CHAPTER 3

DIMENSIONLESS GROUPS AND UNCERTAINTY ANALYSIS

3.1 Dimensionless Groups

The non-dimensional parameters associated with the flow considered here are the jet Reynolds number Re_j based on the mean gas speed V_j at the injection pipe exit and the exit diameter of the injection pipe D_j, the Rayleigh number Ra based on the temperature difference between the heated disk and inlet gas ΔT and the jet-to-disk separation distance H, and the rotational Reynolds number Re_{Ω} based on the disk rotation speed Ω and the radius of the heated rotating disk R_w. They are defined as

$$Re_j = \frac{V_j D_j}{\nu} = \frac{4}{\pi} \frac{Q_j}{\nu D_j}, \tag{3.1}$$

$$Ra = \frac{g\beta(T_f - T_j)H^3}{\alpha\nu} = \frac{g\beta\Delta T H^3}{\alpha\nu} \tag{3.2}$$

and

$$Re_{\Omega} = \frac{\Omega R_w^{\ 2}}{\nu} \tag{3.3}$$

where α is the thermal diffusivity, g is the gravitational acceleration, β is the thermal expansion coefficient, and ν is the kinematic viscosity.

3.2 Uncertainty Analysis

An uncertainty analysis is carried out here to estimate the uncertainty levels in the experiment. Kline and McClintock [52] proposed a formula for evaluating the uncertainty in the result F as a function of independent variables,

$X_1, X_2 \ldots\ldots\ldots\ldots X_n$,

$$F = F(x_1, x_2, x_3, \cdots\cdots x_n) \tag{3.4}$$

The absolute uncertainty of F is expressed as

$$\delta F = \left\{ \left[\left(\frac{\partial F}{\partial X_1} \right) \delta X_1 \right]^2 + \left[\left(\frac{\partial F}{\partial X_2} \right) \delta X_2 \right]^2 + \left[\left(\frac{\partial F}{\partial X_3} \right) \delta X_3 \right]^2 + \ldots \left[\left(\frac{\partial F}{\partial X_n} \right) \delta X_n \right]^2 \right\}^{1/2} \tag{3.5}$$

and the relative uncertainty of F is

$$\frac{\delta F}{F} = \left\{ \left[\left(\frac{\partial \ln F}{\partial \ln X_1} \right) \left(\frac{\delta X_1}{X_1} \right) \right]^2 + \left[\left(\frac{\partial \ln F}{\partial \ln X_2} \right) \left(\frac{\delta X_2}{X_2} \right) \right]^2 + \ldots \left[\left(\frac{\partial \ln F}{\partial \ln X_n} \right) \left(\frac{\delta X_n}{X_n} \right) \right]^2 \right\}^{1/2} \tag{3.6}$$

If $F = x_1^a x_2^b x_3^c \ldots\ldots$, then the relative uncertainty is

$$\frac{\delta F}{F} = \left[\left(a \frac{\delta X_1}{X_1} \right)^2 + \left(b \frac{\delta X_2}{X_2} \right)^2 + \left(c \frac{\delta X_3}{X_3} \right)^2 + \ldots\ldots \right]^{1/2} \tag{3.7}$$

where $(\partial F / \partial X_i)$ and δX_i are, respectively, the sensitivity coefficient and uncertainty level associated with the variable X_i. The values of the uncertainty intervals δX_i are obtained by a root-mean-square combination of the precision uncertainty of the instruments and the unsteadiness uncertainty, as recommended by Moffat [53]. The choice of the variable X_i to be included in the calculation of the total uncertainty level of the result F depends on the purpose of the analysis. The uncertainties for the chosen parameters are calculated as follows:

(1) Uncertainty of the measured temperature difference, $\Delta T = T_f - T_j$

$$\delta(T_f - T_j) = [(\delta T_f)^2 + (\delta T_j)^2]^{1/2} \tag{3.8}$$

(2) The dependence of the air properties k, μ, and ν on the temperature (T in K) [54]

is

$$
\begin{aligned}
& k = 1.195 \times 10^{-6} T^{1.6} / (T + 118) \\
& \mu = 1.448 \times 10^{-6} \mathrm{T}^{1.5} / (\mathrm{T} + 118) \\
& \nu = \mu / \rho
\end{aligned}
\tag{3.9}
$$

The uncertainties of the properties are

$$
\begin{aligned}
\frac{\delta k}{k} &= \frac{T}{k}\frac{\partial k}{\partial T}\frac{\delta T}{T} \\
\frac{\delta \rho}{\rho} &= \frac{T}{\rho}\frac{\partial \rho}{\partial T}\frac{\delta T}{T} \\
\frac{\delta \mu}{\mu} &= \frac{T}{\mu}\frac{\partial \mu}{\partial T}\frac{\delta T}{T}
\end{aligned}
\tag{3.10}
$$

(3) Uncertainty of Rayleigh number, Ra,

$$
\mathrm{Ra} = \frac{g\beta(T_f - T_j)H^3}{\alpha\nu} = \frac{g\beta\Delta T H^3}{\alpha\nu} \tag{3.11}
$$

$$
\frac{\delta \mathrm{Ra}}{\mathrm{Ra}} = \left[\left(\frac{\delta g\beta}{g\beta}\right)^2 + \left(3\frac{\delta H}{H}\right)^2 + \left(\frac{\delta \Delta T}{\Delta T}\right)^2 + \left(\frac{\delta \alpha}{\alpha}\right)^2 + \left(\frac{\delta \nu}{\nu}\right)^2\right]^{1/2} \tag{3.12}
$$

(4) Uncertainty of jet Reynolds number, Re_j,

$$
\mathrm{Re}_j = \frac{V_j D_j}{\nu} = \frac{4}{\pi}\frac{Q_j}{\nu D_j} \tag{3.13}
$$

$$
\frac{\delta \mathrm{Re}_j}{\mathrm{Re}_j} = \left[\left(\frac{\delta \nu}{\nu}\right)^2 + \left(\frac{\delta Q_j}{Q_j}\right)^2 + \left(\frac{\delta D_j}{D_j}\right)^2\right]^{1/2} \tag{3.14}
$$

(5) Uncertainty of rotational Reynolds number, Re_Ω,

$$
\mathrm{Re}_\Omega = \frac{\Omega^2 R_w^{\,2}}{\nu} \tag{3.15}
$$

$$\frac{\delta \mathrm{Re}_\Omega}{\mathrm{Re}_\Omega} = \left[\left(\frac{\delta \nu}{\nu} \right)^2 + \left(2 \frac{\delta R_w}{R_w} \right)^2 + \left(\frac{\delta \Omega}{\Omega} \right)^2 \right]^{1/2} \qquad (3.16)$$

The results from this uncertainty analysis are summarized in Table 3.1

Table 3.1 Summary of uncertainty analysis

Parameter and Estimate Uncertainty	
Parameters	Uncertainty
D_j, R_w, H (m)	± 0.00005 m
T (°C)	± 0.2°C
ΔT (°C)	0.3°C
Q_j (slpm)	± 2%
Ω (rpm)	± 0.2 rpm
μ (Nm/s^2) ρ (kg/m^3) υ (m^2/s)	± 0.05% ± 0.05% ± 0.07%
Ra	± 8.6%
Re_j	± 2.3%
Re_Ω	± 4.0%