國立交通大學

應用數學系數學建模與科學計算碩士班

碩 士 論 文

保角曲面形變的應用—虛擬播報

Application of Conformal Surface Morphing for Virtual Broadcasting

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中 華 民 國 一百零二 年 六 月

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摘要

形變是一個圖像變成另一個圖像的過程。本論文中，我們將著重在人臉曲面的保角形變。根據黎曼映射定理，一個簡單的連通曲面，存在唯一的黎曼保角映射，可將此曲面映射到單位圓盤上。因此我們將三維人臉曲面的對應問題化簡成二維的單位圓盤對應問題。其中我們還透過保角映射與拓樸同倫的概念，進行曲面三角網格結構的統一，再進行曲面的對應以完成形變。本論文的結果，將根據上述的概念，重建三維人臉虛擬播報的影片。

Application of Conformal Surface Morphing for Virtual Broadcasting

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June, 2013

Abstract

Morphing is the process of changing one figure to another. In this study, we focus on the conformal morphing on the facial surface. According to the Riemann mapping theorem, we know that there exists a unique Riemann conformal mapping to map a simply connected surface to a unit disk. Hence, we reduce the 3D facial surface matching problem to a unit disk matching problem. Among the process of morphing, we also use the concept of conformal mapping and homotopy to uniform the structure of triangular mesh on surface, and then complete the surface matching. The result of this study will be based on those concepts to reconstruct the virtual broadcasting film with facial surface.

致謝

在完成碩士班課程與撰寫完本論文後，首先我想要先感謝我的指導老師林松山老師，不僅在學術方面給了我許多學習的機會，也教導了我許多人生觀念，培養好學習態度與習慣，在這裡真的很謝謝老師。另外，也很感謝林文偉老師在此領域中教導我許多理論與方法，並且在論文上給予我指正與意見；再來我還要感謝林瑜堯學姐和樂美亨同學，在研究方法中給了許多想法與討論，並不厭其煩的聆聽我的疑問，幫助我完成畢業論文。最後，要感謝上一屆的吳侑森學長和曾茂清學長，在這過程中給了我建議並且教導我照相機的實做，並協助我完成影片的拍攝。因為有了大家的幫忙，才能讓本論文如期地完成，在此謝謝大家!

黃敏琇

謹誌于交通大學

2013年6月

Application of Conformal Morphing

—Virtual Broadcasting

Min-Shiou Huang

June,2013

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Chapter 1

Introduction

The study of 3D image nowadays is very popular. We can process the 3D mesh with many different ideas and methods. In this study, we foucs on the technique of conformal morphing to extend its application.

Contents

1.1 Morphing

Morphing is the process of changing one figure to another. We can change a cube to a sphere[Fig.1.1(a)], and also can change a surface of a human to a surface of another human[Fig.1.1(b)]. But the process of morphing is not unique. If we use the different parameterization, it could have the different effects[Fig.1.1(c) and (d)].

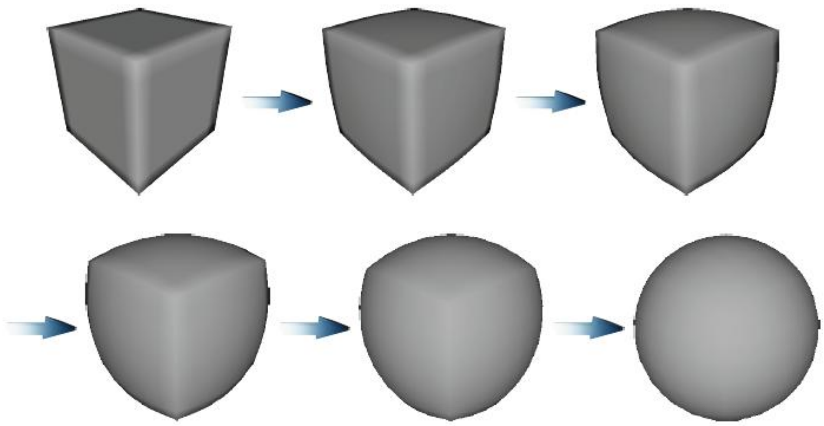


Fig.1.1(a) The process of change a cube to a sphere.

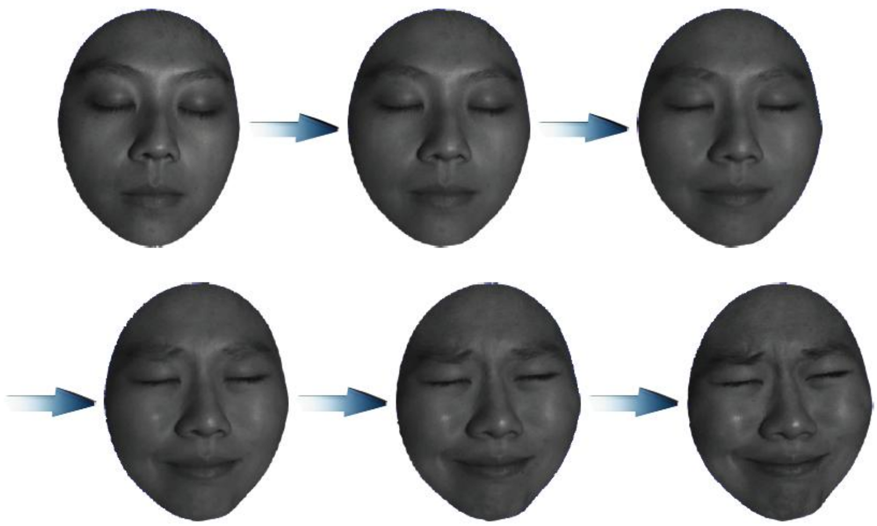


Fig.1.1(b) The process of changing a human face to an another human face.

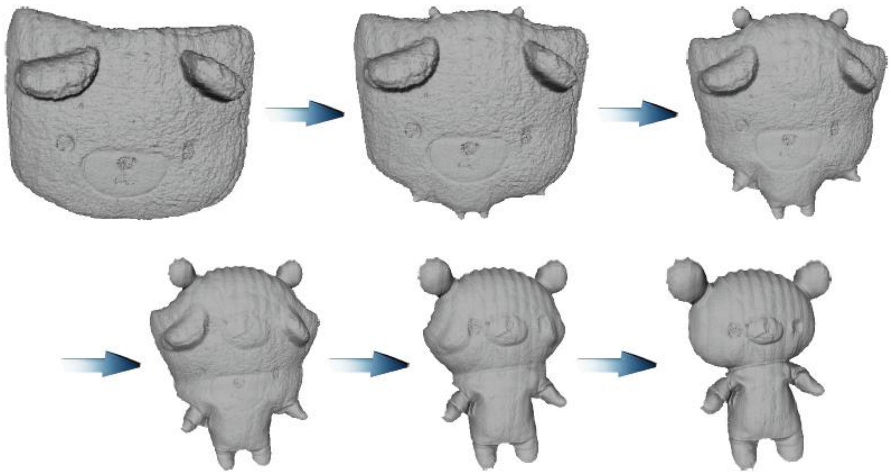


Fig.1.1(c) The process of changing a doll to another.

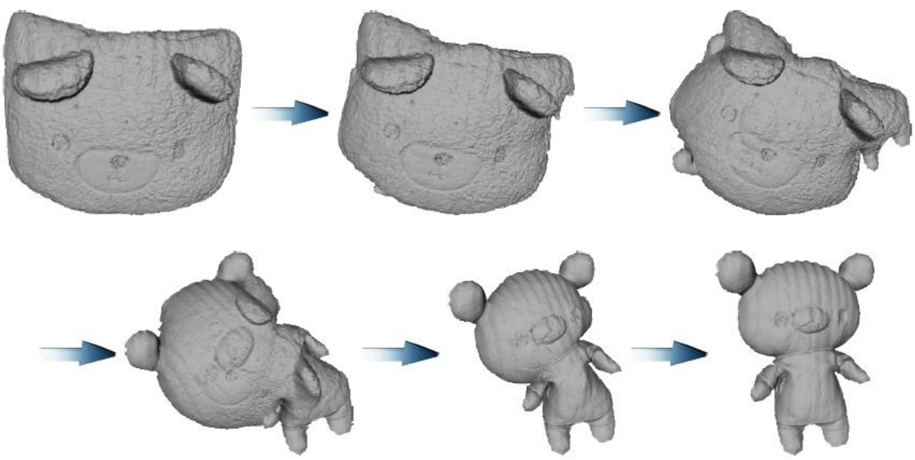


Fig.1.1(d) The process of changing a doll to another with different parameter.

