{h}}^T(n)X(n)\underline{a}\underline{\hat{u}}^T(n)\right]+E\left[\underline{\hat{u}}(n)\underline{a}^T X^T(n)\underline{\tilde{h}}(n)\underline{\tilde{a}}^T(n)\underline{\hat{u}}(n)\underline{\hat{u}}^T(n)\right]\right. \\
& \qquad +E\left[\underline{\hat{u}}(n)\underline{a}^T X^T(n)\underline{\tilde{h}}(n)\underline{\tilde{h}}^T(n)X(n)\underline{a}\underline{\hat{u}}^T(n)\right]+E\left[\underline{\hat{u}}(n)v(n)\underline{\tilde{h}}^T(n)X(n)\underline{a}\underline{\hat{u}}^T(n)\right] \\
& \qquad \left.+E\left[\underline{\hat{u}}(n)\underline{a}^T X^T(n)\underline{\tilde{h}}(n)\underline{\hat{u}}^T(n)v(n)\right]\right\}
\end{aligned}
\qquad (3.4.16)
$$

Eq.(3.4.16) can be rearranged as follows with the assumption of $E\left[\underline{\tilde{h}}(n)\right]=0$, $E\left[\underline{\tilde{a}}(n)\right]=0$ and local noise has zero mean.

$$
R_{\underline{\tilde{a}}}(n+1)\big|_{direct\ \underline{\tilde{h}}(n)\ effect} = \mu_a^2(n)E\left[\underline{\hat{u}}(n)\underline{a}^T X^T(n)\underline{\tilde{h}}(n)\underline{\tilde{h}}^T(n)X(n)\underline{a}\underline{\hat{u}}^T(n)\right] \qquad (3.4.17)
$$

Substitute definition of estimation linear powered input $\underline{\hat{u}}(n)$ into Eq.(3.4.17) and it extends to the following equation:

$$
\begin{aligned}
& \mu_a^2(n)E\left[\underline{\hat{u}}(n)\underline{a}^T X^T(n)\underline{\tilde{h}}\underline{\tilde{h}}^T X(n)\underline{a}\underline{\hat{u}}^T(n)\right] \\
& =\mu_a^2(n)\left\{E\left[\underline{u}(n)\underline{a}^T X^T(n)\underline{\tilde{h}}(n)\underline{\tilde{h}}^T(n)X(n)\underline{a}\underline{u}^T(n)\right]+E\left[X^T(n)\underline{\tilde{h}}(n)\underline{s}^T(u)\underline{\tilde{h}}(n)\underline{\tilde{h}}^T(n)\underline{s}(n)\underline{u}^T(n)\right]\right. \\
& \quad \left.+E\left[\underline{u}(n)\underline{s}^T(n)\underline{\tilde{h}}(n)\underline{\tilde{h}}^T(n)\underline{s}(n)\underline{\tilde{h}}^T(n)X(n)\right]+E\left[X^T(n)\underline{\tilde{h}}(n)\underline{s}^T(u)\underline{\tilde{h}}(n)\underline{\tilde{h}}^T(n)\underline{s}(n)\underline{\tilde{h}}^T(n)X(n)\right]\right\}
\end{aligned}
\qquad (3.4.18)
$$

Due to the estimated linear coefficient error is zero mean, the second and third terms of Eq.(3.4.18) will close to zero. And the fourth term is so small compared to first term that it can be isolated. Eq.(3.4.18) can be simplified to an approximate result.

$$
R_{\underline{\tilde{a}}}(n+1)\big|_{direct\ \underline{\tilde{h}}(n)\ effect} \approx \mu_a^2(n)E\left[\underline{u}(n)\underline{a}^T X^T(n)\underline{\tilde{h}}(n)\underline{\tilde{h}}^T(n)X(n)\underline{a}\underline{u}^T(n)\right] \qquad (3.4.19)
$$

Combining Eq.(3.4.15) and Eq.(3.4.19), Eq.(3.4.14) will be rewritten as:

$$
\begin{aligned}
&R_{\underline{\tilde{a}}}(n+1) \\
&= R_{\underline{\tilde{a}}}(n+1)\Big|_{indirect\ \underline{\tilde{h}}(n)\ effect} + R_{\underline{\tilde{a}}}(n+1)\Big|_{direct\ \underline{\tilde{h}}(n)\ effect} \\
&= R_{\underline{\tilde{a}}}(n) - 2\mu_a(n) R_{\underline{\hat{u}}}(n) R_{\underline{\tilde{a}}}(n) + \mu_a^2(n) E\left[\underline{\hat{u}}(n)\underline{\hat{u}}^T(n)\underline{\tilde{a}}(n)\underline{\tilde{a}}^T(n)\underline{\hat{u}}(n)\underline{\hat{u}}^T(n) \right] + \mu_a^2(n)\sigma_v^2 R_{\underline{\hat{u}}}(n) \\
&\quad + \mu_a^2(n) E\left[\underline{u}(n)\underline{a}^T X^T(n)\underline{\tilde{h}}(n)\underline{\tilde{h}}^T(n) X(n)\underline{a}\underline{u}^T(n) \right] \qquad (3.4.20)
\end{aligned}
$$

Applying trace operation in Eq.(3.4.20), it becomes

$$
\begin{aligned}
&tr\left[R_{\underline{\tilde{a}}}(n+1)\right] \\
&\approx \left[1 - 2\frac{\mu_a}{Q} + \frac{\mu_a^2}{Q}\frac{E\left\{\left\|\underline{\hat{u}}(n)\right\|_2^4\right\}}{E\left\{\left\|\underline{\hat{u}}(n)\right\|_2^2\right\}^2} \right] tr\left[R_{\underline{\tilde{a}}}(n)\right] + \frac{\mu_a^2\sigma_v^2}{E\left\{\left\|\underline{\hat{u}}(n)\right\|_2^2\right\}} + \frac{tr\left[R_{\underline{\tilde{h}}}(n)\right] tr\left\{E\left[\left\|\underline{u}(n)\right\|_2^2 \underline{s}(n)\underline{s}^T(n)\right]\right\}}{L} \\
&= \left[1 - 2\frac{\mu_a}{Q} + \frac{\mu_a^2}{Q}\frac{E\left\{\left\|\underline{\hat{u}}(n)\right\|_2^4\right\}}{E\left\{\left\|\underline{\hat{u}}(n)\right\|_2^2\right\}^2} \right] tr\left[R_{\underline{\tilde{a}}}(n)\right] + \frac{\mu_a^2\sigma_v^2}{E\left\{\left\|\underline{\hat{u}}(n)\right\|_2^2\right\}} + \left\|\underline{h}\right\|_2^2 P_{nl}\, tr\left[R_{\underline{\tilde{h}}}(n)\right] \qquad (3.4.21)
\end{aligned}
$$

When nonlinear order $Q = 3$

$$
P_{nl} = a_1^2\overline{X^4} + \left(a_1^2 + a_2^2 + 2a_1a_3\right)\overline{X^6} + \left(a_1^2 + a_2^2 + a_3^2 + 2a_1a_3\right)\overline{X^8} + \left(a_3^2 + a_2^2 + 2a_1a_3\right)\overline{X^{10}} + a_3^2\overline{X^{12}}
$$

It is similar to the behavior of linear coefficient error variance in Eq.(3.4.12), the current nonlinear coefficient error variance will depend on both linear and nonlinear coefficient error variance of last iteration. By the other way, the iteration variant linear powered input $\underline{\hat{u}}(n)$ also plays an important role in the nonlinear coefficient error behavior equation.

3.5 Modified adaptation strategies

We hope to give some helpful information about nonlinear and linear coefficient adaptation strategies in nonlinear AEC. Because there are two filters, nonlinear polynomial filter and FIR filter, work together in the nonlinear AEC system, the coefficient error variance behavior will affect each other with the "Joint" adaptation strategy, like the approximated derivations showed in Section 3.4. We start from the original "Joint" adaptive update and try to find better adaptation method for nonlinear AEC. Besides "Joint" adaptation, there are four more updating strategies which are described as follows:

1. Joint: The straight forward adaptive method achieved by updating nonlinear and linear coefficients respectively in one iteration.
2. Block Interleave: The linear coefficients are updated in first L iterations (128) (block), then updating the nonlinear coefficients in next L iterations and so on recursively.
3. Select and Go: Joint adaptation is applied in first L iterations, selecting the nonlinear coefficients by the L'th iteration and keep it, only keep updating linear coefficients in following iterations.
4. Select and Go, Joint: The first two phases are the same as Select and Go strategy. Switching to "Joint" strategy for phase III when iteration numbers larger than iteration threshold number (i.e., 5000).
5. Go, Go, Joint: Only updating linear coefficients in phase I, once the iteration number has been satisfied with 1250, switching to phase II, updating nonlinear coefficients only. Do not turn into "Joint" mode (phase III) until nonlinear coefficients are adapted 50 times recursively.

We make comparisons between five strategies with respect to echo power

Converged rate and final converged value. Comparison results are briefly organized in Table 5.1. Simulation results are provided in Section 5.4.

These adaptation strategy simulations indicate that:

- In transient phase, linear coefficients dominate the echo power Converged rate. Therefore, $\hat{\underline{h}}(n)$ should have to converge to some degree first, before the nonlinear coefficient adaptation is enabled.
- Nonlinear coefficient adaptation is an important part for achieving the reasonable small echo power converged value in steady state phase, i.e., they should be updated in steady state phase.
- No matter what strategies are applied in beginning, the reasonable echo power converged value can be approached by applying joint adaptation but it should not be too late.

The simulations results showed that the "Go, Go, Joint" adaptation strategy has the fastest converged rate and smallest converged value over five adaptation strategies. Through that, linear coefficient adaptation needs to be concerned first and nonlinear coefficient adaptation is introduced when the condition is met: the linear coefficients stay around "the almost right place". By the other way, if the nonlinear AEC environment is time-variant, the adaptive procedure needs to repeat again once the echo paths change.

Chapter 4

Stereophonic Acoustic Echo Cancellation

The fundamental problem of stereophonic acoustic echo cancellation (AEC) is the nonuniqueness problem because of remote (far-end) transmission room's multiple high correlated impulse responses [11]. Most methods use the decorrelators to solve the nonuniqueness problem by applying some kinds of nonlinear transformation on far end microphone input signals [12]. Morgan's study [13] also shows that the lower coherence of input signals, the faster convergence and the lower the coefficient misalignment. But it needs some cost on system structure modification and computation complexity for employing nonlinearities in stereophonic AEC.

More recently, using partial update schemes to reduce the coherence in stereophonic AEC is an alternative for solving nonuniqueness problem [14] [15]. In [14], the MMax-NLMS algorithm has been proposed; it employed an efficient technique to determine a tap selection set that gives an approximate joint optimization of maximum absolute sum of the sub-sampled tap-input vectors and minimum inter-channel coherence. By the other way, Hirano proposes a stereophonic AEC without pre-processing; it is achieved by dividing the filter coefficients into two portions (i.e., front and back) and update one part at a time [15].

In mono AEC, Doğançay has proposed a selective partial updates structure to reduce the computation complexity of an adaptive filter by adapting a block of the filter coefficients rather than the entire filter at one iteration [16]. Hirano proposes a stereophonic AEC partial update scheme but he does not give a detail analysis on coefficient error convergence behavior. We are trying to analyze the coefficient error convergence of the partial update stereophonic AEC structure according to Doğançay's mono AEC partial update analysis works.

We will introduce the partial update method of stereophonic AEC in Section 4.1, including notations and recursively updated equations. In Section 4.2, we derive the general iteration formula for the coefficient error variance. Finally, we try to combine the nonlinear loudspeaker with stereophonic AEC to reduce the coherence of input signals and to achieve a better coefficient misalignment result.

4.1 Partial update of stereophonic AEC [14][15]

The general stereophonic AEC scheme had been introduced in Section 2.3. We only define the configurations used in following derivation and introduce the partial update coefficient adaptation equation in this Section.

In Figure 2.3, assuming the talker's signal $s(n)$ is continuous-valued White Gaussian Noise (WGN) with variance σ_s^2 in the following Sections. It is different from the previous configurations in Section 2.3, the far end impulse responses and near end echo path are all assumed to be time invariant. (i.e., $\underline{g}_1(n)=\underline{g}_1$, $\underline{g}_2(n)=\underline{g}_2$ and $\underline{h}_1(n)=\underline{h}_1$, $\underline{h}_2(n)=\underline{h}_2$) for simplicity.

The following notations are necessary for the derivations:

M	$\triangleq$ *number of near end impulse response* $\underline{g}_i$ *coefficients*
N	$\triangleq$ *number of far end impulse response* $\underline{h}_i$ *coefficients*
L	$\triangleq$ *number of adaptive filter* $\underline{\hat{h}}_i(n)$ *coefficients*
B	$\triangleq$ *partial update block number, when divided* int*o front and back*, $B=2$
σ_v^2	$= E[v(n)^2]$, *noise power*
$\underline{g}_i$	$=[g_{i,0},g_{i,1},\cdots,g_{i,M-1}]^T$, $i=1$ *or* 2 ; *far end impulse response*
$\underline{h}_i$	$=[g_{i,0},g_{i,1},\cdots,g_{i,N-1}]^T$, $i=1$ *or* 2 ; *near end impulse response*
$\underline{\tilde{h}}(n)$	$= \underline{h}-\underline{\hat{h}}(n)=[\tilde{h}_0(n),\tilde{h}_1(n),\cdots,\tilde{h}_{L-1}(n)]^T$, *linear coefficient error vector*

$$G_i = \begin{bmatrix} \underline{g}_i^T & 0 & \cdots & \cdots & 0 \\ 0 & \underline{g}_i^T & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \underline{g}_i^T & 0 \\ 0 & \cdots & \cdots & 0 & \underline{g}_i^T \end{bmatrix}_{L\times(M+L-1)} , \text{ far end channel matrix , } i=1 \text{ or } 2$$

$\underline{x}_{1,front}(n)$	$= [x_1(n),x_1(n-1),\cdots,x_1(n-\frac{L}{2}+1)]^T$, *channel* 1 *half front block input*
$\underline{x}_{1,back}(n)$	$= [x_1(n-\frac{L}{2}),\cdots,x_1(n-L+1)]^T$, *channel* 1 *half back block input*
$\mu_1(n)$	$= \dfrac{\mu}{\Vert \underline{x}_{1,i}(n)\Vert_2^2}$, *normalized stepsize of coefficient*, $i=$ *front or back*
$R_{\underline{s}}(n)$	$= \sigma_s^2\cdot I_{M+L-1}$, *tal*ker'*s signal autocorrelation matrix*
$R_{\underline{\tilde{h}}_1}(n)$	$= E[\underline{\tilde{h}}_1(n)\underline{\tilde{h}}_1^T(n)]$, *correlation matrix of coefficient error*

For convenience, only echo cancellation for the one microphone signal will be discussed here. Similar results will apply to the other microphone signal. Introducing the $L \times L$ diagonal matrix A_i [16] to select the half front or half back input block for partial (half) coefficient adaptation [15], we write the partial coefficient updating equation as follows by NLMS algorithm:

Partial (half) coefficient updating equation is defined by:

$$\underline{\hat{h}}_1(n+1) = \underline{\hat{h}}_1(n) + \mu(n) A_i \underline{x}_1(n) e(n) \tag{4.1.1}$$

where $$e(n) = d(n) - \hat{y}(n) \tag{4.1.2}$$

$$A_i = \begin{cases} diag(1, \cdots, 1, 0, \cdots, 0) & i = front, \ \frac{L}{2}\text{'s } 1 \ in \ half \ front \ part \\ diag(0, \cdots, 0, 1, \cdots, 1) & i = back, \ \frac{L}{2}\text{'s } 1 \ in \ half \ back \ part \end{cases} \tag{4.1.3}$$

$$\hat{y}(n) = \underline{\hat{h}}_1^T(n) \underline{x}_1(n) + \underline{\hat{h}}_2^T(n) \underline{x}_2(n) \tag{4.1.4}$$

$$d(n) = \underline{h}_1^T \underline{x}_1(n) + \underline{h}_2^T \underline{x}_2(n) + v(n) \tag{4.1.5}$$

In next section, we try to derive an approximated recursive equation for the variance of coefficient error in the partial update stereophonic AEC scheme.

4.2 Partial update coefficient error convergence analysis

Now, we will derivate the variance convergence behavior of coefficient error in partial update stereophonic AEC. First, express the coefficient error vector as

$$\underline{\tilde{h}}_1(n) = \underline{h}_1 - \underline{\hat{h}}_1(n) \tag{4.2.1}$$

Rearranging Eq.(4.1.1), we can have:

$$\begin{aligned} \underline{\tilde{h}}_1(n+1) = & \left[I - \mu(n) A_i \underline{x}_1(n) \underline{x}_1^T(n) \right] \underline{\tilde{h}}_1(n) - \mu(n) A_i \underline{x}_1(n) \underline{x}_2^T(n) \underline{\tilde{h}}_2(n) \\ & - \mu(n) v(n) A_i \underline{x}_1(n) \end{aligned} \tag{4.2.2}$$

According to the definition of correlation matrix and assumption of error vector and source signal $\underline{s}(n)$ are independent, the coefficient error correlation matrix reads as:

$$
\begin{aligned}
&R_{\underline{\tilde{h}}_1}(n+1)\\
&= R_{\underline{\tilde{h}}_1}(n)-\mu_1(n)A_1R_{\underline{\tilde{h}}_1}(n)G_1R_{\underline{s}}(n)G_1^T-\mu(n)A_1G_1R_{\underline{s}}(n)G_1^TR_{\underline{\tilde{h}}_1}(n)\\
&\quad -\mu_1^2(n)A_1^2G_1E\left[\underline{s}(n)\underline{s}^T(n)G_1^T\underline{\tilde{h}}_1(n)\underline{\tilde{h}}_1^T\,G_1(n)\underline{s}(n)\underline{s}^T(n)\right]G_1^T\\
&\quad -\mu_1(n)A_1R_{\underline{\tilde{h}}_1\underline{\tilde{h}}_2}(n)G_2R_{\underline{s}}(n)G_1^T+\mu_1^2(n)A_1^2G_1E\left[\underline{s}(n)\underline{s}^T(n)G_1^T\underline{\tilde{h}}_1(n)\underline{\tilde{h}}_2^T(n)G_2\underline{s}(n)\underline{s}^T(n)\right]G_1^T\\
&\quad -\mu_1(n)A_1G_1R_{\underline{s}}(n)G_2^TR_{\underline{\tilde{h}}_2\underline{\tilde{h}}_1}(n)+\mu_1^2(n)A_1^2G_1E\left[\underline{s}(n)\underline{s}^T(n)G_2^T\underline{\tilde{h}}_2(n)\underline{\tilde{h}}_1^T(n)G_1\underline{s}(n)\underline{s}^T(n)\right]G_1^T\\
&\quad +\mu_1^2(n)A_1^2G_1E\left[\underline{s}(n)\underline{s}^T(n)G_2^T\underline{\tilde{h}}_2(n)\underline{\tilde{h}}_2^T(n)G_2\underline{s}(n)\underline{s}^T(n)\right]G_1^T+\mu_1^2(n)\sigma_v^2A_1^2G_1R_{\underline{s}}(n)G_1^T
\end{aligned}
\tag{4.2.3}
$$

where $\underline{x}_1(n)=G_1\cdot\underline{s}(n)$ and $\underline{x}_2(n)=G_2\cdot\underline{s}(n)$

