J Optim Theory Appl (2009) 141: 641–659
DOI 10.1007/s10957-008-9488-x

Two-Person Second-Order Games, Part 2: Restructuring Operations to Reach a Win-Win Profile

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Published online: 21 January 2009

Abstract In Part 1 of the paper, using habitual domains theory and finite Markov chain theory, we have introduced a new model for describing the evolution of the states of mind of players over time, the *two-person second-order game*. The concepts of *focal mind profile* as well as the solution concept of *win-win mind profile* have been introduced as solution concepts for these games. In Part 2 of the paper, we address the problem of restructuring a game where the focal profile $(1, 1)$ is not reachable or is not a win-win profile into a game where the profile $(1, 1)$ is a reachable win-win profile. Precisely, under some reasonable assumptions, we derive the *possibility theorem* that it is always possible to reach a win-win mind profile in a two-person second-order game. Moreover, we provide practical operations for restructuring games for reaching a win-win profile.

Keywords Games · Habitual domains · Second-order games · Focal mind profiles · Win-win mind profiles · Markov chains

This research was partially supported by the National Science Council, Taiwan, NSC96-2416-H009-013.

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1 Introduction

In Part 1 of the paper [1], using habitual domains theory and finite Markov chain theory, we have presented a new model for describing the evolution of the states of mind of players over time, the *two-person second-order game*. The concepts of *focal mind profile* as well as the solution concept of *win-win mind profile* have been introduced. Furthermore, we provided the average number of steps needed to reach the focal profile $(1, 1)$ when it is possible to reach it and how to increase the speed to reach it.

In real-world situations, it often happens that the focal profile is not reachable at all or it is not a win-win profile. In such cases, two important problems arise: (i) How to make the focal profile reachable? (ii) How to make a focal profile a win-win profile? In Part 2 of this paper, we address these problems. The resolution of these problems can be achieved by restructuring the game. Note that almost all traditional models of game theory do not consider restructuring games in order to reach better solutions [2]. The idea of restructuring games for reaching better solutions has been introduced by Yu [3], but the games he considered are based on strategies and utility functions not on mind profiles. In this paper, we consider restructuring second-order games in terms of mind profiles, which is more general than that considered in [3]. Indeed, we provide operational techniques for restructuring the game so that a focal profile becomes a reachable win-win profile. These techniques are further expressed in terms of mathematical operations on the transition probability matrix P. Finally, based on these operations, we formulate the *possibility theorem* that, under some reasonable conditions, it is always possible to reach and maintain a win-win mind profile in a two-person second-order game.

Part 2 of the paper is organized as follows. Some preliminaries are presented in Sect. 2. In Sect. 3, using the HD theory, we show how to make a focal mind profile reachable for the players by practical techniques of reframing the game. The corresponding mathematical operations on the transition probability matrix P are also presented. In Sect. 4, the possibility theorem is presented. Section 5 concludes the paper.

2 Preliminaries

Let us consider the second-order two-person game studied in [1],

$$\langle \{\mathrm{I}, \mathrm{II}\}, \{\mathrm{HD}^{\mathrm{I}}, \mathrm{HD}^{\mathrm{II}}\}, \{S^{\mathrm{I}}, S^{\mathrm{II}}\}, P\rangle, \tag{1}$$

where $\{\mathrm{I}, \mathrm{II}\}$ is the set of players; HD^{I} and $\mathrm{HD}^{\mathrm{II}}$ are the habitual domains of Player I and Player II respectively; $S^{\mathrm{I}} = \{1, 2 \dots, s\}$ is the set of states of mind of Player I and $S^{\mathrm{II}} = \{1, 2, \dots, l\}$ is the set of states of mind of Player II; $S = S^{\mathrm{I}} \times S^{\mathrm{II}}$ is the set of mind profiles;

$$P = (P_{(i_1, j_1)(i,j)}), \quad ((i, j), (i_1, j_1)) \in S^{\mathrm{I}} \times S^{\mathrm{II}},$$

is the transition probability matrix. Furthermore, we assume that all the assumptions made in [1] are also valid in the present Part 2 of the paper, in particular, $(1, 1)$

is assumed to be the focal profile of the game (1). For convenience, we recall the definitions of the focal profile and the win-win profile.

