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應用於無線傳感器網路之二步著色

A distance-two coloring with applications to wireless sensor and actor networks

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中華民國一零二年六月

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摘 要

無線感測網路可廣泛的應用在環境監控。一個可以允許感測器與外界溝通的有效方法，是利用一個或多個反應節點作為從無線感測網路中所取得資料的接收者。一個無線傳感器網路是由多個隨機佈署的感測器以及少量的反應節點所組成，而反應節點會組織感測器進而形成一個以其為中心的同心圓形狀網路。定位、路由以及防止碰撞為三個主要的無線傳感器網路問題。本篇論文的主要貢獻為：提出一個新的虛擬結構來做定位以解決防止碰撞問題，同時對於我們所提出的虛擬結構的連接圖給出最佳的(在某些情況下是接近最佳的)二步著色。

關鍵詞：無線傳感器網路，粗質定位，二步著色，防止碰撞。

A distance-two coloring with application to wireless sensor and actor networks

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Abstract

Wireless sensor networks (WSNs) have a wide array of applications in environment and infrastructure monitoring. An efficient solution to allow sensors to communicate with the outside world is making use of one or several actors as the receiver of the data harvested by the WSNs. A wireless sensor and actor network (WSAN) consists of many randomly deployed sensors and a few actors that organize the sensors in their vicinity into an actor-centric network. Localization, routing, and collision avoidance are three fundamental problems in WSANs. The main contribution of this thesis is to solve the collision avoidance problem by proposing a new virtual infrastructure for the localization, and give optimal (in some cases, near-optimal) distance-two colorings for the adjacency graph of our virtual infrastructure.

Keywords: Wireless sensor and actor network, Coarse-grain localization, Distance-two coloring, Collision avoidance

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Contents

List of Figures

List of Tables

1 Introduction

A wireless sensor and actor network (WSAN) [1] consists of massively and randomly deployed tiny *sensors* and a few *actors* that organize the sensors in their vicinity into a short-lived *actor-centric* network to support a specific mission. These tiny and low-cost sensors have small (nonrenewable) energy supply and limited communication range, and, after deployed, are unaware of their location and are unattended. Actors are mobile along the area of deployed sensors to collect the sensed data from sensors within its transmission range and to aggregate and transmit to the outside world. Each actor is equipped with better processing capabilities, higher transmission power to send broadcasts for a distance, and a longer battery life than the sensors. Actor-centric sensor networks have many application in environment and infrastructure monitoring, and can detect emergent, unexpected and coherent behaviors and trends, and find immediate applications in environmental monitoring and homeland security.

In the study of WSANs, there are three fundamental problems: (i) localization, (ii) routing, and (iii) collision avoidance.

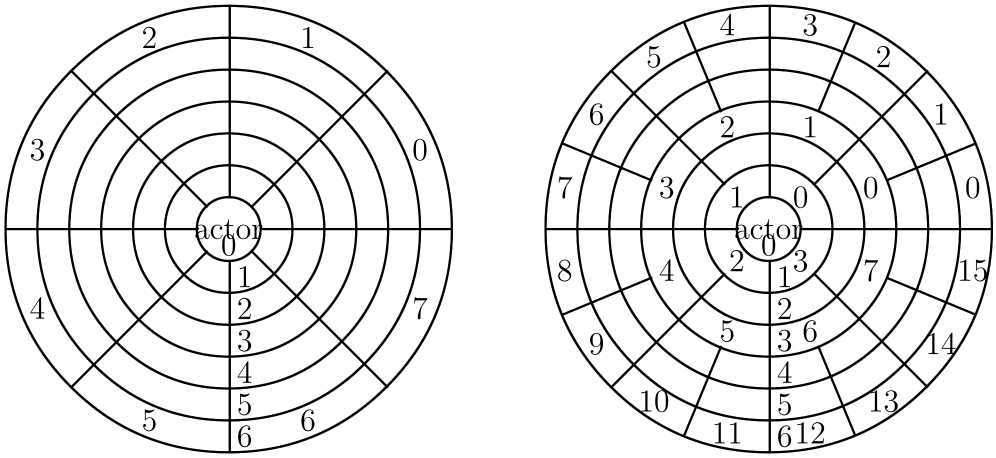
1.1 Localization

Due to the sensors constraints on the cost, size, energy consumption, and implementation environment, most sensor nodes do not know their locations. The localization problem is to determine, for individual sensor nodes, as closely as possible their geographic coordinates in the area of deployment. The sensed data could be meaningless if it is not related to the exact position or at least a sufficiently small region of the monitored area, and position information guiding sensors to transmit data has been studied on many geographic routing protocols. An immediate approach to provide the exact position of each sensor is based on localization systems (e.g., globally positioning system (GPS)), but this approach takes expensive cost and is not suitable for plenty of randomly deployed, tiny and low-cost sensors in many applications. Hence, a coarse-grain location awareness is sufficient for WSANs with a trade-off: that an coarse-grain location awareness is lightweight, but the resulting positioning accuracy is only a rough approximation of the exact geographic location.

Training is referred to the task of allowing each sensor to acquire a coarse-grain location. Wadaa et al. [19] first proposed a training protocol in which each actor trains sensors in its vicinity, namely, the *actor-region*, to associate these sensors with coarse-grain coordinates related to the actor. More precisely, after training, each sensor in the actor-region will acquire two coordinates: the *corona* and the *sector* to which it belongs. A training protocol provides for free a *clustering* of the sensors and a virtual infrastructure, where a *cluster* consists of all sensors having the same coordinates.

The resulted virtual infrastructure of training protocols proposed in [2, 3, 4, 14, 17, 18] are identical (see Figure 1(a)); one consequence of these training protocols is that: the number of sectors in each corona are the same. By contrast, Navarra and Pinotti [12] presented a new virtual infrastructure in which the number of sectors is doubled at each corona i, for i is a power of 2; see Figure 1(b). The papers [7, 13] also used the same virtual infrastructure as [12]. One interesting result of [12] is that the ratio given by the area spanned by two clusters is at most 2. Notice that in [4, 12, 13, 17, 18, 19] the terminology *sink-centric network* was used instead of actor-centric network.

