Analysis of pumping test data for determining unconfined-aquifer parameters: Composite analysis or not?

Hund-Der Yeh · Yen-Chen Huang

Abstract Recently, composite analysis (CA), which simultaneously analyzes all drawdown data from multiple observation wells, has been applied to determine the hydraulic parameters of an unconfined aquifer. Moench (1994) claimed that the value of specific yield (S_y) determined from non-composite analysis (nonCA) is sometimes unrealistically low as compared with that obtained by water-balance calculation, and results from CA are better representative of aquifer properties than those from nonCA. To examine the validity of this assertion, the drawdown data from a pumping test conducted at Cape Cod, Massachusetts, USA, were analyzed using both nonCA and CA methods. The results show that the mean estimates of hydraulic conductivity and S_y determined from CA are close to those determined from nonCA. In some cases the analysis based on CA also results in low estimates of S_y as compared with those determined based on nonCA. A hypothetical case study is presented, which examines the effect of measurement errors on the estimated parameters. The results indicate that the CA method also gives poorer estimates of S_y than the nonCA method if the pumping test data contain measurement errors. Moench AF (1994) Specific yield as determined by type-curve analysis of aquifer-test data. Ground Water, 32(6):949–957.

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Introduction

Groundwater hydrologists often conduct pumping tests and data analyses to obtain aquifer parameters which are necessary information for quantitative groundwater studies. For an unconfined aquifer system, the analysis of pumping test data is slightly complicated because the transient drawdown curve exhibits three segments in response to the pumping. During the first segment, which occurs at early pumping time, water is released from storage instantaneously. During the second segment, the vertical gradient near the water table induces drainage of the porous matrix and, consequently, the decline rate of the hydraulic head slows down and may even stop after a period of time. Finally, when the flow is essentially horizontal, most of the pumping is supplied by the specific yield, S_y, in the third segment. Boulton (1954, 1963) developed an analytical solution for the unconfined-aquifer flow equation by introducing the concept of delayed yield. Prickett (1965) described a systematic approach for the determination of hydraulic parameters using a graphical procedure based on Boulton's method. Cooley and Case (1973) indicated that Boulton's equation described a flow system with a rigid phreatic aquitard on top of the main aquifer where the effect of the unsaturated zone above the phreatic surface was neglected. Neuman (1972, 1974) developed a solution that considers the effects of elastic storage and anisotropy of aquifers on the drawdown behavior. Neuman (1975) also gave a graphical type-curve match procedure to determine the hydraulic parameters of unconfined aquifers. Moench (1995) combined the Boulton and Neuman models for flow toward a well in an unconfined aquifer. He used a non-physical parameter α_1 to represent the delayed decline of the water table during pumping. Grimestad (2002) reanalyzed transient drawdown data from two pumping tests conducted in unconfined aquifers, one at Cape Cod, Massachusetts, USA (Moench 1994) and the other at Borden, Texas, USA (Nwankwor et al. 1984). Grimestad (2002) concluded that a portion of the water pumped from the aquifers was derived from other sources. Zhan and Zlotnik (2002) discussed how a solution for flow to a horizontal or slanted well in an unconfined aquifer can be obtained. Hunt (2006) used a meaningful aquifer parameter instead of an empirical constant in the equation of Zhan and Zlotnik (2002) to describe flow to a well when a number

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of overlying aquitards exist between the pumped aquifer and free surface.

In the past, the pumping drawdown data obtained from a single observation well were commonly used for the aquifer parameter determination. Based on Boulton's solution for large-time data, Mania and Sucche (1978) employed the least-squares approach to determine the unconfined-aquifer parameters. Followed the concept of Ferris et al. (1962), Moench (1994) employed the composite analysis (CA) and graphical type-curve method (called composite plot) to determine the specific yield. He pointed out that the S_y determined from analyzing the drawdown data from a single observation well were sometimes unrealistically low as compared with those determined by water-balance calculations from field data or controlled laboratory experiments performed on samples of aquifer material. He interpreted that the unrealistically low values of S_y were generally caused by (1) improper procedures, (2) bad data, or (3) aquifer heterogeneity. Moreover, he showed that the effect of partial penetration should be included in the analysis of the drawdown data and the composite plot has to be used with a single match point for all measured drawdown data. Finally, Moench concluded that the determination of S_y using CA is consistent with that obtained by field water-balance calculation in a relatively homogeneous, unconfined aquifer. Based on the drawdown data from multiple observation wells, Heidari and Moench (1997) determined the best-fit parameters using the nonlinear least squares approach instead of the composite plot.

Meier et al. (1998) used Cooper and Jacob's solution (Cooper and Jacob 1946) to determine effective transmissivity values based on the analysis of synthetic drawdown data in a confined aquifer with heterogeneous transmissivity and homogeneous storativity. The results indicated that the transmissivity values determined from the analysis of simulated drawdowns from individual observation wells are all very close to the effective transmissivity value. Wu et al. (2005) presented two approaches (distance drawdown and spatial moment analyses) in determining effective transmissivity and storage coefficient in a synthetically heterogeneous confined aquifer. The results indicated that the estimate of transmissivity needed long pumping time to converge its geometric mean. Wu et al. (2005) concluded that the analyzed results using the drawdown data from a single observation well may be difficult to interpret because of the heterogeneity of the aquifers. Illman and Neuman (2001, 2003) and Illman and Tartakovsky (2005) analyzed a series of cross-hole air injection tests conducted in unsaturated fractured tuffs. The type-curve, steady-state, and asymptotic analyses were used to determine the equivalent permeability and porosity of fractured tuffs. The results showed that the geometric mean of the permeability obtained by analyzing cross-hole measurements is larger by a factor 50 than that obtained by single-hole pneumatic injection tests.

Chen and Ayers (1998) determined four parameters of the unconfined aquifer based on both Neuman's (1974) and Moench's (1995) analytical solutions. They first analyzed the drawdown data from individual wells (also called nonCA in this study), and the data sets from two or more wells were then combined into a large data set and analyzed simultaneously. In the Chen and Ayers (1998) study, observation wells along a line or randomly chosen were analyzed by CA. The results indicated that the parameter α_1 in Moench's solution (1995) for representing the delay yield might be not important and difficult to determine properly in the analysis of their pumping test data. Moreover, they found that the value of S_y determined from CA might be lower than the normal range for sand and gravel of the test site. Kollet and Zlotnik (2005) presented the analysis of transient drawdown data from a pumping test. The results showed that highly uncertain and physically unrealistic estimates of S_y and vertical hydraulic conductivity (K_z) might be due to the heterogeneity of the aquifer and the return flow of the test. They suggested that both the nonCA and CA analyses were necessary for analyzing the pumping test data in examining the consistency and reliability of parameter estimates. Tartakovsky and Neuman (2007) developed an analytical solution for the delayed response process characterizing flow to a partially penetrating well in an unconfined aquifer. The solution generalized those of Neuman (1972, 1974) by accounting for unsaturated flow above the water table. The field data from Cape Cod (Moench et al. 2001) were analyzed using both CA and nonCA. The results indicate that the estimates of S_y and storage are respectively smaller and larger than those of Moench et al. (2001).

Simulated annealing (SA) is a stochastic technique for solving optimization problems. SA was first proposed by Kirkpatrick et al. (1983) as a method for solving combinatorial optimization problems. Subsequently, utilization of SA in optimization problems has been widely applied in hydrological engineering. For example, this method has been employed to design the strategies of groundwater remediation (Dougherty and Marryott 1991; Marryott et al. 1993), determine the hydraulic parameters of aquifers (Yeh et al. 2007a; Yeh and Chen 2007), and identify the groundwater contamination source (Yeh et al. 2007b).

