

Modeling heat extraction from hot dry rock in a multi-well system

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ABSTRACT

A mathematical model is developed for describing the heat energy extracted from a hot dry rock in a multi-well system. The solutions for the water temperature, accounting for a geothermal gradient in a geothermal reservoir, are given in the Laplace domain and computed by numerical inversion, the modified Crump method. The results show that the heat extraction effectiveness is affected significantly by the well spacing, well radius, reservoir thickness, and pumped flow rate in a multi-well system. The water temperature decreases with increasing pumping rate and increases with the well spacing, well radius, and reservoir thickness. The geothermal gradient affects only the early time heat extraction effectiveness significantly and has direct impact on the water temperature all the time if the vertical thickness of geothermal reservoir is large. The present solution is useful for designing and simulating the heat extraction project of geothermal energy exploitation in a multi-well system.

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1. Introduction

The extraction of geothermal energy from a hot dry or wet rock in a multi-well system has been received much attention recently. The economic use of geothermal energy is currently limited to a small number of the conventional geothermal steam and hot water reservoirs. Limited by the low thermal conductivity of the rock, the geothermal exploitation must have a sufficiently large heat transfer area for an economic use of geothermal energy. This new source of energy encourages developing a technique for extracting the geothermal energy from a hot rock in a deep geologic system, which involves drilling two or more wells to a suitable depth, creating permeable fractures in the rock, and injecting cold water into the rock fracture.

Gringarten et al. [1] presented a theory for heat extraction from a fractured hot dry rock based on the assumption of the infinite and uniform apertures of parallel vertical fractures. Cold water enters through the bottom of each fracture and the heat conductivity transfers from the hot rock to the fractured rock via the water flow. Their solution could be used in determining the water temperature and the ratio of the amount of heat extracted by means of the water flow within the fractures. Gringarten and Sauty [2] presented a mathematical model for describing the transient behavior of the pumped water and aquifer temperatures during re-injection of a fluid. Their solution for the water temperature within a stream channel was obtained from Lauwerier's one-dimensional solution [3]. Their results were helpful for designing a geothermal heat

extraction system in an aquifer with a uniform regional flow. Kolditz [4] examined the heat transfer in the fractured rocks when mining the heat energy from a hot dry rock. He investigated the dimensional effect of heat diffusion in the rock matrix and its impact on the heat conductivity where the heat is transferred from a hot dry rock. The dimensional effect is investigated by comparison of a two and half dimensional to a three-dimensional (3D) analytical model. The fully 3D model reflects the nature of the spatial heat propagation in the geological formations and can predict the minimum borehole separations. Cheng et al. [5] developed an analytical solution for the heat extraction from a fractured hot dry rock, assuming one-dimensional heat conduction perpendicular to the fracture. They modeled the multi-dimensional heat flow in a reservoir based on an integral equation. Carotenuto et al. [6] used the finite-element method to simulate the heat flow in an aquifer, which can be withdrawn by the downhole heat exchangers (DHE) from a geothermal aquifer. Their simulations are used to investigate the effect of the main properties of the aquifer and the optimization position of the tube casing slotted section for the geothermal wells in the DHE. Under the low-temperature geothermal resources, Tenma et al. [7] applied the finite element heat and mass transfer (FEHM) code [8] to study the model of the thermal storage system. Ghassemi et al. [9] presented an analytical solution for 3D heat flow in a geothermal reservoir using the same approach [5]. Afterward, Ghassemi et al. [10] formulated a 3D integral equation to evaluate the thermal stress during the heat extraction in an enhanced geothermal reservoir. Fujii et al. [11] developed the mass and head transfer model to simulate the behavior of the large-scale ground-couple heat pump (GCHP) system in the Akita Plain, northern Japan. Their model was used to study how to optimize

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operation schemes and to estimate the maximum of heat extraction rate in the GCHP system. Nalla et al. [12] conducted a parametric sensitivity analysis in determining the operating and design variables in the wellbore heat exchangers. They indicated that the heat energy extraction is apparently affected by the fluid residence time, heat transfer contact area, and formation thermal properties. Considering the closed-loop of pipes buried in inclined boreholes, Cui et al. [13] developed an analytical solution to a transient 3D heat conduction model for describing the temperature response in the ground heat exchangers (GHEs). Applying the superimposition of temperature excesses resulted from individual boreholes, the heat transfer in the GHEs with multiple boreholes is then studied.