Definition 2.1 A profile $(i, j) \in S^{\mathrm{I}} \times S^{\mathrm{II}}$ is called a *focal profile* of the game (1) if it is a profile that has some special appeal to the players and it is desirable to reach it and make it stable.

Definition 2.2 A profile $(i, j) \in S^{\mathrm{I}} \times S^{\mathrm{II}}$ is called *win-win profile* of the game (1) if

(a) (i, j) is a focal profile.
(b) (i, j) is an *absorbing* profile or equivalently $P_{(i,j)(i,j)} = 1$.

3 Reaching the Focal Profile (1, 1): General Case

In this section, we will explore restructuring techniques that can make a game with a transition probability P, which has no win-win profile, into a new one with P^*, which has a win-win profile as to solve the game. After a general description of techniques of restructuring games in practice based on HD theory, we summarize them into three operations on P with illustration on the three Examples 2.1–2.3 presented in [1].

3.1 Restructuring Games in Practice

As P reflects an actual domain of a situation which is only a small part of the potential domain of the game, P can be changed. There are many parameters including the rule of the game, the form of interaction of the players, the information input and output and the HD of the players, that can be subject to change or manipulation. A *superior strategist* uses the changes of parameters to create a game situation in which all players can declare victory. An *ordinary strategist* tries to find an optimal strategy within a fixed set of parameters. For more details on this, see the books of Yu [4, 5].

Yu provides three sets of tools for expanding and enriching HD so that new ideas, including alternatives, criteria, perceptions, and outcomes can be generated as to solve challenging decision problems. The three sets of tools can also allow us to get into the depth of our potential domains as to gain creative and good ideas to solve our challenging game problems. To avoid distraction we list a self-explanatory set of basic tools in Appendix A.1.

There are four important steps for effective restructuring games [4, 5]. (i) Understand the game situation, (ii) identify solvable game situations, (iii) create charge for moving towards solvable situations, and (iv) build enthusiasm for the desired actions. Each step, directly or indirectly, involves (a) expansion and enrichment of HDs of players, (b) effective suggestion of new ideas to catch the players' attention, and (c) effective integration of the new ideas with the core of the HDs of players. For (a) we provide Tool Box 1 of Appendix A.1. For (b), we list five important considerations for new ideas to capture our attention in Tool Box 2 of the Appendix A.1. For (c) in Tool Box 3 of the same Appendix we list five steps for integrating new ideas or concepts with the core of our HD, which are important in practice to transform the focal profile into the win-win profile. For the details and more of these tool boxes and beyond, please see [4, 5].

For illustration, let us consider the following second-order game [3]

$$\langle \{\mathrm{I}, \mathrm{II}\}, \{X_t^{\mathrm{I}}, X_t^{\mathrm{II}}\}, \{\mathrm{C}_t^{\mathrm{I}}, \mathrm{C}_t^{\mathrm{II}}\}, \{F_t^{\mathrm{II}}, F_t^{\mathrm{II}}\}, \{D_t^{\mathrm{I}}, D_t^{\mathrm{II}}\}, \{I_t^{\mathrm{I}}, I_t^{\mathrm{II}}\}\rangle \tag{2}$$

where the decision elements $X_t^{\mathrm{I}}, \mathrm{C}_t^{\mathrm{I}}, F_t^{\mathrm{I}}, D_t^{\mathrm{I}}, I_t^{\mathrm{I}}$ are the *set of strategies, the criteria, the payoffs measured in terms of criteria, the preference and the external information input*, respectively, at time t for the ith player, $i = \mathrm{I}, \mathrm{II}$. As the game evolves over time, all of its components (parameters of the game) are subject to change and control. The game (2) is more general than the traditional games of normal form since it allows the players to restructure the game by self suggestion or with the help of the external world through the information input I_t^{I}, $i = \mathrm{I}, \mathrm{II}$. In the game (2) the evaluation of the situation at any time is done through the payoff functions and preferences, which implies that the charge levels of players are evaluated by payoffs. In [3, 5], the following techniques for restructuring the game (2) are discussed in details: (i) reframing X_t^{I}, (ii) reframing C_t^{I}, (iii) reframing F_t^{I}, (iv) reframing D_t^{I} and (v) utilizing I_t^{I}.