The objective of this study is to examine whether aquifer parameter determination using CA with different spatial distribution and types of observation wells gives a better or higher estimate of S_y than nonCA in analyzing pumping test data obtained from unconfined aquifers. The computer method based on simulated annealing and Neuman's solution (Neuman 1974) (also called SANS) (Huang et al. 2008) is used to determine the unconfined-aquifer parameters of the field case and hypothetical case studies using both CA and nonCA approaches. In the field case study, the drawdown data sets from each of 20 observation wells are first analyzed using nonCA and then these wells are classified into seven groups to examine whether the spatial distribution and the type of observation wells affect the results of CA. In the hypothetical case study, a pumping test which has one pumping well and two observation wells is conducted. The synthetic transient drawdown data are generated based on Neuman's solution (Neuman 1974). In each observation well, 20 sets of noise generated from normal distribution with zero mean and variance of 10^{-4} are first added to the drawdown data and then the nonCA and CA are employed to determine the hydraulic parameters. The purpose of this case study is to investigate the effect of measurement errors on the parameter estimation. This article provides extensive case studies which may be helpful in choosing the right method, CA or nonCA, for analyzing field pumping test data.

Methodology and case studies

Methodology of non-composite and composite analyses

The aquifer parameters can be determined by the least-squares approach when minimizing the sum of square residuals between the observed and predicted drawdowns. That is

$$\text{Minimize} \sum_{i=1}^{n} (Oh_i - Ph_i)^2 \tag{1}$$

where Oh_i and Ph_i are respectively the observed and predicted drawdowns at different time steps and n is the total number of observed drawdown data. Heidari and Moench (1997) suggested using observed drawdown data obtained from multiple wells to simultaneously determine the best-fit aquifer parameters. The objective function they used for a CA is defined as

$$\text{Minimize} \sum_{j=1}^{\mathrm{nw}} \sum_{i=1}^{n} \left(Oh_{i,j} - Ph_{i,j}\right)^2 \tag{2}$$

where nw is the number of observation wells and n is the number of observed drawdown data at each well.

The computer method based on simulated annealing and Neuman's solution (SANS)

Simulated annealing (SA) is known as an optimization algorithm for simulating a material crystallized in the process of annealing. The arrangement of the material molecules is initially disordered at high temperature. The system is gradually cooled; meanwhile, the arrangement becomes more ordered and the system approaches a thermodynamic equilibrium. Based on this concept, the solution, which may not be the best one, is accepted to avoid the solution becoming trapped in a local optimum during the optimization procedure. The details of the SA can be found in Huang et al. (2008).

The analytical solution (Neuman 1974), considering the effects of delayed yield and well partial penetration describing the groundwater flow system in an unconfined aquifer, is

$$s(r,z,t) = \frac{q}{4\pi T} \int_0^\infty 4yJ_0\left(y\beta^{1/2}\right) \left[u_0(y) + \sum_{n=1}^{\infty} u_n(y)\right] dy \tag{3}$$

where q is pumping rate, J_0 (x) is the zero order Bessel function of the first kind, $\beta = K_z r^2 / K_r b^2$ is a dimensionless parameter, K_r is the horizontal hydraulic conductivity, r is the distance between pumping well and observation well, b is the thickness of the aquifer, y is a dummy variable. The functions $u_0(y)$ and $u_\mathrm{n}(y)$ are respectively defined as

$$u_0(y) = \frac{\left\{1-\exp\left[-t_\mathrm{s}\beta\left(y^2-r_0^2\right)\right]\right\}\cosh(r_0 z_\mathrm{D})}{\left[y^2+(1+\sigma)r_0^2-\left(y^2-r_0^2\right)^2/\sigma\right]\cosh(r_0)} \cdot \frac{\sinh[r_0(1-d_\mathrm{D})]-\sinh[r_0(1-l_\mathrm{D})]}{(l_\mathrm{D}-d_\mathrm{D})\sinh(r_0)} \tag{4}$$

$$u_\mathrm{n}(y) = \frac{\left\{1-\exp\left[-t_\mathrm{s}\beta\left(y^2+r_\mathrm{n}^2\right)\right]\right\}\cos(r_\mathrm{n} z_\mathrm{D})}{\left[y^2-(1+\sigma)r_\mathrm{n}^2-\left(y^2+r_\mathrm{n}^2\right)^2/\sigma\right]\cos(r_\mathrm{n})} \cdot \frac{\sin[r_\mathrm{n}(1-d_\mathrm{D})]-\sin[r_\mathrm{n}(1-l_\mathrm{D})]}{(l_\mathrm{D}-d_\mathrm{D})\sin(r_\mathrm{n})} \tag{5}$$

where $t_s = Tt/Sr^2$ denotes the dimensionless time since pumping started, T is transmissivity which equals $K_r \times b$, S is the storage coefficient, t represents the pumping time, $d_\mathrm{D}=d/b$ represents the dimensionless vertical distance between the top of perforation in the pumping well and the initial position of the water table, and $l_\mathrm{D}=l/b$ is the dimensionless vertical distance between the bottom of perforation in the pumping well and the initial position of the water table. The variables of r_0 and r_n are respectively the roots of the following two equations

$$\sigma r_0 \sinh(r_0) - \left(y^2 - r_0^2\right)\cosh(r_0) = 0,\ r_0^2 < y^2 \tag{6}$$

and

$$\sigma r_\mathrm{n} \sin(r_\mathrm{n}) + \left(y^2 + r_\mathrm{n}^2\right)\cos(r_\mathrm{n}) = 0,\ (2n-1)(\pi/2) < r_\mathrm{n} < n\pi \tag{7}$$

The SANS approach was employed to determine the hydraulic parameters of the unconfined aquifer when using Eqs. (1) and (3) for nonCA and Eqs. (2) and (3) for the CA.

Description of the field case study

The site of Cape Cod shown in Fig. 1 was selected for the study. The aquifer system consists of unconsolidated glacial outwash sediments deposited during the recession, 14,000–15,000 years BP, of the late Wisconsinan continental ice sheet (Moench et al. 2001). The depth of the pumping well is 24.4 m below the ground surface. The top and bottom of the screen are located 4.0 and 18.3 m, respectively, below the initial water table, which is ~5.8 m below ground surface. The aquifer-saturated thickness is ~48.8 m. The total number of the observation wells is 20, which includes three sets of well clusters. The clusters F504 and F505 both have three observation wells each where the piezometers are separately located at shallow, mid-depth, and deep-depth. Details with regard to the