The objective of this study is to propose a mathematical model describing heat extraction from a hot dry rock in a multi-well system. The Laplace-domain solutions for dimensionless temperatures of the water and rock are developed and their corresponding time-domain solutions are evaluated by the numerical inverse algorithm of the modified Crump method [14,15]. The model presented in this paper can be used to predict the water temperature within an extraction well and the heat extraction effectiveness from a hot dry rock in a multi-well system.

2. Mathematical model

A series of pumping wells with a uniform aperture are installed in the reservoir of a hot dry rock. The cool water is injected into the fracture which is located at the bottom of the hot dry rock. The heat energy is transferred from the hot dry rock to the water flow within an extraction well. The heat energy is then extracted by pumping the hot water from the extraction wells throughout the reservoir of a hot dry rock. Fig. 1 shows a schematic representation of an open well from which the heat energy is extracted by pumping the hot water.

To characterize the heat extraction from a hot dry rock in a multi-well system, the assumptions inherent in this study are: (1) the rock is a homogeneous, isotropic, and impermeable; (2) the physical densities and specific heats of both the water and rock and the thermal conductivity of the rock are constant; (3) the heat is transferred horizontally by conduction in the rock and vertically by convection within an extraction well; (4) the temperature of both the water in the extraction well and the rock are initially the same; (5) there is no heat flux across the boundary at $r = r_E$ where r_E is a half distance between two adjacent extraction wells. Cheng et al. [5] mentioned that under a typical laminar flow condition, the heat transfer is dominated by advection within the well and diffusion

in the reservoir. Accordingly, the effects of the heat dispersion and heat storage are negligible in this study.

The governing equation describing the water temperature in a radial system can be formulated based on the principle of the energy balance as

$$(\pi b^2) \rho_w c_w \left[v \frac{\partial T_w(z, t)}{\partial z} \right] = (2\pi b) K_R \frac{\partial T_R(r, z, t)}{\partial r} \Big|_{r=b}, \quad (1)$$

where $T_w(z, t)$ is the exit temperature at the top of an extraction well, $T_R(r, z, t)$ is the temperature of the rock, K_R is the thermal conductivity of the rock, b is the radius of the extraction well, ρ_w is the water density, c_w is the specific heat of the water, r is the radial distance from the centerline of extraction well, z is the vertical distance along the extraction well, and t is the time from the start of pumping water. Note that the properties of the rock and water are assumed temperature-invariant. The assumption is valid when the change of temperatures in the rock and water flow is not very large [16]. The average velocity of water flow, v , is $Q/(\pi b^2)$, where Q is the flow rate from an extraction well.

The heat conduction equation for a rock temperature can be written as

$$\frac{\partial^2 T_R(r, z, t)}{\partial r^2} + \frac{1}{r} \frac{\partial T_R(r, z, t)}{\partial r} = \frac{\rho_R c_R}{K_R} \frac{\partial T_R(r, z, t)}{\partial t}, \quad (2)$$

where ρ_R and c_R are the density and specific heat of the rock, respectively.

The temperatures of the rock and the water within an extraction well are initially the same, that is

$$\begin{aligned} T_R(r, 0, t) = T_w(0, t) = T_{R0}, \quad t < 0, \\ T_R(r, 0, t) = T_w(0, t) = T_{W0}, \quad t \geq 0, \end{aligned} \quad (3)$$

where T_{R0} and T_{W0} are the initial temperatures of the rock and the water within an extraction well, respectively. Assume that the cool water is injected into the fracture under the constant temperature condition. The heat transfer from reservoir to water within the injection well and the fracture is negligible due to the velocity of water flow within the injection well and fracture is faster than that within an extraction well. Under this circumstance, the water temperature at the inlet of extraction well is constant and denoted as T_{R0} . Taking the geothermal gradient into account, the rock temperature of the reservoir is dependent on the vertical distance z from the surface of the earth. The temperature distribution along the z -axis can be expressed as the initial rock temperature T_{R0} minus the product of vertical distance z above the injection point

$$T_R(r, z, t) = T_w(z, t) = T_{R0} - wz, \quad t < z/v, \quad (4)$$

where the geothermal gradient w is assumed constant. The continuity of the temperature along the extraction well in a reservoir at $r = b$ requires