The effects of the far end impulse response are introduced by matrix G_i; it is defined in Section 4.1, which contains $\underline{g}_i$ and performs convolution between $\underline{s}(n)$ and $\underline{g}_i$. Eq.(4.2.3) can be further simplified by defining $\underline{\tilde{f}}_1(n)=G_1^T\underline{\tilde{h}}_1(n)$, $\underline{\tilde{f}}_2(n)=G_2^T\underline{\tilde{h}}_2(n)$ and using Gaussian assumption [see also Appendix C]. It becomes:

$$
\begin{aligned}
&R_{\underline{\tilde{h}}_1}(n+1)\\
&=R_{\underline{\tilde{h}}_1}(n)-\mu_1(n)\sigma_s^2A_1\left[R_{\underline{\tilde{h}}_1}(n)G_1G_1^T+G_1G_1^TR_{\underline{\tilde{h}}_1}(n)+R_{\underline{\tilde{h}}_1\underline{\tilde{h}}_2}(n)G_2G_1^T+G_1G_2^TR_{\underline{\tilde{h}}_2\underline{\tilde{h}}_1}(n)\right]\\
&\quad+2\mu_1^2(n)\sigma_s^4A_1G_1\left[R_{\underline{\tilde{f}}_1}(n)+R_{\underline{\tilde{f}}_1\underline{\tilde{f}}_2}(n)+R_{\underline{\tilde{f}}_2\underline{\tilde{f}}_1}(n)+R_{\underline{\tilde{f}}_2}(n)\right]G_1^T\\
&\quad+\mu_1^2(n)\sigma_s^4\left[tr\left(R_{\underline{\tilde{f}}_1}(n)\right)+tr\left(R_{\underline{\tilde{f}}_1\underline{\tilde{f}}_2}(n)\right)+tr\left(R_{\underline{\tilde{f}}_2\underline{\tilde{f}}_1}(n)\right)+tr\left(R_{\underline{\tilde{f}}_2}(n)\right)\right]A_1G_1G_1^T\\
&\quad+\mu_1^2(n)\sigma_v^2\sigma_s^2A_1G_1G_1^T
\end{aligned}
\tag{4.2.4}
$$

Define $K_{ij}=G_iG_j^T\ \ i,j=1\ or\ 2$, Rewrite Eq.(4.2.4) as follows:

$$
\begin{aligned}
&R_{\underline{\tilde{h}}_1}(n+1)\\
&=R_{\underline{\tilde{h}}_1}(n)-\mu_1(n)\sigma_s^2A_1\left[R_{\underline{\tilde{h}}_1}(n)K_{11}+K_{11}R_{\underline{\tilde{h}}_1}(n)+R_{\underline{\tilde{h}}_1\underline{\tilde{h}}_2}(n)K_{21}+K_{12}R_{\underline{\tilde{h}}_2\underline{\tilde{h}}_1}(n)\right]\\
&\quad+2\mu_1^2(n)\sigma_s^4A_1\left[K_{11}R_{\underline{\tilde{h}}_1}(n)K_{11}+K_{11}R_{\underline{\tilde{h}}_1\underline{\tilde{h}}_2}(n)K_{21}+K_{12}R_{\underline{\tilde{h}}_2\underline{\tilde{h}}_1}(n)K_{11}+K_{12}R_{\underline{\tilde{h}}_2}(n)K_{21}\right]\\
&\quad+\mu_1^2(n)\sigma_s^4\left[tr\left(K_{11}R_{\underline{\tilde{h}}_1}(n)\right)+tr\left(K_{21}R_{\underline{\tilde{h}}_1\underline{\tilde{h}}_2}(n)\right)+tr\left(K_{12}R_{\underline{\tilde{h}}_2\underline{\tilde{h}}_1}(n)\right)+tr\left(K_{22}R_{\underline{\tilde{h}}_2}(n)\right)\right]A_1K_{11}\\
&\quad+\mu_1^2(n)\sigma_s^2\sigma_v^2A_1K_{11}
\end{aligned}
\tag{4.2.5}
$$

Eq.(4.2.5) showed that coefficient error variance of adaptive filter $\underline{\hat{h}}_1(n)$ is determined by the interaction of four major terms: its previous coefficient error variance $R_{\underline{\tilde{h}}_1}(n)$, the other channel coefficient error variance $R_{\underline{\tilde{h}}_2}(n)$, cross-correlation of two channel coefficient error $R_{\underline{\tilde{h}}_1\underline{\tilde{h}}_2}(n)$ and the far end impulse response effect K_{ij}.

For simplicity, we make another assumption that $\underline{\tilde{h}}_1(n)$ and $\underline{\tilde{h}}_2(n)$ are uncorrelated, i.e., $R_{\underline{\tilde{h}}_1\underline{\tilde{h}}_2}(n)=0$. Eq.(4.2.5) can be further simplified as follows:

$$\begin{aligned}
&R_{\underline{\tilde{h}}_1}(n+1)\\
&=R_{\underline{\tilde{h}}_1}(n)-\mu_1(n)\sigma_s^2 A_i\left[R_{\underline{\tilde{h}}_1}(n)K_{11}+K_{11}R_{\underline{\tilde{h}}_1}(n)\right]+2\sigma_s^4\mu_1^2(n)A_i\left[K_{11}R_{\underline{\tilde{h}}_1}(n)K_{11}+K_{12}R_{\underline{\tilde{h}}_2}(n)K_{21}\right]\\
&\quad+\mu^2(n)\sigma_s^4\left[tr\left(K_{11}R_{\underline{\tilde{h}}_1}(n)\right)+tr\left(K_{22}R_{\underline{\tilde{h}}_2}(n)\right)\right]A_iK_{11}+\mu_1^2(n)\sigma_v^2\sigma_s^2A_iK_{11} \qquad (4.2.6)
\end{aligned}$$

Applying the trace operation on both sides and $tr(A_iK_{11})=\frac{L}{B}\left\|\underline{g}_1\right\|_2^2$, B is the partial update block number as mentioned in Section 4.2.1. It is given by

$$\begin{aligned}
&tr\left(R_{\underline{\tilde{h}}_1}(n+1)\right)\\
&=tr\left(R_{\underline{\tilde{h}}_1}(n)\right)-\mu_1(n)\sigma_s^2\left[tr\left(K_{11}A_iR_{\underline{\tilde{h}}_1}(n)\right)+tr\left(A_iK_{11}R_{\underline{\tilde{h}}_1}(n)\right)\right]+2\sigma_s^4\mu_1^2(n)\left[tr\left(K_{11}A_iK_{11}R_{\underline{\tilde{h}}_1}(n)\right)+tr\left(K_{21}A_iK_{12}R_{\underline{\tilde{h}}_2}(n)\right)\right]\\
&\quad+\sigma_s^4\mu_1^2(n)\frac{L}{B}\left\|\underline{g}_1\right\|_2^2\left[tr\left(K_{11}R_{\underline{\tilde{h}}_1}(n)\right)+tr\left(K_{22}R_{\underline{\tilde{h}}_2}(n)\right)\right]+\sigma_v^2\sigma_s^2\mu_1^2(n)\frac{L}{B}\left\|\underline{g}_1\right\|_2^2 \qquad (4.2.7)
\end{aligned}$$

Assume that the diagonal elements of $K_{11}R_{\underline{\tilde{h}}_1}(n)$, $K_{11}R_{\underline{\tilde{h}}_1}(n)K_{11}$ and $K_{12}R_{\underline{\tilde{h}}_2}(n)K_{21}$ are uniform distributed, due to the fact that A_i's effect can be replaced by a simple ratio $\frac{1}{B}$ in 2^{nd}, 3^{rd}, 4^{th} and 5^{th} terms of Eq.(4.2.7). To further simplify K_{ij}, denote the SVD decomposition of the far end channel matrix G_i as:

$$G_i=Q_i\Sigma_iV_i^T \quad , i=1\ or\ 2 \qquad (4.2.8)$$

then

$$K_{ii} = G_i G_i^T = Q_i D_i Q_i^T \quad i = 1 \ or \ 2 \tag{4.2.9}$$

where

$$Q_i : \ orthogonal \ similarity \ matrix \ , \ Q_i Q_i^T = I_{L\times L}$$

$$D_i = diag\{\lambda_{i,1}, \lambda_{i,2}, ..., \lambda_{i,L}\}, eigen \ values \ of \ K_{ii}$$

Eq.(4.2.7) can be further represented as follows:

$$\begin{aligned}
& tr\left[R_{\underline{d}_1}(n+1)\right] \\
&= tr\left[R_{\underline{d}_1}(n)\right] - \frac{2\mu_1(n)\sigma_s^2}{B} tr\left[D_1 R_{\underline{d}_1}(n)\right] + \frac{2\sigma_s^4 \mu_1^2(n)}{B}\left\{ tr\left[D_1^2 R_{\underline{d}_1}(n)\right] + tr\left[K_{21}K_{12}R_{\underline{\tilde{h}}_2}(n)\right]\right\} \\
&\quad + \sigma_s^4 \mu_1^2(n)\frac{L}{B}\left\|\underline{g}_1\right\|^2 \left\{ tr\left[D_1 R_{\underline{d}_1}(n)\right] + tr\left[K_2 R_{\underline{\tilde{h}}_2}(n)\right]\right\} + \sigma_v^2\sigma_s^2\mu_1^2(n)\frac{L}{B}\left\|\underline{g}_1\right\|_2^2
\end{aligned} \tag{4.2.10}$$

where $\underline{d}_1(n) = Q_1^T \underline{\tilde{h}}_1(n)$ and $R_{\underline{d}_1}(n) = Q_1^T R_{\underline{\tilde{h}}_1}(n) Q_1$

Substituting $\mu_1(n) = \dfrac{B\mu}{L\sigma_s^2\left\|\underline{g}_1\right\|_2^2}$ into Eq.(4.2.10) and re-organizing it, we have:

$$\begin{aligned}
& tr\left[R_{\underline{d}_1}(n+1)\right] \\
&= tr\left[R_{\underline{d}_1}(n)\right] + \frac{B\mu^2 - 2\mu}{L\left\|\underline{g}_1\right\|_2^2} tr\left[D_1 R_{\underline{d}_1}(n)\right] + \frac{2B\mu^2}{L^2\left\|\underline{g}_1\right\|_2^4} tr\left[D_1^2 R_{\underline{d}_1}(n)\right] + \frac{2B\mu^2}{L^2\left\|\underline{g}_1\right\|_2^4} tr\left[K_{12}R_{\underline{\tilde{h}}_2}(n)K_{21}\right] \\
&\quad + \frac{B\mu^2}{L\left\|\underline{g}_1\right\|_2^2} tr\left[K_2 R_{\underline{\tilde{h}}_2}(n)\right] + \frac{B\mu^2}{L\left\|\underline{g}_1\right\|_2^2}\frac{\sigma_v^2}{\sigma_s^2}
\end{aligned} \tag{4.2.11}$$

Substituting Eq.(4.2.8) in to $K_{ij} = G_i G_j^T \ i, j = 1 \ or \ 2$ and $i \neq j$, the 4th term in Eq.(4.2.11) can be represented in $R_{\underline{d}_2}(n)$ as follows:

$$\begin{aligned}
tr\left(K_{12}R_{\underline{\tilde{h}}_2}(n)K_{21}\right) &= tr\left(G_1^T G_1 G_2^T R_{\underline{\tilde{h}}_2}(n) G_2\right) \\
&= tr\left(V_1 D_1 V_1^T V_2 \textstyle\sum_2^T R_{\underline{d}_2}(n) \sum_2 V_2^T\right) \\
&= tr\left(\textstyle\sum_2 V_2^T V_1 D_1 V_1^T V_2 \sum_2^T R_{\underline{d}_2}(n)\right)
\end{aligned} \tag{4.2.12}$$

and $tr\left[K_2 R_{\underline{\tilde{h}}_2}(n)\right] = tr\left[D_2 R_{\underline{d}_2}(n)\right]$, the final iterative formula for the variance of the first channel coefficient error reads as:

$$tr\left[R_{\underline{d}_1}(n+1)\right]$$
$$=tr\left[R_{\underline{d}_1}(n)\right]+\frac{B\mu^2-2\mu}{L\left\|\underline{g}_1\right\|_2^2}tr\left[D_1R_{\underline{d}_1}(n)\right]+\frac{2B\mu^2}{L^2\left\|\underline{g}_1\right\|_2^4}tr\left[D_1^2R_{\underline{d}_1}(n)\right]$$
$$+\frac{2B\mu^2}{L^2\left\|\underline{g}_1\right\|_2^4}tr\left[\Sigma_2V_2^TV_1D_1V_1^TV_2\Sigma_2^TR_{\underline{d}_2}(n)\right]+\frac{B\mu^2}{L\left\|\underline{g}_1\right\|_2^2}tr\left[D_2R_{\underline{d}_2}(n)\right]+\frac{B\mu^2}{L\left\|\underline{g}_1\right\|_2^2}\frac{\sigma_v^2}{\sigma_s^2} \quad (4.2.13)$$

According to Eq.(4.2.13), in order to find the values of $tr\left[D_1R_{\underline{d}_1}(n)\right]$ and $tr\left[D_1^2R_{\underline{d}_1}(n)\right]$, it needs to know the matrix information of $R_{\underline{d}_1}(n)$. $R_{\underline{d}_2}(n)$ is also needed for determining the values of $tr\left[\Sigma_2V_2^TV_1D_1V_1^TV_2\Sigma_2^TR_{\underline{d}_2}(n)\right]$ and $tr\left[D_2R_{\underline{d}_2}(n)\right]$. We can not know the variance behavior of single channel coefficient error only by its last iteration variance. We need to know the iterative matrix information, including $R_{\underline{d}_1}(n)$ and $R_{\underline{d}_2}(n)$. Reorganize Eq.(4.2.6), $R_{\underline{d}_1}(n)$ is given iteratively by:

$$R_{\underline{d}_1}(n+1)$$
$$=R_{\underline{d}_1}(n)-\frac{B\mu}{L\left\|\underline{g}_1\right\|_2^2}A_1\left[R_{\underline{d}_1}(n)D_1+D_1R_{\underline{d}_1}(n)\right]+\frac{2B^2\mu^2}{L^2\left\|\underline{g}_1\right\|_2^4}A_1\left[D_1R_{\underline{d}_1}(n)D_1+Q_1^TK_{12}Q_2R_{\underline{d}_2}(n)Q_2^TK_{21}Q_1\right]$$
$$+\frac{B^2\mu^2}{L^2\left\|\underline{g}_1\right\|_2^2}\left[tr\left(D_1R_{\underline{d}_1}(n)\right)+tr\left(D_2R_{\underline{d}_2}(n)\right)\right]A_1D_1+\frac{\sigma_v^2}{\sigma_s^2}\frac{B^2\mu^2}{L^2\left\|\underline{g}_1\right\|_2^4}A_1D_1 \quad (4.2.14)$$

Similarly, $R_{\underline{d}_2}(n)$ is written as:

$$R_{\underline{d}_2}(n+1)$$
$$=R_{\underline{d}_2}(n)-\frac{B\mu}{L\left\|\underline{g}_2\right\|_2^2}A_1\left[R_{\underline{d}_2}(n)D_2+D_2R_{\underline{d}_2}(n)\right]+\frac{2B^2\mu^2}{L^2\left\|\underline{g}_2\right\|_2^4}A_1\left[D_2R_{\underline{d}_2}(n)D_2+Q_2^TK_{21}Q_1R_{\underline{d}_1}(n)Q_1^TK_{12}Q_2\right]$$
$$+\frac{B^2\mu^2}{L^2\left\|\underline{g}_2\right\|_2^4}\left[tr\left(D_2R_{\underline{d}_2}(n)\right)+tr\left(D_1R_{\underline{d}_1}(n)\right)\right]A_1D_2+\frac{\sigma_v^2}{\sigma_s^2}\frac{B^2\mu^2}{L^2\left\|\underline{g}_2\right\|_2^4}A_1D_2 \quad (4.2.15)$$

Finally, Eq.(4.2.14) and Eq.(4.2.15) are working together iteratively to represent the two joint channel coefficient error behavior. We have tried to give efforts on transforming the coefficient error behaviors in another domain to make a simpler analysis in this section. Unfortunately, these two joint error variance equations can not be further derived into a simple form. Even that it can not be further simplified, some meaningful information are given in the convergence analysis equation. The convergence iterative value is decided by (respect to $R_{\underline{d}_1}(n+1)$): (1) previous convergence value $R_{\underline{d}_1}(n)$, (2) local noise variance σ_v^2, (3) previous convergence value of another channel $R_{\underline{d}_2}(n)$ and the most complicated part of (4) multiplication between transmission room effect(K_{12}, K_{21})and previous convergence value of another channel $R_{\underline{d}_2}(n)$. The (1) and (2) effects are similar with the result of monophonic AEC convergence analysis [16]. But the local noise variance σ_v^2 will be isolated easily, if it is too small to compare with the norm of far end impulse response. By the other way, (3) and (4) are the main difference between monophonic and stereophonic AEC convergence equations. Especially, (4) highlights the effect of transmission room impulse responses in a stereophonic AEC convergence analysis.

4.3 Nonlinear loudspeaker and stereophonic AEC

Applying nonlinearities on the far end input signal is one of the effective methods to overcome the nonuniqueness problem in stereophonic AEC. In spite of many studies discussing various applications of nonlinear operation in stereophonic AEC system, the nonlinearities also exist in loudspeakers of teleconferencing system. In this section, we try to verify that the introduction of a nonlinear loudspeaker (NLS), simulated by memoryless polynomial filter, will provide a better result on echo power behavior than the original stereophonic AEC without NLS. In Figure 4.2, we replace the linear loudspeakers and adaptive filters with the nonlinear loudspeaker and preprocess scheme (see also Section 3.2) separately, i.e., the NLS / NAF mode. The far end signal $x_i(n)$ will undergo the "nonlinear operation" before they go through echo path $\underline{h}_i(n)$, this procedure is similar to the nonlinearities introduced in other researches.

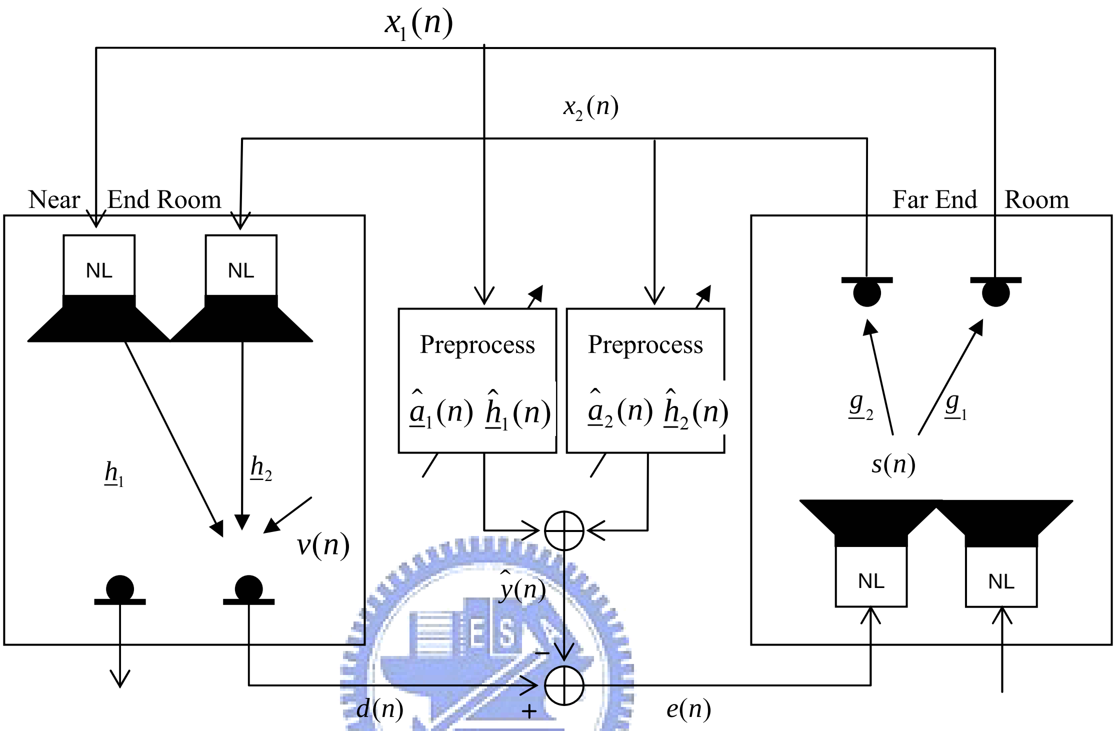

Figure 4.1 Stereophonic AEC with NLS and Preprocessor

First, we demonstrate the echo power behavior in three "channel/adaptive-filter" combination modes (simulations see also Section 5.7). Stereophonic AEC with NLS have better echo power behavior than that without NLS. The nonlinearities in loudspeaker will provide "nonlinear operation" effect on far end input signals to achieve a better converged result.