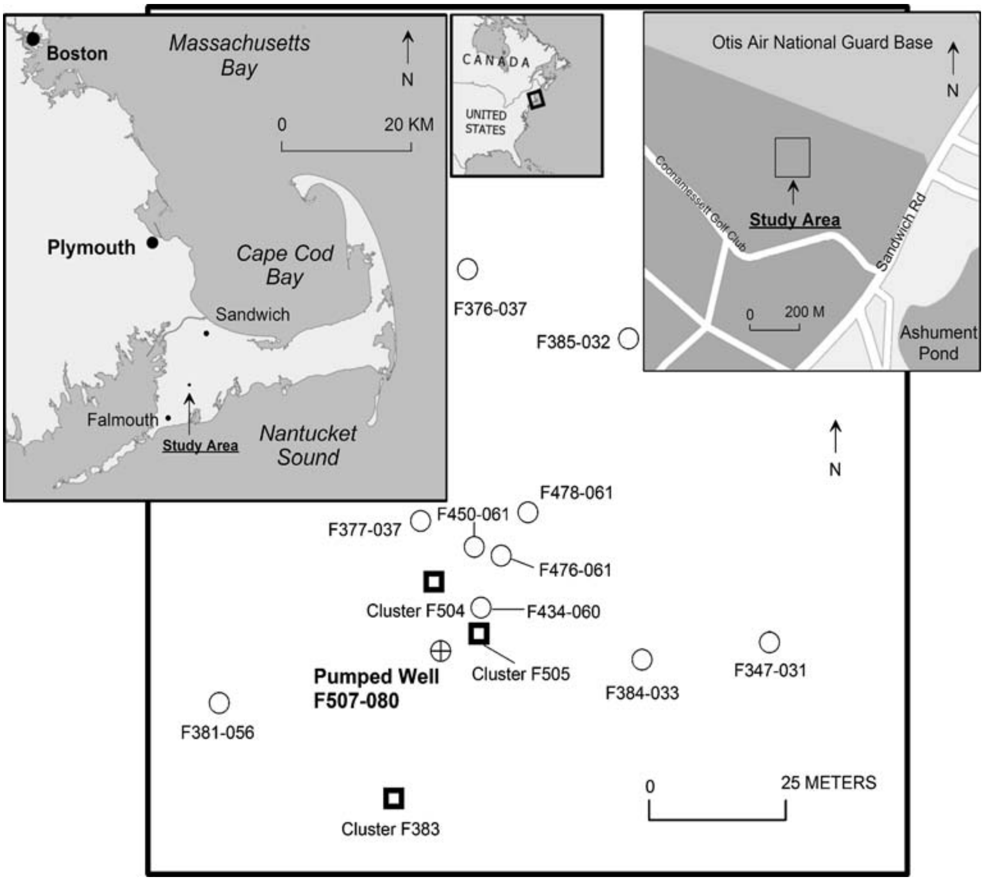

Fig. 1 The study site in Cape Cod, MA, USA

exact radial and vertical positions, the lengths of the well screens, and the characteristics of the observation wells are given in Table 1 (Moench et al. 2001). The terms PW, SP, MP, DP, LS, SDT, and LDT represent the pumping well, shallow-depth piezometer, mid-depth piezometer, deep-depth piezometer, long-screened well, short radial distance between pumping well and observation well, and long radial distance between pumping well and observation well, respectively. The farthest observation well (F376-037) is located 69.37 m from the pumping well. The structure and distribution of the piezometers in the observation wells were listed in a table of Moench et al. (2001). Well F507-080 was pumped (1.21 m^3/min) for 72 h. In the case study, 20 observation wells are classified into the following seven groups based on the number of wells used, the length of well screen, and the distance from the pumping well to the observation well:

1. Two wells which are randomly chosen from 20 observation wells
2. A well cluster which consists of two or three observation wells
3. Three or four wells with different radial distances from the pumping well
4. Three or four wells with long screens
5. Three or four wells with different depth of piezometers
6. Two or three well clusters
7. Twenty wells (global analysis)

The type of observation for the hydraulic head of the aquifer includes the piezometer and the observation well with different length of screen. Accordingly, there are 50, 3, 2, 2, 5, and 4 cases for groups 1–6, respectively, analyzed by the present method of CA.

The hypothetical case study

A synthetic pumping data set generated based on Neuman's solution (Neuman 1974) was used to explore the effect of measurement errors on the estimated parameters. The pumping test was conducted with a constant pumping rate of 1,000 m^3/day in an unconfined aquifer of 10 m thickness. Two observation wells were installed, observation well one (OW1) was located at 10 m away from the pumping well, while the second (OW2) was located 30 m. The depth of the pumping well was 10 m and the screen length was 5 m where the top of the screen was 5 m below the initial water table. The depth of the two observation wells was 6 m and the screen length was 1 m where the top of the screen was the same as that of the pumping well. The pumping period was 1,000 min and the total number of drawdown data was 48 in each observation well. The "true" parameter values are: $K_r=10^{-3}$ m/s, $K_z=10^{-4}$ m/s, $S=10^{-4}$, and $S_y=10^{-1}$. Forty sets of noise were first generated by the routine RNNOF of IMSL (1997), which produces normally distributed random numbers

Table 1 Locations of the observation wells

Well No.	Radial distance[a] (m)	Depth[b] (m)	Number of observed data	Screen length (m)	Type of well[c]
F507-080	0.10	4.02	31	14.33	PW
F505-032	7.28	3.26	32	0.61	SP, SDT
F505-059	5.94	9.33	32	2.74	MP, SDT
F505-080	6.58	17.80	33	0.61	DP, SDT
F504-032	14.20	2.93	31	0.61	SP, SDT
F504-060	15.18	9.14	33	2.74	MP
F504-080	16.18	17.53	32	0.61	DP
F377-037	25.94	4.05	30	0.61	SP
F383-032	28.35	3.69	18	0.61	SP
F383-061	28.32	12.16	24	0.61	MP
F383-082	28.90	18.84	18	0.61	DP
F383-129	29.47	32.92	17	0.61	DP
F384-033	41.85	4.82	23	0.61	SP
F381-056	48.71	6.10	20	0.61	SP, LDT
F347-031	68.79	4.51	18	0.61	SP, LDT
F434-060	11.77	0.61	15	11.89	LS
F450-061	20.21	0.52	16	11.89	LS
F476-061	19.99	0.67	16	11.89	LS
F478-061	30.88	0.67	16	11.89	LS
F385-032	68.46	3.05	17	0.61	SP, LDT
F376-037	69.37	4.02	20	0.61	SP, LDT

[a] Distance from the center of pumping well

[b] Depth below the initial water table to the top of the screen

[c] *PW* pumping well; *SP* shallow-depth piezometers; *MP* mid-depth piezometers; *DP* deep-depth piezometers; *LS* long-screened well; *SDT* short radial distance between pumping well and observation well; *LDT* long radial distance between pumping well and observation well

with zero mean and unit variance. They were then divided by 100 to represent the measurement errors with the magnitude on the order of centimeter. In each observation well, twenty sets of noise were added into the drawdown data and the SANS was employed to determine the hydraulic parameters based on nonCA. Then the drawdown data from two observation wells were analyzed based on CA.

Table 2 The parameters determined from the SANS using nonCA (20 cases)

Well No.	K_r (m/min) $\times 10^{-2}$	K_z (m/min) $\times 10^{-2}$	S $\times 10^{-3}$	S_y $\times 10^{-1}$	SEE $\times 10^{-3}$ (m)
F505-032	8.58	0.98	8.38	0.65	7.06
F505-059	7.66	3.25	5.23	1.76	8.55
F505-080	7.42	3.40	5.39	2.01	7.02
F504-032	8.99	1.51	8.42	0.91	5.02
F504-060	8.31	2.99	4.80	1.29	3.80
F504-080	8.12	3.27	5.29	1.60	3.54
F377-037	9.11	2.17	8.66	1.44	2.65
F383-032	9.46	1.08	34.70	1.90	3.64
F383-061	7.96	2.76	7.34	1.89	3.34
F383-082	7.99	2.16	9.45	1.67	2.05
F383-129	8.05	1.88	5.11	1.11	1.82
F384-033	8.59	1.72	7.14	1.96	2.82
F381-056	9.77	1.67	3.87	1.47	3.36
F347-031	9.61	1.05	26.80	1.73	1.53
F434-060	7.90	2.26	61.80	1.60	4.02
F450-061	8.90	1.74	41.50	1.52	3.41
F476-061	8.85	1.74	57.30	1.36	4.36
F478-061	9.43	1.60	51.10	1.51	2.21
F385-032	11.79	0.94	2.29	1.73	2.34
F376-037	13.10	0.96	5.90	1.32	7.06
Mean	8.98×10^{-2}	1.96×10^{-2}	1.80×10^{-2}	1.52×10^{-1}	
SD	1.38×10^{-2}	8.10×10^{-3}	1.98×10^{-2}	3.49×10^{-2}	
CV	0.15	0.41	1.10	0.23	
Mean ($\log_{10}$)	−1.05	−1.75	−1.97	−0.83	
SD ($\log_{10}$)	0.06	0.19	0.44	0.12	

SEE standard error of estimated; *SD* standard deviation; *CV* coefficient of variation