$$T_w(z, t) = T_R(b, z, t) \quad (5)$$

and the no-heat flux boundary condition at $r = r_E$ is

$$\frac{\partial T_R(r, z, t)}{\partial r} \Big|_{r=r_E} = 0. \quad (6)$$

The parameters used hereafter are defined as

$$\begin{aligned} T_{RD} &= \frac{T_{R0} - T_R(r, z, t)}{T_{R0} - T_{W0}}, \quad T_{WD} = \frac{T_{R0} - T_w(z, t)}{T_{R0} - T_{W0}}, \\ T'_{RD} &= T_{RD} - \frac{w^* z}{H}, \quad \alpha = \frac{2\pi b K_R}{\rho_w c_w Q}, \quad \beta = \frac{K_R}{\rho_R c_R}, \quad w^* = \frac{wH}{T_{R0} - T_{W0}}, \end{aligned} \quad (7)$$

where H is the vertical thickness of a reservoir. Using the above dimensionless groupings, Eqs. (1)–(6) can be rewritten as

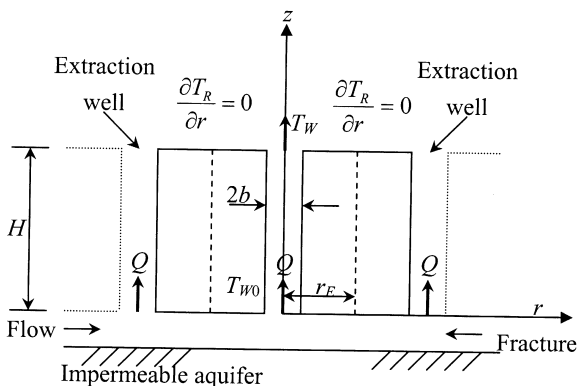


Fig. 1. A schematic diagram of heat extraction from a hot dry rock in a multi-well system.

$$\frac{\partial T_{WD}(z, t)}{\partial z} = \alpha \left. \frac{\partial T'_{RD}(r, z, t)}{\partial r} \right|_{r=b}, \quad (8)$$

$$\frac{\partial^2 T'_{RD}(r, z, t)}{\partial r^2} + \frac{1}{r} \frac{\partial T'_{RD}(r, z, t)}{\partial r} = \frac{1}{\beta} \frac{\partial T'_{RD}(r, z, t)}{\partial t}, \quad (9)$$

$$\lim_{t \rightarrow 0} T'_{RD}(r, z, t) = \lim_{t \rightarrow 0} T_{WD}(z, t) = 0, \quad (10)$$

$$T_{WD}(0, t) = 1, \quad (11)$$

$$T_{WD}(z, t) = T'_{RD}(b, z, t) + \frac{w^* z}{H}, \quad (12)$$

and

$$\left. \frac{\partial T'_{RD}(r, z, t)}{\partial r} \right|_{r=r_E} = 0. \quad (13)$$

Applying the Laplace transforms with respect to time to Eqs. (8)–(13), the Laplace-domain solution for the dimensionless rock temperature is given as (the detailed development is presented in Appendix A)

$$\begin{aligned} \bar{T}_{RD} = & \left[\frac{A_1(r, s)}{sA_1(b, s)} - \frac{w^* A_1(r, s)}{s\alpha q H A_2(b, s)} \right] \exp \left\{ \frac{\alpha q A_2(b, s)}{A_1(b, s)} z \right\} \\ & + \frac{w^* A_1(r, s)}{s\alpha q H A_2(b, s)} + \frac{w^* z}{sH} \end{aligned} \quad (14)$$

with

$$A_1(r, s) = K_1(qr_E)I_0(qr) + I_1(qr_E)K_0(qr) \quad (15)$$

and

$$A_2(r, s) = K_1(qr_E)I_1(qr) - I_1(qr_E)K_1(qr), \quad (16)$$

where s is the Laplace variable [17] and $q = \sqrt{s/\beta}$. The functions of $I_0(\cdot)$ and $K_0(\cdot)$ are the modified Bessel functions of the first and second kinds of order 0, respectively; and $I_1(\cdot)$ and $K_1(\cdot)$ are the modified Bessel functions of the first and second kinds of order 1, respectively. Note that detailed numerical calculations for those modified Bessel functions can be found in Yang and Yeh [18]. Using the continuity of temperature along the extraction well in a reservoir at $r = b$, the dimensionless water temperature obtained via substituting Eq. (14) into Eq. (12) in Laplace domain is

$$\bar{T}_{WD} = \left[\frac{1}{s} - \frac{w^* A_1(b, s)}{s\alpha q H A_2(b, s)} \right] \exp \left\{ \frac{\alpha q A_2(b, s)}{A_1(b, s)} z \right\} + \frac{w^* A_1(b, s)}{s\alpha q H A_2(b, s)} + \frac{w^* z}{sH}. \quad (17)$$