Secondly, we present the comparisons of misalignment (or echo power) in different nonlinear degree (see also Figure 5.7.6). The more significant nonlinearity is the smaller convergence value is.

Chapter 5

Computer Simulations

In this Chapter, computer simulations are introduced to verify the algorithm and describe some observations discussed in Chapter 3 and Chapter 4. First, we will define some parameters and measure equations used in following simulations In Section 5.1. Second, when the other coefficients are perfect and fixed, the linear or nonlinear coefficient error convergence analysis will be verified separately in Section 5.2. Third, the coefficient error convergence analysis will be demonstrated again without perfect coefficient assumption in Section 5.3. Fourth, we introduce and compare the five adaptive strategies for nonlinear AEC in Section 5.4. Last, there are some supplemental simulations for nonlinear AEC in Section 5.5.

For stereophonic AEC, verifications of coefficient error convergence analysis are given in Section 5.6. Combining stereophonic AEC with nonlinear loudspeaker, three different “stereophonic channel/adaptive-filter” modes will be introduced and compared in Section 5.7. White Gaussian noise is used as input signal without any additional statement in following Sections of this Chapter, and speech signals will also be used for verifying some results in Section 5.5.

5.1 Simulation parameters and room impulse responses

The near end room echo path responses $\underline{h}$ ($\underline{h}_1$ in stereophonic channel) and $\underline{h}_2$ shown in Figure 5.1.1 and Figure 5.1.2 separately, are generated by computer simulations. Similarly, far end room impulse responses $\underline{g}_1$ and $\underline{g}_2$ used in stereophonic AEC simulation are illustrated in Figure 5.1.3 and Figure 5.1.4. Figure 5.1.5 shows the speech signal with sampling rate 8K Hz, this speech signal will be used to verify the nonlinear AEC scheme in Section 5.5. The nonlinear memoryless polynomial channel is defined by the vector $\underline{a}$ with nonlinear order $Q = 3$; the three nonlinear coefficients are fixed by:

$$\underline{a} = [a_1, a_2, a_3]^T = [1, 0.1, 0.3]^T \tag{5.1.1}$$

The first element can be thought as the weight will multiply to the linear response, it is normalized to be 1; the second and third nonlinear harmonic weight are decided by the harmonic distortion comparisons in [4].

In following simulations, two main remarks are used to compare the performance: one is residual error power $e^2(n)$; the other is misalignment (normalized tap coefficients error) $\varepsilon(n)$ or its square value $\varepsilon^2(n)$. The misalignment is defined by following equation:

$$\varepsilon(n) \triangleq \frac{\left\|\underline{h} - \hat{\underline{h}}(n)\right\|_2}{\left\|\underline{h}\right\|_2}, \quad \left\|\cdot\right\|_2 : 2-norm \tag{5.1.2}$$

Residual error power is a value used to measure the echo reduction performance of AEC. It is equivalent to misalignment in monophonic AEC; in contrast, it does not provide information about misalignment in stereophonic AEC. By the way, misalignment represents system tracking ability of AEC for room impulse response (RIR).

And the system signal to noise ratio is defined as:

$$SNR \triangleq 10\log_{10}\frac{p_x}{p_v} \tag{5.1.3}$$

where p_x and p_v are the average power of far end signal $x(n)$ and local noise $v(n)$.

We assume both numbers of linear filter tap and length of RIR ($\underline{h}$) equal to 128. If there are not any other statements, the following parameters are used in simulation:

SNR: 20 dB

NLMS Linear Step Size: $\mu_h = 0.05$

NLMS Nonlinear Step Size: $\mu_a = 0.01$

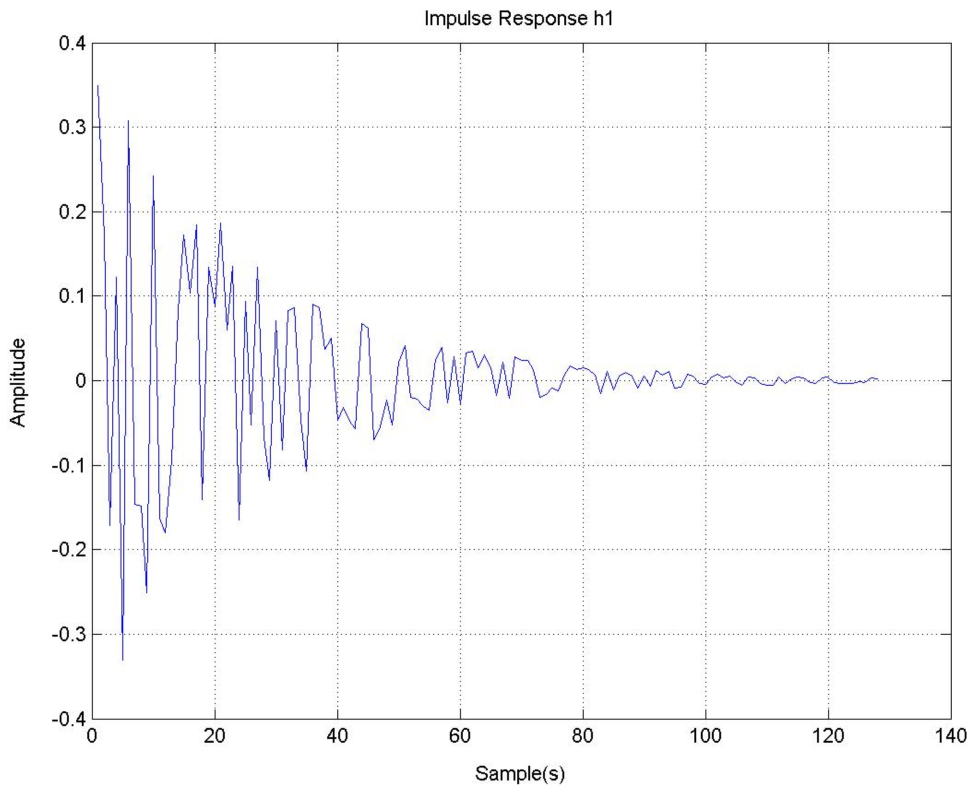

Figure 5.1.1 Near end room impulse response $\underline{h}$ ($\underline{h}_1$)

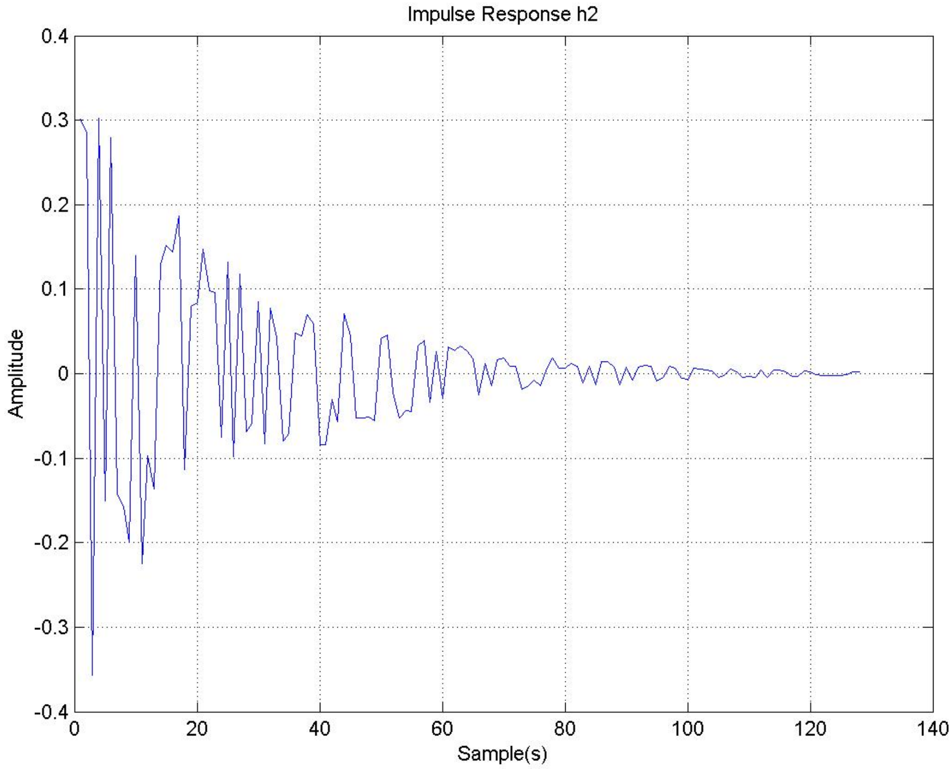

Figure 5.1.2 Near end room impulse response $\underline{h}_2$

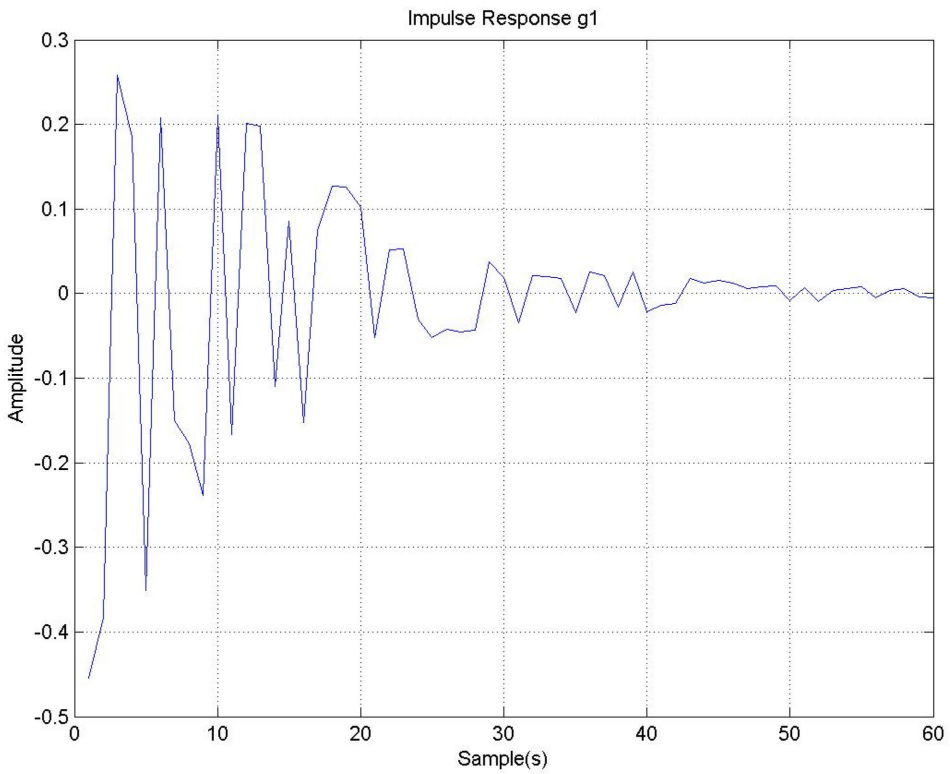

Figure 5.1.3 Far end room impulse response $\underline{g}_1$

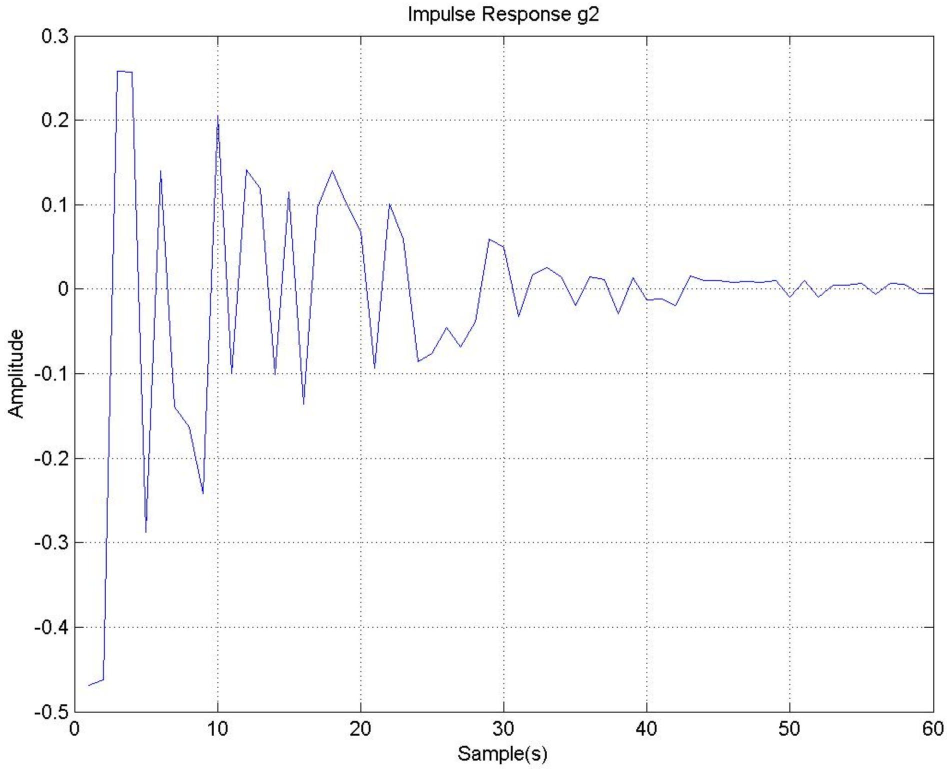

Figure 5.1.4 Far end room impulse response $\underline{g}_2$

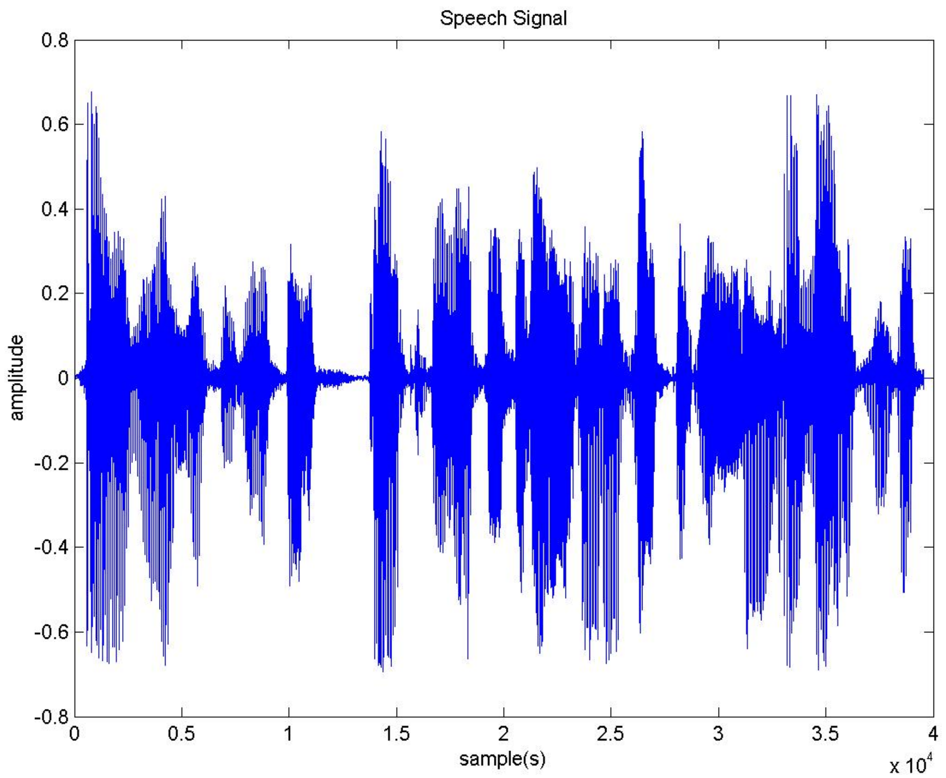

Figure 5.1.5 8K Hz sampling speech signal

5.2 Individual coefficient error convergence

5.2.1 Linear coefficient error convergence

According to the assumption of nonlinear coefficients are perfect, the behavior derivation (i.e., Eq.(3.2.9)) of linear coefficient error convergence will be verified by the computer simulation in this Section. Coefficients of the linear filter in echo canceller are initialized to zero. Nonlinear coefficients are perfect and fixed.

The comparison of theoretical derivation and simulation is shown in Figure 5.2.1.

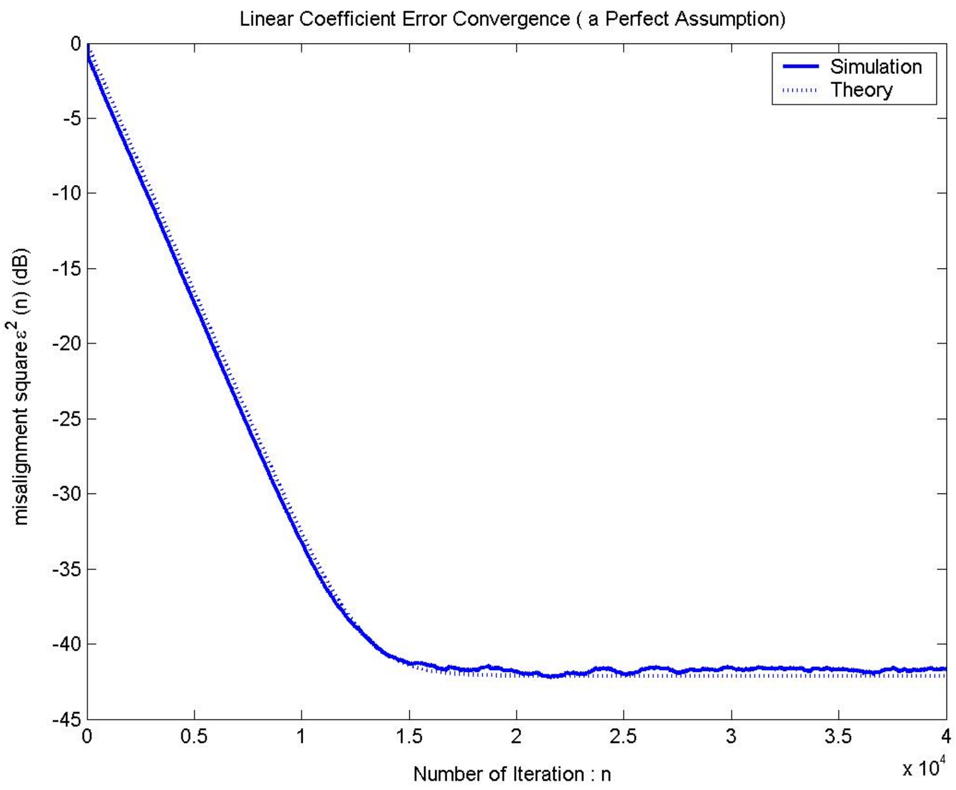

Figure 5.2.1 Linear coefficient error convergence simulation and theory

The converged line drew by theoretical equation Eq.(3.2.9) almost matches the simulation result in Figure 5.2.1. It indicates that the analysis of linear coefficient error convergence in Section 3.2 is correct and appropriate.