In this study, we hope we can keep some features of the given surfaces during the morphing process in order to achieve the virtual effect on facial surface. For achieving our expected result, the surface morphing is very important. Then, the problem of surface matching will come out. We expect that we can use some methods and techniques for surface matching to display the morphing effects that we hope.

1.2 Dealing with the 3D Mesh

The 3D mesh is captured from the 3D camera. In order to use the Riemann mapping theroem, we have to reduce the 3D mesh into a simply connected surface. Here we propose an automatic way to deal with the 3D mesh by three steps: Smoothing Boundary, Canonical Boundary Cut and Holes Filling[Fig.1.2].

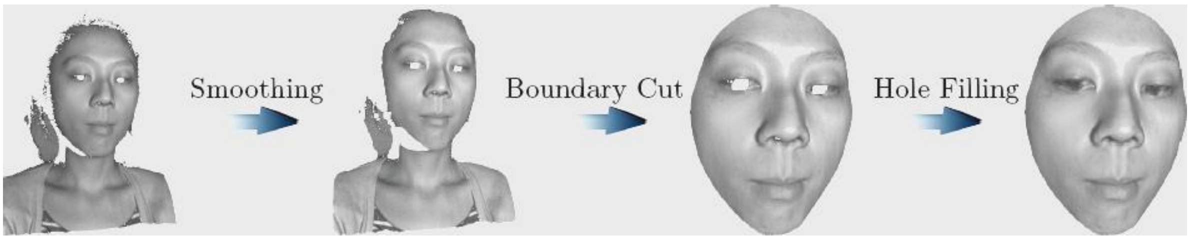


Fig.1.2 The process of reducing the 3D mesh into a simply connected surface.

1.3 Morphing of Conformal Surface

In recent years, surface morphing is very important, but it is difficult. The surface morphing of facial face produces the problem of surface matching, because the ratio of facial features is different from one person into another. According to the Riemann mapping theorem, we can reduce an R^3 surface matching problem into a unit disk matching problem and the process is conformal.

1.4 The Goal in this Study

Virtual broadcasting is an application of conformal morphing, and our goal is to make a film of virtual broadcasting with facial surface. In order to achieve this goal, we may use the Riemann mapping theorem, the idea of homotopy and the method of surface matching.etc. We may hope the virtual broadcasting film simulate to the original film.

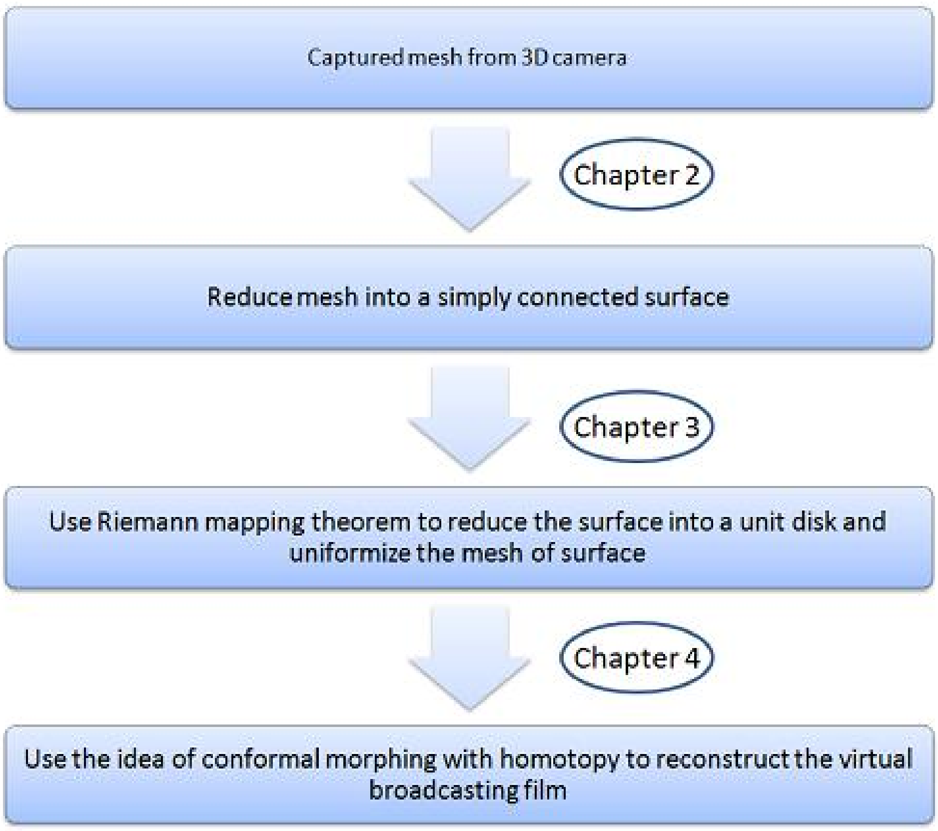


Fig.1.4 The process of how to reconstruct the film.

Chapter 2

Dealing with 3D Mesh of Human Face

In this chapter, we propose an automatic way to deal the 3D mesh in order to reduce those mesh into the simply connected surface efficiently.

Contents

2.1 Mesh Captured from the 3D Camera

In this study, our goal is to make a virtual broadcasting film with the 3D mesh of human face. In order to complete this film, we use the Riemann Mapping Theorem to reduce an R^3 surface problem to an unit disk problem. (We will introduce the Riemann Mapping Theorem in Chapter 3.) Because the Riemann Mapping Theorem is defined on a simply connected surface, we have to let the mesh have one boundary without holes.

The mesh data captured from the 3D camera[Fig.2.1(a) and (b)] is full of noise and holds. Therefore, we deal the mesh with three steps: Boundary Smoothing, Canonical Boundary Cut and Holes Filling to get a simply connected surface.

Fig.2.1(a) The back of the 3D camera.

Fig.2.1(b) The front of the 3D camera.

2.2 Boundary Smoothing

The mesh captured from the camera is triangular structure[Fig.2.2], and most of the noise vertices have the property that the number of reference by faces is less than 3. For example, the number of reference with an isolated vertex is 0 and the number of reference with an isolated face is 1. Denote the number of reference of vertex v as $r_f(v)$. We use this property to construct a simple method to remove those noise vertices.

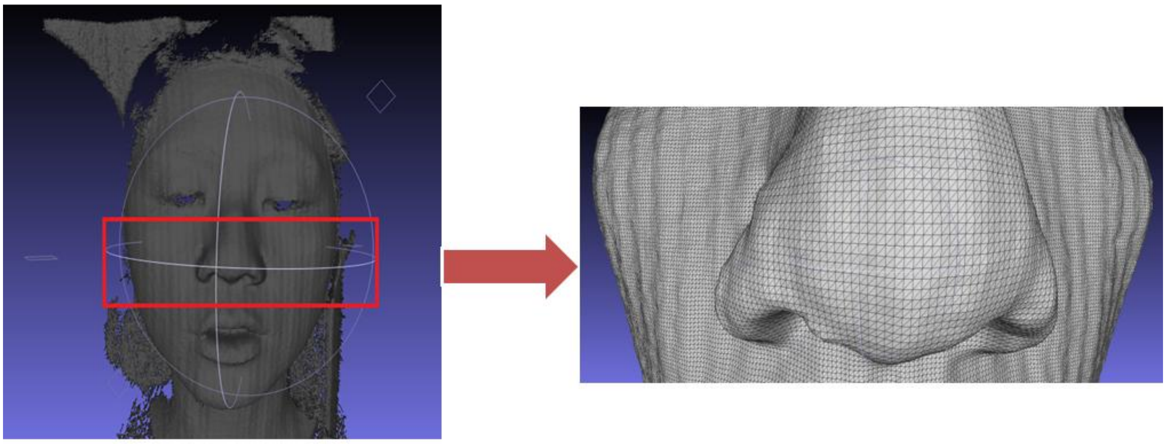


Fig.2.2 The mesh is triangular structure.