In [2], Bertossi et al. proposed two scalable energy-efficient training protocols for sensor networks. Navarra, et al. [13] proposed the protocol, called Cooperative. This protocol is the fastest training algorithm for asynchronous sensors, and it matches the running time of the fastest known training algorithm for synchronous sensors. Other training protocols for WSNs have been proposed in the literature [3, 4, 19].



(a) (b)

Figure 1: (a) The virtual infrastructure with 7 coronas and 8 sectors proposed in [2, 3, 4, 14, 17, 18, 19]; the number of sectors in each corona will be the same. (b) The virtual infrastructure with 7 coronas and 4 sectors in corona 1 proposed in [7, 12, 13]; the number of sectors is doubled at coronas 2 and 4.

1.2 Routing

In a trained actor-centric network, the routing can be easily performed as followed: the message can be trivially routed inward within a single sector to the actor or routed following several paths consisting of subpaths within a sector or within a corona (clockwise or counterclockwise, depending on which is the shortest path) and a subpath toward the actor within a sector. In addition, to help the actor to locate an event that has occurred in the network, each sensor can add on its coordinates to the sensed data before delivering the messages to the actor.

1.3 Collision avoidance

A wireless sensor network can be modeled as a graph with sensor nodes as vertices and the communication link, if it exists, between any two nodes as an edge. An ordinary coloring assigns each vertex a color such that two adjacent vertices receive distinct colors. Our graph-theoretic terminologies are standard; see [5, 21]. During data transmission, packet collisions (i.e., radio interference) may occur and lead to packet losses and retransmissions, which result in an overhead on energy consumption and transmission latency, and therefore shorten the network lifetime. There are two major types of collisions: the *direct* and the *hidden collisions* [6, 20]. The former occurs when a node simultaneously delivers and receives packets, and the latter occurs when a node simultaneously receives packets from more than one node. An ordinary coloring could solve the direct collision by scheduling two sensor nodes with a link to transmit in distinct time-slots (or channels or frequencies). However, an ordinary coloring could not solve the hidden collision. Therefore, instead of using an original coloring, a *distance-two coloring* is needed, which assigns each vertex a color in such a way that two vertices receive distinct colors if they are of distance at most 2.

1.4 Our contribution

Throughout this thesis, we will follow the convention (see [12]) that ℓ is an integer and $\ell \geq 3$. In [12], Navarra and Pinotti defined the *adjacency graph* G_ℓ for their virtual infrastructure, which is: each vertex corresponds to a cluster and two vertices are adjacent if their corresponding clusters share the boundary of a corona or a sector, where ℓ is the number of sectors imposed in corona 1. They gave an optimal distance-two coloring for G_3 and a quasioptimal one for G_4. Then, Das et al. [7] gave a distance-two coloring of G_ℓ with 2ℓ colors, and Navarra et al. [13] improved the distance-two coloring algorithms of G_ℓ. We now list the best previous known results in Table 1.

G_ℓ	# of colors	lower bound	optimal coloring
$\ell = 3 \cdot 2^i$, $i \geq 0$	6	6	Yes [13]
$\ell = 4$	7	7	Yes [13]
$\ell = 5$	7	7	Yes [13]
$\ell = 4 \cdot i$, $i \geq 2$	8	6	No [13]
$\ell \geq 7$	9	6	No [13]

Table 1: The best previous distance-two colorings for G_ℓ.

Let $|i|_j$, where i is an integer and j is a positive integer, denote the non-negative remainder of the integer division of i by j.

In this article, we propose a new virtual infrastructure and distance-two colorings for the adjacency graph H_ℓ of our virtual infrastructure. We now list our results in Table 2.

H_ℓ	# of colors	lower bound	optimal coloring
$\ell = 3 \cdot i$, $i \geq 1$	6	6	Yes
$\ell = 4$	7	7	Yes
$\ell = 5$	7	7	Yes
$\ell = 8 \cdot i$, $i \geq 1$	6	6	Yes
$\ell = 10$ or $\ell = 20$	7	6	No
$\ell \in \{m, 2m, 4m\}$, odd $m \geq 7$ and $3 \nmid m$	8	6	No [9]

Table 2: The performance of our distance-two colorings for H_ℓ.

The remaining part of this thesis is organized as follows. Section 2 gives our virtual infrastructure, basic definitions, and lower bounds for distance-two coloring. Section 3 proposes our distance-two colorings. Section 4 discusses the leader election problem for our virtual infrastructure. Concluding remarks are given in the final section.

2 Our virtual infrastructure, basic definitions, and lower bounds

We first describe the WSAN model. In a WSAN, all sensors possess three basic capabilities: sensory, computation, and wireless communication; and operate subject to the following constraints:

1. Each sensor is *asynchronous* — it wakes up for the first time according to its internal clock and it is not engaged in an explicit synchronization protocol, neither with the actor nor with the other sensors;

2. Individual sensors are *unattended* — once deployed, it is neither feasible nor practical to devote attention to individual sensors;

3. No sensor has global information about the network topology, but each sensor can receive transmissions from the sink;

4. The sensors are *anonymous* — they are not associated with unique IDs;

5. Each sensor has a modest non-renewable energy budget and a limited transmission range;

6. Sensors can transmit and receive on multiple frequency channels. Moreover, the number of channels and frequencies are the same for all the sensors.

A training protocol imposes a virtual coordinate system onto the sensor networks by establishing:

1. *Coronas* : The actor-region is divided into k coronas $C_0, C_1, \ldots, C_{k-1}$ determined by k concentric circles of radii $r_1 < r_2 < \cdots < r_k$ centered at the actor.

2. *Sectors* : The actor-region is divided into h equiangular sectors $S_0, S_1, \ldots, S_{h-1}$, originated at the actor, each having a width of $\frac{2\pi}{h}$ radians.

For convenience, the coronas and sectors are referred by specifying their numbers; thus, corona C_c and sector S_s will be referred to as corona c and sector s, respectively. In a built virtual coordinate system, a *cluster* is the intersection between a corona c and a sector s. All sensors in a cluster acquire the same coordinates, denoted by (c, s). For convenience, the radii r_i's are considered as $r_i = i$ for $i = 1, 2, \ldots, k$.