5.2.2 Nonlinear Coefficient Error Convergence

We verify nonlinear coefficient error convergence analysis Eq.(3.3.8) by computer simulation in this Section. The nonlinear coefficients are initialized to be $[1,0,0]^T$, it is equivalent to only linear adaptation in the beginning. Linear coefficients are perfect and fixed. The comparison of theoretical derivation and simulation is shown in Figure 5.2.2.

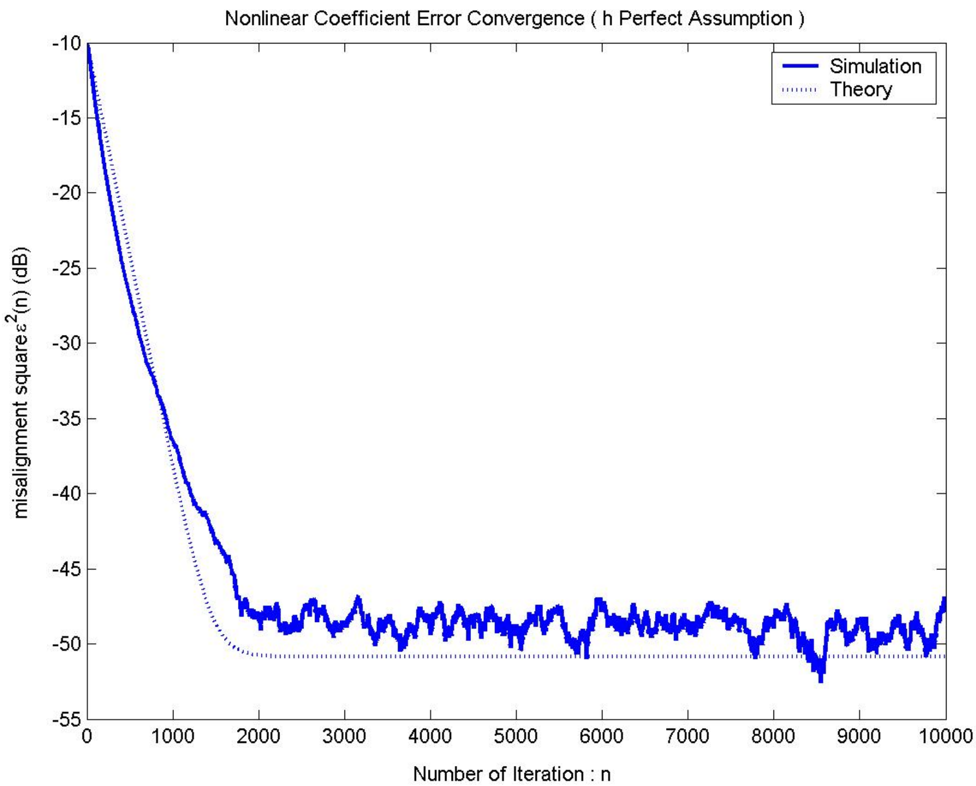

Figure 5.2.2 Nonlinear coefficient error convergence of simulation and theory

The theoretical convergence analysis Eq.(3.3.8) is an approximated result, it keeps 2~3 dB away from simulated line in the converged phase.

5.3 Joint Coefficient Error Convergence

Without the assumption: linear or nonlinear coefficients are perfect, we have derived the two analysis equations (i.e., Eq.(3.4.12) and Eq.(3.4.21)) for the joint coefficient error convergence in the situation of linear and nonlinear coefficients error affect each other. Because the interaction of these two approximated convergence analysis equations will lead the convergence behavior to a bigger inaccuracy state. We compromise between pure theory and simulation: in stead of depending all the iteration of two analysis equations, we take the n'th simulated nonlinear (linear) coefficient misalignment as the parameter for calculating the $n+1$'th linear (nonlinear) coefficient misalignment. We call it "Semi-Theory" mode. Here are the verifications for linear coefficients analysis equation (Eq.3.4.12) for different SNR values in Figure 5.3.1(20dB), Figure 5.3.2(10dB) and Figure 5.3.3(5dB).

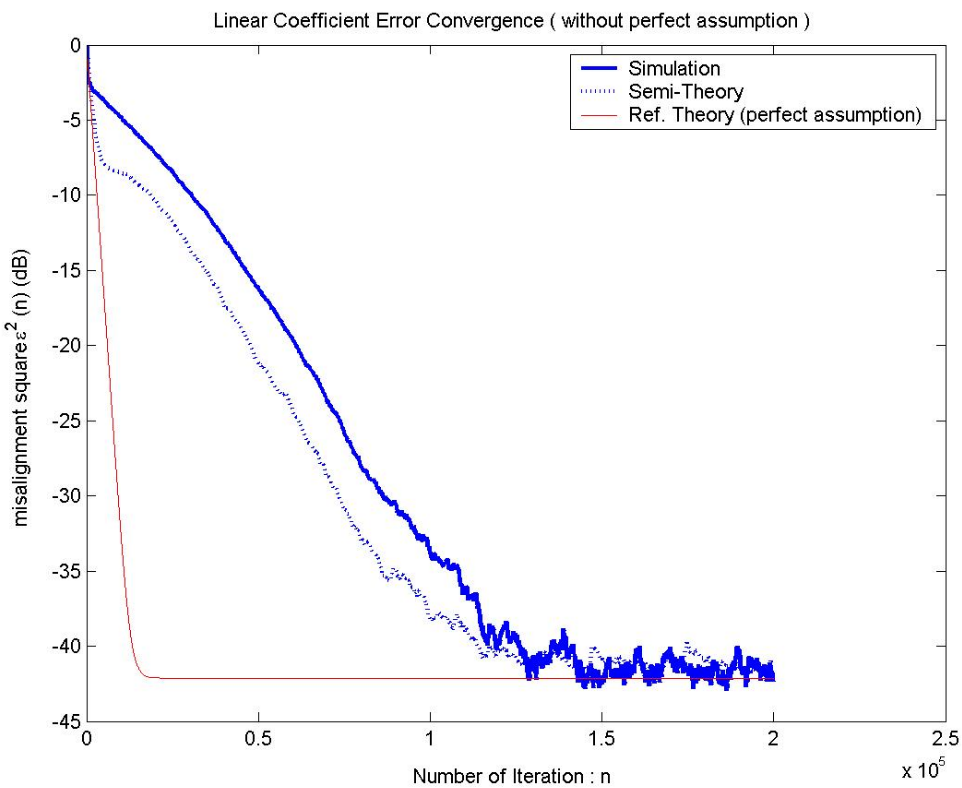

Figure 5.3.1 Linear coefficient error convergence in semi-theory (SNR=20dB)

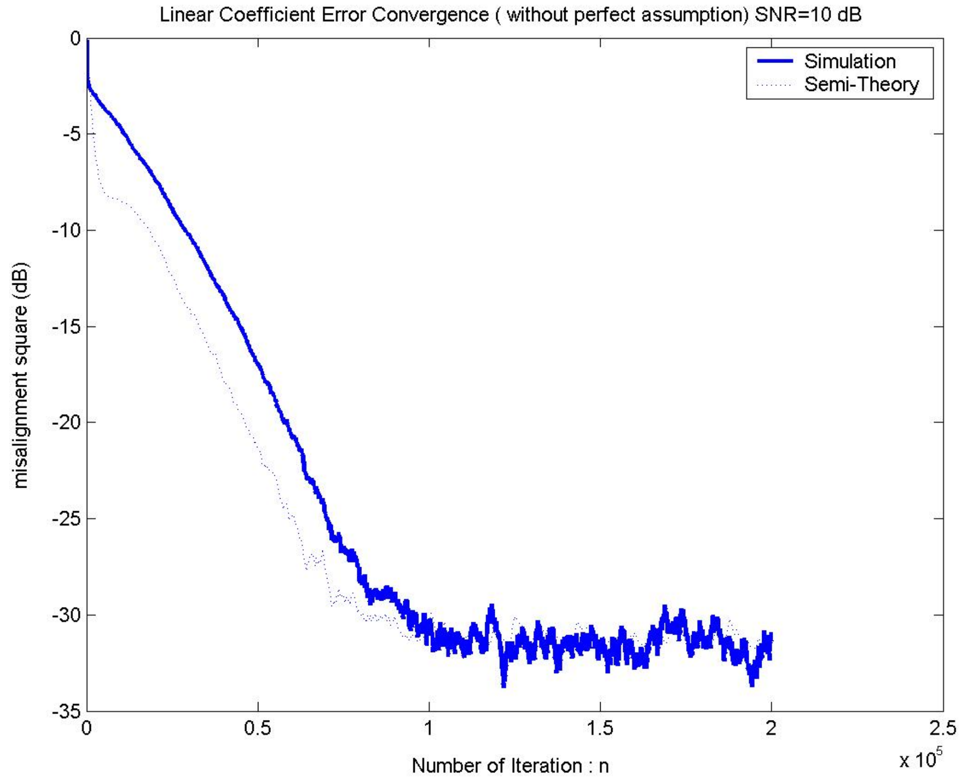

Figure 5.3.2 Linear coefficient error convergence in semi-theory (SNR=10dB)

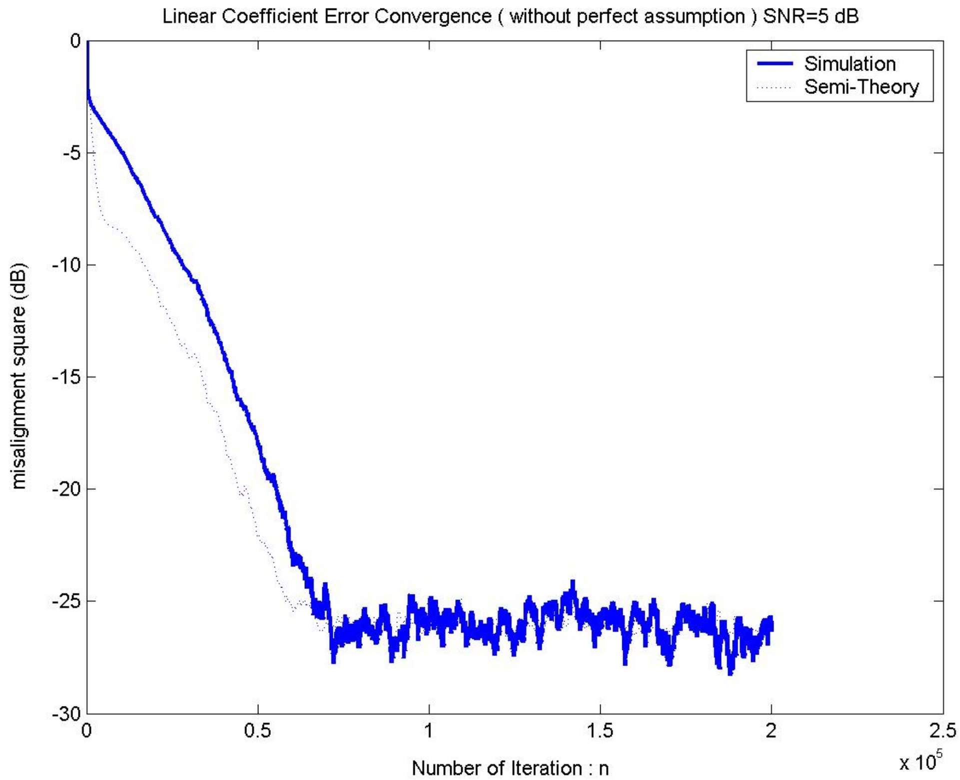

Figure 5.3.3 Linear coefficient error convergence in semi-theory (SNR=5dB)

Unlike the linear coefficient convergence behavior under perfect nonlinear coefficients, the “Semi-Theory” line (dot-line) can match the slower converged trend of simulation and has the almost same value for final convergence results for different SNR values in Figure 5.3.1~5.3.3. By the way, the smaller SNR value, the higher coefficient error convergence value we have. The increased coefficient error convergence value is almost equal to the SNR decrease value.

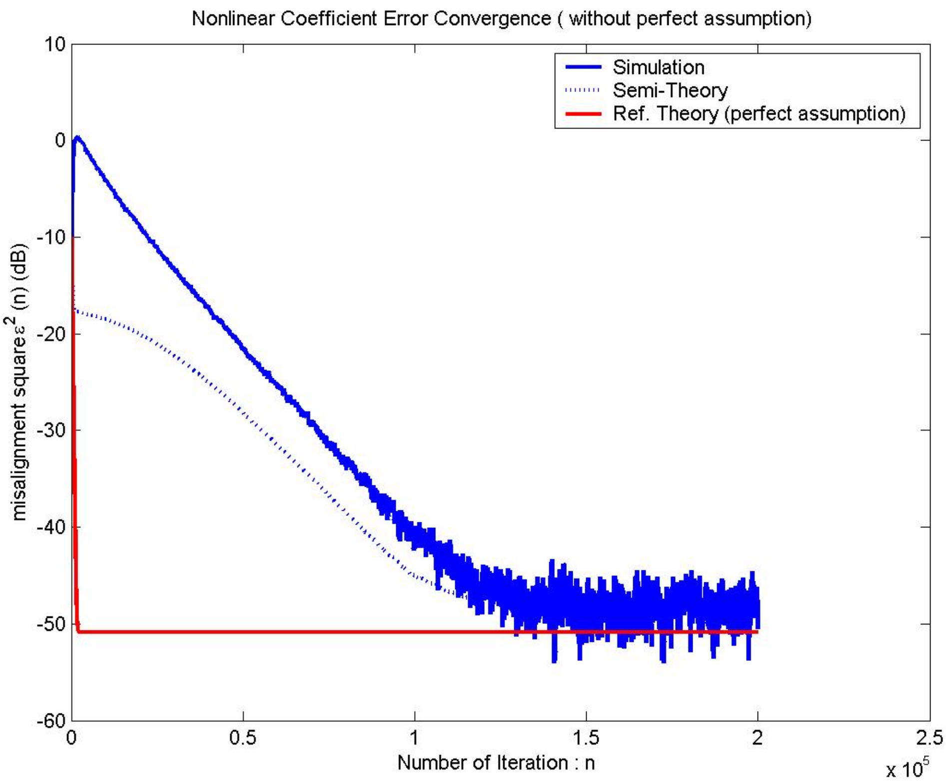

Figure 5.3.4 Nonlinear coefficient error convergence in semi-theory (SNR=20dB)

Although it has a big inaccuracy in the beginning in Figure 5.3.4, the more iterations the smaller inaccuracy can be obtained in the rest part of nonlinear coefficient error convergence. Similarly, both “Semi-Theory” and simulation have the similar convergence value in the back part of convergence. Nonlinear coefficient error convergence for different are shown in Figure 5.3.5~5.3.6.

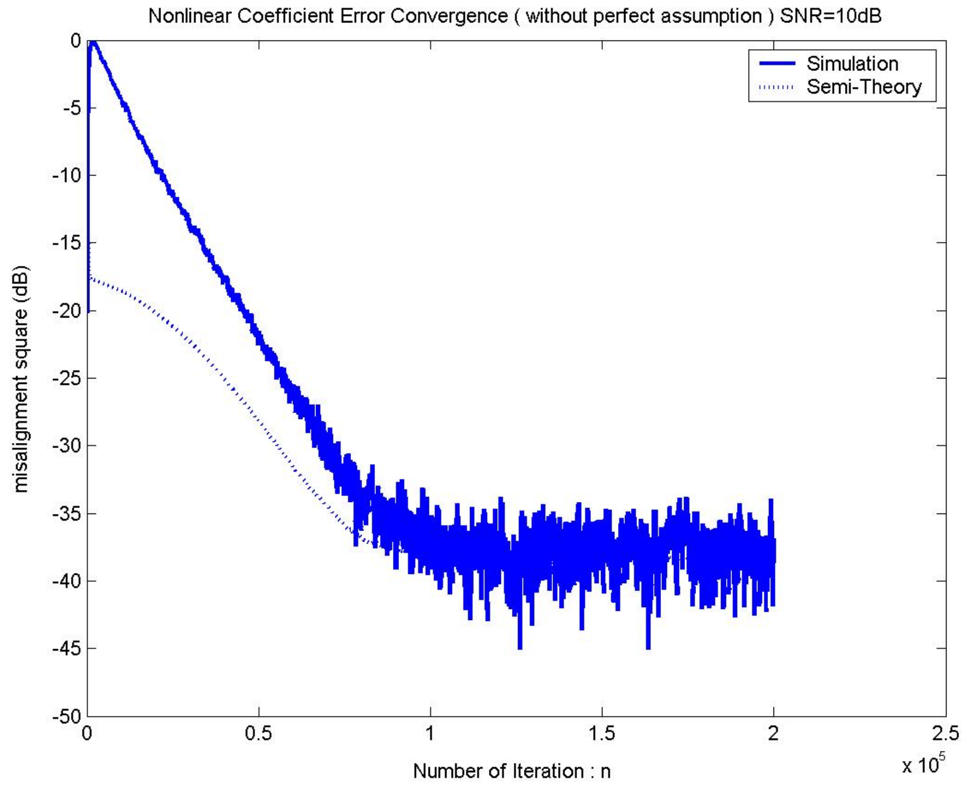

Figure 5.3.5 Nonlinear coefficient error convergence in semi-theory (SNR=10dB)

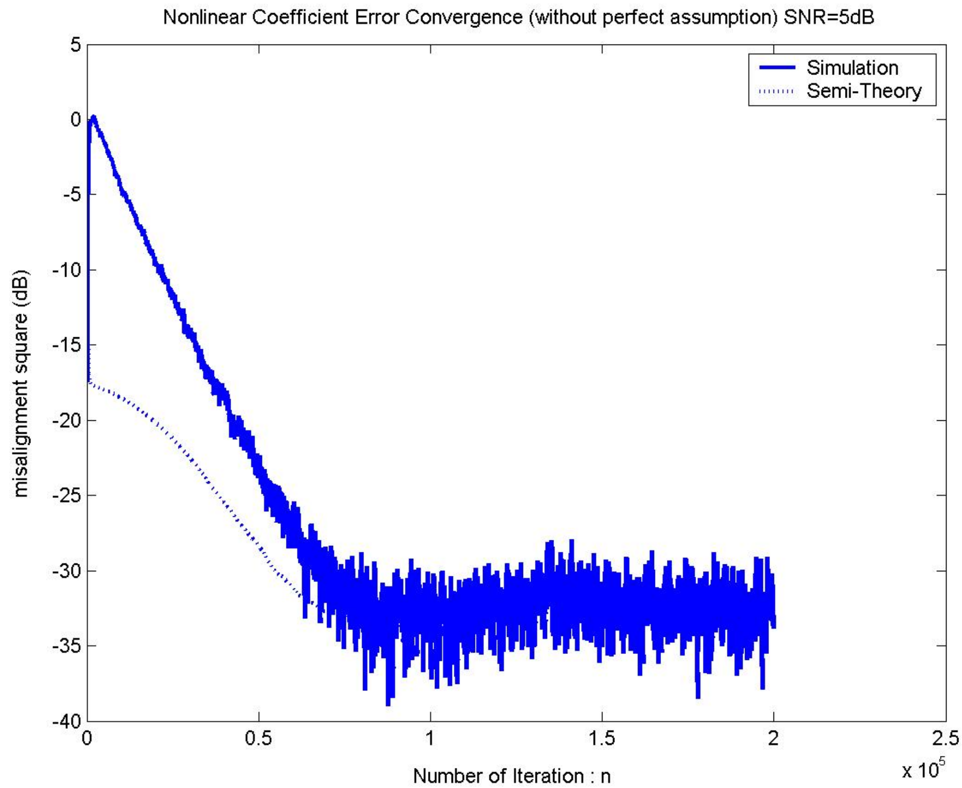

Figure 5.3.6 Nonlinear coefficient error convergence in semi-theory (SNR=5dB)

5.4 Adaptation strategies for nonlinear AEC

5.4.1 Five adaptation strategies

Five major adaptation strategies (operation descriptions see also Section 3.5) will be compared in this section. These strategies are obtained and modified by observations of simulation results. The first starting strategy is the original adaptation method: joint update, nonlinear and linear coefficients are updated iteratively in cascade order by the preprocess scheme. First of all, we demonstrate the residual error power behaviors of five different adaptation strategies. Then, the description will be emphasized on how we find the two faster strategies: “Select and Go, Joint” and “Go, Go, Joint”.