2.1 Boundary Smoothing

Input: the triangular mesh (V , F)

Output: the teiangular mesh (V' , F')

for i=1 to |V| **do**

computer r_i = the number of reference of V_i

if $r_i < 3$ **then**

delete V_i and the faces reference to the inddex i

edd if

end for

2.3 Canonical Boundary Cut

We can observe that usually the point at the nose has the maximum value over z-axis[Fig.2.3(a)]. It is easy to pick the point as the center of ellipsoid automatically or manually. Then we find out the distance α between nose and chin is as the radius of semi-major of the ellipsoid and the distance β between nose and cheek is as the radius of semi-minor of the ellipsoid. In the end, we cut-off those points which is outside of the sphere.[Fig. 2.3(b)]

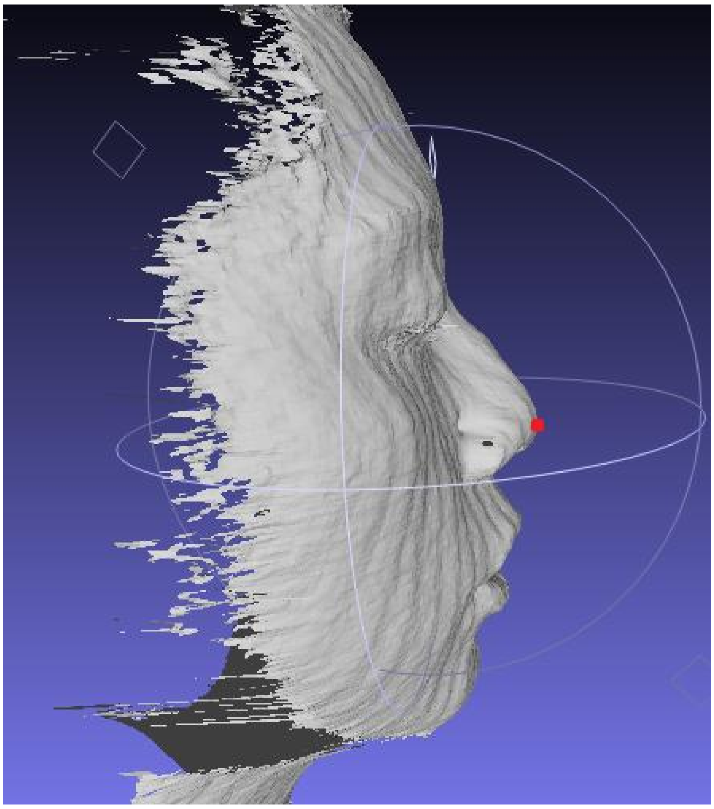


Fig.2.3(a) The maximum value over z-axis is on the nose.

Algorithm 2.2 Canonical Boundary Cut

Input: the triangular mesh (V , F)

Output: the teiangular mesh (V' , F') with canonical boundary

Determine an ellipsoid E with center c, the semi-major α,and the semi-minor β

for i = 1 to |V| **do**

if V_i is outside of E **then**

delete V_i and the faces reference to the index i

end if

end for

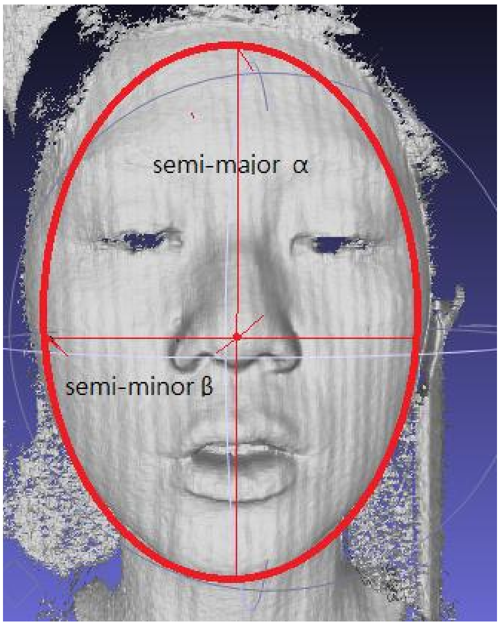


Fig.2.3(b) Cut-off those points which is outside of the sphere.

2.4 Holes Filling

The principle of the 3D camera is using the structure light to get the information of depth by reflex light. If the mesh has some areas which do not reflect the structure light, the camera will regard it as a hole. For filling the holes,which are usually not large,we use the linear interpolation by boundary points to construct piecewise linear patches to fill the holes. We first smooth the the boundary of hole several times such that the shape of the hole is nearly convex. Then we compute the center by the boundary points of the hole. Next, we separate the hole's boundary points into 6 parts and compute the mean of each part to obtain a hexagon with center point. Last, connect each hole's boundary vertex with the nearest vertex of the hexagon.

2.3 Hole Filling by linear interpolation

Input: the triangular mesh (V , F)

Output: the teiangular mesh (V' , F') with no hole

Smooth the boundary several times so that each boundary of holes is near a convex shape.

for each hole **do**

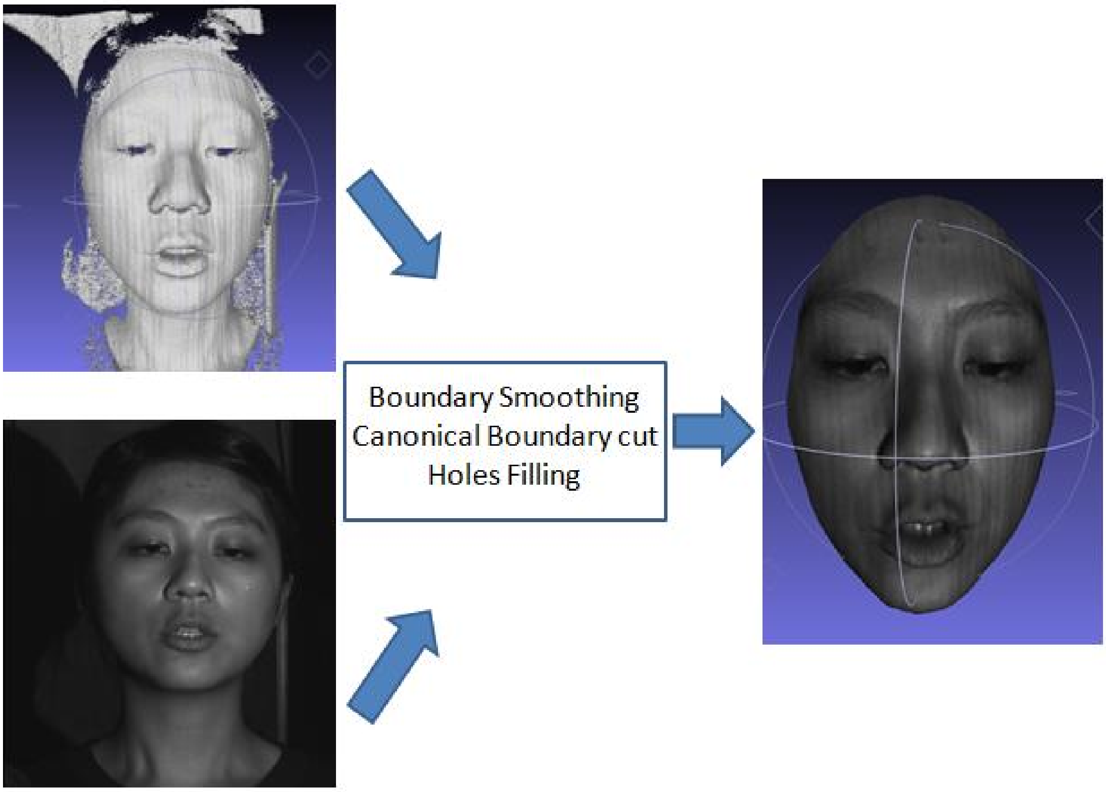
Compute the center of the hole

Construct a neighborhood of the center by using the linear interpolation of the center and the boundary points.

Construct the faces by using the boundary points, the center point and the points of neighborhood.

end for

To sum up the processing as following:



Chapter 3

Conformal mapping

Recently, 3D scanning technology is developing extremely fast. The 3D scanning can capture the dynamic facial expression in real time. It is a challenge to process the huge amount of geometric data efficiently. The main method is to use conformal mapping to transform 3D surface to canonical 2D domains. Therefore, we have some applications of conformal mapping with 3D meshes in this study.

Contests

3.1 Introduction of Conformal Mapping

A conformal map between two surfaces preserves angles. The Riemann mapping theorem states that any simply connected surface with a single boundary can be conformal to the unit disk.