2.1 Our virtual infrastructure

We now propose a new virtual infrastructure with ℓ sectors imposed in corona 1 and the number of sectors is doubled at each corona c, where $c = 2p$, for $p = 1, 2, \ldots, \lfloor\frac{k-1}{2}\rfloor$. Set

$$h_c = \ell \cdot 2^{\lfloor\frac{c}{2}\rfloor}$$

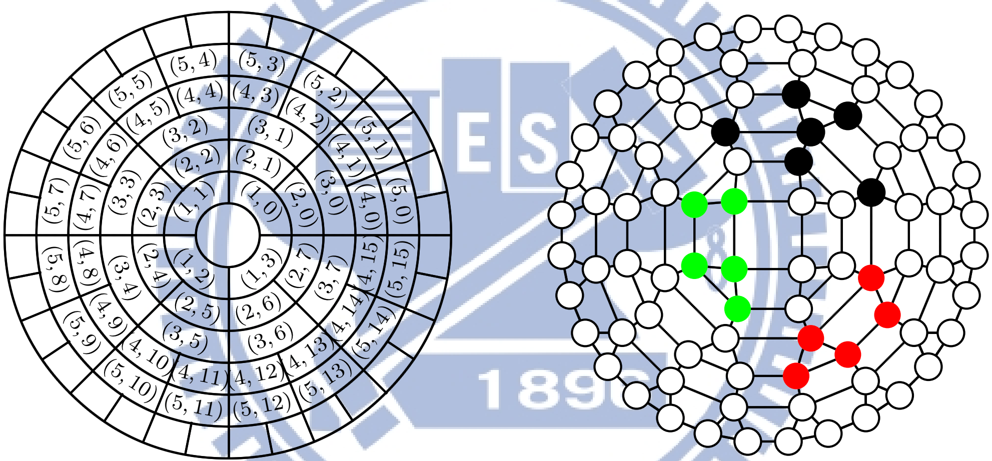
for easy writing, which is the number of sectors in corona c. The formulated definition of the adjacency graph of our virtual infrastructure is given in the following definition.

Definition 1. *The adjacency graph H_ℓ has one vertex (c, s), where $1 \le c \le k-1$ and $0 \le s < h_c$, for each cluster in the virtual infrastructure. Two vertices (c, s) and (c', s'), with $c \ge c'$, are adjacent if*

1. $c = c'$ and $|s - s'| \equiv 1 \pmod{h_c}$, or

2. $c = c' + 1$ is odd and $s = s'$, or

3. $c = c' + 1$ is even and $s' = \lfloor \frac{s}{2} \rfloor$. (See Figure 2 for an illustration.)



(a) (b)

Figure 2: (a) The virtual infrastructure that starts with $\ell = 4$ sectors. (b) Its corresponding adjacency graph H_4; the six black, the five green, and the five red vertices denote $S_{(3,1)}$, $T_{(2,4)}$, and $T_{(4,13)}$, respectively.

2.2 Basic definitions and lower bounds for distance-two coloring

It is easy to see that in adjacency graphs G_ℓ and H_ℓ, a vertex corresponds to a cluster and two vertices are adjacent if their corresponding clusters share the boundary of a corona or a sector. For the rest of our discussion, we will not consider corona 0 since the sensors in it can retrieve the information with the actor by itself, so the scheduling of communication in it is not necessary. Equivalently, it could be assumed that the transmission reaches the actor when it reaches corona 0 (see also [7, 12, 13]). We now give the definition of distance-two coloring.

Definition 2. *A distance-two coloring of a graph G is an assignment of a color to each of the vertices of G in such a way that two vertices are assigned different colors whenever they are at distance one or two (i.e., they are adjacent or have a common neighbor). If the colors are chosen from a set of d colors, then the coloring is called a distance-two d-coloring.*

Before going further, we introduce two notations that will be used in later discussion. For odd $c \geq 3$, define $S_{(c,s)}$ be a 6-element subset of the vertex set of H_ℓ such that

$$S_{(c,s)} = \{(c-1, s),\ (c, |s-1|_{h_c}),\ (c, s),\ (c, |s+1|_{h_c}),\ (c+1, |2s|_{h_{c+1}}),\ (c+1, |2s+1|_{h_{c+1}})\}.$$

For even $c \geq 2$, define $T_{(c,s)}$ be a 5-element subset of the vertex set of H_ℓ such that

$$T_{(c,s)} = \{(c-1, ||\lfloor \tfrac{s-1}{2} \rfloor|_{h_{c-1}}),\ (c-1, ||\lfloor \tfrac{s+1}{2} \rfloor|_{h_{c-1}}),\ (c, |s-1|_{h_c}),\ (c, s),\ (c, |s+1|_{h_c})\}.$$

For $S_{(c,s)}$ or $T_{(c,s)}$, the vertex (c, s) will be called its *center*. See Figure 3 for an illustration. Also, the six black, the five green, and the five red vertices shown in Figure 2(b) denote $S_{(3,1)}$, $T_{(2,4)}$, and $T_{(4,13)}$, respectively.

Immediately, we observe the following fact.

Lemma 2.1. *All the vertices in $S_{(c,s)}$ have a pairwise distance of at most two. This is also true for $T_{(c,s)}$.*

This lemma is obvious and its proof is omitted. Do notice that it is impractical to consider a virtual infrastructure with three (or fewer) coronas. Therefore, all the adjacency graphs G_ℓ's and H_ℓ's are assumed to have at least 4 coronas. This assumption is crucial to the lower bound of the number of colors required by a distance-two coloring. In particular,