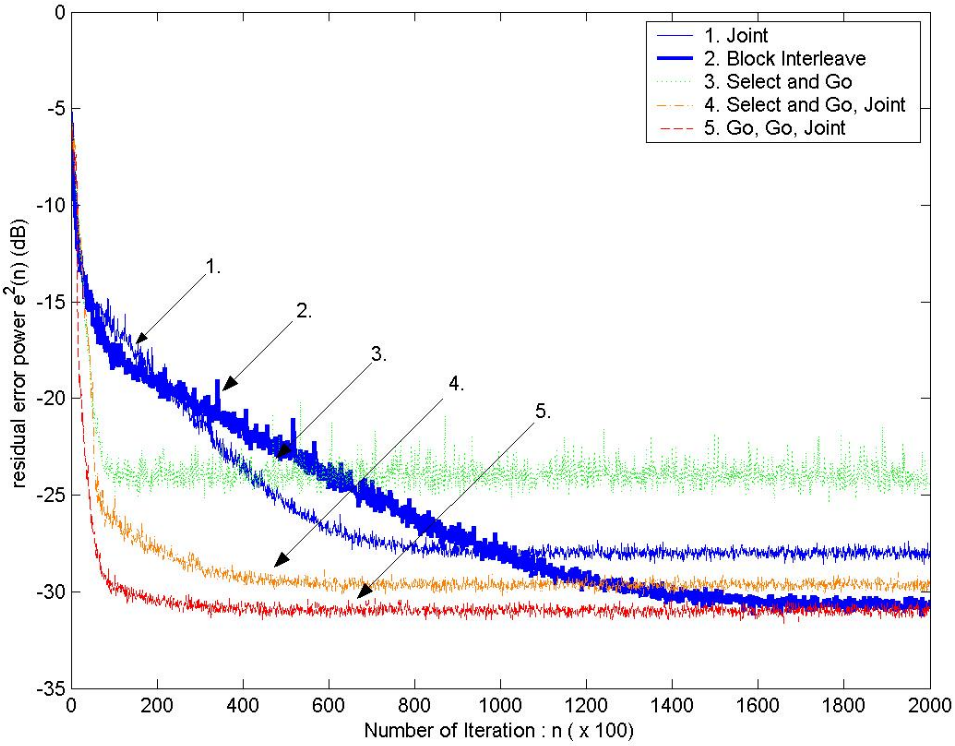

Figure 5.4.1 Residual error power of five adaptation strategies

We will recommend the "Go, Go, Select" adaptation is the appropriate strategy under the considerations of convergence rate (fastest) and converged value (smallest). In Figure 5.4.1, "Block Interleave" also has a small converged value but it takes a long time (140,000 samples, i.e., 17.5 seconds for 8K samples) to converge to the steady state. Comparison results are briefly organized in Table 5.1.

Adaptation Strategy	**Converged rate (Transient Phase)**	**Converged value (Steady State Phase)**
Joint	Slow	-27 dB
Block Interleave	Slowest	-30 dB
Select and Go	Middle	-24 dB
Select and Go, Joint	Fast	-29 dB
Go, Go, Joint	Fastest	-31 dB

Table 5.1 Adaptation strategies comparisons

5.4.2 Behaviors of nonlinear coefficient misalignment

The adaptive procedure "Select" both in "Select and Go" and "Select and Go, Joint" strategies is to select the L'th nonlinear coefficients and keep it for a while; this method is modified from the phenomena that the nonlinear coefficient misalignment will have a respective smallest value in the first L iterations.

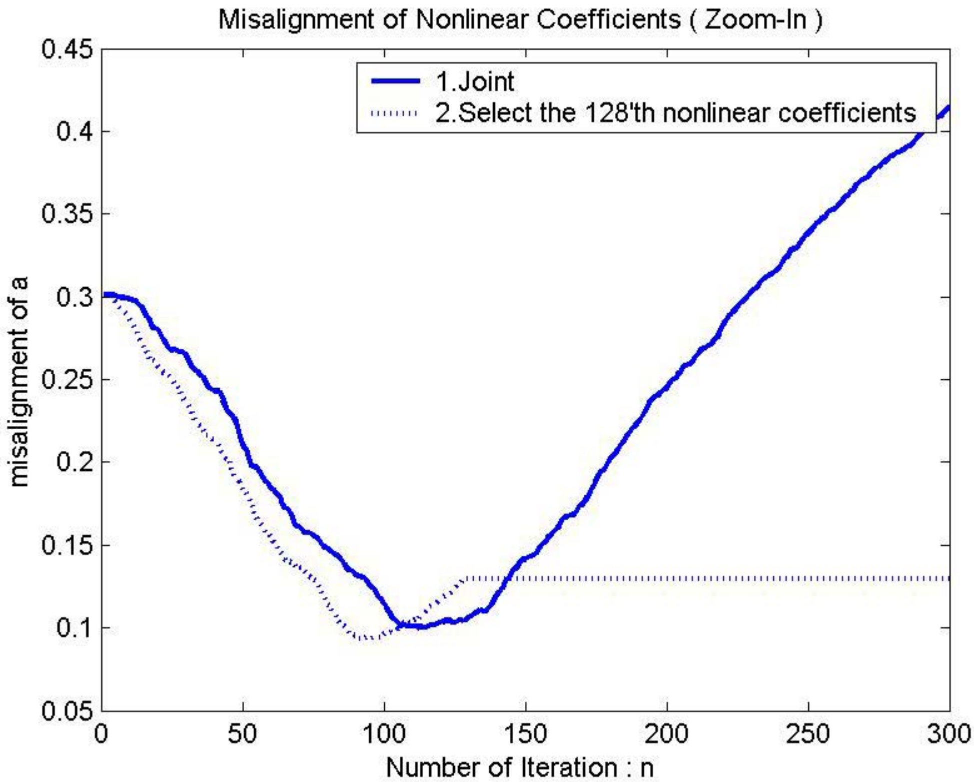

Figure 5.4.2 Nonlinear coefficient misalignment in a zoom-in scope

"Select and Go" strategy keeps the selected nonlinear coefficients and only updates linear coefficients in the following adaptation. It is different from "Select and Go, Joint" stragy, "Select and Go, Joint" strategy restores "Joint" adaptation when iteration numbers is greater than the desired threshold iterations: 5000. Because of nonlinear coefficients can be further achieved to a smaller misalignment, the "Select and Go, Joint" strategy can have a better residual error power behavior than "Select and Go" strategy, like the simulations in Figure 5.4.1.

5.4.3 "Go, Go, Joint" adaptation strategy

The main idea of "Go, Go, Joint" strategy is giving more weight on linear coefficient in transient phase for achieving better residual error power behavior. Only the linear coefficients are adapted first, turn into nonlinear coefficient adaptation when the adaptation numbers threshold (1250) is satisfied; finally, applying "Joint" adaptation if nonlinear coefficient adaptation number is greater than 50. The residual error power behavior of "Go, Go, Joint" adaptation strategy is shown in Figure 5.4.1. But it does not give a clear view to tell the different adaptive phases. Three different adaptive phases can be recognized more clearly in Figure 5.4.3.

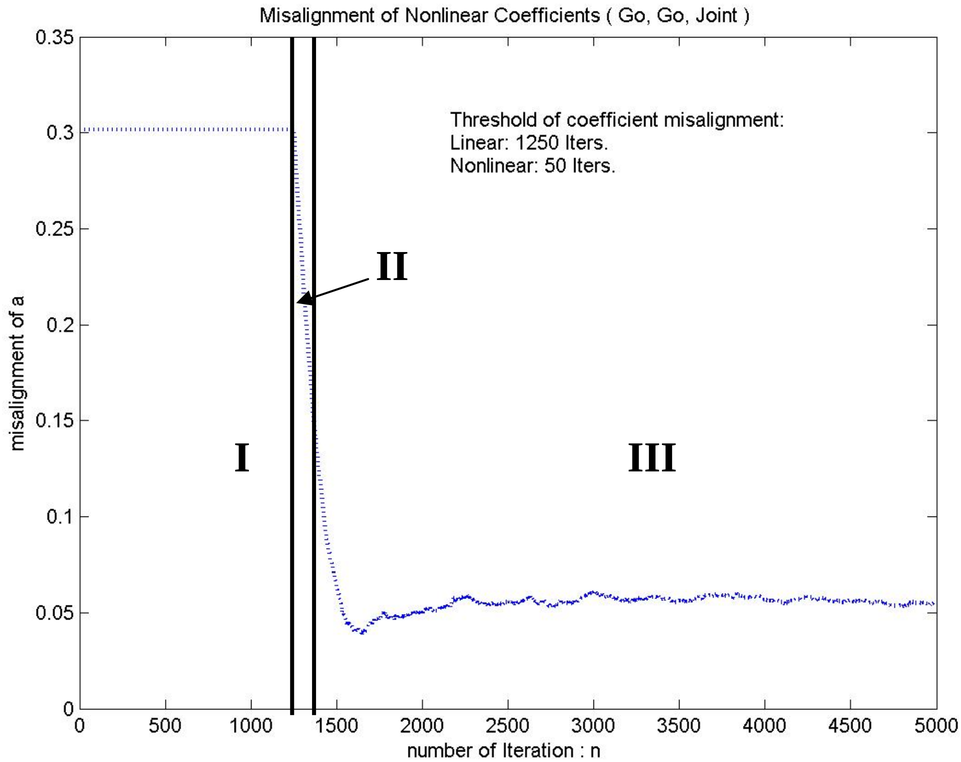

Figure 5.4.3 Three phases of "Go, Go, Joint" strategy in nonlinear coefficient view

In phase I, only the linear coefficients are updated, the nonlinear coefficients keep fixed. The nonlinear coefficients begin to update and the linear part are kept fixed in phase II; this phase has the fewest iterations among the three phases because the linear coefficient misalignment had been adjusted to some correct degree. It is more efficient for making the nonlinear coefficients approach to the right direction than that without any linear coefficient adaptation first. In phase III, the "Joint" adaptation are applied; linear and nonlinear coefficients are adapted together in a more correct sense than applying “Joint” strategy directly in beginning iteration. According to the simulations, the linear threshold plays an important role in this strategy in first phase. Can we have first phase iteration threshold as long as we want and get a better convergence result? The answer is “No”, simulations are shown in Figure 5.4.4 and 5.4.5.

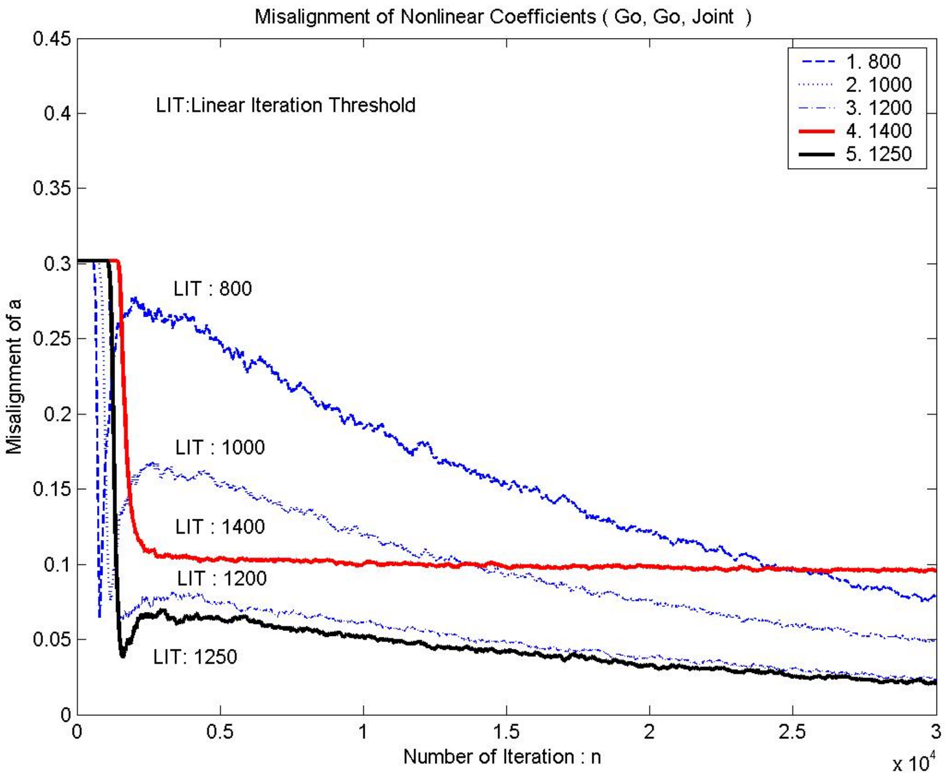

Figure 5.4.4 Misalignment of nonlinear coefficients in different linear iteration threshold

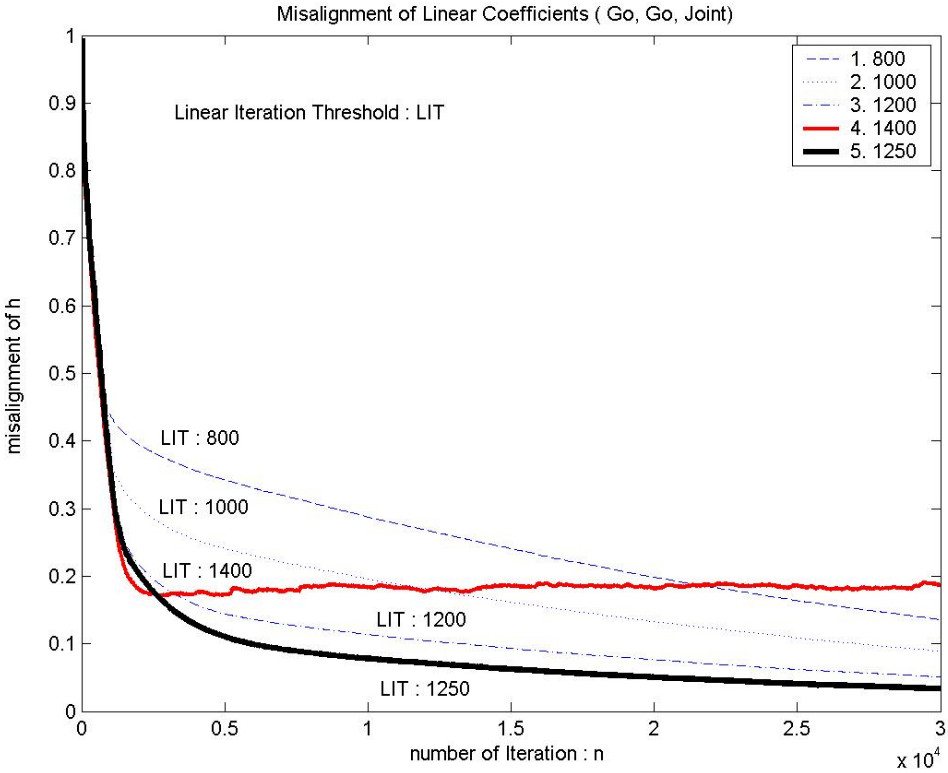

Figure 5.4.5 Misalignment of linear coefficients in different linear iteration threshold

When the linear iteration threshold is longer, we can not obtain a better misalignment behavior both in nonlinear and linear coefficients. In stead of showing the residual error power in different linear threshold; here, we take the coefficient misalignment for demonstration because it has the equivalent but more direct effect than residual error power representation.

In Figure 5.4.1, the “Select and Go, Joint” and “Go, Go, Joint” strategies have the similar behavior both in transient and steady state phases. Can we have a better convergence behavior than that in the “Go, Go, Joint” by tuning the only parameter: linear coefficient only adaptation numbers (5000), in the “Select and Go, Joint”?

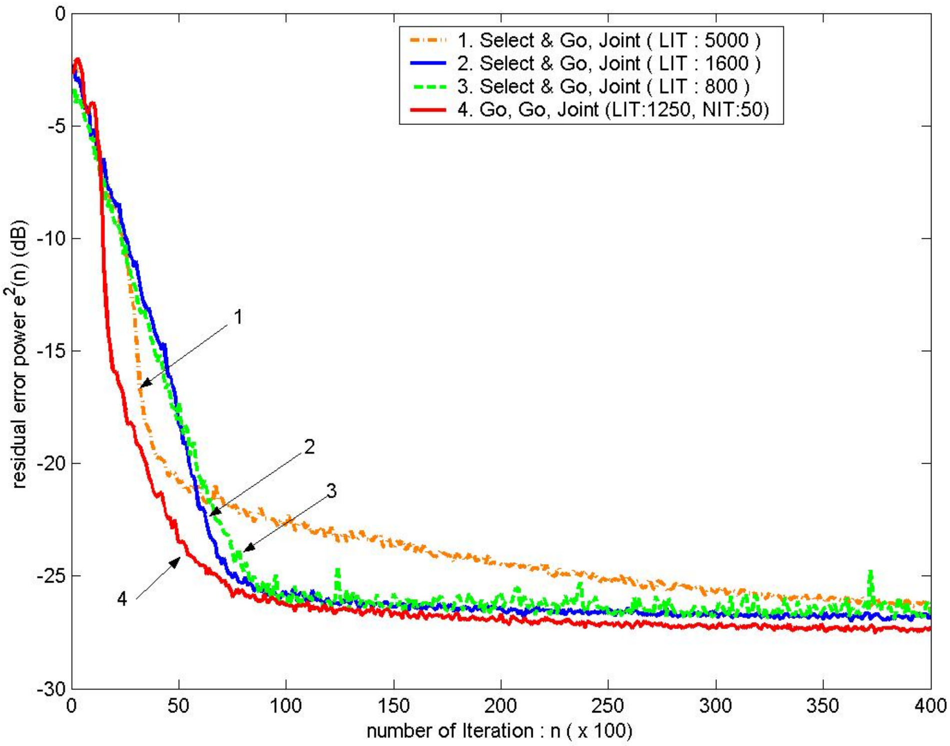

Figure 5.4.6 Echo power comparisons of two adaptation strategies

In Figure 5.4.6, it shows that no matter what linear coefficient only adaptation numbers are used in the "Select and Go, Joint" strategy, the "Go, Go, Joint" strategy still has the better convergence behavior than "Select and Go, Joint".