For example[Fig.3.1], a human surface S has the single boundary and map to the unit disk D. There exists a conformal mapping $\phi : S \to D$. Suppose r_1 and r_2 are any two curve on the surface S, ϕ maps them to $\phi(r_1)$ and $\phi(r_2)$. If the intersection angle between r_1 and r_2 is θ, then the intersection angle between $\phi(r_1)$ and $\phi(r_2)$ is also θ. Therefore, we say ϕ is a conformal map, which means angle preserving.

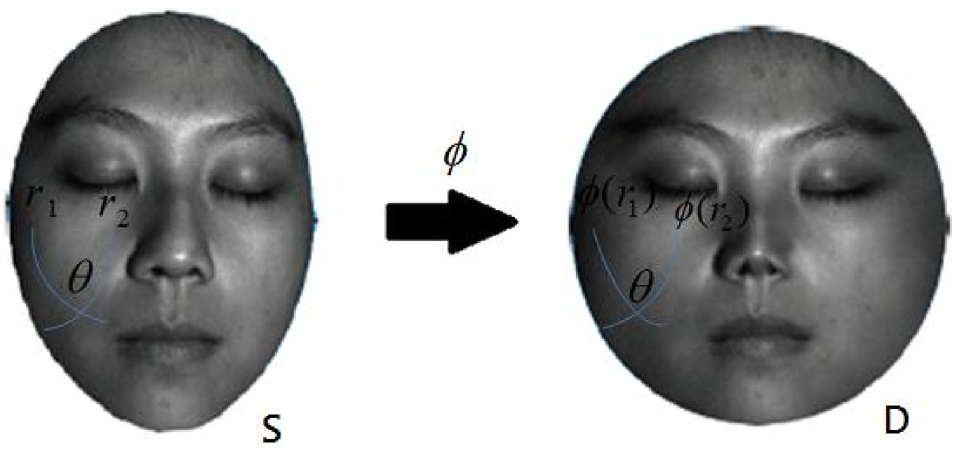


Fig.3.1 The Riemann mapping from a human face to the unit disk is conformal.

The conformal map between two planar domain is the conventional analytic function or holomorphic function. From the point of view, conformal mappings are the generalizations of holomorphic functions, and the Riemann surfaces are the generalizations of complex plane. Holomorphic differential means the derivative of an analytic function, and it can be defined on surface. We can locally map the surface to plane by integrating the holomorphic differentials.

3.2 Theory

Here, we give the definitions of harmonic function, holomorphic function, conformal mapping and the Riemann mapping theorem.

Definition 1 *(Harmonic function)*

Suppose $u : D \to R$ is a real valued function defined on a domain $D \subseteq \mathbb{C}$, and $u \in C^2(D)$. For any $z \in D$, $z = x + iy$, we have

$$\Delta u(z) = \frac{\partial^2 u(z)}{\partial x^2} + \frac{\partial^2 u(z)}{\partial y^2} = 0$$

then we call $u(z)$ is a harmonic function on D, where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplace operator.

Note that $\begin{cases} z = x + iy \\ \overline{z} = x - iy \end{cases} \Longrightarrow \begin{cases} x = \frac{1}{2}(z + \overline{z}) \\ y = \frac{i}{2}(\overline{z} - z) \end{cases}$

and $\begin{cases} dz = dx + idy \\ d\overline{z} = dx - idy \end{cases} \Longrightarrow \begin{cases} \frac{\partial}{\partial z} = \frac{1}{2}(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}) \\ \frac{\partial}{\partial \overline{z}} = \frac{1}{2}(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}) \end{cases}$

then we have $\langle dz, \frac{\partial}{\partial z} \rangle = \langle d\overline{z}, \frac{\partial}{\partial \overline{z}} \rangle = 1$.Therefore, the Laplace operator becomes as

$$\triangle = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z \partial \overline{z}}$$

Definition 2 *(Holomorphic function)*

A function $f : \mathbb{C} \to \mathbb{C}$ is a complex fuction, $f : (x, y) \to (u(x, y), v(x, y))$ is holomorphic if it satisifies the Cauchy-Riemann equation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad and \; \frac{\partial f}{\partial \bar{z}} = 0$$

Note that, if a holomorphic function $f : D \to \mathbb{C}$ is bijective and f^{-1} is also holomorphic, then f is called to be biholomorphic or conformal mapping.

Definition 3 *(conformal mapping)*

Suppose M and N are two Riemann surface. A mapping $f : M \to N$ is called a conformal mapping if $\forall p \in M, \widetilde{p} \in N, f(p) = \widetilde{p}$, for any local parameter chart (U, ϕ) and $(\widetilde{U}, \widetilde{\phi})$, $z = \phi(p)$, $\widetilde{z} = \widetilde{\phi}(\widetilde{p})$, under local parameters $\widetilde{z} = \widetilde{\phi} \circ f \circ \phi^{-1}(z)$ is holomorphic in U.[Fig.3.2]

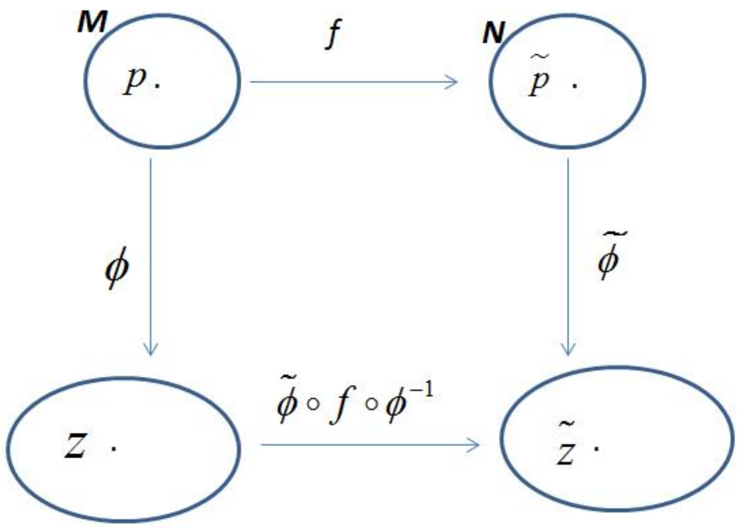


Fig.3.2 An expression of conformal mapping.

We know that a mapping $f : S \to \widetilde{S}$ is a conformal mapping between two surface if and only if there exists a positive number λ, such that at the correspondings point we have $\widetilde{I} = \lambda^2 I$, where $\widetilde{I}$ is the first fundamental form on the surface in $\widetilde{S}$, and I is the first fundamental form on the surface in S.

Now, suppose $f : \mathbb{C} \to \mathbb{C}$ is a holomorphic function, $w = f(z)$, then

$$dw = df(z) = \frac{\partial f(z)}{\partial z} dz + \frac{\partial f(z)}{\partial \bar{z}} d\bar{z}$$

but $\frac{\partial f(z)}{\partial \bar{z}} = 0$, therefore we have $dw = \frac{\partial f(z)}{\partial z} dz$, then

$$dw d\overline{w} = |\frac{\partial f(z)}{\partial z}|^2 dz d\bar{z}$$

$$I_w = \lambda^2 I_z$$

where $\lambda = \frac{\partial f(z)}{\partial z}$, and I_w, I_z are the first fundamental form.

Hence, f is a conformal map.

We can conclude that if $f : D \to \mathbb{C}$ is holomorphic and $f'(z_0) \neq 0$, $\forall z_0 \in D$, then f is conformal.

Therefore, a holomorphic function between planar domains preserves angles. Similary, holomorphic mappings between Riemann surfaces with metrics also preserve angles.

Theorem 1 *(Riemann mapping theorem)*

Suppose D is a simply connected domain on the complex plane, $z_0 \in D$ is an arbitrary interior point, then there exist a unique holomorphic map $f : D \to \triangle$ from D to the unit disk $\triangle = \{w = f(z) \mid |w| < 1\}$ with $f(z_0) = 0$ and $f'(z_0) > 0$.

In order words, according to the Riemann mapping theorem, there exists a conformal mapping from a simply connected surfce to the unit disk.

3.3 Compute the Conformal Mapping

Because the conformal mapping is an important technology of conformal morphing, we will introduce how to compute the conformal mapping from a simply connected surface to the unit disk in this section.