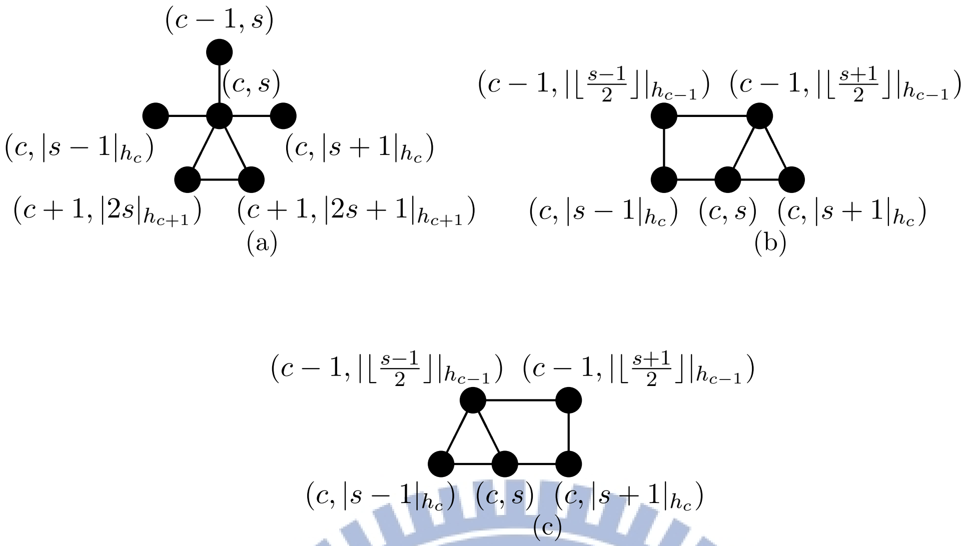


Figure 3: (a) $S_{(c,s)}$; (b) $T_{(c,s)}$ for even s; (c) $T_{(c,s)}$ for odd s.

Navarra et al. [13] proved that any distance-two coloring of G_ℓ requires at least 6 colors; and we prove the following lemma

Lemma 2.2. *Any distance-two coloring of H_ℓ requires at least 6 colors.*

Proof. By definition, $S_{(3,1)} = \{(2,1),(3,0),(3,1),(3,2),(4,2),(4,3)\}$. Since $S_{(3,1)}$ has 6 vertices, it follows from Lemma 2.1 that any distance-two coloring of H_ℓ requires at least 6 colors. ∎

The above lower bound can be sharpened for H_4 and H_5; see the following theorem.

Theorem 2.3. *[9] Any distance-two coloring of H_4 or H_5 requires at least 7 colors.*

In [13], Navarra et al. also proved that any distance-two coloring of G_4 or G_5 requires at least 7 colors. Theorem 2.3 provides a much simpler proof for such a result since the subgraph of H_ℓ induced by vertices in coronas 1 to 4 is isomorphic to the subgraph of G_ℓ induced by vertices in the same coronas. Before ending this section, we give two interesting results. The first one uses the well-known Brooks' Theorem, which says that if

G is a connected graph other than a complete graph or an odd cycle, then the chromatic number of G is at most the maximum degree of G.

Lemma 2.4. *The minimum number of colors required by a distance-two coloring of H_ℓ is between 6 and 16.*

Proof. A distance-two coloring of a graph G can be obtained from a coloring of the square of G (i.e., G^2). Since the maximum degree of H_ℓ^2 is at most 16, by Lemma 2.2 and Brooks' Theorem, we have this lemma. ∎

Lemma 2.5. *If H_ℓ has a distance-two d-coloring, then so does $H_{2\ell}$.*

Proof. This lemma follows from the fact that $H_{2\ell}$ is an induced subgraph of H_ℓ ($H_{2\ell}$ can be obtained form H_ℓ by removing vertices in coronas 1 and 2). ∎

3 Our distance-two coloring algorithms

In this section, we propose algorithms $OPT3$, $OPT8$, $OPT4$, $OPT5$, and COL to color H_ℓ for $\ell = 3 \cdot i$, $\ell = 8 \cdot i$, $\ell = 4$, $\ell = 5$, and $\ell \geq 3$ (the general case), respectively. We will prove that the first four algorithms (i.e., $OPT3$, $OPT8$, $OPT4$, and $OPT5$) give optimal distance-two colorings, and the last algorithm COL gives a near-optimal one.

3.1 Optimal coloring for H_ℓ with $\ell = 3 \cdot i$

Let $M(c,s)$ denote the value of the (c,s) entry in a matrix M. The idea of our coloring algorithm is to design a 4-by-3 matrix with the following three properties (Ψ_1, Ψ_2, and Ψ_3) and to use this matrix to perform coloring.

Ψ_1: For c, s, and s', we always have $M(|c|_4, |s'|_3) \neq M(|c|_4, |s|_3)$ if $|s'|_3 \neq |s|_3$.

Ψ_2: For c, s, and s', we always have $M(|c+1|_4, |s|_3) \neq M(|c|_4, |s'|_3)$.

Ψ_3: For c and s, we always have $M(|c+2|_4, |s|_3) = M(|c|_4, |2s+2|_3)$.

We now design a 4-by-3 matrix

$$A = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} 6 & 5 & 4 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 1 \end{pmatrix} \end{array}.$$

Then $A(0,0) = 6$, $A(0,1) = 5$, $A(0,2) = 4$, etc. It is easy to verify that matrix A is designed with properties Ψ_1, Ψ_2, and Ψ_3. We now give a coloring algorithm for H_ℓ with $\ell = 3 \cdot i$, $i \geq 1$; see Figure 4 for an illustration of this algorithm.

Algorithm 1 *OPT*3 (As Executed At Every Vertex)

1: vertex (c, s) gets the color $A(|c|_4, |s|_3)$;

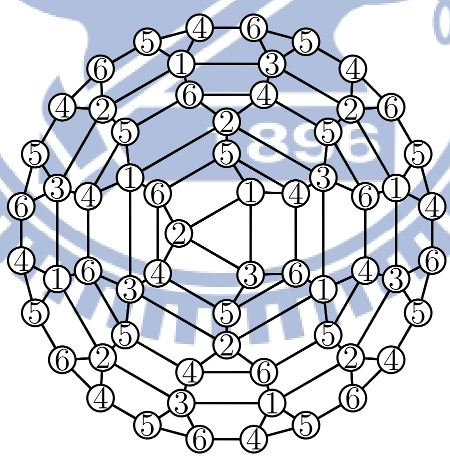


Figure 4: The optimal distance-two 6-coloring for H_3 produced by $OPT3$.

Theorem 3.1. *Algorithm OPT3 is distributed, takes constant time, and produces an optimal distance-two 6-coloring for H_ℓ with $\ell = 3 \cdot i$, $i \geq 1$.*

Proof. It is obvious that $OPT3$ is distributed (a vertex could look up matrix A and obtain its own color independently) and takes constant time. Since $\ell = 3 \cdot i$ and $i \geq 1$ and

$h_c = \ell \cdot 2^{\lfloor \frac{c}{2} \rfloor}$, we have $3 \mid h_c$. Let f be the coloring produced by $OPT3$. We now verify that f is a distance-two coloring. Suppose (c, s) and (c', s') are two distinct vertices that are of distance at most 2 and $c \le c'$. Then $c' - c \le 2$ and there are three cases.