5.5 Extended simulations for nonlinear AEC

5.5.1 Insufficient order of nonlinear coefficient

Tow sets of simulation comparison are provided in this sub-section: one is the residual error power comparison between the linear adaptive filter and the sufficient order (nonlinear order $Q = 3$) nonlinear adaptive filter; the other is the comparison of residual error power performance in sufficient and insufficient order nonlinear adaptive filter. In Figure 5.5.1, the order of nonlinear channel $Q = 3$; we use linear adaptive filter and sufficient order nonlinear adaptive filter to cancel the echo signal separately.

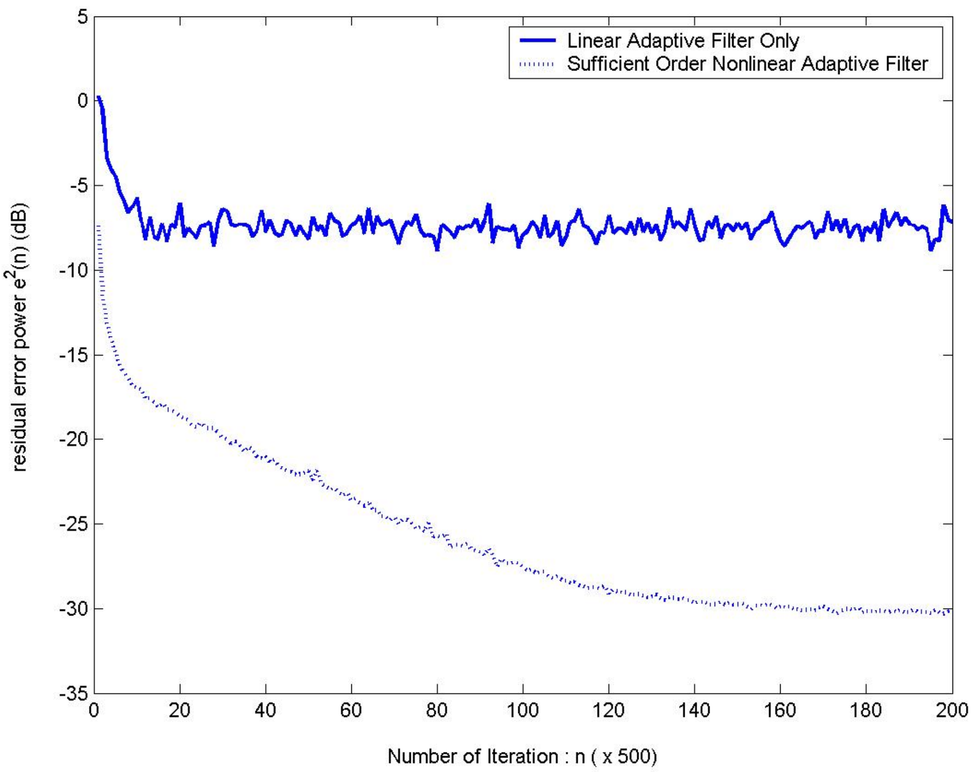

Figure 5.5.1 Linear and nonlinear adaptation convergence for $Q = 3$ nonlinear channel

Figure 5.5.1 tells us that there existing a big difference if only applying linear adaptive filter on nonlinear channel instead of applying nonlinear adaptive filter. The convergence behavior of sufficient order nonlinear adaptive filter has not only the faster converged rate in the beginning but also a significant difference in final residual error power converged value.

In Figure 5.5.2, the order of nonlinear channel is changed to $Q = 5$; besides the linear adaptive filter and sufficient order nonlinear adaptive filter, we add another nonlinear adaptive filter order $Q = 3$ to simulate the insufficient order nonlinear adaptive filter and compare the performance.

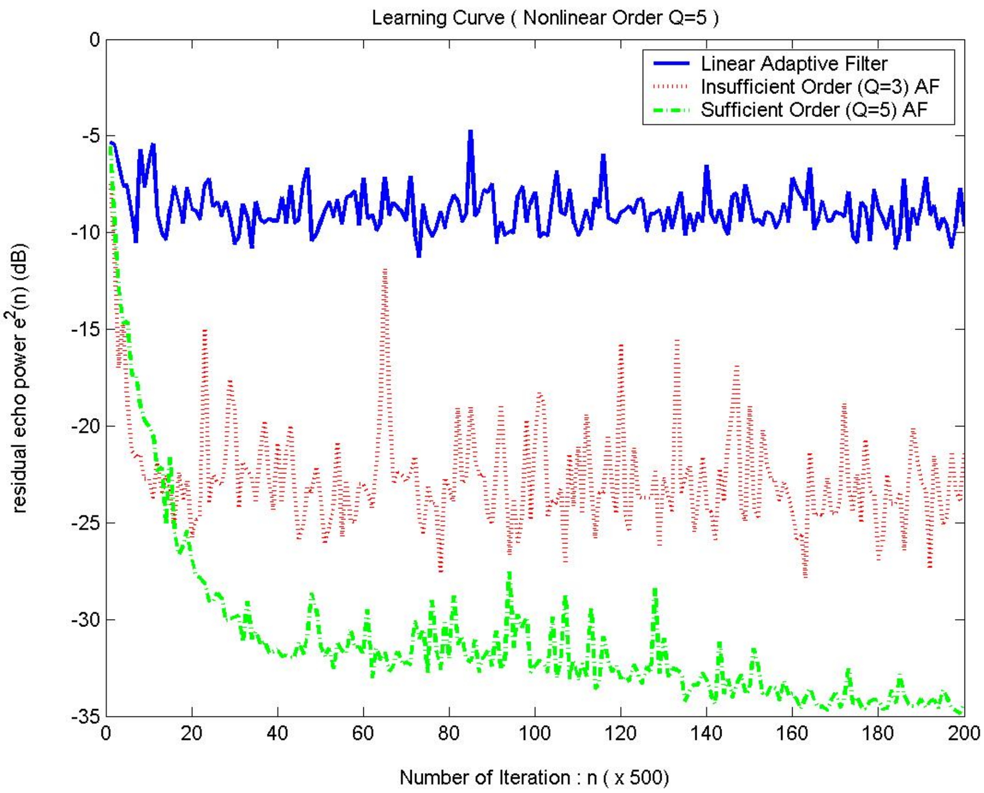

Figure 5.5.2 Linear and nonlinear adaptation convergence for $Q = 5$ nonlinear channel

The residual error power behavior of linear adaptive filter maintains at the same level with that one in Figure 5.5.1. Even the nonlinear order is insufficient; the $Q = 3$ nonlinear adaptive filter also has better performance than linear adaptive filter in this simulation. It is reasonable; the sufficient order nonlinear adaptive filter has the best performance.

5.5.2 Speech verification

We replace the original WGN far end input signal $x(n)$ with the speech signal introduced in Section 5.1 and verify the echo cancellation performance in the nonlinear adaptive filter. The speech signal parameters are:

Sample Rate: 8K Hz, 16 bits / sample

Total Length: 39846 samples (4.98 seconds)

For convenient, the ERLE measurement is used in this performance verification. The definition of ERLE is as :

$$ERLE \triangleq 10\log\frac{E\left[d^2(n)\right]}{E\left[e^2(n)\right]} \tag{5.5.1}$$

According to the definition, it is obvious that the larger ERLE value the better performance of echo cancellation we have. Nonlinear adaptive filter order is sufficient ($Q = 3$) and the simulation results are demonstrated in Figure 5.5.3, Figure 5.5.4 and Figure 5.5.5.

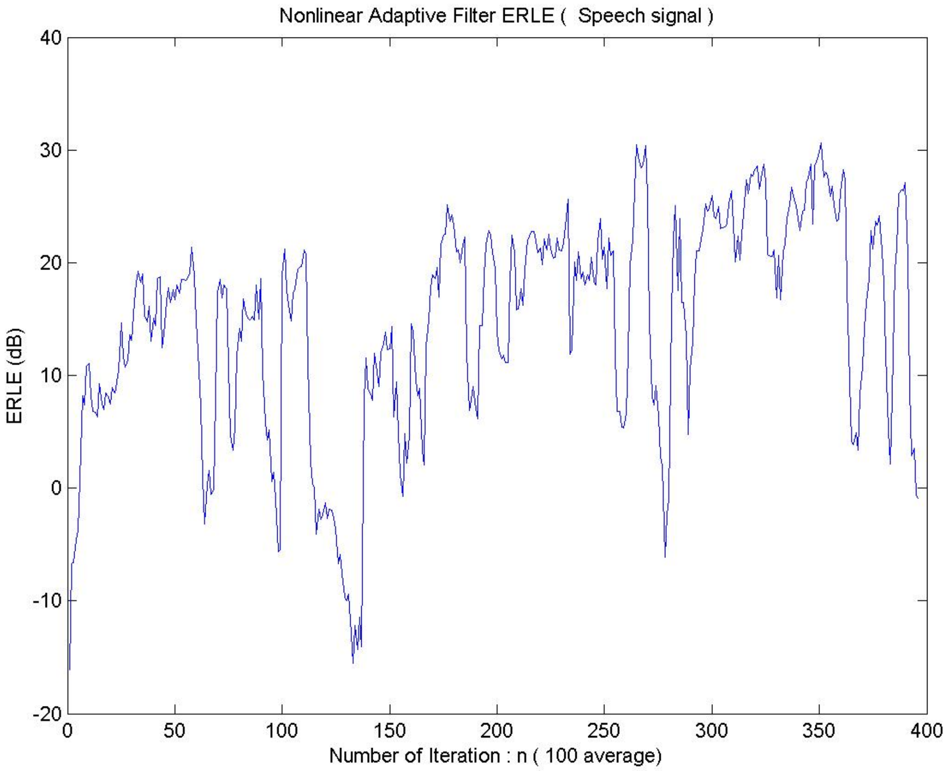

Figure 5.5.3 ERLE of speech input nonlinear adaptive filter

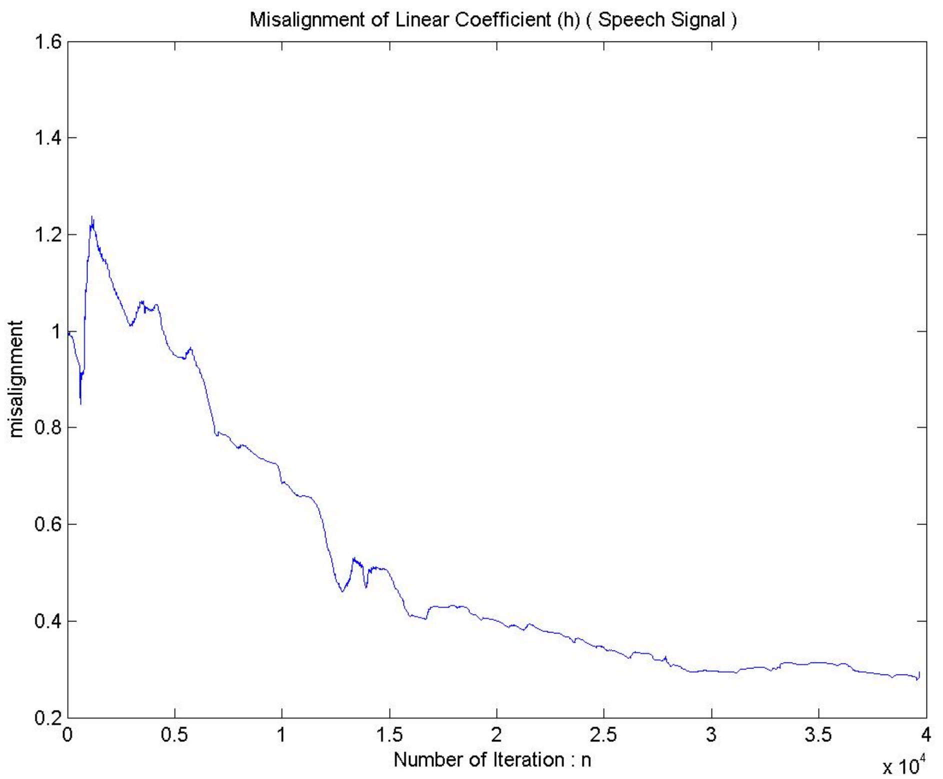

Figure 5.5.4 Linear coefficient misalignment of speech input nonlinear AEC

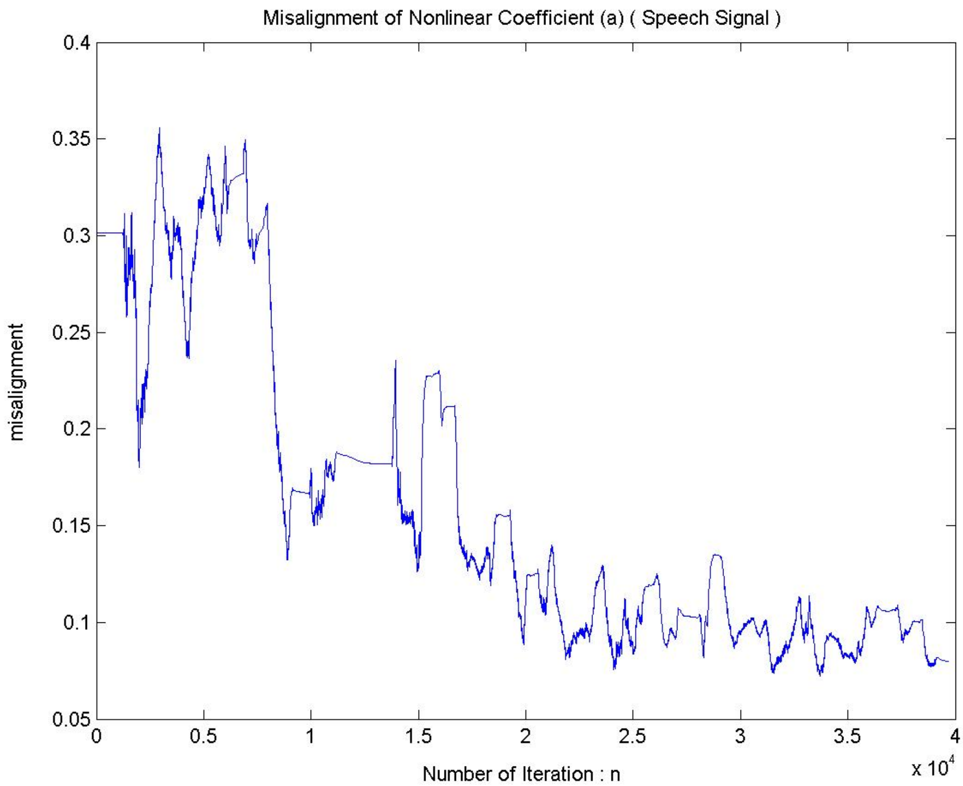

Figure 5.5.5 Nonlinear coefficient misalignment of speech input nonlinear AEC

The two coefficient misalignment figures show that the nonlinear adaptive filter worked properly for this speech input signal. We have a not good begging and few instable behaviors in the ERLE figure but it returns to a better performance in final signal part.

5.6 Convergence analysis of partial update stereophonic AEC

Partial update stereophonic AEC coefficient error convergence analyzed equations Eq.(4.2.14) and Eq.(4.2.15) will be verified by simulation in this section. Without loss of generality, we let $A_i = I_L$, the full coefficient updating is one case of partial coefficient updating. The simulation configurations are like follows:

SNR: 20 dB

NLMS Step Size: $\mu = 0.01$

Far End Impulse Responses ($\underline{g}_1$ and $\underline{g}_2$ see also Section 5.1) Numbers: $M = 128$

Near End Impulse Responses ($\underline{h}_1$ and $\underline{h}_2$ see also Section 5.1) Numbers: $N = 128$

Adaptive Filter ($\hat{\underline{h}}_1$) Tap Numbers: $L = 64$

In stereophonic AEC, coefficients misalignment could represent the echo cancellation performance more confidently than residual error power. The simulated result is demonstrated in square of misalignment ($\varepsilon^2(n)$) and is shown in Figure 5.6.1. Without loss of generality, this simulation comparison is presented by one coefficient error convergence ($\tilde{\underline{h}}_1(n)$), the other ($\tilde{\underline{h}}_2(n)$) is similar.

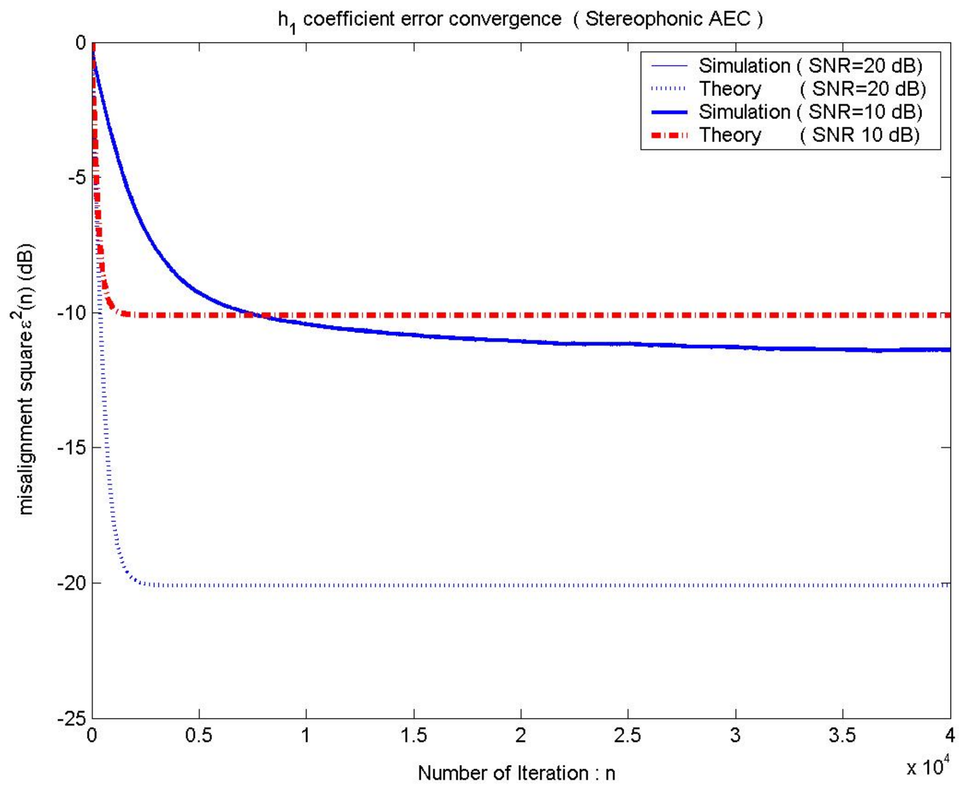

Figure 5.6.1 Coefficient error convergence of stereophonic AEC in different SNR

In Figure 5.6.1, the simulated lines are almost the same in SNR=10 dB and 20 dB. When SNR =10 dB, we have a small inaccuracy (1~1.5 dB) between the simulation and theory convergence behaviors. When SNR=20 dB, there is 9-dB difference between the simulation and theory final converged value. It is not a small inaccuracy that can be isolated. Although big inaccuracy exists in final coefficient error converged value for high SNR, but both convergence behavior trend are similar. It indicates that the major direction of derivation is not wrong, but some assumptions and simplifications need to be reconsidered for larger SNR condition. We have a better prediction for low SNR condition.

5.7 Nonlinear loudspeaker and stereophonic AEC

In this section, we will demonstrate the effect of stereophonic AEC with nonlinear loudspeaker in two methods: one is presented in comparisons of three different "channel/adaptive-filter" combinations; the other shows how much the nonlinearities affect stereophonic AEC in different degrees.