First, we know that the harmonic map between a topological sphere and the canonical unit sphere is automtically conformal, but it is not necessarily conformal between a topological disk and the unit disk, so we compute the double covering $\overline{M}$ of the topological disk M, which is a topological sphere, then compute a harmonic map f between the doubled surface and the unit sphere S^2 with non-linear heat diffusion method, such that each copy of the topological disk is mapped to a hemisphere. Then we use stereographic to project the unit sphere onto the whole plane, the lower hemi-sphere is mapped to the unit disk. Therefore, it induces the mapping from the surface to the unit disk, and the map is conformal.[Fig.3.3]

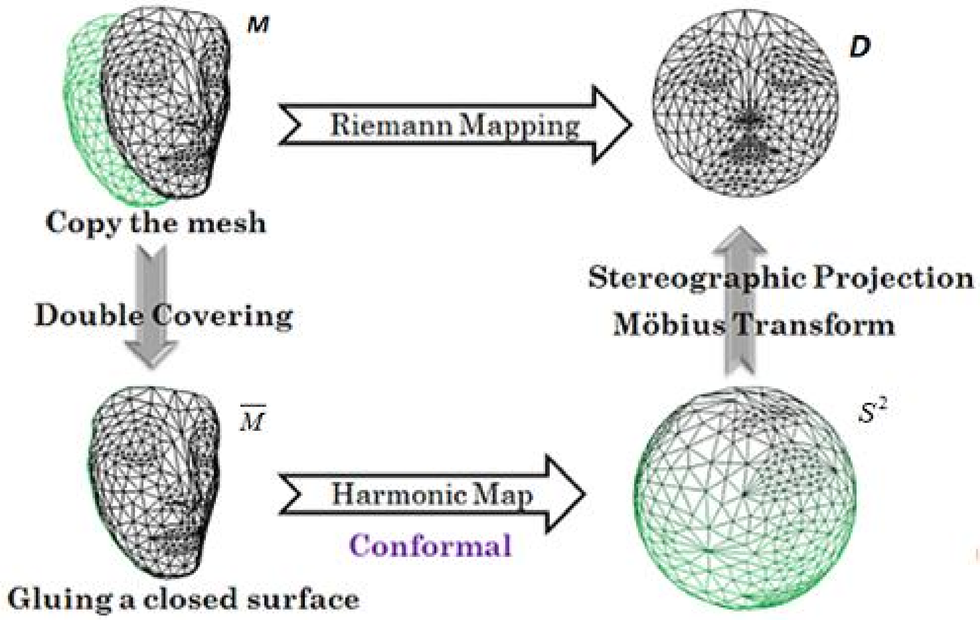


Fig.3.3 The expression of computing the conformal mapping.

In the process, we reduce the double surface to the unit sphere by using the non-linear heat diffusion method to evolve the map and compute the harmonic energy. Suppose $f : \overline{M} \to S^2$, where $\overline{M}$ is a closed surface and the genus of $\overline{M}$ is zero. We have

$$\frac{df}{dt} = -(\triangle f)^{||}$$

Evolve f to minimize the harmonic energy until it becomes a harmonic map. Given a vertex $v \in \overline{M}$, where $f(\overline{M}, t) \in S^2$,denoted $n(f(v,t))$ be the normal vector on the tangent surface at $f(v,t)$. Then the components of $\triangle f$ are defined as

$$\triangle f(v,t)^{\perp} = \langle \triangle f(v,t), n(f(v,t)) \rangle n(f(v,t))$$

$$
\begin{aligned}
\triangle f(v,t)^{\parallel} &= \triangle f(v,t) - \triangle f(v,t)^{\perp} \\
&= \triangle f(v,t) - \langle \triangle f(v,t), n(f(v,t))\rangle n(f(v,t))
\end{aligned}
$$

Then the nonlinear heat diffusion equation becomes as:

$$
\frac{df}{dt} = -(\triangle f)^{\parallel} = -(\triangle f(v,t) - \langle \triangle f(v,t), n(f(v,t))\rangle n(f(v,t)))
$$

If the target surface is the unit sphere, then $n(f(v,t)) = f(v,t)$. We simplify the equation as:

$$
\frac{df}{dt} = -(\triangle f - \langle \triangle f, f\rangle f)
$$

The iteration of the formular is using the **Explicit Eular Method,** as following:

$$
\begin{aligned}
f^{(m+1)}(v_i,t) &= f^{(m)}(v_i\,,t) - \delta t[\triangle f(v,t)^{\parallel}] \\
&= f^{(m)}(v_i\,,t) - \delta t[\triangle f^{(m)}(v_i\,,t) - \triangle f^{(m)}(v_i\,,t)^{\perp}] \\
&\approx f^{(m)}(v_i\,,t) - \frac{\delta t}{h}[K - D^{(m)}]f^{(m)}(v_i\,,t)
\end{aligned}
$$

where h denote the mesh size, K is the Laplacian matrix and $D^{(m)}$ is a diagonal matrix with

$$
D_{ii}^{(m)} = \langle\ Kf^{(m)}(v_i\,,t)\ ,\ f^{(m)}(v_i\,,t)\ \rangle
$$

In this study, we use the **Quasi-implicit Eular Method** to iterate f, because the convergent speed of Explicit Eular Method is slow. The formular has been improved by professor Tsung-Ming Huang and professor Wen-Wei Lin.

The Quasi-implicit Eular Method is defined as:

$$[I + \delta t^{(m)}\ (K - D^{(m)})]\ f^{(m+1)}(v_i, t) = f^{(m)}(v_i\ , t)$$

Then normalize $f^{(m+1)}(v_i, t)$ such that the mass center is at the sphere center and compute its harmonic energy, denoted as $\varepsilon_H(f^{(m+1)})$. To iterate f until the difference between $\varepsilon_H(f^{(m+1)})$ and $\varepsilon_H(f^{(m)})$ is small enough. Therefore, we find out the spherical conformal mapping form a topological sphere to the canoncial unit sphere.

Use Mőbius transform to adjust the conformal map such that ∂M map to the equator of the sphere. Finally, compute the stereographic projection, which map the unit sphere onto the plane. In order words, we map the half-sphere to the unit disk conformally. Sum up the process of mapping the simply connected surface to the unit disk as following:

Input: Simply connected surface M with a single boundary

Output: A conformal map $h : M \to D^1$

(1) Double covering M to $\overline{M}$, where $\overline{M}$ is a topological sphere

(2) Compute a harmonic(or conformal) map $f : \overline{M} \to S^2$ by heat flow diffusion

(3) Select $v_0,\ v_1,\ v_2 \in \partial M$.

Use Mőbius transform $\tau : (v_0,\ v_1,\ v_2) \longrightarrow (0,\ 1,\ i)$ by $w = \frac{az+b}{cz+d}$, such that $ad - bc = 1$, $a, b, c, d \in \mathbb{C}$

(4) Comput sterographic projection $\phi : S^2 \to \mathbb{C}$ such that $h = \phi \circ \tau \circ f$ is a conforml map.

3.4 Mesh Uniformization

Every surface captured from the 3D camera have the different number of vertex and face. In order to matching surface in the morphing process efficently, we uniform the number of vertex and face on each surface. First, we pick the feature points on surface in sequence and use the Riemann mapping theorem to reduce the surface into a unit disk[Fig.3.4(a)]. Creat some points on the boundary of unit disk by unitizing vector from the origin to the feature point, which we choose.[Fig.3.4(b)].

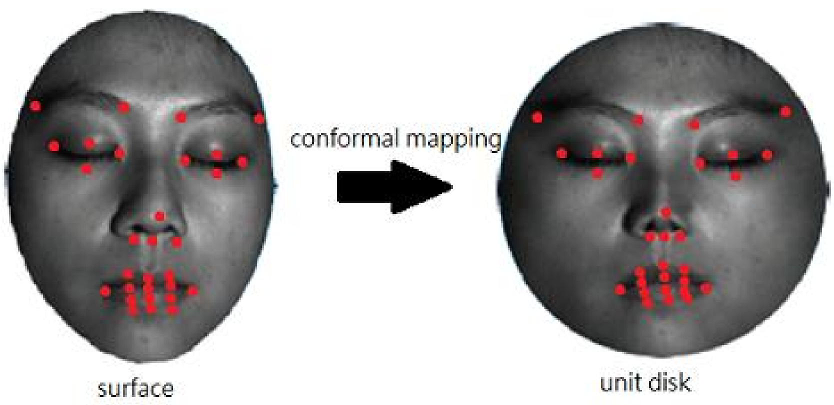


Fig.3.4(a) Pick the feature points on surface in sequence and map the surface into a unit disk.