The Bessel functions of $I_1(u)$ and $K_1(u)$ are ∞ and 0, respectively, as u approaches ∞ . As such, one can obtain $\lim_{r_E \rightarrow \infty} A_1(b, s)/A_2(b, s) = -K_0(qb)/K_1(qb)$. When $r_E \rightarrow \infty$, the heat extraction system can be considered as a single-well system. Under this condition, Eq. (17) leads to

$$\bar{T}_{WD} = \left[\frac{1}{s} + \frac{w^* K_0(qb)}{s\alpha q H K_1(qb)} \right] \exp \left\{ -\frac{\alpha q K_1(qb)}{K_0(qb)} z \right\} - \frac{w^* K_0(qb)}{s\alpha q H K_1(qb)} + \frac{w^* z}{sH}, \quad (18)$$

which presents the dimensionless temperature of the water flow through a single-well system.

In the thin layer of the hot dry rock, the effect of geothermal gradient in a heat reservoir is negligible. Then, Eq. (17) reduces to

$$\bar{T}_{WD} = \left[\frac{1}{s} \right] \exp \left\{ \frac{\alpha q A_2(b, s)}{A_1(b, s)} z \right\}, \quad (19)$$

where the rock temperature is taken as the average temperature along the vertical thickness of the reservoir.

3. Numerical calculations

Due to the product of the Bessel and exponential functions, the time-domain solution of Eq. (17) may not be tractable. However,

Eq. (17) can be numerically inverted with four significant figures by the routine INLAP of IMSL [19]. This routine was originally developed by Crump [14] and later modified by de Hoog et al. [15]. Note that this routine has been successfully applied in some groundwater problems, see, e.g. [20,21].

4. Results and discussion

This study presents a linear model involving a series of equidistant and vertical extraction wells in a homogeneous, isotropic, and impermeable hot dry rock. There is no heat transfer by radiation within the extraction well or by conduction in the vertical direction of the extraction well or formation. The heat within an extraction well is transferred by the conduction between the water and rock. The behavior of heat extraction as a function of the well spacing, well radius, reservoir thickness, and pumped flow rate is investigated as follows. The initial rock temperature, T_{R0} , is taken as an average of the rock temperature throughout the zone. The data used in the following simulations are $\rho_w = 1000 \text{ kg/m}^3$, $c_w = 1000 \text{ cal/kg } ^\circ\text{C}$ ($4185.5 \text{ J/kg } ^\circ\text{C}$), $\rho_r = 2650 \text{ kg/m}^3$, $c_r = 250 \text{ cal/kg } ^\circ\text{C}$ ($1046.4 \text{ J/kg } ^\circ\text{C}$), and $K_r = 0.62 \text{ cal/ms } ^\circ\text{C}$ ($2.6 \text{ J/ms } ^\circ\text{C}$) [5]. The dimensionless water temperature (T_{WD}) is defined as $[T_{R0} - T_w(z, t)] / (T_{R0} - T_{W0})$, which defines a 100% heat extraction effectiveness if $T_{WD} = 0$ and the exhaustion of heat extraction if $T_{WD} = 1$. Therefore, the T_{WD} is high when the outlet temperature of pumped water is low; that is, the heat extraction effectiveness is also low.

Fig. 2 shows the plots of dimensionless temperature (T_{WD}) versus time (t) for $b = 0.05 \text{ m}$, $H = 200 \text{ m}$, $Q = 1 \times 10^{-3} \text{ m}^3/\text{s}$, and $w = 0$ and $0.03 \text{ } ^\circ\text{C/m}$ when $r_E = 1, 2, 5 \text{ m}$ or infinity, where $r_E \rightarrow \infty$ corresponds to the case of a single extraction well. In Fig. 2, the dashed line presents dimensionless water temperature for the case where the geothermal gradient effect ($w = 0 \text{ } ^\circ\text{C/m}$) is negligible and the solid line denotes the dimensionless water temperature for the case where the geothermal gradient effect ($w = 0.03 \text{ } ^\circ\text{C/m}$) is included. This figure indicates that T_{WD} increases significantly with r_E for the same w and time. The effect of the well spacing on T_{WD}