Case 1: $c' = c$. Then since $3 \mid h_c$, we have $1 \le |s' - s|_{h_c} \le 2$ and hence $1 \le |s' - s|_3 \le 2$, which implies $|s'|_3 \ne |s|_3$. By Ψ_1, $f(c', s') - f(c, s) = A(|c|_4, |s'|_3) - A(|c|_4, |s|_3) \ne 0$.

Case 2: $c' = c + 1$. By Ψ_2, $f(c', s') - f(c, s) = A(|c+1|_4, |s|_3) - A(|c|_4, |s'|_3) \ne 0$.

Case 3: $c' = c + 2$. Then either $s' = 2s$ or $2s + 1$ occurs. In the former case, by Ψ_3 and then Ψ_1, $f(c', s') - f(c, s) = A(|c+2|_4, |2s|_3) - A(|c|_4, |s|_3) = A(|c|_4, |2(2s)+2|_3) - A(|c|_4, |s|_3) = A(|c|_4, |s+2|_3) - A(|c|_4, |s|_3) \ne 0$. In the latter case, again by Ψ_3 and then Ψ_1, $f(c', s') - f(c, s) = A(|c+2|_4, |2s+1|_3) - A(|c|_4, |s|_3) = A(|c|_4, |2(2s+1)+2|_3) - A(|c|_4, |s|_3) = A(|c|_4, |s+1|_3) - A(|c|_4, |s|_3) \ne 0$.

Therefore, f is a distance-two coloring. It is obvious that f uses 6 colors. Thus by Lemma 2.2, $OPT3$ is optimal and we have this theorem. ∎

3.2 Optimal coloring for H_ℓ with $\ell = 8 \cdot i$

First we define seven permutations on colors $1, 2, \ldots, 6$: $p_0 = (3,5)$, $p_1 = (1,3)$, $p_2 = (2,6)$, $p_3 = (2,5)$, $p_4 = (3,4)$, $p_5 = (3,6)$, and $p_6 = (1,2)$, where a permutation (x, y) exchanges colors x and y in a coloring (i.e., replaces x with y, and y with x), and for a color c we denote the operator $\circ$ by

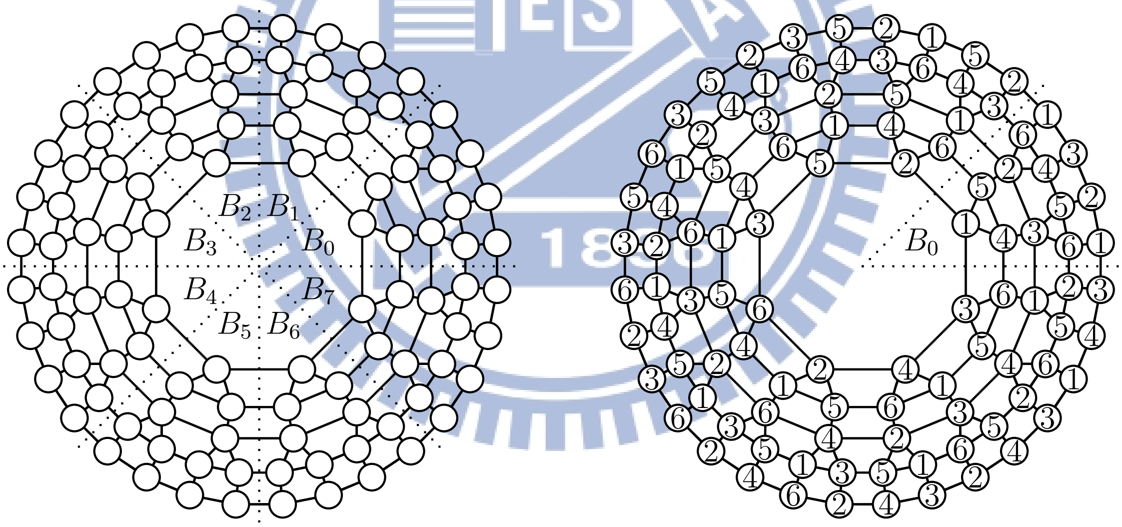
$$c \circ (x, y) = \begin{cases} y & \text{if } c = x; \\ x & \text{if } c = y; \\ c & \text{otherwise,} \end{cases} \quad \text{and} \quad c \circ (x, y)(x', y') = (c \circ (x, y)) \circ (x', y').$$

We now give a coloring algorithm for H_ℓ with $\ell = 8 \cdot i$, $i \ge 1$. Imagine that we partition the vertices of H_ℓ into eight subsets (we also call them *blocks*) $B_0, B_1, \ldots, B_7$, where $B_b = \{(c, s) : \left\lfloor \frac{8s}{\ell \cdot 2^{\lfloor \frac{c}{2} \rfloor}} \right\rfloor = b\}$. See Figure 5(a). The idea of our algorithm is to color

H_ℓ by using OPT3; when $3 \mid \ell$, we are done, and when $3 \nmid \ell$, we change the colors of vertices in B_b by using the permutation $\prod_{j<b} p_j = p_0 p_1 \ldots p_{b-1}$. See Figure 5(b) for an illustration of this algorithm.

Algorithm 2 *OPT*8 (As Executed At Every Vertex)

if $|\ell|_3 = 0$ **then** // $3 \mid \ell$.
    vertex (c, s) gets the color $A(|c|_4, |s|_3)$;
else
    let $b = \left\lfloor \frac{8 \cdot s}{\ell \cdot 2^{\lfloor \frac{c}{2} \rfloor}} \right\rfloor$;
    if $|\ell|_3 = 1$ **then**
        vertex (c, s) gets the color $A(|c+2|_4, |s|_3) \circ \prod_{j<b} p_j$;
    else // $|\ell|_3 = 2$.
        vertex (c, s) gets the color $A(|c|_4, |s|_3) \circ \prod_{j<b} p_j$;
    end if
end if



(a) (b)

Figure 5: (a) The 8 blocks of H_8. (b) The distance-two 6-coloring for H_8 produced by *OPT*8.