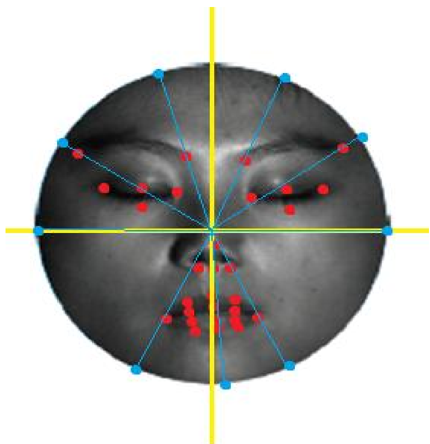


Fig.3.4(b) Unitizing vector from the origin to the feature point.

Afterwards,we construct a basic triangular mesh with feature points and the boundary points on the unit disk[Fig3.4 (c)]. Second, we have to refine the triangular mesh by connecting the midpoints,which are on the edge of each mesh. For example, if a single triangular mesh refined once, it becomes four triangular mesh; if it refined twice, it becomes sixteen triangular mesh[Fig.3.4(d)]. In other words, if there exist m triangular mesh and refine it n times, it becomes $m \times 4^n$ triangular mesh. Then find out the boundary points on the reconstructed mesh and unitise them to the boundary of unit disk. Repeat the process several times[Fig3.4(e) and (f)], we can recieve a reconstructed triangular mesh on the unit disk.

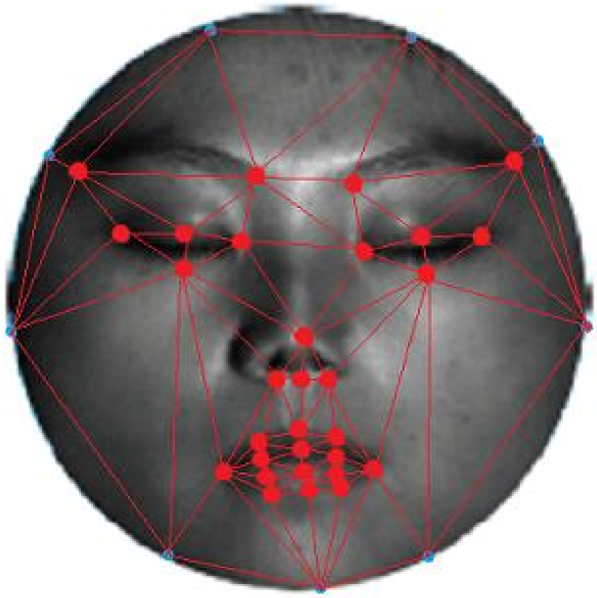


Fig.3.4(c) Construct a basic triangular mesh with feature points and the boundary points.

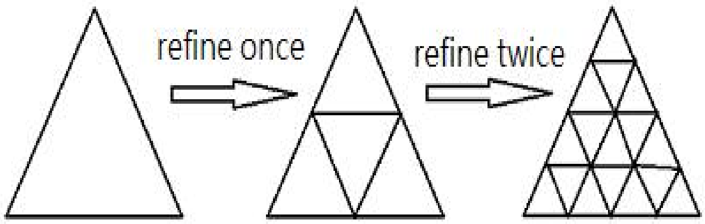


Fig.3.4(d) The process of refining a single triangular mesh twice.

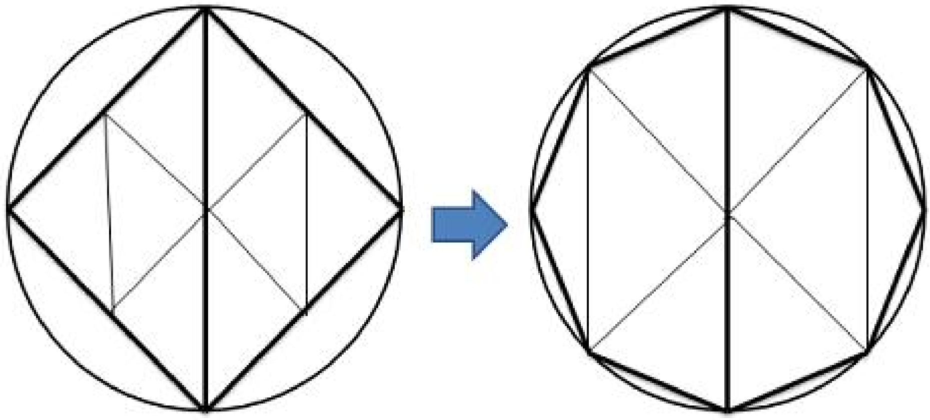


Fig.3.4(e) Refind once and unitise the points to the boundary on the unit disk.

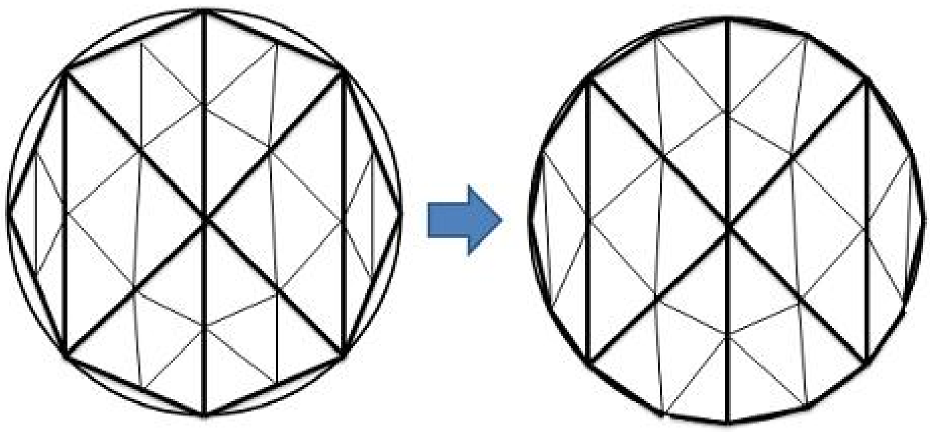


Fig.3.4(f) Refind twice and unitise the points to the boundary on the unit disk.

Last, the points which be refined on the unit disk, have to pull back to the surface. The expression of $[v_0,\ v_1,\ v_2]$ denotes the triangular mesh composed of the vertex $v_0,\ v_1$ and v_2. Suppose p_i, $i = 1, 2, 3$, be the vertex on a original triangular mesh and there exists a conformal mapping f such that $f(p_i) = q_i$, $i = 1, 2, 3$. Suppose a refined point q_0 is in the mesh $[q_1,\ q_2,\ q_3]$ on the unit disk. We compute the area A_i such that

$$A_1 = area[q_0,\ q_1,\ q_2], A_2 = area[q_0,\ q_2,\ q_3] and A_3 = area[q_0,\ q_1,\ q_3].$$

Then use the ratio of A_i to find out the point p_0 in the mesh $[p_1,\ p_2,\ p_3]$ such that

$$A_1 :\ A_2 :\ A_3 = area[p_0,\ p_1,\ p_2]\ :\ area[p_0,\ p_2,\ p_3]\ :\ area[p_0,\ p_1,\ p_3]$$

We can pull back the point q_0 on the unit disk to the point p_0 on the surface.

Therefore, we uniform the structure of mesh and the number of vertex on each mesh.

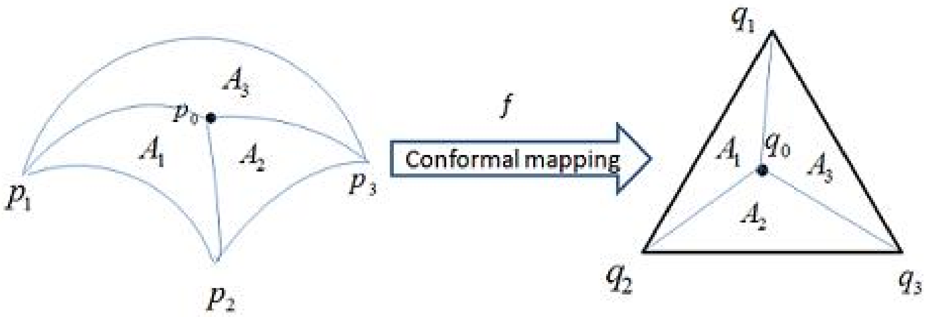


Fig.3.4(e) The expression of computing the area of mesh.

Chapter 4

Application of conformal morphing

Morphing is the process of changing one figure into another and surface morphing has many applications. In this chapter, I will show how to make a film with 3D mesh by conforml morphing.