For illustration, let us see how the games of Examples 2.1–2.3 of [1] are solved by restructuring them. In Example 2.1 (prisoner dilemma), by changing the rule of the game, introducing a penalty (via a contract) that whoever deviates from the strategy RP has to pay 9 units to the other player [5], the game in payoff form will become

$$\begin{array}{c} \\ \mathrm{RP} \\ \mathrm{SP} \end{array} \begin{array}{c} \begin{array}{cc} \mathrm{RP} & \mathrm{SP} \end{array} \\ \begin{pmatrix} (11, 11) & (10, 6) \\ (6, 10) & (5, 5) \end{pmatrix} \end{array}.$$

In the new payoff matrix, both players declare victory by using the strategy RP and a win-win profile is obtained. For more general restructuring see [6].

In Example 2.2 (game of silence), the wife went to the depth of her PD (potential domain) as to find the good idea of opening and closing drawers and approaching her husband who was watching TV. This action made the husband ask: what are you looking for? She attentively answered “your voice”. The husband felt love from wife. They hugged each other and solved the game. Observe that the hypotheses H1–H8 are used effectively by the wife.

In Example 2.3 (Alinsky strategy), by looking into the depth of the PD, the organization could not find how to make the city authorities cooperate. They finally used H8 (external information input). By consulting with Alinsky, a great social movement leader, who came out with the idea of mobilizing interested people to legally (and patiently) occupy all the toilets of Chicago airport. This idea created a high level of charge and made the city authorities quickly come to cooperate with the organization. Alinsky went to the depth of PD to come out with this great strategy to restructure the game as to solve the conflict. In the new game all players can declare victory without even a fight.

For all these three examples, we provide mathematical illustration in Sect. 4, after we introduce the three basic operations in the next section.

3.2 Mathematical Operations for Restructuring Games

As the transition probability matrix P reflects the structure of the game, any change in the structure of the game affects the matrix P. By knowing the matrix P, one can identify the status of the game and determine how to restructure the game so that it would be possible to reach the focal profile $(1,1)$ and/or to make it a win-win profile. For this purpose, we introduce three basic mathematical operations on the transition probability matrix P. The effective execution of these mathematical operations is practically supported by the techniques of restructuring games presented in Sect. 3.1.

Definition 3.1

(i) Let (k,t) and (e,f) be two profiles such that $P_{(k,t)(e,f)}=0$. When the operation of transforming the probability $P_{(k,t)(e,f)}$ into a positive number, denoted by $(k,t)\overset{(+)}{\to}(e,f)$, is performed, we say that we have connected (k,t) with (e,f).

(ii) Let D and D' be two subsets of the set of profiles S such that $P_{(k,t)(e,f)}=0$, for all $(k,t)\in D$ and $(e,f)\in D'$. If for some profile $(r,q)\in D$ and some profile $(g,h)\in D'$, the operation $(r,q)\overset{(+)}{\to}(g,h)$ is performed, we say that we have connected D with D', which is denoted by $D\overset{(+)}{\to}D'$. If $D\overset{(+)}{\to}D'$ and $D'\overset{(+)}{\to}D$ are performed, we will denote this fact by $D'\overset{(+)}{\leftrightarrow}D$.

(iii) Let (k,t) and (e,f) be two profiles such that $P_{(k,t)(e,f)}>0$. When the operation of transforming the probability $P_{(k,t)(e,f)}$ into zero, denoted by $(k,t)\overset{(-)}{\to}(e,f)$, is performed, we say that we have *disconnected* (k,t) with (e,f).

Remark 3.1 Note that, since the transition probability matrix is stochastic, the operation $(k,t)\overset{(+)}{\to}(e,f)$ will make it necessary to adjust some or all of the probabilities $P_{(k,t)(u,v)}$, $(u,v)\in S\backslash\{(e,f)\}$ by decreasing them. We shall not stop to elaborate this adjustment when connection is involved. Making a connection involves suggesting, implanting, nurturing, habituating, and repeating good ideas from PD or external world. The toolboxes mentioned before are useful. H1–H8 are important to create changes so that good ideas can be accepted and the connection can be established. In practice, to make a disconnection, we could change the parameters of the game so that the move from a profile to another profile would create a high charge level, which would make the players not like to move. For instance, in the prisoner dilemma, by introducing the penalty, the players would not like to deviate from the strategy RP, because any deviation will create high level of charge from the view point of self and collective interest.