Results and discussion

Field case study

Non-composite analysis

Table 2 lists four estimated parameters for the pumping test data obtained from those 20 observation wells based on nonCA. Note that the standard error of the estimate (SEE) listed in the last column of Table 2 is defined as $\left(\sum_{j=1}^{n} e_j^2/v\right)^{1/2}$, where e_j represents the difference between the observed drawdown and the drawdown predicted by Neuman's solution (Neuman 1974) with the estimated parameters, and v, the degree of freedom, is equal to the number of observed data points minus the number of unknowns (Yeh 1987). The estimated K_r ranges from 7.42×10^{-2} to 1.31×10^{-1} m/min. The K_z is about half or one order of magnitude less than K_r. Note that the estimated S at observation wells with long screen length (F434-060, F450-061, F476-061, and F478-061) is about one order larger than that of other observation wells. The lowest S_y (6.50×10^{-2}) is obtained from the well F505-032 and the highest one (2.01×10^{-1}) is from the well F505-080 as shown in Table 2. In addition, the estimated S_y is larger than 0.1 for all wells except the wells F505-032 and F504-032. The SEE values range from 1.53×10^{-3} to 8.55×10^{-3} m, indicating that the nonCA can accurately determine the hydraulic parameters. The last five rows of Table 2 display the mean (i.e., arithmetic mean), standard deviation (SD), coefficient of variation (CV), mean of logarithmic parameter values (Mean($\log_{10}$)), and the SD of logarithmic parameter values, SD ($\log_{10}$), of each parameter. The CV, defined as the ratio of the SD to the mean, is a measure of dispersion of a probability distribution. The SDs of estimated parameters demonstrate that the estimate of parameter S has high variation at the test site. Figure 2 displays the estimated parameters K_r, K_z, S, and S_y versus the radial distance between pumping well and observation well. Those results indicate that estimated parameters are quite uncorrelated with the distance between pumping well and observation well. Figure 3 shows the observed drawdown and the predicted drawdown curve drawn based on the parameters of K_r= 7.42×10^{-2} m/min, K_z=3.40×10^{-2} m/min, S=5.39×10^{-3}, and S_y=2.01×10^{-1} obtained from analyzing the data of well

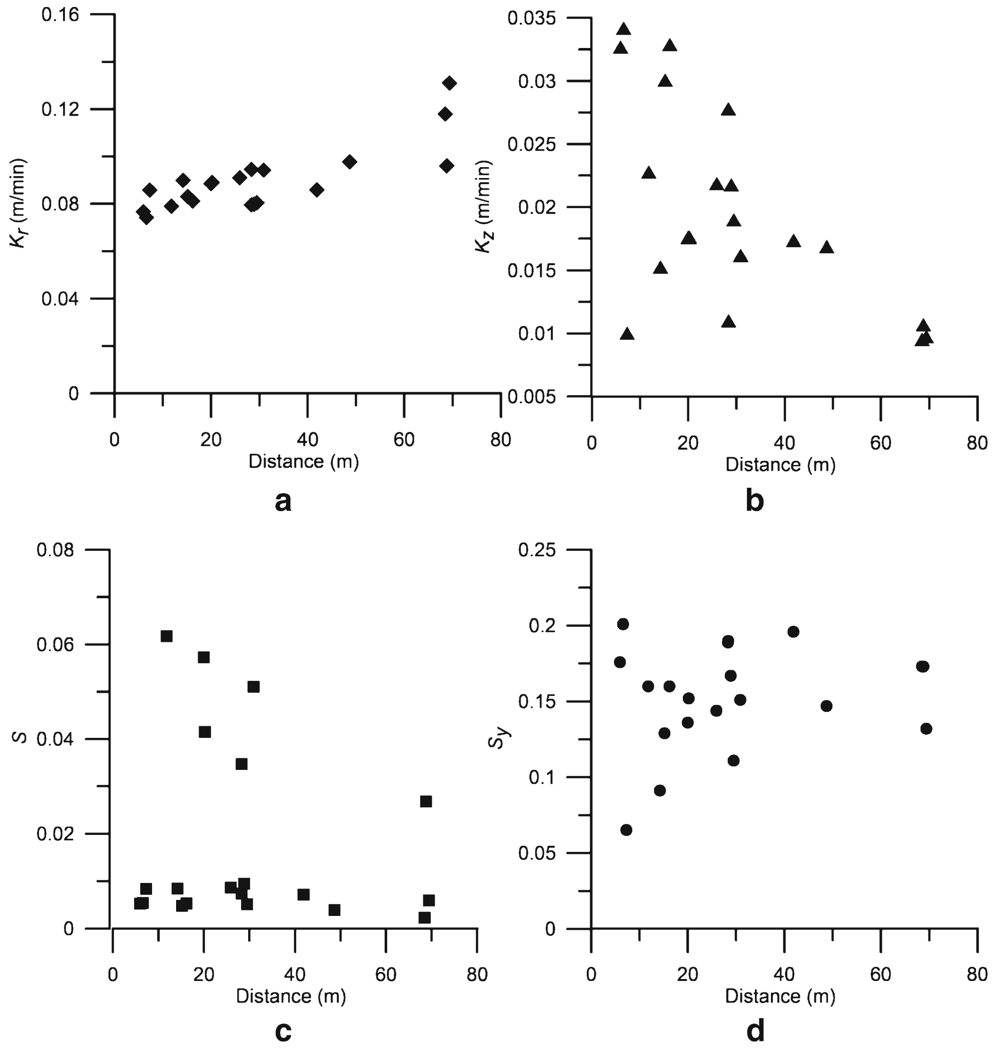

Fig. 2 The estimated parameters of **a** K_r **b** K_z **c** S and **d** S_y versus the radial distance between pumping and observation wells

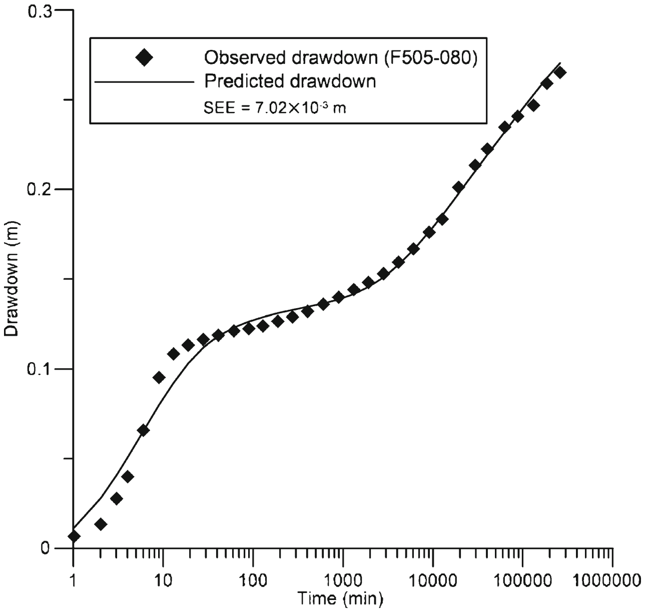

Fig. 3 The drawdown curve predicted by Neuman's solution (Neuman 1974) with parameters determined from the nonCA for well F505-080

F505-080 using the SANS of nonCA. This figure demonstrates that the predicted drawdown curve fits the observed data quite well and thus the aquifer parameters are accurately determined.

Composite analysis

The drawdown data of the seven groups were employed to investigate whether the spatial distribution and the type of observation wells affect the results of CA. Table 3 shows 50 cases which are divided into three sets for group 1 (two observation wells which are randomly chosen from 20 observation wells) and their drawdown data were simultaneously analyzed based on CA. Sets 1–3 represent situations where the S_y determined by the CA is smaller than, falls between, and larger than S_y of nonCA, respectively. The total number of cases analyzed in each of these three sets is respectively 2, 38, and 10 and the estimated parameters are displayed in Tables 4, 5, and 6, for sets 1–3, respectively. Note that the statistics of nonCA results are also listed at the bottom of Tables 4–6 for the comparison of nonCA and CA results. The means of estimated K_r determined from CA listed in Tables 4–6 are 9.31×10^{-2}, 8.30×10^{-2}, and 8.10×10^{-2} m/min, respectively. Those values are very close to that determined from nonCA, i.e., 8.98×10^{-2} m/min. This may be due to the fact that the aquifer is quite homogeneous in the horizontal direction (Masterson et al. 1997). Note that the SD is not given in Table 4 since there are only two case results. In these three sets, the estimated S_y ranges from 7.40×10^{-2} to 2.27×10^{-1} and the mean of each set is 1.36×10^{-1}, 1.46×10^{-1}, and 1.60×10^{-1}, respectively. The S_y determined from nonCA shown in Table 2 ranges from 6.50×10^{-2} to 2.01×10^{-1} with a mean of 1.52×10^{-1}. Obviously, the mean of S_y obtained from nonCA is not significantly different from those of three sets determined from CA. Similarly, the means and SDs of logarithmic parameter values obtained from the results of CA listed in Tables 4–6 are also close to those determined from the results of nonCA. These results demonstrate that the average values of parameters K_r and S_y analyzed by the SANS of CA and nonCA are close. Notice that the results