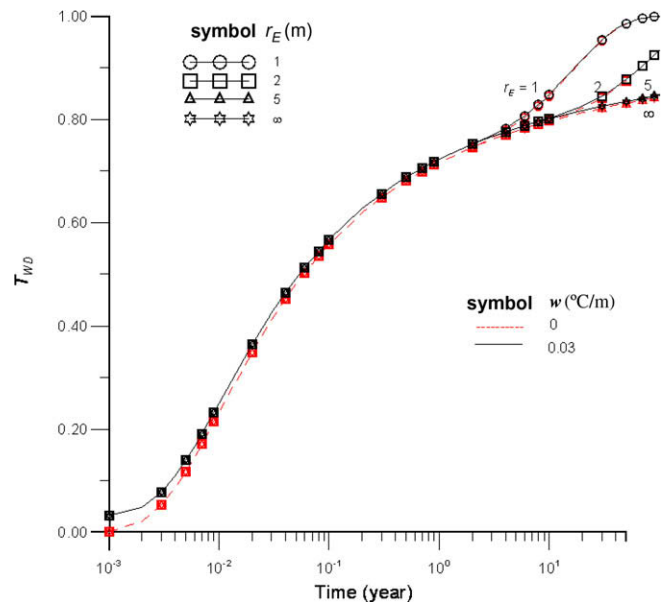


Fig. 2. Plots of T_{WD} versus t for $b = 0.05 \text{ m}$, $H = 200 \text{ m}$, and $Q = 1 \times 10^{-3} \text{ m}^3/\text{s}$ when $r_E = 1, 2, 5 \text{ m}$ or infinity. The dashed line presents dimensionless water temperature when neglecting the geothermal gradient effect ($w = 0 \text{ } ^\circ\text{C/m}$) and the solid line denotes dimensionless water temperature when considering the geothermal gradient effect ($w = 0.03 \text{ } ^\circ\text{C/m}$).

also increases significantly as $t = 3$ years for $r_E = 1$ m and $t = 10$ years for $r_E = 2$ m. The dimensionless water temperature for the $r_E = 5$ m case agrees with that for the infinity case. Those results illustrate that the temperature of pumped water decreases with increasing distance of adjacent extraction wells. The heat extraction effectiveness decreases with the distance between two adjacent extraction wells. The desired distance between the adjacent extraction wells is suggested to be 5 m under this case. The impact of the geothermal gradient on dimensionless temperature in the multi-well system is also investigated herein. The values of T_{WD} for $w = 0.03$ °C/m are higher than those without considering the effect of geothermal gradient (i.e., $w = 0$ °C/m). In addition, the impact of geothermal gradient on dimensionless water temperature is most profound at early pumping time. In other words, the geothermal gradient impact decreases with pumping time and is negligible at large time, indicating that the geothermal gradient affects only the early time heat extraction effectiveness.

The T_{WD} within an extraction well is plotted in Fig. 3 as a function of time for $r_E = 5$ m, $H = 200$ m, $Q = 1 \times 10^{-3}$ m³/s, and $w = 0$ and 0.03 °C/m when $b = 0.025, 0.05$ or 0.10 m. It shows that the T_{WD} decreases with increasing extraction-well radius b . This result indicates that the introduction of a larger-radius extraction well leads to a significant increase of heat extraction effectiveness from a reservoir. However, the introduction of the extraction well of a larger radius may cause difficulties in construction and a higher installation cost as well. The figure also shows that the values of T_{WD} for $w = 0.03$ °C/m are higher than those for $w = 0$ °C/m. In addition, the difference in the values of T_{WD} is larger at early pumping time and negligible if the pumping time is large.

Fig. 4 shows the plot of T_{WD} versus t for $b = 0.05$ m, $r_E = 5$ m, $Q = 1 \times 10^{-3}$ m³/s, and $w = 0$ and 0.03 °C/m when $H = 100, 200$ or 500 m. For fixed values of the pumping time and geothermal gradient, the dimensionless temperature, T_{WD} , decreases with increasing vertical thickness of the reservoir, H , except when $t \leq 0.1$ year. Thus, a thicker reservoir of a hot dry rock yields a higher heat extraction. The impact of the geothermal gradient on T_{WD} increases with the reservoir thickness and decreases with the pumping time. If the heat effectiveness is required greater than 80% during the