First, three residual error power behaviors of "channel/adaptive-filter" modes are showed in Figure 5.7.1; "channel" means the simulated channel with or without nonlinear loudspeaker (NLS and LLS); and "adaptive filter" includes the linear or nonlinear adaptive filter (i.e., NAF , see also joint update in section 3.5). Here, we try to integrate stereophonic AEC with the preprocessor scheme mentioned in Chapter 3. Three "channel/adaptive-filter" modes are NLS/NAF, NLS/Linear Adaptive Filter (LAF) and without NLS/LAF.

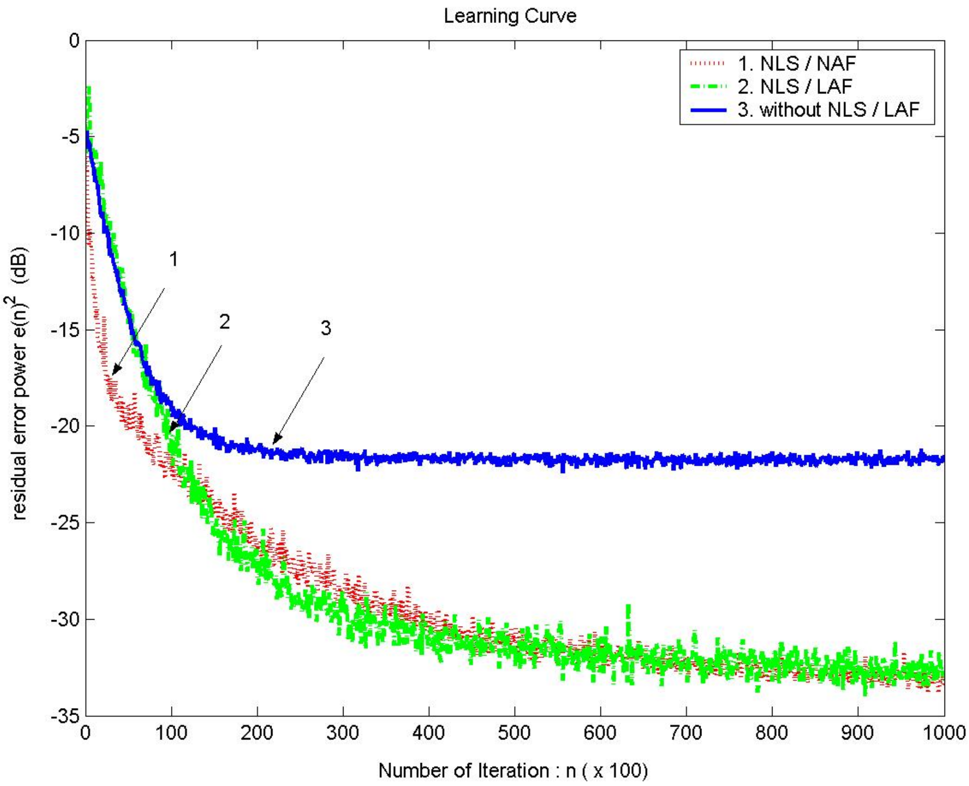

Figure 5.7.1 Residual error power of three stereophonic AEC modes

Modes	Convergence Value
NLS / NAF	-33 dB
NLS / LAF	-33 dB
Without NLS / LAF (Original SAEC)	-22 dB

Table 5.2 Convergence value of three Modes

The converged value comparison results are shown in Table 5.2.The simulation results indicate that stereophonic AEC with nonlinear loudspeaker (1. and 2.) have better residual error power behavior than that without NLS. Although the reprocess integrated mode has the faster convergence rate than the NLS/LAF mode in first

10000 iterations, it does not give any obvious difference in final convergence value. Misalignments of two linear coefficients are shown in Figure 5.7.2 and Figure 5.7.3.

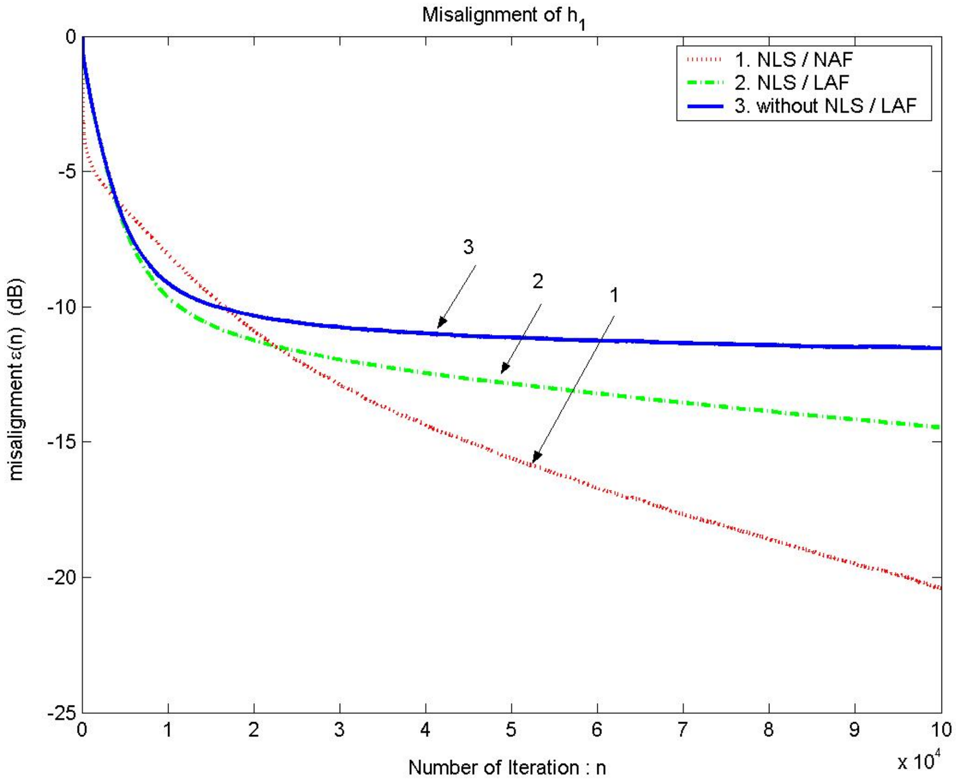

Figure 5.7.2 Misalignment of echo path $\underline{h}_1$

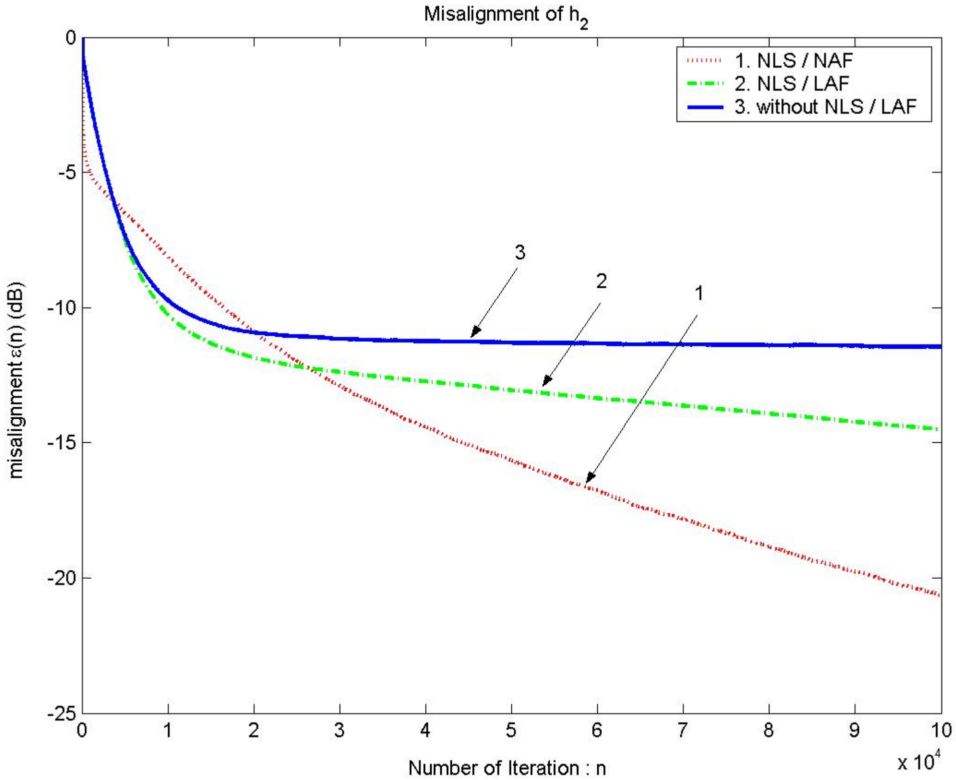

Figure 5.7.3 Misalignment of echo path $\underline{h}_2$

It also shows that it has better convergence behavior by applying NLS than LLS in Figure 5.7.2 and 5.7.3.

In order to verify the nonlinear adaptive filter scheme also works well in stereophonic AEC, nonlinear coefficient misalignments (with two different nonlinear coefficient sets, i.e., NLS: $\underline{a}_1 = (1, 0.1, 0.33)$ and $\underline{a}_2 = (1, 0.12, 0.3)$) are also given in Figure 5.7.4 and Figure 5.7.5 individually.

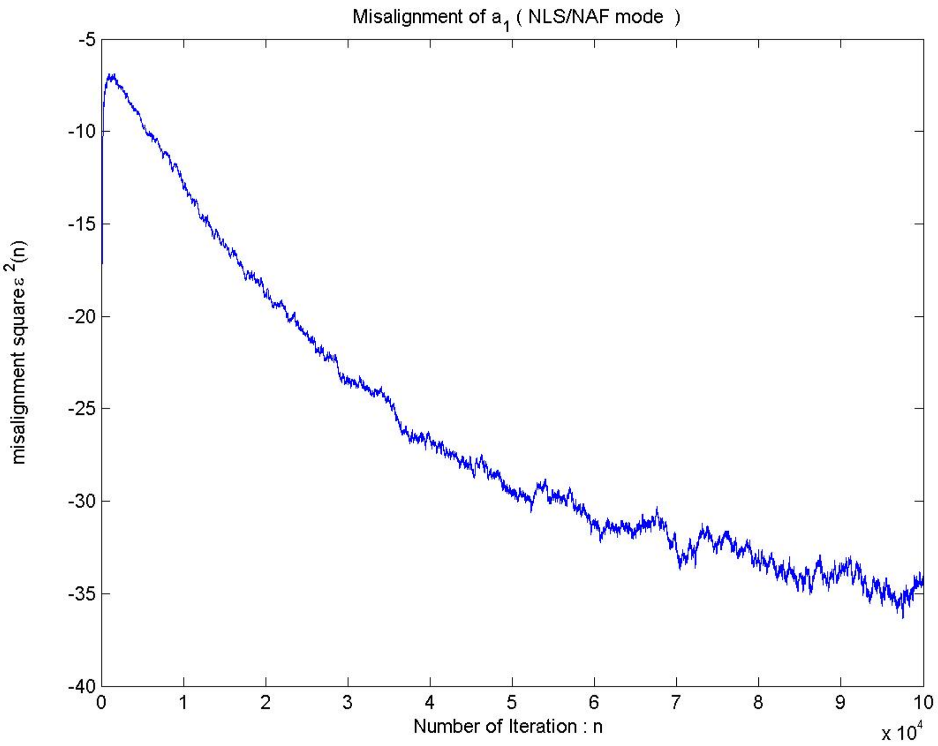

Figure 5.7.4 Misalignment of nonlinear coefficients $\underline{a}_1$ in NLS / NAF mode

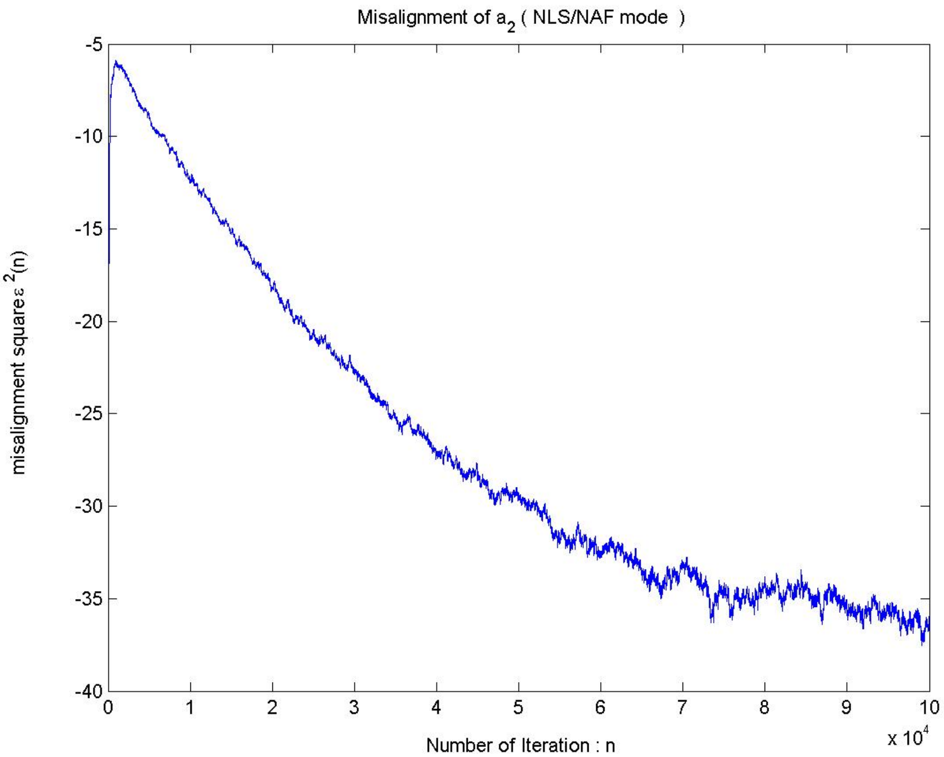

Figure 5.7.5 Misalignment of nonlinear coefficients $\underline{a}_2$ in NLS / NAF mode

For simplicity, the degree of nonlinearity is controlled by the Sigmoid function parameters (α, β) (the larger (α, β) pair the higher degree of nonlinearity is) in this simulation. Sigmoid function (like Figure 5.7.6, for $\alpha = 1,\ 2$ and 5, $\beta = 1$) defined as follow: The larger α the more linearity is.

$$f(x) = (\frac{2}{1 - \exp(-\alpha x)} - 1)\beta \tag{5.7.1}$$

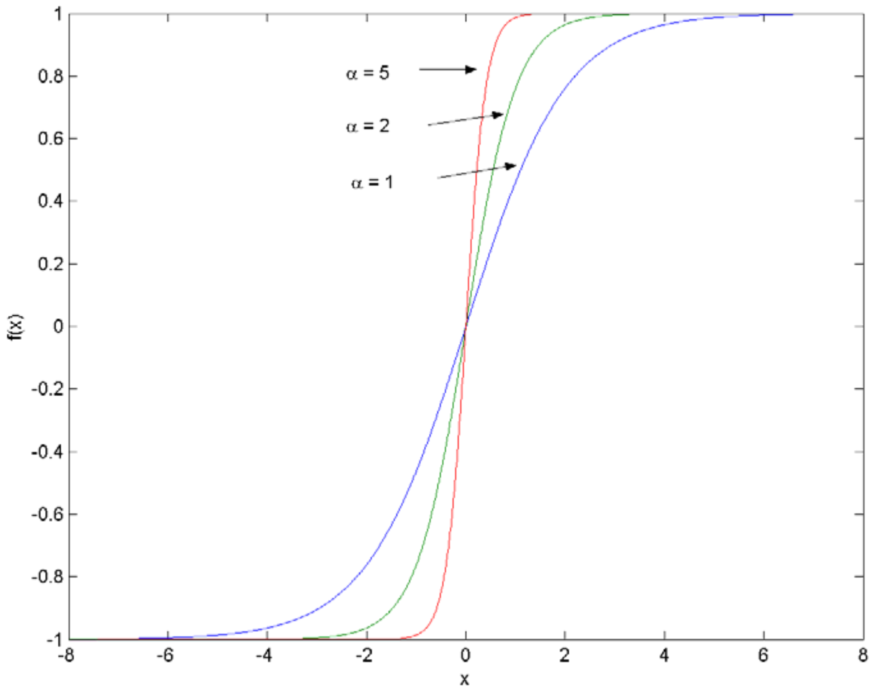

Figure 5.7.6 Sigmoid function ($\alpha = 1,\ 2$ and 5, $\beta = 1$)

Finally, the misalignment convergence of different nonlinear degree is shown as Figure 5.7.7.

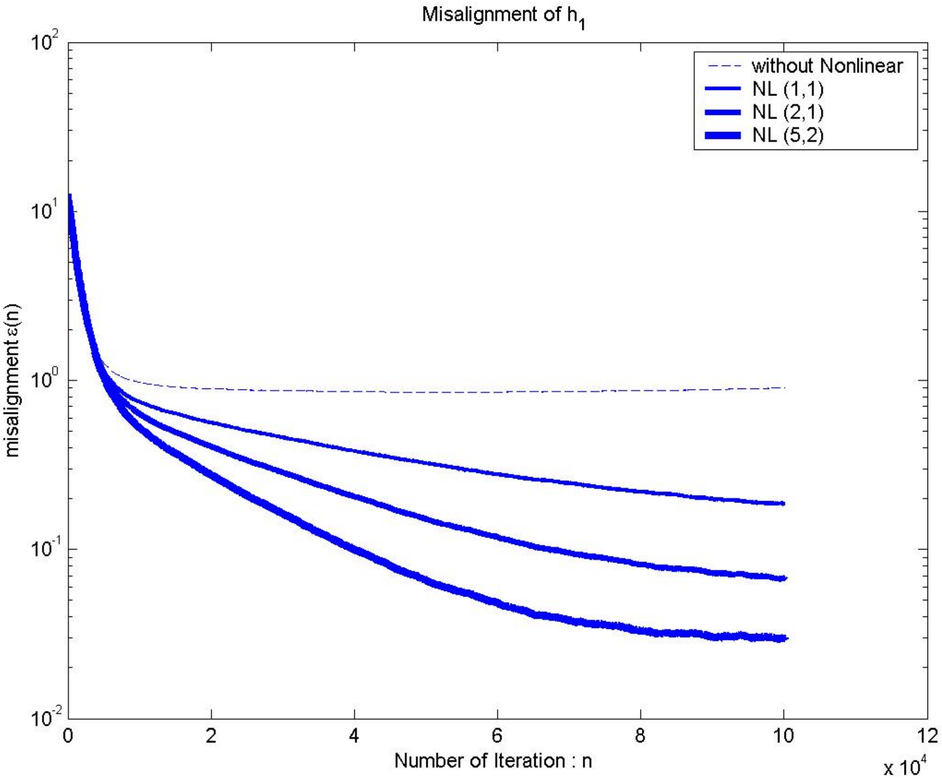

Figure 5.7.7 Misalignment of echo path in different nonlinear degree

Figure 5.7.7 tells us that the higher nonlinear degree the better misalignment convergence behavior we can have. But, there is one thing need to note for applying nonlinearities: nonlinear operation will generate annoying audible distortion to listener in near end room if the level of nonlinearities is too significant. The trade off between far end input signal coherence and audio quality is necessary when applying nonlinearities in stereophonic AEC.