Theorem 3.2. *Algorithm OPT8 is distributed, takes constant time, and produces an optimal distance-two 6-coloring for H_ℓ with $\ell = 8 \cdot i$, $i \geq 1$.*

Proof. It is obvious that *OPT*8 is distributed and takes constant time. If $|\ell|_3 = 0$, then

$OPT8$ performs in the same way as $OPT3$; by Theorem 3.1, $OPT8$ produces an optimal distance-two 6-coloring. In the remaining proof, we consider $|\ell|_3 \neq 0$.

For convenience, let $p_7 = (2,4)$, $A_0 = A$, and $A_1, A_2, \ldots, A_7$ be matrices such that $A_b(c,s) = A(c,s) \circ \prod_{j<b} p_j$. Then:

$$A_0 = \begin{array}{cc} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} 6 & 5 & 4 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 1 \end{pmatrix} \end{array}, \quad A_1 = \begin{array}{cc} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} 6 & 3 & 4 \\ 1 & 2 & 5 \\ 4 & 3 & 6 \\ 5 & 2 & 1 \end{pmatrix} \end{array}, \quad A_2 = \begin{array}{cc} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} 6 & 1 & 4 \\ 3 & 2 & 5 \\ 4 & 1 & 6 \\ 5 & 2 & 3 \end{pmatrix} \end{array}, \quad A_3 = \begin{array}{cc} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} 2 & 1 & 4 \\ 3 & 6 & 5 \\ 4 & 1 & 2 \\ 5 & 6 & 3 \end{pmatrix} \end{array},$$

$$A_4 = \begin{array}{cc} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} 5 & 1 & 4 \\ 3 & 6 & 2 \\ 4 & 1 & 5 \\ 2 & 6 & 3 \end{pmatrix} \end{array}, \quad A_5 = \begin{array}{cc} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} 5 & 1 & 3 \\ 4 & 6 & 2 \\ 3 & 1 & 5 \\ 2 & 6 & 4 \end{pmatrix} \end{array}, \quad A_6 = \begin{array}{cc} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} 5 & 1 & 6 \\ 4 & 3 & 2 \\ 6 & 1 & 5 \\ 2 & 3 & 4 \end{pmatrix} \end{array}, \quad A_7 = \begin{array}{cc} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} 5 & 2 & 6 \\ 4 & 3 & 1 \\ 6 & 2 & 5 \\ 1 & 3 & 4 \end{pmatrix} \end{array}.$$

Since A_0 is exactly A, it clearly has the properties Ψ_1, Ψ_2, and Ψ_3. For $b = 1, 2, \ldots, 7$, A_b is obtained by renaming the colors in A. Thus $A_1, A_2, \ldots, A_7$ also have the properties Ψ_1, Ψ_2, and Ψ_3.

Let f be the coloring produced by $OPT8$. Then

$$f(c,s) = \begin{cases} A_b(|c+2|_4, |s|_3) & \text{if } |\ell|_3 = 1; \\ A_b(|c|_4, |s|_3) & \text{if } |\ell|_3 = 2, \end{cases} \quad for (c,s) \in B_b.$$

We now verify that f is a distance-two coloring. Suppose $(c,s) \in B_b$ and $(c',s') \in B_{b'}$ are two distinct vertices that are of distance at most 2. Then $|b'-b|_8 \leq 2$ and there are three cases.

Case 1: $b' = b$. Then (c,s) and (c',s') belong to the same block and therefore get their colors from the same matrix A_b, which has properties Ψ_1, Ψ_2, and Ψ_3. Thus, using an argument similar to the one used in Theorem 3.1, we have

$$f(c',s')-f(c,s)=\begin{cases} A_b(|c'+2|_4,|s'|_3)-A_b(|c+2|_4,|s|_3)\neq 0 & \text{if } |\ell|_3=1;\\ A_b(|c'|_4,|s'|_3)-A_b(|c|_4,|s|_3)\neq 0 & \text{if } |\ell|_3=2.\end{cases}$$

Case 2: $b'=|b+2|_8$. This case occurs only when $\ell=8$ and $c'=c=1$ and $|s'-s|_8=2$. By checking the coloring in corona 1 of Figure 5(b), $f(c',s')-f(c,s)\neq 0$ holds.

Case 3: $b'=|b+1|_8$. Then (c,s) and (c',s') belong to two adjacent blocks and there are two subcases.

Subcase 3-1: $c'=c$. In this subcase, $1\leq |s'-s|_{h_c}\leq 2$. When $b=0,1,\ldots,6$, we have $|s'|_3\neq |s|_3$, and we observe that if the color $A_{b+1}(|c|_4,|s'|_3)$ is not indicated in p_b, then $A_{b+1}(|c|_4,|s'|_3)=A_b(|c|_4,|s'|_3)\neq A_b(|c|_4,|s|_3)$ by Ψ_1; otherwise, for some s'', $A_{b+1}(|c|_4,|s'|_3)=A_b(|c+1|_4,|s''|_3)\neq A_b(|c|_4,|s|_3)$ by Ψ_2. When $b=7$, we have $|s'+h_c|_3\neq |s|_3$, and we observe that if the color $A_0(|c|_4,|s'|_3)$ is not indicated in p_7, then $A_0(|c|_4,|s'|_3)=A_7(|c|_4,|s'+h_c|_3)\neq A_7(|c|_4,|s|_3)$ by Ψ_1; otherwise, for some s'', $A_0(|c|_4,|s'|_3)=A_7(|c+1|_4,|s''|_3)\neq A_7(|c|_4,|s|_3)$ by Ψ_2. Thus, we have

$$f(c',s')-f(c,s)=\begin{cases} A_{|b+1|_8}(|c'+2|_4,|s'|_3)-A_b(|c+2|_4,|s|_3)\neq 0 & \text{if } |\ell|_3=1;\\ A_{|b+1|_8}(|c'|_4,|s'|_3)-A_b(|c|_4,|s|_3)\neq 0 & \text{if } |\ell|_3=2.\end{cases}$$

Subcase 3-2: $c'\neq c$. Since (c,s) and (c',s') belong to two adjacent blocks, in this subcase, (c',s') and (c,s) are of distance exactly two and $|c'-c|=|s'-s|_{h_c}=1$. So we only need to check the colors used on the boundary of two adjacent blocks (i.e., the boundary of B_0 and B_1, the boundary of B_1 and B_2, …, the boundary of B_7 and B_0). Here we list the colors used on these boundaries for $|\ell|_3=1$

and $|\ell|_3 = 2$:

$\lvert\ell\rvert_3 = 1$	B_0	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_0
$c = 1$	$3 \cdots 2$	$1 \cdots 5$	$2 \cdots 3$	$5 \cdots 6$	$3 \cdots 2$	$6 \cdots 4$	$2 \cdots 3$	$4 \cdots 1$	$3 \cdots$
$c = 2$	$6 \cdots 6$	$3 \cdots 3$	$4 \cdots 4$	$2 \cdots 2$	$1 \cdots 1$	$3 \cdots 3$	$5 \cdots 5$	$2 \cdots 2$	$6 \cdots$
$c = 3$	$1 \cdots 1$	$2 \cdots 2$	$5 \cdots 5$	$3 \cdots 3$	$6 \cdots 6$	$2 \cdots 2$	$4 \cdots 4$	$3 \cdots 3$	$1 \cdots$
$c = 2$	$4 \cdots 5$	$6 \cdots 4$	$1 \cdots 6$	$4 \cdots 1$	$5 \cdots 4$	$1 \cdots 5$	$6 \cdots 1$	$5 \cdots 6$	$4 \cdots$
$c = 1$	$3 \cdots 2$	$1 \cdots 5$	$2 \cdots 3$	$5 \cdots 6$	$3 \cdots 2$	$6 \cdots 4$	$2 \cdots 3$	$4 \cdots 1$	$3 \cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

$\lvert\ell\rvert_3 = 2$	B_0	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_0
$c = 1$	$1 \cdots 1$	$2 \cdots 2$	$5 \cdots 5$	$3 \cdots 3$	$6 \cdots 6$	$2 \cdots 2$	$4 \cdots 4$	$3 \cdots 3$	$1 \cdots$
$c = 2$	$4 \cdots 5$	$6 \cdots 4$	$1 \cdots 6$	$4 \cdots 1$	$5 \cdots 4$	$1 \cdots 5$	$6 \cdots 1$	$5 \cdots 6$	$4 \cdots$
$c = 3$	$3 \cdots 2$	$1 \cdots 5$	$2 \cdots 3$	$5 \cdots 6$	$3 \cdots 2$	$6 \cdots 4$	$2 \cdots 3$	$4 \cdots 1$	$3 \cdots$
$c = 4$	$6 \cdots 6$	$3 \cdots 3$	$4 \cdots 4$	$2 \cdots 2$	$1 \cdots 1$	$3 \cdots 3$	$5 \cdots 5$	$2 \cdots 2$	$6 \cdots$
$c = 5$	$1 \cdots 1$	$2 \cdots 2$	$5 \cdots 5$	$3 \cdots 3$	$6 \cdots 6$	$2 \cdots 2$	$4 \cdots 4$	$3 \cdots 3$	$1 \cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

From the above lists, two vertices get different colors if they are on the boundary of two adjacent blocks and of distance exactly two.

From the above, f is a distance-two coloring. It is obvious that f uses 6 colors. Thus by Lemma 2.2, $OPT8$ is optimal and we have this theorem. ∎

3.3 Optimal coloring for H_4

We define a matrix

$$M_4 = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{pmatrix} 6 & 4 & 1 & 5 & - & - & - & - \\ 1 & 3 & 7 & 2 & 6 & 3 & 7 & 2 \end{pmatrix} \end{array}$$

where "–" means the corresponding item is not used. By Lemma 2.5, H_8 is a subgraph of H_4. Thus one way to color H_4 is to extend a coloring of H_8 and this leads to Algorithm $OPT4$. See Figure 6 for an illustration of this algorithm.

Algorithm 3 *OPT*4 (As Executed At Every Vertex)

if $c \leq 2$ **then**
  vertices (c, s) get the color $M_4(c, s)$;
else // $c \geq 3$.
  if $(c, s) = (3, 0) \mid\mid (c, s) = (3, 4)$ **then**
    vertex (c, s) gets the color 7;
  else
    let $b = \left\lfloor \frac{2 \cdot s}{2^{\lfloor \frac{c}{2} \rfloor}} \right\rfloor$;
    vertex (c, s) gets the color $A(|c+2|_4, |s|_3) \circ \prod_{j<b} p_j$; // use the $|\ell|_3 = 1$ case in *OPT*8.
  end if
end if

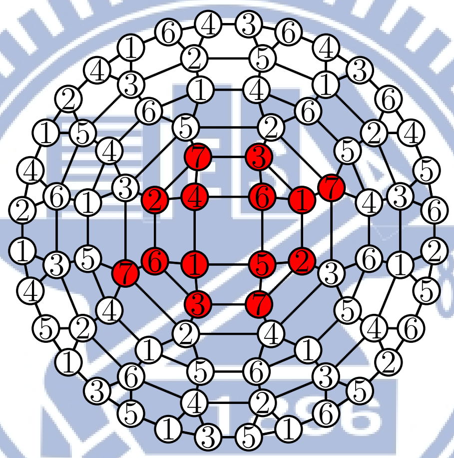


Figure 6: The distance-two 7-coloring for H_4 produced by $OPT4$; all the vertices are colored by using A (along with permutations $p_0, p_1, \ldots, p_6$) except that those highlighted are colored by using M_4.

Theorem 3.3. *Algorithm OPT4 is distributed, takes constant number of steps, and produces an optimal distance-two 7-coloring for* H_4.