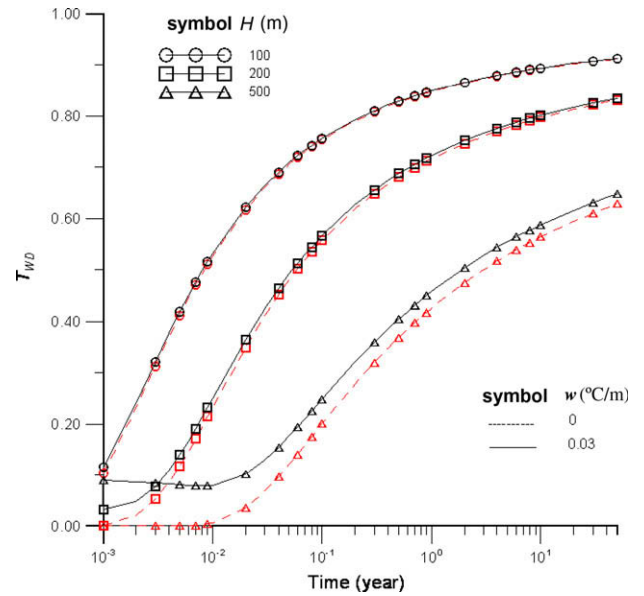


Fig. 4. Plots of T_{WD} versus t for $b = 0.05$ m, $r_E = 5$ m, $Q = 1 \times 10^{-3}$ m³/s, and $w = 0$ and 0.03 °C/m when $H = 100, 200$ or 500 m. The dashed line presents dimensionless water temperature when neglecting the geothermal gradient effect ($w = 0$ °C/m) and the solid line denotes dimensionless water temperature when considering the geothermal gradient effect ($w = 0.03$ °C/m).

operation period, the reservoir thickness of the hot dry rock must be larger than 200 m. The differences in dimensionless water temperature between the cases of $w = 0$ and 0.03 °C/m are shown in Fig. 5. The differences in dimensionless water temperature increase with reservoir thickness. In addition, the differences in dimensionless water temperature decrease with the pumping time and approach a constant value when the pumping time is large. The figure shows that the differences in dimensionless water temperature between these cases are 0.0016 for $H = 100$ m, 0.0059 for $H = 200$ m, and 0.0342 for $H = 500$ m when $t = 1$ year and 0.0008 for $H = 100$ m, 0.0032 for $H = 200$ m, and 0.0187 for $H = 500$ m when $t = 50$ years. Apparently, the reservoir thickness affects T_{WD}

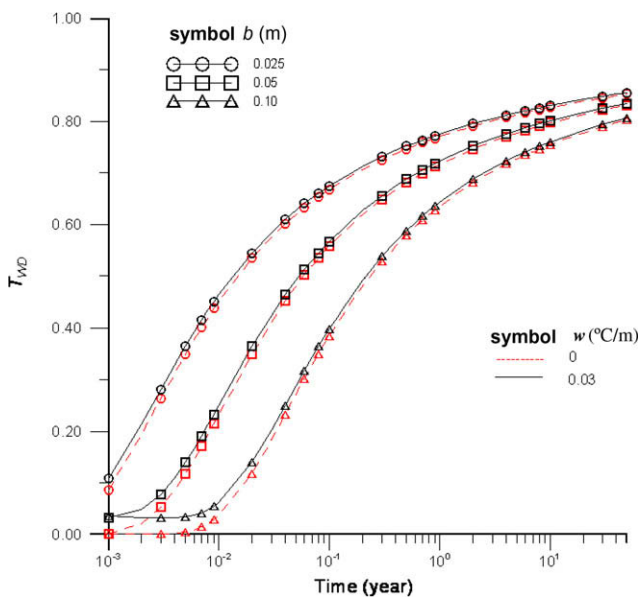


Fig. 3. Plots of T_{WD} versus t for $r_E = 5$ m, $H = 200$ m, $Q = 1 \times 10^{-3}$ m³/s, and $w = 0.03$ °C/m when $b = 0.025, 0.05$ or 0.10 m. The dashed line presents dimensionless water temperature when neglecting the geothermal gradient effect ($w = 0$ °C/m) and the solid line denotes dimensionless water temperature when considering the geothermal gradient effect ($w = 0.03$ °C/m).