Table 3 The estimated S_y in well group 1 determined from CA in comparison to those from nonCA

Composite wells		
Set 1[a], number of cases: 2		
F377-037, F381-056	F476-061, F376-037	
Set 2[b], number of cases: 38		
F505-032, F505-080	F505-032, F383-129	F434-060, F476-061
F381-056, F347-031	F505-032, F384-033	F450-061, F476-061
F383-032, F383-061	F505-032, F377-037	F476-061, F478-061
F383-032, F384-033	F505-032, F450-061	F504-060, F434-060
F383-032, F505-080	F504-032, F504-060	F504-060, F450-061
F383-061, F383-082	F504-060, F383-032	F505-059, F434-060
F384-033, F381-056	F504-060, F504-080	F377-037, F505-080
F385-032, F376-037	F504-060, F505-080	F381-056, F505-080
F504-032, F383-129	F505-032, F434-060	F505-080, F504-080
F504-032, F504-080	F505-032, F504-060	F505-059, F450-061
F505-032, F383-032	F505-032, F505-059	F505-059, F476-061
F505-032, F383-061	F505-059, F505-080	F505-059, F504-060
F505-032, F383-082	F434-060, F450-061	
Set 3[c], number of cases: 10		
F383-129, F504-060	F505-059, F376-037	F504-060, F476-061
F385-032, F505-080	F434-060, F478-061	F504-060, F478-061
F505-032, F376-037	F450-061, F478-061	F505-059, F478-061
F505-032, F504-032		

[a] S_y is smaller than that of two nonCA cases

[b] S_y falls between that of two nonCA cases

[c] S_y is larger than that of two nonCA cases

Table 4 The estimated parameters of set 1 in Table 3 (two cases)

	K_r (m/min) $\times 10^{-2}$	K_z (m/min) $\times 10^{-2}$	$S \times 10^{-3}$	$S_y \times 10^{-1}$
F377-037, F381-056	9.35	1.96	8.67	1.42
F476-061, F376-037	9.27	1.53	57.90	1.29
Mean	9.31×10^{-2}	1.75×10^{-2}	3.33×10^{-2}	1.36×10^{-1}
CV	6.08×10^{-3}	1.74×10^{-1}	1.05	6.78×10^{-2}
Mean ($\log_{10}$)	−1.03	−1.76	−1.65	−0.87
Statistics of nonCA results				
Mean	8.98×10^{-2}	1.96×10^{-2}	1.80×10^{-2}	1.52×10^{-1}
SD	1.38×10^{-2}	8.10×10^{-3}	1.98×10^{-2}	3.49×10^{-2}
CV	0.15	0.41	1.10	0.23
Mean ($\log_{10}$)	−1.05	−1.75	−1.97	−0.83
SD ($\log_{10}$)	0.06	0.19	0.44	0.12

SD standard deviation; *CV* coefficient of variation

Table 5 The estimated parameters of set 2 in Table 3 (38 cases)

Well No.	K_r (m/min) $\times 10^{-2}$	K_z (m/min) $\times 10^{-2}$	$S \times 10^{-3}$	$S_y \times 10^{-1}$
F505-032, F505-080	8.46	2.67	5.04	0.74
F381-056, F347-031	9.65	1.43	8.12	1.64
F383-032, F383-061	8.38	2.47	8.71	1.87
F383-032, F384-033	9.18	14.30	12.30	1.94
F383-032, F505-080	7.55	3.21	5.35	2.10
F383-061, F383-082	8.04	2.65	7.91	1.70
F384-033, F381-056	9.20	1.63	6.20	1.71
F385-032, F376-037	12.50	9.06	6.17	1.50
F504-032, F383-129	8.77	1.98	6.54	0.89
F504-032, F504-080	8.75	3.04	5.24	0.90
F505-032, F383-032	7.50	1.97	7.25	1.85
F505-032, F383-061	7.68	3.01	5.68	1.30
F505-032, F383-082	7.94	1.85	7.20	1.20
F505-032, F383-129	8.65	1.24	7.71	0.64
F505-032, F384-033	7.61	1.77	7.44	1.55
F505-032, F377-037	7.70	2.64	5.60	1.34
F505-032, F450-061	7.83	3.29	5.57	1.20
F504-032, F504-060	8.61	3.00	4.80	0.94
F504-060, F383-032	8.14	2.75	5.20	1.77
F504-060, F504-080	8.38	2.99	5.18	1.36
F504-060, F505-080	7.85	3.02	5.13	1.59
F505-032, F434-060	7.89	3.53	5.52	1.06
F505-032, F504-060	8.08	3.35	4.94	0.95
F505-032, F505-059	8.25	3.12	5.05	0.82
F505-059, F505-080	7.59	3.28	5.26	1.86
F434-060, F450-061	8.23	2.01	55.30	1.58
F434-060, F476-061	8.26	2.01	59.40	1.50
F450-061, F476-061	8.86	1.77	47.70	1.44
F476-061, F478-061	8.98	1.63	59.40	1.47
F504-060, F434-060	7.98	3.11	4.80	1.50
F504-060, F450-061	8.34	2.86	4.96	1.47
F505-059, F434-060	7.68	3.25	5.18	1.71
F377-037, F505-080	7.81	3.01	5.20	1.72
F381-056, F505-080	7.83	2.98	5.20	1.63
F505-080, F504-080	7.49	3.35	5.35	2.10
F505-059, F450-061	7.82	3.07	5.29	1.75
F505-059, F476-061	7.83	3.06	5.35	1.71
Mean	8.30×10^{-2}	3.12×10^{-2}	1.12×10^{-2}	1.46×10^{-1}
SD	8.78×10^{-3}	2.23×10^{-2}	1.55×10^{-2}	3.77×10^{-2}
CV	0.11	0.71	1.38	0.26
Mean ($\log_{10}$)	−1.08	−1.56	−2.13	−0.85
SD ($\log_{10}$)	0.04	0.19	0.31	0.13
Statistics of nonCA results				
Mean	8.98×10^{-2}	1.96×10^{-2}	1.80×10^{-2}	1.52×10^{-1}
SD	1.38×10^{-2}	8.10×10^{-3}	1.98×10^{-2}	3.49×10^{-2}
CV	0.15	0.41	1.10	0.23
Mean ($\log_{10}$)	−1.05	−1.75	−1.97	−0.83
SD ($\log_{10}$)	0.06	0.19	0.44	0.12

SD standard deviation; *CV* coefficient of variation

Table 6 The estimated parameters of set 3 in Table 3 (10 cases)

	K_r (m/min) $\times 10^{-2}$	K_z (m/min) $\times 10^{-2}$	$S \times 10^{-3}$	$S_y \times 10^{-1}$
F383-129, F504-060	8.34	2.99	4.84	1.27
F385-032, F505-080	7.39	3.38	5.43	2.27
F505-032, F376-037	7.78	1.45	8.48	1.40
F505-032, F504-032	8.36	1.70	7.27	0.93
F505-059, F376-037	7.71	3.13	5.33	1.87
F434-060, F478-061	8.15	1.97	62.30	1.67
F450-061, F478-061	9.02	1.65	47.70	1.56
F504-060, F476-061	8.34	2.91	4.95	1.42
F504-060, F478-061	8.19	2.82	5.06	1.66
F505-059, F478-061	7.68	3.13	5.30	1.97
Mean	8.10×10^{-2}	2.51×10^{-2}	1.57×10^{-2}	1.60×10^{-1}
SD	4.68×10^{-3}	7.32×10^{-3}	2.10×10^{-2}	3.79×10^{-2}
CV	0.06	0.29	1.34	0.24
Mean ($\log_{10}$)	−1.09	−1.62	−2.05	−0.81
SD ($\log_{10}$)	0.02	0.14	0.42	0.11
Statistics of nonCA results				
Mean	8.98×10^{-2}	1.96×10^{-2}	1.80×10^{-2}	1.52×10^{-1}
SD	1.38×10^{-2}	8.10×10^{-3}	1.98×10^{-2}	3.49×10^{-2}
CV	0.15	0.41	1.10	0.23
Mean ($\log_{10}$)	−1.05	−1.75	−1.97	−0.83
SD ($\log_{10}$)	0.06	0.19	0.44	0.12

SD standard deviation; *CV* coefficient of variation

of sensitivity analysis performed by Huang and Yeh (2007) indicated that the drawdown in an unconfined aquifer was more sensitive to parameters K_r and S_y than the other two parameters in response to pumping. Accordingly, the drawdown curve predicted based on the parameters obtained by the CA and nonCA will be similar because the parameters K_r and S_y dominate the drawdown behavior in response to the pumping.