Chapter 6

Conclusions

We have developed the linear and nonlinear coefficient error convergence analyses based on condition of perfect estimated coefficients for nonlinear AEC. The developed convergence analyses are well suited for nonlinear application where the scheme is a cascade mode in a memoryless polynomial filter and FIR filter order. We also have analyzed the joint coefficient error convergence analysis of nonlinear AEC in a semi-theory method. Although the results keep a small difference away from the simulated results in transient phase, they meet well in final convergence value of steady state fortunately. We have shown several adaptation strategies in nonlinear AEC. According to the result of comparisons between adaptation strategies, we make a recommendation for nonlinear AEC adaptation: linear coefficients have converged to some degree, before the nonlinear coefficients adaptation is enabled.

In stereophonic AEC partial update scheme, convergence analysis clarifies the behavior for correct echo path identification, which is decided not only by the interaction of two estimated linear filters but also by the transmission room impulse responses effect. The convergence analysis of stereophonic AEC partial update gives a well predict to simulated result in low SNR condition. We also have demonstrated the nonlinear loudspeaker effect in a stereophonic AEC and applied the nonlinear adaptive scheme in stereophonic AEC. It shows that the coherence can be reduced by the nature nonlinearity of loudspeaker and nonlinear adaptive scheme works well in nonlinear-stereophonic AEC situation.

The future work can be followings: (i) verify the cascaded polynomial nonlinear AEC scheme in a realistic nonlinear channel, i.e., real loudspeaker, (ii) modify the joint coefficient error convergence analysis of nonlinear AEC by a pure theory configuration, (iii) propose more accurate convergence analysis in stereophonic AEC partial update scheme when the SNR is higher, (iv) modify both schemes (nonlinear and partial adaptation stereophonic AEC) for real speech signals.

Appendix A

Approximation of $R_{\hat{\underline{s}}}(n)$

In this appendix, we will give the mathematical derivation for finding a simple form of $R_{\hat{\underline{s}}}(n)$. Let's start from the definition of $R_{\hat{\underline{s}}}(n)$.

$$R_{\hat{\underline{s}}}(n) = E\left[\hat{\underline{s}}(n)\hat{\underline{s}}^{T}(n)\right] \tag{A.1}$$

where
$$\hat{\underline{s}}(n) = X(n)\hat{\underline{a}}(n) = \begin{bmatrix} \hat{s}_0(n) \\ \hat{s}_1(n) \\ \vdots \\ \hat{s}_{L-1}(n) \end{bmatrix} \tag{A.2}$$

$\hat{\underline{s}}(n)$ is the nonlinear power input vector, produced by production between power input matrix $X(n)$ and estimated nonlinear coefficients $\hat{\underline{a}}(n)$. Owing to Eq.(A.1) and (A.2), correlation matrix of $\hat{\underline{s}}(n)$ (i.e. $R_{\hat{\underline{s}}}(n)$) is consists of same elements product part (diagonal elements) and different elements product part (other elements). For generality, the same elements product part can be represented as follows.

$$E\left[\hat{s}_i^2(n)\right] = E\left[\left(\hat{a}_1 x(n-i) + \hat{a}_2 x^2(n-i) + \cdots + \hat{a}_O x^O(n-i)\right)^2\right] \tag{A.3}$$

For simplicity consideration, let the nonlinear order O =3 which is the same as the nonlinear reference channel order. Eq.(A.3) can be rewritten as follows.

$$
\begin{aligned}
E\left[\hat{s}_i^2(n)\right] &= E\left[\left(\hat{a}_1(n)x(n-i)+\hat{a}_2(n)x^2(n-i)+\hat{a}_3(n)x^3(n-i)\right)^2\right] \\
&= E\left[\hat{a}_1^2(n)\right]E\left[x^2(n-i)\right]+E\left[\hat{a}_2^2(n)\right]E\left[x^4(n-i)\right]+E\left[\hat{a}_3^2(n)\right]E\left[x^6(n-i)\right] \\
&\quad +E\left[\hat{a}_1(n)\hat{a}_3(n)\right]E\left[x^4(n-i)\right] \\
&= a_1^2\overline{X^2}+\left(a_2^2+2a_1a_3\right)\overline{X^4}+a_3^2\overline{X^6} \\
&\quad +E\left[\tilde{a}_1^2(n)\right]\overline{X^2}+E\left[\tilde{a}_2^2(n)\right]\overline{X^4}+E\left[\tilde{a}_3^2(n)\right]\overline{X^6}
\end{aligned}
\tag{A.4}
$$

$where\ \hat{\underline{a}}(n)=\underline{a}+\tilde{\underline{a}}(n)\ \ and\ \ \overline{X^k}\triangleq E\left[x^k(n)\right]$

Eq.(A.4) can be easily gotten through the assumption $x(n)$ is zero mean and WGN signal. It also means that the diagonal terms of $R_{\hat{\underline{s}}}(n)$ are the same value given by Eq.(A.4). Similarly, the different elements product part will have the same value and it can be written as follows.

$$
\begin{aligned}
E\left[s_i(n)s_j(n)\right] = E\Big[&\left(\hat{a}_1x(n-i)+\hat{a}_2x^2(n-i)+\cdots+\hat{a}_Ox^O(n-i)\right) \\
&\times\left(\hat{a}_1x(n-j)+\hat{a}_2x^2(n-j)+\cdots+\hat{a}_Ox^O(n-j)\right)\Big] \quad i\neq j
\end{aligned}
\tag{A.5}
$$

Rearranging the Eq.(A.5) bye the assumption of the nonlinear order O =3.

$$
\begin{aligned}
E\left[s_i(n)s_j(n)\right] &= E\Big[\left(\hat{a}_1x(n-i)+\hat{a}_2x^2(n-i)+\hat{a}_3x^3(n-i)\right) \\
&\qquad \times\left(\hat{a}_1x(n-j)+\hat{a}_2x^2(n-j)+\hat{a}_3x^3(n-j)\right)\Big] \\
&= E\left[\hat{a}_2^2\right]E\left[x^2(n-i)\right]E\left[x^2(n-j)\right] \\
&= a_2^2\left(\overline{X^2}\right)^2+E\left[\tilde{a}_2^2(n)\right]\left(\overline{X^2}\right)^2
\end{aligned}
\tag{A.6}
$$

Eq.(A.6) is also the result of zero mean and WGN $x(n)$ assumption. Comparing diagonal terms in $R_{\hat{\underline{s}}}(n)$ (Eq.(A.4)) with non-diagonal terms in $R_{\hat{\underline{s}}}(n)$ (Eq.(A.6)), non-diagonal terms could be ignored under the conditions of $a_2 << a_1$ and $\left(\overline{X^4}\right)\geq\left(\overline{X^2}\right)^2$. Ignoring the correlation matrix of $\hat{\underline{s}}(n)$, $R_{\hat{\underline{s}}}(n)$ can be written as follows.

$$
\begin{aligned}
R_{\hat{\underline{s}}}(n) &\approx E\begin{bmatrix} \hat{s}_0^2(n) & 0 & 0 & \cdots & 0 \\ 0 & \hat{s}_1^2(n) & 0 & \cdots & 0 \\ 0 & 0 & \hat{s}_2^2(n) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \hat{s}_{L-1}^2(n) \end{bmatrix} \\
&= E\left[\hat{s}_i^2(n)\right] \cdot \boldsymbol{I}_L \qquad 0 \le i \le L-1 \\
&= \sigma_{\hat{s}}^2 \cdot \boldsymbol{I}_L
\end{aligned}
\tag{A.7}
$$

Eq.(A.7) is the simplified result of correlation matrix $R_{\hat{\underline{s}}}(n)$. In section 3.2, the nonlinear coefficients is assumed to be perfect (i.e. $\hat{\underline{a}}(n) = \underline{a}$ *and* $\tilde{\underline{a}}(n) = 0$). We can get the special case of Eq.(A.7) in section 3.2 as follows.

$$R_{\underline{s}}(n) = E\left[s_i^2(n)\right] \cdot \boldsymbol{I}_L = \sigma_s^2 \cdot \boldsymbol{I}_L \tag{A.8}$$

where $E\left[s_i^2(n)\right] = \sigma_s^2 = a_1^2\overline{X^2} + \left(a_2^2 + 2a_1a_3\right)\overline{X^4} + a_3^2\overline{X^6}$ (A.9)

Appendix B

Approximated Values of $E\left[\|\underline{u}(n)\|_2^2\right]$ and $E\left[\|\underline{u}(n)\|_2^4\right]$

Let's start from the definition of $\underline{u}(n)$.

$$\underline{u}(n) = X^T(n)\underline{h} = \begin{bmatrix} u_1(n) \\ u_2(n) \\ \vdots \\ u_O(n) \end{bmatrix} \tag{B.1}$$

and the 2-norm of $\underline{u}(n)$ can be written as

$$E\left[\|\underline{u}(n)\|_2^2\right] = E\left[\left\|X^T(n)\underline{h}\right\|_2^2\right] = E\left[u_1^2(n) + u_2^2(n) + \cdots + u_O^2(n)\right] \tag{B.2}$$

and

$$\begin{aligned} E\left[u_1^2(n)\right] &= E\left[\left(h_0 x(n) + h_1 x(n-1) + \cdots + h_{L-1} x(n-L+1)\right)^2\right] \\ &= \|\underline{h}\|_2^2 \, E\left[x^2(n)\right] \end{aligned} \tag{B.3}$$

In Eq.(B.3) the cross product terms have been eliminated through the $x(n)$'s assumptions of zero mean and WGN. Similarly, we can derivate the other terms like follows (O is set to be 3 which is equal to the reference channel nonlinear order.):

$$\begin{aligned} E\left[u_2^2(n)\right] &= E\left[\left(h_0 x^2(n) + h_1 x^2(n-1) + \cdots + h_{L-1} x^2(n-L+1)\right)^2\right] \\ &= \|\underline{h}\|_2^2 \, E\left[x^4(n)\right] + \sum_{i=0}^{L-1}\sum_{j\neq i}^{L-1} h_i h_j E^2\left[x^2(n)\right] \end{aligned} \tag{B.4}$$

and

$$E\left[u_3^2(n)\right]=E\left[\left(h_0x^3(n)+h_1x^3(n-1)+\cdots+h_{L-1}x^3(n-L+1)\right)^2\right]$$
$$=\|\underline{h}\|_2^2\,E\left[x^6(n)\right] \tag{B.5}$$

Substituting Eq.(B.3), Eq.(B.4) and Eq.(B.5) into Eq.(B.2) with the knowledge of O=3, we have:

$$E\left[\|\underline{u}(n)\|_2^2\right]=\left(\overline{X^2}+\overline{X^4}+\overline{X^6}\right)\|\underline{h}\|_2^2+\sum_{i=0}^{L-1}\sum_{j\neq i}^{L-1}h_ih_j\left(\overline{X^2}\right)^2 \tag{B.6}$$

$$where\quad \overline{X^k}\triangleq E\left[x^k(n)\right]$$

The summation of different linear coefficients cross product, $\sum_{i=0}^{L-1}\sum_{j\neq i}^{L-1}h_ih_j$, would be close to zero since the echo response has the exponential decayed characteristic. We can approximate the expected value of 2-norm linear power input vector like this:

$$E\left[\|\underline{u}(n)\|_2^2\right]\approx\left(\overline{X^2}+\overline{X^4}+\overline{X^6}\right)\|\underline{h}\|_2^2 \tag{B.7}$$

Now, take advantage of $x(n)$'s zero mean and WGN properties and the results mentioned above, we can find the approximate value of $E\left[\|\underline{u}(n)\|_2^4\right]$. Be definition:

$$E\left[\|u(n)\|_2^4\right]$$
$$=E\left[\left(u_1^2(n)+u_2^2(n)+u_3^2(n)\right)^2\right]$$
$$=E\left[u_1^4(n)+u_2^4(n)+u_3^4(n)+2\left(u_1^2(n)u_2^2(n)+u_1^2(n)u_3^2(n)+u_2^2(n)u_3^2(n)\right)\right] \tag{B.8}$$

According to Eq.(B.8), we only need to calculate the six value individually.

$$E\left[u_1^4(n)\right]=\left(3\sum_{i=0}^{L-1}h_i^4+3\sum_{i=0}^{L-1}\sum_{j\neq i}^{L-1}h_i^2h_j^2\right)\sigma_x^4 \tag{B.9}$$

$$E\left[u_2^4(n)\right]=\left[105\sum_{i=0}^{L-1}h_i^4+60\sum_{i=0}^{L-1}\sum_{j\neq i}^{L-1}h_i^3h_j+27\sum_{i=0}^{L-1}\sum_{j\neq i}^{L-1}h_i^2h_j^2+36(\prod_{i=0}^{L-1}h_i)(\sum_{j=0}^{L-1}h_j)\right]\sigma_x^8 \tag{B.10}$$

$$E\left[u_3^4(n)\right]=\left(10395\sum_{i=0}^{L-1}h_i^4+675\sum_{i=0}^{L-1}\sum_{j\neq i}^{L-1}h_i^2h_j^2\right)\sigma_x^{12} \tag{B.11}$$

$$E\left[u_1^2(n)u_2^2(n)\right]=\left[15\sum_{i=0}^{L-1}h_i^4+3\sum_{i=0}^{L-1}\sum_{j\neq i}^{L-1}h_i^2h_j^2+6\sum_{i=0}^{L-1}\sum_{j\neq i}^{L-1}h_i^3h_j+2\left(\sum_{j=0}^{L-1}h_j\right)\left(\prod_{i=0}^{L-1}h_i\right)\right]\sigma_x^6 \tag{B.12}$$

$$E\left[u_1^2(n)u_3^2(n)\right]=\left(105\sum_{i=0}^{L-1}h_i^4+33\sum_{i=0}^{L-1}\sum_{j\neq i}^{L-1}h_i^2h_j^2\right)\sigma_x^8 \tag{B.13}$$

$$E\left[u_2^2(n)u_3^2(n)\right]=\left[945\sum_{i=0}^{L-1}h_i^4+210\sum_{i=0}^{L-1}\sum_{j\neq i}^{L-1}h_i^3h_j+45\sum_{i=0}^{L-1}\sum_{j\neq i}^{L-1}h_i^2h_j^2+30\left(\sum_{j=0}^{L-1}h_j\right)\left(\prod_{i=0}^{L-1}h_i\right)\right]\sigma_x^{10} \tag{B.14}$$

Substituting Eq.(B.9)~ Eq.(B.14) into Eq.(B8) and rearranging it, we have:

$$\begin{aligned}E\left[\left\|\underline{u}(n)\right\|_2^4\right]=&\left(3\sigma_x^4+105\sigma_x^6+210\sigma_x^8+945\sigma_x^{10}+10395\sigma_x^{12}\right)\left(\sum_{i=0}^{L-1}h_i^4\right)\\&+\left(3\sigma_x^4+3\sigma_x^6+612\sigma_x^8+45\sigma_x^{10}+675\sigma_x^{12}\right)\left(\sum_{i=0}^{L-1}\sum_{j\neq i}^{L-1}h_i^2h_j^2\right)\\&+\left(6\sigma_x^6+60\sigma_x^8+210\sigma_x^{10}\right)\left(\sum_{i=0}^{L-1}\sum_{j\neq i}^{L-1}h_i^3h_j\right)\\&+\left(36\sigma_x^6+2\sigma_x^8+30\sigma_x^{10}\right)\left(\sum_{i=0}^{L-1}h_i\right)\left(\prod_{j=0}^{L-1}h_j\right)\end{aligned} \tag{B.15}$$

Similarly, the third and fourth term close to zero, and through this $E\left[\left\|u(n)\right\|_2^4\right]$ can be approximated by follows:

$$\begin{aligned}E\left[\left\|\underline{u}(n)\right\|_2^4\right]\approx&\left(3\sigma_x^4+105\sigma_x^6+210\sigma_x^8+945\sigma_x^{10}+10395\sigma_x^{12}\right)\left(\sum_{i=0}^{L-1}h_i^4\right)\\&+\left(3\sigma_x^4+3\sigma_x^6+612\sigma_x^8+45\sigma_x^{10}+675\sigma_x^{12}\right)\left(\sum_{i=0}^{L-1}\sum_{j\neq i}^{L-1}h_i^2h_j^2\right)\end{aligned} \tag{B.10}$$

Appendix C

Gaussian Assumption

In this appendix, we are trying to give a detail description on Gaussian assumption.

$$\mathbf{B} = E[\underline{x}(n)\underline{x}^T(n)\mathbf{A}\underline{x}(n)\underline{x}^T(n)] \tag{C.1}$$

where $\mathbf{A}$ represents the symmetric matrix $\underline{v}(n)\underline{v}^T(n)$.

The Gaussian assumption: If z_1, z_2, z_3, z_4 are real zero-mean, Gaussian random variables then

$$E[z_1 z_2 z_3 z_4] = E[z_1 z_2]E[z_3 z_4] + E[z_1 z_3]E[z_2 z_4] + E[z_1 z_4]E[z_2 z_3] \tag{C.2}$$

Using subscripts to denote the components of the vectors $\underline{x}(n)$ and $\underline{v}(n)$, where the dependency upon (n) is momentarily suppressed, we compute the kl th term in Eq.(C.1):

$$\begin{aligned} b_{kl} = \sum_{p=0}^{L-1}\sum_{q=0}^{L-1} E(x_k x_p a_{pq} x_q x_l) &= \sum_{p=0}^{L-1}\sum_{q=0}^{L-1} E[a_{pq}][E(x_k x_p)E(x_q x_l) \\ &\quad + E(x_k x_q)E(x_p x_l) + E(x_k x_l)E(x_p x_q)] \\ &= E[a_{kl}]\sigma_x^4 + E[a_{lk}]\sigma_x^4 + \delta[k-l]\sum_{p=0}^{L-1} E(a_{pp})\sigma_x^4 \end{aligned} \tag{C.3}$$

where $\sigma_x^2 = E[x_k^2]$. Note that x_k and a_{pq} are assumed to be independent, and $E(x_p x_q)$ is zero for $p \neq q$ since $\underline{x}(n)$ is WGN. We can obtain the matrix form of

Eq.(C.3) :

$$\mathbf{B} = \sigma_x^4[2\mathbf{R}_v(n) + trace(\mathbf{R}_v(n)) \cdot \mathbf{I}] \tag{C.4}$$

According to Eq.(C.4) we can get that:

$$E\left\{ \underline{s}(n)\underline{s}^T(n)\underline{\tilde{f}}_1(n)\underline{\tilde{f}}_1^T(n)\underline{s}(n)\underline{s}^T(n) \right\} = \sigma_s^4 \left\{ 2R_{\underline{\tilde{f}}_1}(n) + tr\left(R_{\underline{\tilde{f}}_1}(n) \right) \cdot I \right\} \tag{C.5}$$

where $\underline{\tilde{f}}_1(n)\underline{\tilde{f}}_1^T(n)$ is symmetric matrix.

$\underline{s}(n)$ and $\underline{\tilde{f}}_1(n)$ are independent , $s(n)$ is *WGN* signal.

By the other way, $\underline{\tilde{f}}_1(n)\underline{\tilde{f}}_2^T(n)$ is not symmetric matrix:

$$E\left\{ \underline{s}(n)\underline{s}^T(n)\underline{\tilde{f}}_1(n)\underline{\tilde{f}}_2^T(n)\underline{s}(n)\underline{s}^T(n) \right\} = \sigma_s^4 \left\{ R_{\underline{\tilde{f}}_1\underline{\tilde{f}}_2}(n) + R_{\underline{\tilde{f}}_2\underline{\tilde{f}}_1}(n) + tr\left(R_{\underline{\tilde{f}}_1\underline{\tilde{f}}_2}(n) \right) \cdot I \right\} \tag{C.6}$$

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