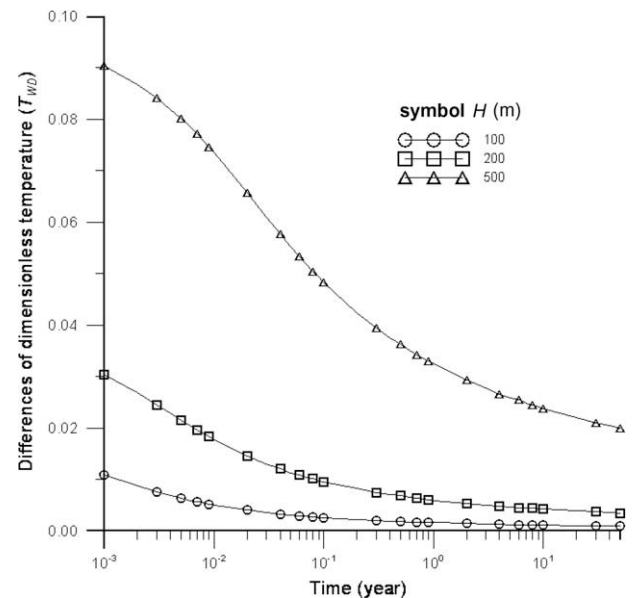


Fig. 5. The difference in T_{WD} between the cases of $w = 0$ and 0.03 °C/m versus t for $b = 0.05$ m, $r_E = 5$ m, and $Q = 1 \times 10^{-3}$ m³/s when $H = 100, 200$ or 500 m.

significantly for a thicker hot dry rock. In addition, the T_{WD} will be underestimated if neglecting the geothermal gradient of the reservoir. Thus, the effect of geothermal gradient of the reservoir must be considered in the evaluation of water temperature if the hot dry rock is thick.

The effect of the pumping rate on the T_{WD} is demonstrated in Fig. 6 for $b = 0.05$ m, $r_E = 5$ m, $H = 200$ m, and $w = 0.03$ °C/m when $Q = 5 \times 10^{-4}$, 1×10^{-3} or 2×10^{-3} m³/s. This figure shows that the T_{WD} increases with the pumping rate for a specific time; on the other hand, the dimensionless temperature of pumped water decreases with the pumping rate. The over-exploitation for heat from a hot dry rock reduces the heat extraction effectiveness and, more seriously, results in the exhaustion of heat energy.

Those results demonstrate that the present model can be used to assess the effects of the well spacing, well radius, reservoir thickness, geothermal gradient, and pumping rate on the water temperature in a multi-well system. This model can be used as a tool to design a heat extraction system for geothermal energy in a hot dry rock.

5. Conclusions

A mathematical model is presented for describing heat extraction through the water flow from a hot dry rock in a multi-well system. The solutions for dimensionless water temperature are given in the Laplace domain and their corresponding time-domain solutions are obtained by the modified Crump method. The following major conclusions can be drawn from this study.

1. The present model can be used to assess the effects of the well spacing, well radius, reservoir thickness, geothermal gradient, and pumping rate on the water temperature in a multi-well system. In addition, it can also estimate the temperatures of the reservoir and water and the heat extraction effectiveness in a multi-well system. It is useful in designing and simulating a heat extraction system for the exploitation of geothermal energy.
2. The pumped water temperature increases with the distance between two adjacent extraction wells. The introduction of a larger well spacing yields a less heat extraction effectiveness

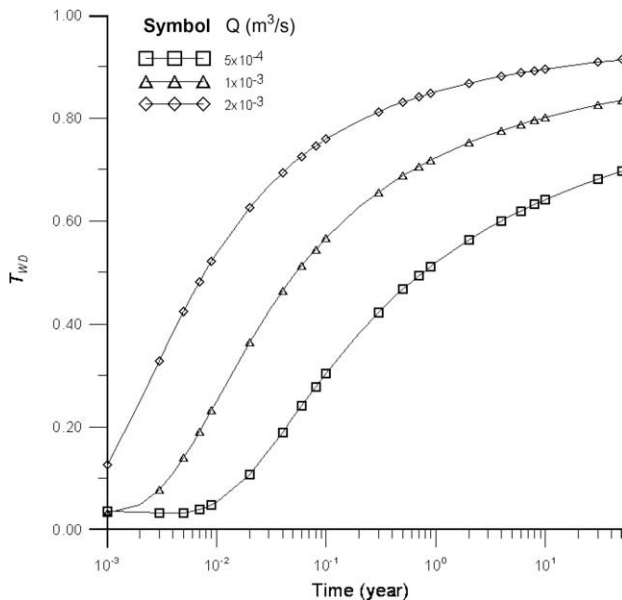


Fig. 6. The effect of pumped flow rate (Q) on T_{WD} for $b = 0.05$ m, $r_E = 5$ m, $H = 200$ m, and $w = 0.03$ °C/m when $Q = 1 \times 10^{-4}$, 5×10^{-4} , 1×10^{-3} or 2×10^{-3} m³/s.