Table 7 lists the results of groups 2–4 and Table 8 shows the results of groups 5 and 6 when applying CA. Also, the statistics of the nonCA results are given at the bottom of these two tables. The estimated S_y in group 2 from CA always falls between the lowest and highest ones of those of the related nonCA cases. For example, the estimated S_y of cluster F505 is 9.30×10^{-2} and the nonCA results of wells F505-032, F505-059, and F505-080 are 6.50×10^{-2}, 1.76×10^{-1} and 2.01×10^{-1}, respectively. Thus, the lowest value is 6.50×10^{-2} and the highest one is 2.01×10^{-1}. In group 3, the mean is 7.50×10^{-2} m/min for K_r and 1.83×10^{-1} for S_y, respectively. The mean of K_r is slightly less than that of the nonCA cases. Oppositely, the mean of S_y is larger than that from the nonCA. In group 4, the estimates of S for these two cases are about one order larger than those of other groups. Notably, these results are similar to those in Table 2. The statistics of estimated K_r and S_y in this table are close to those obtained by nonCA.

Table 8 displays the results of well groups 5 and 6. For group 5, the results are obtained by analyzing the drawdown data from three or four piezometers installed at deep, middle, or shallow depths. The mean values of parameters K_r and S_y are 7.83×10^{-2} m/min and 1.56×10^{-1}, respectively. The mean of K_r is slightly less than that of the nonCA cases; on the other hand, the mean of K_z is larger than that from the nonCA. The small SDs of each parameter imply that the estimated parameters will be close despite using the data from the observation wells with different depths of piezometers. In this table, the means of estimated parameters are close to that determined from the nonCA. Table 9 shows four statistics of estimated parameters determined from nonCA and CA based on the data of groups 1–6. The means of estimated K_r and S_y are close and those of K_z and S show little variation. Table 10 displays the results of analyzing all the data simultaneously based on three different analytical solutions (i.e., Moench 1997; Tartakovsky and Neuman 2007; and Neuman, 1974). The result shown on the first row are reported by Moench et al. (2001), the second ones are adopted from Tartakovsky and Neuman (2007; Table 3), and the last ones are obtained from group 7 by the SANS. Moench et al. (2001) gives the largest value of S_y (0.26) and smallest value of S (2.21×10^{-3}) while Tartakovsky and Neuman (2007) yields the largest value of K_z (4.88×10^{-2} m/min) and smallest value of K_r (6.12×10^{-2} m/min) and the SANS produces the largest value of K_r (7.77×10^{-2} m/min) and smallest value of S_y (0.17). Overall, the differences among those three sets of estimated parameters are minor. The statistics of nonCA results are listed at the bottom of this table. The means of estimated K_r and S_y by the nonCA are slightly larger and smaller than those obtained by the SANS, respectively. This is reasonable because a greater K_r accompanied by a smaller S_y can produce a similar drawdown curve to that produced by a smaller K_r with a greater S_y. Note that both works of Moench (1997) and Tartakovsky and Neuman (2007) contain some additional parameters in their solutions which are not included in Neuman's solution (Neuman 1974) of SANS and thus are not listed.

Figure 4 displays the comparison between the observed drawdown data in wells F505-080 and F505-032 and the predicted drawdown based on the estimated parameters

Table 7 The estimated parameters determined from well groups 2, 3, and 4 based on CA

Wells for CA	Estimated parameters			
	K_r (m/min) $\times10^{-2}$	K_z (m/min) $\times10^{-2}$	$S\times10^{-3}$	$S_y\times10^{-1}$
Group 2 (an observation well cluster which consists of two or three observation wells)				
Cluster F505	8.17	3.13	5.06	0.93
Cluster F504	8.66	3.01	5.10	0.99
Cluster F383	8.07	3.20	5.40	1.65
Mean	8.30×10^{-2}	3.11×10^{-2}	5.19×10^{-3}	1.19×10^{-1}
SD	3.16×10^{-3}	9.61×10^{-4}	1.86×10^{-5}	4.00×10^{-2}
Mean ($\log_{10}$)	−1.08	−1.51	−2.29	−0.94
SD ($\log_{10}$)	0.02	0.01	0.02	0.14
Group 3 (three or four observation wells with different radial distances from the pumping well)				
F505-032, F384-033, F376-037	7.51	1.74	7.43	1.76
F505-032, F383-032, F381-056, F376-037	7.49	1.96	6.49	1.89
Mean	7.50×10^{-2}	1.85×10^{-2}	6.96×10^{-3}	1.83×10^{-1}
Mean ($\log_{10}$)	−1.12	−1.73	−2.16	−0.74
Group 4 (three or four observation wells with long screens)				
F434-060, F476-061, F478-061	8.35	1.87	61.40	1.58
F434-060, F450-061, F476-061, F478-061	8.46	1.84	56.90	1.57
Mean	8.41×10^{-2}	1.86×10^{-2}	5.92×10^{-2}	1.58×10^{-1}
Mean ($\log_{10}$)	−1.08	−1.73	−1.23	−0.80
Statistics of nonCA results				
Mean	8.98×10^{-2}	1.96×10^{-2}	1.80×10^{-2}	1.52×10^{-1}
SD	1.38×10^{-2}	8.10×10^{-3}	1.98×10^{-2}	3.49×10^{-2}
Mean ($\log_{10}$)	−1.05	−1.75	−1.97	−0.83
SD ($\log_{10}$)	0.06	0.19	0.44	0.12

SD standard deviation

Table 8 The estimated parameters determined from well groups 5 and 6 based on CA

Wells for CA	Estimated parameters			
	K_r (m/min) $\times10^{-2}$	K_z (m/min) $\times10^{-2}$	$S\times10^{-3}$	$S_y\times10^{-1}$
Group 5 (three or four observation wells with different depth of piezometers)				
F505-080, F504-080, F383-082	7.71	3.20	5.30	1.78
Three deep piezometers				
F505-080, F504-080, F383-082, F383-129	7.76	3.17	5.26	1.71
Four deep piezometers				
F505-059, F504-060, F383-061	7.86	3.03	5.19	1.68
Three mid piezometers				
F505-032, F504-032, F377-037	8.05	2.14	6.45	1.23
Three shallow piezometers				
F505-032, F504-032, F377-037, F383-032	7.78	2.83	5.34	1.41
Four shallow piezometers				
Mean	7.83×10^{-2}	2.87×10^{-2}	5.51×10^{-3}	1.56×10^{-1}
SD	1.33×10^{-3}	4.36×10^{-3}	5.29×10^{-4}	2.33×10^{-2}
Mean ($\log_{10}$)	−1.11	−1.55	−2.26	−0.81
SD ($\log_{10}$)	0.01	0.07	0.04	0.07
Group 6 (two or three well clusters)				
Clusters F504, F505	8.06	3.15	5.07	1.13
Clusters F505, F383	7.72	3.16	5.39	1.55
Clusters F504, F383	8.26	3.01	5.16	1.36
Clusters F504, F505, F383	7.85	3.17	5.15	1.43
Mean	7.97×10^{-2}	3.12×10^{-2}	5.19×10^{-3}	1.37×10^{-1}
SD	2.37×10^{-3}	7.54×10^{-4}	1.38×10^{-4}	1.77×10^{-2}
Mean ($\log_{10}$)	−1.10	−1.51	−2.28	−0.87
SD ($\log_{10}$)	0.01	0.01	0.01	0.06
Statistics of nonCA results				
Mean	8.98×10^{-2}	1.96×10^{-2}	1.80×10^{-2}	1.52×10^{-1}
SD	1.38×10^{-2}	8.10×10^{-3}	1.98×10^{-2}	3.49×10^{-2}
Mean ($\log_{10}$)	−1.05	−1.75	−1.97	−0.83
SD ($\log_{10}$)	0.06	0.19	0.44	0.12