Contents

4.1 Goal

We may apply the conformal morphing to make a series of motion pictures, which means an animation constructed by a series of moving images. The 3D camera can capture 30 mesh figures and the correspond texture images per second. Suppose we capture a film with one minute, then we have about 1,800 mesh figures. The file size of an oringinal figure which captured from our 3D camera is about 30 megabytes, so the file size of original film is too large. Our goal is using fewer figures to reconstruct a film to simulate the original film. That is, we can reduce the file size of the film and reduce the storage space.

For example, if we capture a 3D film from our 3D camera, the film consist of 1,000 frames. We choose 10% of the frames to reconstruct the film, and deal those frames with uniformization and compute the conformal mapping. Then, we reconstruct each mesh frame by using the feature points on the face and obtain a series of coarser meshes. We use the method of homotopy and conformal morphing to create the missing frames between two consecutive frames. Note that the number of points of each mesh, which we reconstructed, is approximately 1/6 of the number of the original mesh. That is, we use only 1.6% of the original data to reconstruct the original film. Now, we make a virtual broadcasting film by this idea.

4.2 To Construct the Database

4.2.1 Choosing Syllable

For our goal, I decide to make a virtual broadcasting film about one minute, and the content of the film is a form of classical poetry, which is created by a chinese poet.

The poetry is as following :

"春花秋月何時了，往事知多少。
小樓昨夜又東風，故國不堪回首月明中。
雕欄玉砌應猶在，只是朱顏改。
問君能有幾多愁，恰似一江春水向東流。"

Use the 3D camera to capture the film of foregoing poetry, which contains approximately 1,300 mesh figures and the corresponding texture images. Each word selected three figures to represent the syllable, and the syllables file name be stored by the syllable chart [5]. Note that, in chinese words, each syllable has fore different tone, but the shape of the mouth are exact resemblance. By the way, the pronunciations of Chinese characters are composed of 411 syllables. Then we deal those meshes, which we choose, with the method in chapter 2 to receive the simply connected surface. All of the meshes, which we selected, have the different number of vertex and face, so we pick the feature points on each mesh to uniform the mesh. The selecting of feature points will be intrduced in next subsection.

4.2.2 Pick the Feature Points

In order to do mesh uniformization and surface morphing, we have to pick feature points on each face. For the good effect of conformal morphing, we pick 30 feature points through extreme changes in facial expressions[Fig.4.2.2(a)]. Seeing the [Fig.4.2.2(b)], the mouth has the most number of feature points to as accurate as possible display the changes of the lips when we talk. Also, we can choose some of feature point,which from NO.31 to

NO.33, to make the relative position of the two figures a more accurate. Note that, for the process of mesh uniformization, the sequence of the feature points of each mesh must be the same.

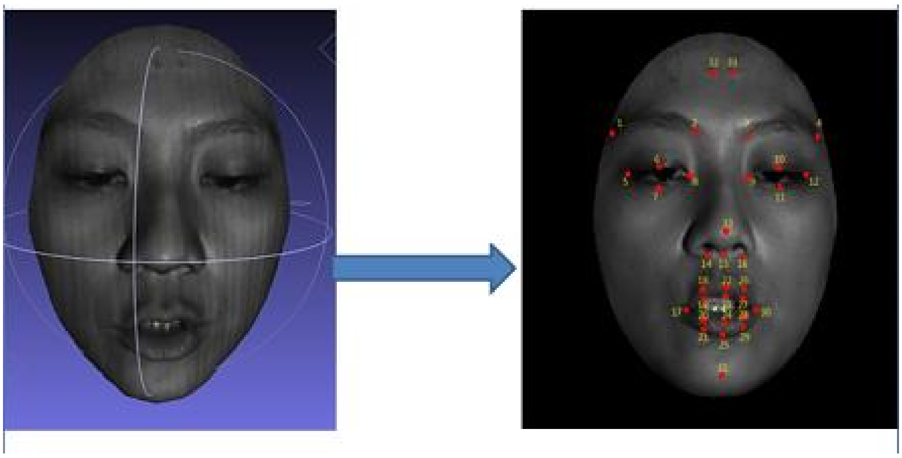


Fig.4.2.2(a) Pick the feature points on the surface

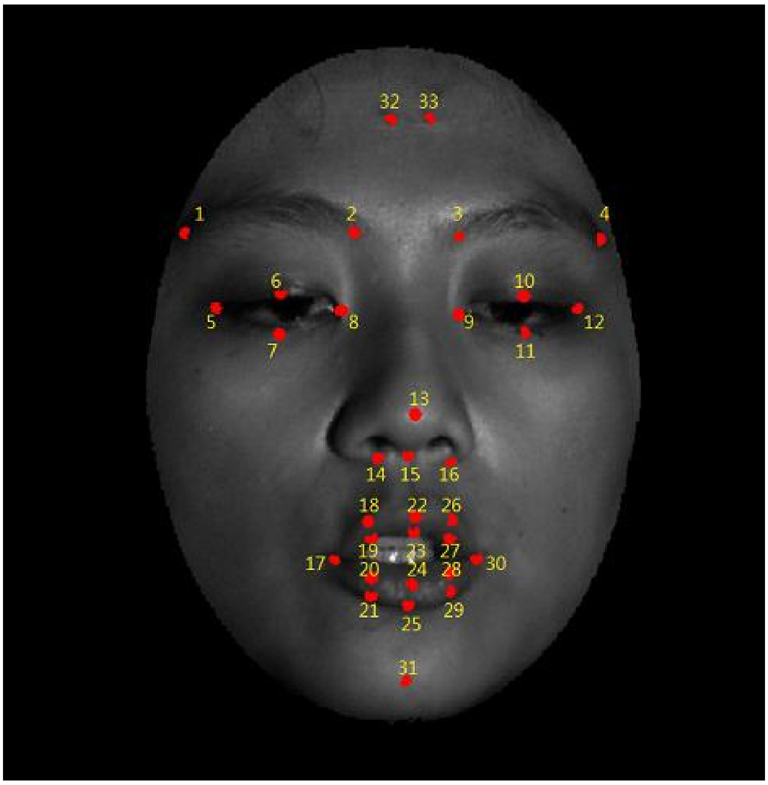


Fig. 4.2.2 (b) The number of feature points (Enlargement)

Uniform each mesh with the method of mesh uniformization in section 3.4, so each mesh has the same number of vertices and the face structure.The number sequence of vertices is the same on each mesh, so we can matching two vertex,which on different mesh, with the same number of vertex.

4.3 Surface Matching with Linear Homotopy and Cubic Spline Homotopy

Solving nonlinear systems is a fundamental problem which occurs frequently in various models of science and engineering like computational geometry. Therefour, mostly numerical methods sould be employed. For using local numerical methods,like Newton-Raphson method, the proper information about the locations of the roots is very important. The Newton-Raphson method need initial guss, sufficiently close to the roots being sought. For this reason, we need gloable numerical methods, which are concerning the start values of the iteration. Homotopy continuation proved to be as a reliable and efficient method to solve nonlinear systems, and it does not need accurate initial guess and can be used to locate all geometrically isolated solutions of a square polynomial system.

Homotopy is a continuous transformation from one function to another. Two continuous maps f_0, f_1: $M \to N$ are said to be homotopic if there is an intermediate family of continuous maps $f_t : M \to N$, for $0 \leqslant t \leqslant 1$ which vary contiously with respect to t. Now, we give the formal definition of homotopy:

Definition 4 *(Homotopy)*

Two continuous maps f_0, f_1: $M \to N$ are said to be homotopic if there is a continuous map $F : M \times I \to N$ such that $F(\cdot , 0) = f_0$ and $F(\cdot , 1) = f_1$. The map F is called a homotopy between f_0 and f_1, denoted as $F : f_0 \simeq f_1$. For each $t \in [0,1]$, we denote $F(\cdot , t)$ by $f_t : M \to N$, where f_t is a continuous map.

For getting the surfaces of the morphing process, We use the linear homotopy:

$$F(\cdot\ , t) = (1 - t)I + tf \quad , \ t \in [0, 1]$$

where I is the identity map.

Hence, we construct the V_t of the morphing process figure between two figures :

$$V_t = (1 - t)V_0 + tV_1$$

where V_0 is the vertex of the source figure and V_1 is the vertex of the target figure.