Proof. It is obvious that $OPT8$ is distributed and takes constant time. Let f be the coloring produced by $OPT4$. We now verify that f is a distance-two coloring. Suppose (c, s) and (c', s') are two distinct vertices that are of distance at most 2. If at least one of (c, s) and (c', s') is highlighted (see Figure 6), then $f(c, s) \neq f(c', s')$ can be verified by a brute-force checking. If both of (c, s) and (c', s') are not highlighted, then $OPT4$ performs in the same way as the $|\ell|_3 = 1$ case of $OPT8$; hence $f(c,s) \neq f(c',s')$ by Theorem 3.2. From the above, f is a distance-two coloring. It is obvious that f uses 7 colors. Thus by Theorem 2.3, $OPT4$ is optimal and we have this theorem. ∎

3.4 Optimal coloring for H_5

We first define a matrix $M_5 =$

$$
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c}
\begin{array}{cccccccccccccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 \end{array} \\
\begin{pmatrix}
1 & 2 & 5 & 4 & 3 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
4 & 5 & 6 & 4 & 1 & 7 & 6 & 1 & 5 & 7 & - & - & - & - & - & - & - & - & - & - \\
3 & 2 & 7 & 5 & 6 & 3 & 2 & 7 & 4 & 1 & - & - & - & - & - & - & - & - & - & - \\
6 & 5 & 4 & 6 & 3 & 4 & 2 & 1 & 4 & 2 & 1 & 4 & 5 & 1 & 3 & 5 & 2 & 6 & 5 & 2 \\
1 & 2 & 7 & 1 & 2 & 5 & 3 & 7 & 5 & 3 & 6 & 2 & 7 & 6 & 2 & 4 & 3 & 7 & 4 & 3
\end{pmatrix}
\end{array}
$$

where "–" means the corresponding item is not used. By Lemma 2.5, H_{40} is a subgraph of H_5. Thus one way to color H_5 is to extend a coloring of H_{40} and this leads to Algorithm $OPT5$. See Figure 7 for an illustration of this algorithm.

Algorithm 4 $OPT5$ (As Executed At Every Vertex)

---

if $c \leq 5$ **then**
  vertices (c,s) get the color $M_5(c,s)$;
else // $c \geq 6$.
  let $b = \lfloor \frac{8 \cdot s}{5 \cdot 2^{\lfloor \frac{c}{2} \rfloor}} \rfloor$;
  vertex (c,s) gets the color $A(|c|_4, |s|_3) \circ \prod_{j<b} p_j$; // use the $|\ell|_3 = 2$ case in $OPT8$.
end if

Theorem 3.4. *Algorithm $OPT5$ is distributed, takes constant number of steps, and produces an optimal distance-two 7-coloring for H_5.*

Proof. It is obvious that $OPT8$ is distributed and takes constant time. Let f be the coloring produced by $OPT5$. We now verify that f is a distance-two coloring. Suppose

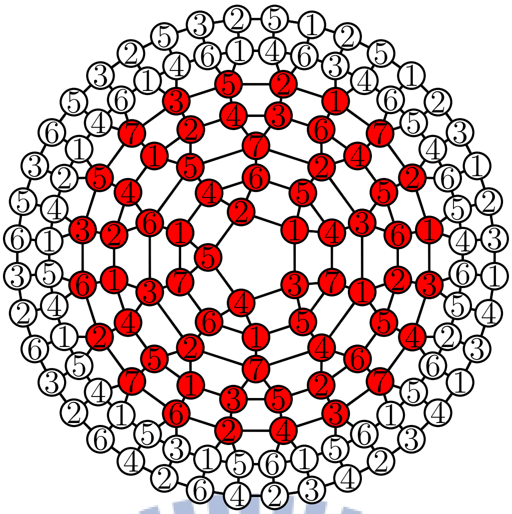


Figure 7: The distance-two 7-coloring for H_5 produced by $OPT5$; all the vertices are colored by using A (along with permutations $p_0, p_1, \ldots, p_6$) except that those highlighted are colored by using M_5.

(c, s) and (c', s') are two distinct vertices that are of distance at most 2. If at least one of (c, s) and (c', s') is highlighted (see Figure 7), then $f(c, s) \neq f(c', s')$ can be verified by a brute-force checking. If both of (c, s) and (c', s') are not highlighted, then $OPT5$ performs in the same way as the $|\ell|_3 = 2$ case of $OPT8$; hence $f(c, s) \neq f(c', s')$ by Theorem 3.2. From the above, f is a distance-two coloring. It is obvious that f uses 7 colors. Thus by Theorem 2.3, $OPT5$ is optimal and we have this theorem. ∎

4 The leader election problem

The *leader election* problem is to select a leader (from the sensors in a cluster) to perform certain tasks on each cluster. Because sensor networks contain many sensed data of the local environment, leader election can be used to combine or aggregate the data into meaningful information. More precisely, leader election has applications to coordination and data fusion, the latter is also called data aggregation and can be used to reduce the number of data to be communicated between the sensor node and the actor so that to avoid information overload. Leaders paly the most important role of each cluster. Thus an efficient process for the election of a cluster leader (or data aggregator node) is essential.

In [13], the authors mentioned that they use the uniform leader election for radio networks protocol in [15] (abbreviated as ULERNP) to select a leader for each cluster. Unfortunately, we find that this is incorrect. In ULERNP, the network has to be a single-hop network (i.e., every two nodes can communicate directly). Therefore to use ULERNP to select a leader for each cluster in the virtual infrastructure G_ℓ, the nodes in each cluster have to form a complete graph; however, it is usually impossible that every two nodes in a cluster can communicate directly. Furthermore, when the nodes are very dense, ULERNP usually produces dramatic communication overhead.

In [8], a hybrid approach that combines the energy conservation with the simplicity was introduced. This approach is based on four selection parameters: (1) the available energy, (2) the number of neighbouring sensor nodes, (3) the distance from the current group leader, and (4) the level of trust; for details, please refer to [8]. This approach can be used in leader election for G_ℓ and H_ℓ. However, nodes may produce a lot of communication overhead since G_ℓ and H_ℓ are usually multi-hop networks. For other leader election protocols, please see [10, 16].

Before closing this section, we propose an idea of how to perform leader election in a multi-hop network like G_ℓ and H_ℓ. We will only consider the parameter (1) and the distance from the candidate node to the other nodes in the cluster (the leader should be easy accessed from the other nodes). If more than one node can be selected, we randomly select one of them as the leader.

5 The concluding remarks

In this thesis, we propose a virtual infrastructure called H_ℓ and an distance-two coloring algorithm for H_ℓ. Our virtual infrastructure H_ℓ provides a coarse-grained location to the sensors in a network and allows geographic routing. Our distance-two coloring algorithm can be used to assign the frequency channels (or colors) in a fully distributed manner and our algorithm uses fewer channels than the previous work [13]. In the future, we intend to determine an appropriate way for the leader election problem, because choosing the right leader can help enhancing the network lifetime and can make routing more easier. In real world applications, the environment may have obstruction in it. Thus it is also challenging to find a virtual infrastructure for such an environment.

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