SD standard deviation

Table 9 The statistics of estimated parameters based on nonCA and CA for data of groups 1–6

Parameter	K_r (m/s) × 10^{-2}	K_z (m/s) × 10^{-2}	S × 10^{-2}	S_y × 10^{-1}	K_r (m/s)	K_z (m/s)	S	S_y
Statisitics	Mean				SD			
NonCA	8.98	1.96	1.80	1.52	1.38	8.10	1.98	3.49
Group 1								
Set 1	9.31	1.75	3.33	1.36	NA	NA	NA	NA
Set 2	8.30	3.12	1.12	1.46	8.78×10^{-3}	2.23×10^{-2}	1.55×10^{-2}	3.77×10^{-2}
Set 3	8.10	2.51	1.57	1.60	4.68×10^{-3}	7.32×10^{-3}	2.10×10^{-2}	3.79×10^{-2}
Group 2	8.30	3.11	0.52	1.19	3.16×10^{-3}	9.61×10^{-4}	1.86×10^{-5}	4.00×10^{-2}
Group 3	7.50	1.85	0.70	1.83	NA	NA	NA	NA
Group 4	8.41	1.86	5.92	1.58	NA	NA	NA	NA
Group 5	7.83	2.87	0.55	1.56	1.33×10^{-3}	4.36×10^{-3}	5.29×10^{-4}	2.33×10^{-2}
Group 6	7.97	3.12	0.52	1.37	2.37×10^{-3}	7.54×10^{-4}	1.38×10^{-4}	1.77×10^{-2}
	Mean ($\log_{10}$)				SD ($\log_{10}$)			
NonCA	−1.05	−1.75	−1.97	−0.83	0.06	0.19	0.44	0.12
Group 1								
Set 1	−1.03	−1.76	−1.65	−0.87	2.64×10^{-3}	0.08	0.58	0.03
Set 2	−1.08	−1.56	−2.13	−0.85	NA	NA	NA	NA
Set 3	−1.09	−1.62	−2.05	−0.81	0.02	0.14	0.42	0.11
Group 2	−1.08	−1.51	−2.29	−0.94	0.02	0.01	0.02	0.14
Group 3	−1.12	−1.73	−2.16	−0.74	NA	NA	NA	NA
Group 4	−1.08	−1.73	−1.23	−0.80	NA	NA	NA	NA
Group 5	−1.11	−1.55	−2.26	−0.81	0.01	0.07	0.04	0.07
Group 6	−1.10	−1.51	−2.28	−0.87	0.01	0.01	0.01	0.06

NA means that the SD is not available since there are only two cases in the group and the SD has no meaning here

determined from both nonCA (dashed line) and CA (solid line). Judging from the curve fitting (SEE values) to the observed data, the analysis of data based on nonCA gives more precisely parameter estimation (SEE=7.02×10^{-3} m for well F505-080 and 7.06×10^{-3} m for well F505-032) than that based on CA (SEE=9.51×10^{-3} m).

In fact, the mathematical model describing the response of an aquifer to well pumping generally assumes that the aquifer material is homogeneous. Mathematically, the analysis of data based on CA suggests that the geology of the observation well near the pumping well has greater weight than that far away from the pumping well in a multiple observation well system. In other words, the geological properties of the observation wells located near the pumping well will have greater influence on the parameter estimation than geological properties far away from the pumping well. If the aquifer is fairly homogeneous, the estimated parameters based on the CA and nonCA should be about the same. However, in a heterogeneous aquifer, the application of CA to the determination of parameters is very likely to give biased estimate of the hydraulic parameters. In addition, the principle of least squares requires that the measurement errors are mutually independent (Neter et al. 1996). For sampling points installed very close together or located at the same place but at different depths, there is a tendency for neighboring observations to be correlated if the data are collected sequentially. Under those circumstances, the assumption that the measurement errors are mutually independent is very likely violated. In other words, autocorrelation may exit in the drawdown data of multiple observation wells. Thus, the application of CA may give a biased estimation for the hydraulic parameters from a statistical viewpoint (Berthouex and Brown 2002).

Hypothetical case study

Tables 11–13 display the results of the hypothetical case study when analyzing the drawdown data from OW1, OW2, and both of these two observation wells, respectively. From these tables, the means of estimated parameters are very close to the target values and the differences

Table 10 The global analysis obtained by three different approaches

	K_r (m/min) × 10^{-2}	K_z (m/min) × 10^{-2}	S × 10^{-3}	S_y
Moench et al. (2001)	7.02	4.27	2.21	0.26
Tartakovsky and Neuman (2007)	6.12	4.88	5.10	0.18
SANS	7.77	3.14	5.28	0.17
Statistics of nonCA results				
Mean	8.98×10^{-2}	1.96×10^{-2}	1.80×10^{-2}	1.52×10^{-1}
SD	1.38×10^{-2}	8.10×10^{-3}	1.98×10^{-2}	3.49×10^{-2}
Mean ($\log_{10}$)	−1.05	−1.75	−1.97	−0.83
SD ($\log_{10}$)	0.06	0.19	0.44	0.12

SD standard deviation

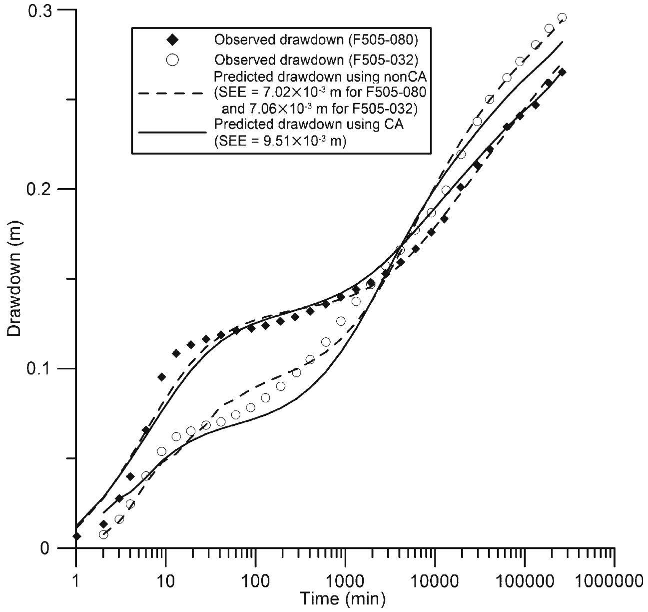

Fig. 4 The drawdown curves predicted by Neuman's solution (Neuman 1974) with parameters determined from nonCA and CA for the wells F505-032 and F505-080

in estimated parameters are also minor when compared with the target ones. This result may be due to the fact that there is plenty of observed drawdown data so that the effect of measurement errors, which are normally distributed with zero mean, is minor. The SEE value is about 0.01 which is equal to the SD of the measurement errors. In Table 13, the categories A, B, and C represent the situation whereby the S_y determined from the CA is smaller than, falls between, and is greater than S_y of nonCA, respectively. Most of the sets (17 sets) belong to the category B and this result indicates that the CA generally gives an average estimate of S_y. However, the total number of categories A and C is 3 and this result indicates that there is 15% chance that the CA gives larger or smaller estimates of S_y than those of the nonCA cases in this hypothetical case study. Obviously, the parameter determination obtained from the CA still gives poorer estimate of S_y than that determined from nonCA if the drawdown data contain measurement errors.