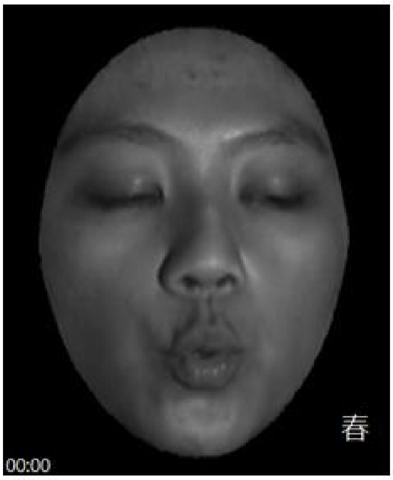
Then, we construct the missing frames between two consecutive frames. In order to complete the process of morphing, we have to find the matching function with linear homotopy.

A spline is a smooth polynomail function that is piecewise-defined, and possesses a high degree of smoothness at the place where the polynomial pieces connect, which be called knots. Splines are curves, which need to be smooth and continuous and it defined as piecewise polynomials of degree n with function values and first (n-1) derivatives that agree at the knots. Note that splines with no konts are generally smoother than spline with knots. In interpolation problem, spline interpolation is usually referred to as polynomial interpolation, because it produces the similar results. But spline interpolation is preferred over polynomial interpolation, because the interpolation error can be made small when using low degree polynomials for the spline. The most commonly used splines are cubic spline, which is a spline constructed of piecewise third-order polynomials. We use cubic spline to be the matching fuction, and then compute V_t by piecewise cubic spline homotopy between V_i pointwisely for each time step.

4.4 The Result of Virtual Broadcasting

According to the step and technique in anterior sections, we complete the virtual broadcasting film,which is the poetry in section 4.1.1, with about 50 second. The original film have approximatelly 1,400 frames, but we only use merely 145 frames to reconstruct the film, which simulate the oringinal film. Note that, we also can choose the closed-eye figures in database,then it can make the virtual report naturlly. Last, accompany the film with sound to complete the virtual report. To tape the sound with the poetry, we can mix the film with sound by using the vedio software "Corel VideoStudio Pro". It can load a trial version for free on their official website.

Here, I intercept some images from the film,which we reconstructed on the next page: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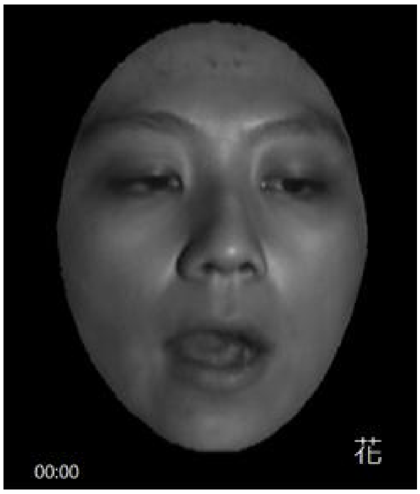
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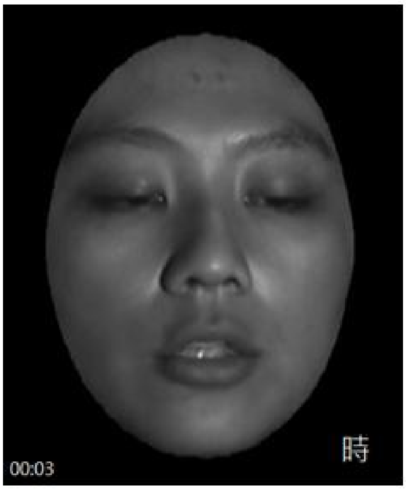
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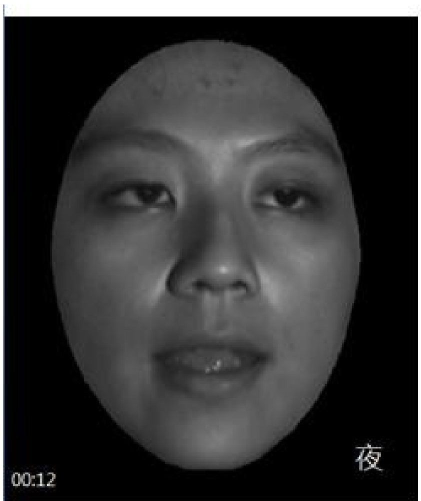
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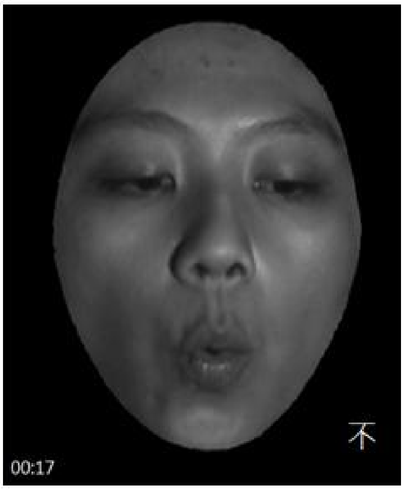
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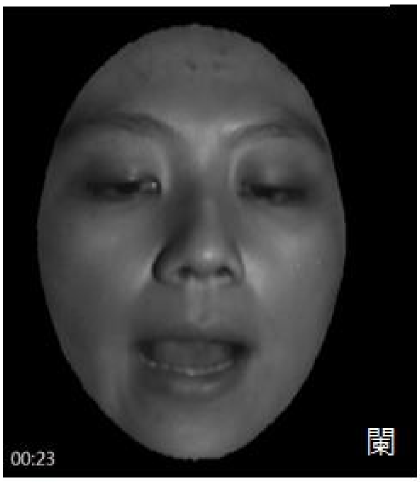
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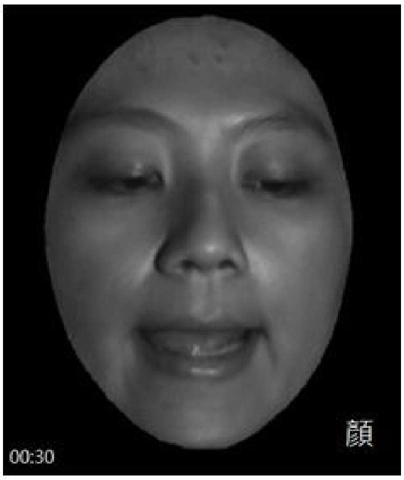
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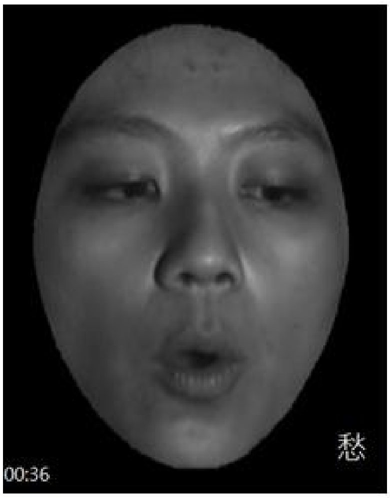
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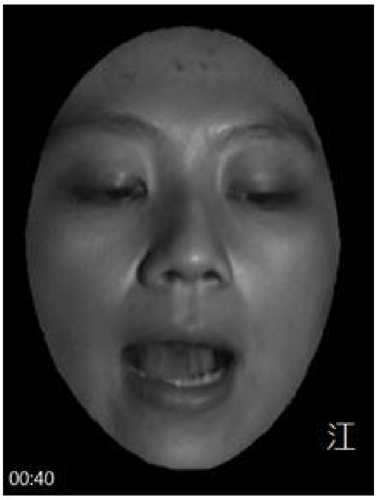
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Chapter 5

Conclusion

In this study, we first give some introduction of conformal morphing, which is an important technology in recent years. In order to compute the conformal map, we must to deal with the 3D mesh which captured from the 3D camera such that the surface is a simply connected surface. Then we introduce the conformal mapping and how to compute it, because we can use the Riemann mapping theorem to reduce the complicated problem from the surface of three dimension to the unit disk.

Second, we uniform the structure of mesh and the number of vertex on each surface in order to match two surfaces by cubic spline homotopy. Therefore, we can create surface between two real surfaces for procuring the process of conformal morphing.

The technology of conformal morphing has many applications. In this study, we use fewer surface to reconstruct a film as like as the original film. It can reduce the film size and the space of storage. The film, which we reconstructed, achieve the effect of virtual broadcasting. The complete film has been put on youtube, and the web address is http://www.youtube.com/watch?v=vFZtAZx-bak.

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