per area. In addition, the introduction of the pumping well of a larger radius leads to a significant increase of the heat extraction effectiveness from a geothermal reservoir. The dimensions of well spacing and radius in a multi-well system are subject to the constraints of the available drilling technique and well installation cost.

3. The thicker reservoir of a hot dry rock yields the higher heat extraction effectiveness. The geothermal gradient affects only the early-time heat extraction effectiveness. The geothermal gradient has direct impact on the water temperature all the time if the vertical thickness of the reservoir is large. In addition, the water temperature may also increase with the decreasing pumping rate.

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Appendix A. Derivation of Eq. (14)

The Laplace transforms is applied to Eqs. (8)–(13) to remove the independent time variable and the results, respectively, give to

$$\frac{d\bar{T}_{WD}(z, s)}{dz} = \alpha \frac{d\bar{T}_{RD}(r, z, s)}{dr} \Big|_{r=b}, \quad (A1)$$

$$\frac{d^2\bar{T}_{RD}(r, z, s)}{dr^2} + \frac{1}{r} \frac{d\bar{T}_{RD}(r, z, s)}{dr} = q^2\bar{T}_{RD}(r, z, s), \quad (A2)$$

$$\bar{T}_{WD}(0, s) = \frac{1}{s}, \quad (A3)$$

$$\bar{T}_{WD}(z, s) = \bar{T}_{RD}(b, z, s) + \frac{1}{s} \frac{w^*z}{H}, \quad (A4)$$

and

$$\frac{d\bar{T}_{RD}(r, z, s)}{dr} \Big|_{r=r_E} = 0, \quad (A5)$$

where $q^2 = s/\beta$.

Applying the method of separation of variables leads to the solution in the form of

$$\bar{T}_{RD}(r, z, s) = R(r, s) \cdot Z(z, s). \quad (A6)$$

Substituting Eq. (A6) into Eqs. (A1)–(A5) yields

$$\frac{d\bar{T}_{WD}(z, s)}{dz} = \alpha R'(b, s) \cdot Z(z, s), \quad (A7)$$

$$R''(r, s) + \frac{1}{r} R'(r, s) = q^2 R(r, s), \quad (A8)$$

$$\bar{T}_{WD}(z, s) = R(b, s) \cdot Z(z, s) + \frac{1}{s} \frac{w^*z}{H}, \quad (A9)$$

and

$$R'(r_E, s) = 0, \quad (A10)$$

where $R'(r, s)$ and $R''(r, s)$ are the first and second derivatives of R , respectively. Then, substituting Eq. (A3) into Eq. (A9) produces

$$R(b, s) \cdot Z(0, s) = \frac{1}{s}. \quad (A11)$$

From Eqs. (A8), (A10) and (A11), one can obtain the solution, $R(r, s)$, as

$$R(r, s) = \frac{A_1(r, s)}{sZ(0, s)A_1(b, s)}, \quad (\text{A12})$$

where

$$A_1(r, s) = K_1(qr_E)I_0(qr) + I_1(qr_E)K_0(qr). \quad (\text{A13})$$

Differentiating Eq. (A9) and substituting it into Eq. (A7) results in

$$\dot{Z}(z, s) - \frac{\alpha R'(b, s)}{R(b, s)}Z(z, s) + \frac{1}{sR(b, s)} \frac{w^*}{H} = 0, \quad (\text{A14})$$

where $R'(r, s)$ and $\dot{Z}(z, s)$ are the first derivatives with respect to r and z , respectively. The following result is obtained from Eqs. (A14) and (A3):

$$Z(z, s) = \left[Z(0, s) - \frac{w^*}{s\alpha HR'(b, s)} \right] \exp \left[\frac{\alpha R'(b, s)}{R(b, s)} z \right] + \frac{w^*}{s\alpha HR'(b, s)}. \quad (\text{A15})$$

Finally, the dimensionless rock temperature equation (14) is obtained by substituting Eqs. (A12) and (A15) into Eq. (A6).

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