Conclusions

This study uses a computer method named SANS developed by Huang et al. (2008) to analyze the drawdown data of real and hypothetical cases for determining the hydraulic parameters of unconfined aquifers. In the field cases, the drawdown data of the pumping tests are taken from the famous experimental site of Cape Cod, Massachusetts for the comparison of parameter estimation based on nonCA and CA approaches. On the other hand, the hypothetical case is designed to explore the effect of measurement errors on the estimated parameters.

The drawdown data obtained from 20 observation wells at the Cape Cod site were analyzed separately based on the present method of nonCA. In addition, the drawdown data from those observation wells were classified into seven groups and analyzed by the present method of CA. Those seven groups are: (1) two wells randomly chosen from 20 observation wells; (2) a well cluster consisting of two or three observation wells; (3) three or four wells with different radial distances from the

Table 11 The results of nonCA using the data from OW1

Noise set	K_r (m/s) × 10^{-3}	K_z (m/s) × 10^{-4}	S × 10^{-4}	S_y × 10^{-1}	SEE (m) × 10^{-2}
1	1.014	0.985	0.991	0.958	1.067
2	1.001	1.033	0.947	1.011	1.115
3	0.991	1.024	1.006	1.035	1.071
4	0.968	1.052	1.014	1.118	1.025
5	1.049	0.976	0.917	0.819	0.967
6	1.007	1.013	0.984	0.918	0.925
7	0.975	1.015	1.062	1.092	0.936
8	1.026	0.950	0.972	0.853	1.018
9	1.005	0.986	1.029	0.944	1.120
11	0.979	1.010	0.980	1.163	0.979
12	0.996	1.013	0.928	1.022	0.856
13	1.097	0.934	1.007	0.658	0.985
14	0.999	1.002	1.005	1.001	1.003
15	1.044	0.935	1.059	0.824	0.979
16	0.985	0.985	0.900	1.097	1.078
17	1.021	0.958	0.997	0.922	0.869
18	0.979	1.040	1.077	1.126	1.023
19	0.999	0.997	0.944	1.020	0.902
20	0.980	1.061	1.012	1.167	0.887
Mean	1.008×10^{-3}	9.956×10^{-4}	9.903×10^{-5}	9.792×10^{-2}	
SD	3.173×10^{-5}	3.792×10^{-6}	4.751×10^{-6}	1.344×10^{-2}	

SD standard deviation

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Table 12 The results of nonCA using the data from OW2

Noise set	K_r (m/s) × 10^{-3}	K_z (m/s) × 10^{-4}	S × 10^{-4}	S_y × 10^{-1}	SEE (m) × 10^{-2}
1	0.989	0.933	1.294	1.071	0.883
2	1.045	0.987	1.109	0.907	1.005
3	1.023	1.065	0.928	0.950	0.934
4	1.027	1.022	0.948	0.927	1.198
5	1.001	1.035	1.038	0.979	0.964
6	0.963	0.993	0.927	1.109	0.982
7	1.063	0.925	1.160	0.903	0.912
8	0.959	0.912	1.259	1.147	0.966
9	0.968	0.944	0.733	1.139	0.907
11	0.992	0.978	1.110	1.005	1.143
12	0.904	0.895	0.992	1.275	0.959
13	0.950	0.933	1.287	1.093	1.065
14	1.047	1.010	0.759	0.911	1.013
15	0.973	0.916	1.188	1.067	0.959
16	0.988	0.951	1.026	1.036	1.122
17	1.021	0.992	1.040	0.944	1.127
18	1.015	0.991	1.039	0.961	1.160
19	0.996	1.020	1.107	0.995	0.959
20	0.997	0.954	1.117	0.985	0.980
Mean	9.961×10^{-4}	9.696×10^{-5}	1.070×10^{-4}	1.022×10^{-1}	
SD	3.740×10^{-5}	4.647×10^{-6}	1.649×10^{-5}	9.703×10^{-3}	

SD standard deviation

pumping well; (4) three or four wells with long screen; (5) three or four wells with different depth of piezometers; (6) two or three well clusters; and (7) all wells (global analysis). The results were used to examine whether the spatial distribution and the type of observation well affect the determination of parameters when applying the present method of CA. For well group 1, 50 cases were analyzed and the S_y determined by the CA can be divided into three categories; i.e., the S_y is smaller than, in between, and larger than those of the counterpart, nonCA. The means of K_r and S_y in these three categories are very close to that determined from the nonCA. This result indicates that both CA and nonCA give similar results since the drawdown behavior is much more sensitive to these two parameters than the other two parameters in response to the pumping. For well groups 2–6, the estimated S_y always falls between the lowest and highest values of S_y from nonCA when analyzing the drawdown data from

Table 13 The results of CA using the data from OW1 and OW2

Noise set	K_r (m/s) ×10^{-3}	K_z (m/s) ×10^{-4}	S × 10^{-4}	S_y × 10^{-1}	SEE (m) × 10^{-2}	Category
1	1.003	0.977	1.002	1.043	0.936	B
2	1.017	1.007	0.960	0.958	1.043	B
3	0.996	1.026	0.991	1.014	0.993	B
4	0.996	1.020	0.993	0.993	1.099	B
5	1.015	1.015	0.946	0.944	0.957	B
6	0.985	1.026	0.988	1.028	0.951	B
7	1.003	0.964	1.065	1.005	0.974	B
8	0.985	0.973	1.018	1.040	1.029	B
9	0.986	0.990	1.005	1.055	1.047	B
11	1.008	0.985	0.982	0.999	1.056	A
12	0.994	0.993	0.940	1.029	0.959	B
13	1.026	0.988	1.078	0.894	1.092	B
14	1.006	0.994	0.972	0.986	1.030	B
15	1.010	0.956	1.093	0.959	0.958	B
16	1.000	0.969	0.909	1.019	1.084	B
17	1.009	0.972	1.005	0.970	0.978	C
18	1.009	1.005	1.059	0.983	1.071	B
19	1.000	1.003	0.965	1.001	0.910	B
20	1.023	1.006	1.013	0.948	0.953	A
Mean	1.004×10^{-3}	9.916×10^{-5}	1.000×10^{-4}	9.930×10^{-2}		
SD	1.176×10^{-5}	2.150×10^{-6}	4.735×10^{-6}	3.961×10^{-3}		

Category A is the estimated S_y using CA is smaller than those of two nonCA cases; category B is the estimated S_y using CA falls between those of two nonCA cases; category C is the estimated S_y using CA is larger than those of two nonCA cases; *SD* standard deviation three and four observation wells under different data selection strategies. For well group 7, the estimated parameters have the largest value of K_r and the smallest value of S_y when compared with those analyzed by different analytical solutions in the previous studies.

In the hypothetical case, two sets of pumping drawdown data are generated. One set is from observation well 1 (OW1) and the other is from observation well 2 (OW2). Forty sets of normally distributed noise were generated to represent the measurement errors with the magnitude on the order of centimeter and two 20 noise sets were added to the drawdown data of OW1 and OW2, respectively. The SANS was first used to determine the hydraulic parameters based on nonCA using the drawdown data from OW1 or OW2. Then the drawdown data from two observation wells were analyzed based on CA. In this case study, the estimates of S_y are larger or smaller than those determined from nonCA in 3 out of total 20 cases. Obviously, the parameter determination using CA also gives poorer estimates of S_y than that determined using nonCA when the data contain measurement errors.

Based on the results of field and hypothetical case studies, the parameter determination based on the CA does not always give better result than those obtained based on the nonCA. The biased estimates of S_y are probably attributed to one or more of problems, including improper procedures, bad data, and aquifer heterogeneity (Moench 1994) and have nothing to do with the use of CA or nonCA.

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