Operation 1 Dissolving Irrelevant Recurrent Classes or Removing Decision Traps. According to Remark 4.2, Case 2, in [1], when the Markov chain of the game has more than one recurrent class, the process may not reach the focal profile $(1,1)$. Therefore, the presence of more than one recurrent class is not desirable. Thus, an operation that dissolves recurrent classes is necessary for making the focal profile $(1,1)$ reachable.

(O1) For any recurrent class R such that $(1,1) \notin R$, choose another recurrent class R' and perform $R \overset{(+)}{\to} R'$, i.e. connect R with R'.

Operation (O1) should be performed under the following conditions: (i) The transition probabilities within $S \backslash R$ and from $S \backslash R$ to R should remain constant. (ii) Positive transition probabilities within R should be kept positive.

Remark 3.2 In terms of habitual domain theory [5], a recurrent class not containing the focal profile $(1,1)$ may be regarded as a *decision trap*. Indeed, when players do not interact and do not exchange information between them and the external world, they may be trapped in a subset of profiles that does not contain the focal profile $(1,1)$. Such decision traps occur very often in real life. For instance, between wife and husband (see Example 2.2 in [1]), between two companies, between two countries, etc. The formation of decision traps is based on hypotheses H1–H8. Indeed, the fact that the players are not able to get out of the decision trap is due to (H1) a strong circuit pattern related to the states of mind involving the decision trap; by (H4) analogy and association and (H5) goal setting and state evaluation, their stay in the decision trap can be perpetuated. That is, the players stick to the same states of mind according to their perception and evaluation of the game. When the players are trapped in a decision trap, the charge level of at least one player may increase with time, as to create drive for action. Once the charge level of one or both players reaches a certain level, the states of mind of the players start to change and, according to H5–H7, some actions can be taken by one or both players to get out of the decision trap. This action and change of mind yield a change in the game and in the transition probability matrix that supports Operation (O1).

Operation (O1) is justified by the following proposition.

Proposition 3.1 *When Operation* (O1) *is performed on a recurrent class* R, R *will be dissolved as a recurrent class. That is,* (O1) *makes the profiles of* R *transient.*

Proof See Appendix A.2. □

Operation 2 Integrating the Focal Profile (1, 1) with a Recurrent Class.

(O2) (i) Making the focal profile $(1,1)$ communicate with the profiles of a recurrent class.

Choose one of the existing recurrent classes, say R. Perform $(1,1) \overset{(+)}{\leftrightarrow} R$. The transformation $(1,1) \overset{(+)}{\to} R$ is not necessary if $(1,1) \to (k,t)$ for some $(k,t) \in R$.

(O2) (ii) Transforming the subset $R \cup \{(1,1)\}$ into a recurrent class.

Perform $(1,1) \overset{(-)}{\to} (k,t)$ for all $(k,t) \notin R$ with $P_{(1,1)(k,t)} > 0$.

Operation (O2) should be performed under the following conditions: (a) Without transforming into zero any positive transition probability among the profiles of R, and without transforming into positive number any transition probability from R to any profile $(i, j) \notin R \cup \{(1, 1)\}$; (b) the transition probabilities within $S \backslash R \cup \{(1, 1)\}$ and from $S \backslash R \cup \{(1, 1)\}$ to $R \cup \{(1, 1)\}$ should remain constant; (c) if $P_{(1,1)(k,t)} = 0$ for some $(k, t) \in S \backslash R$ then it should be kept equal to zero.

When the profile $(1, 1)$ is transient, it may not be reached. Operation (O2) can be used to transform it into a recurrent profile. Precisely, Operation (O2) has the following consequence.

Proposition 3.2 *By performing Operation* (O2), *the focal profile* $(1, 1)$ *is transformed into a recurrent profile of the recurrent class* $R \cup \{(1, 1)\}$.

Proof See Appendix A.2. □

Remark 3.3 The conditions imposed on Operation (O2) can be interpreted as follows. Condition (a) means that, for all profiles $(k, t), (e, f) \in R$, such that $P_{(k,t)(e,f)} > 0$ before Operation (O2), we keep their positivity also after (O2) is performed; and for all $(i, j) \notin R \cup \{(1, 1)\}$ and $(e, f) \in R$, we maintain $P_{(e,f)(i,j)} = 0$, after (O2) is performed. Note that, since R was a recurrent class before (O2), for all $(i, j) \notin R \cup \{(1, 1)\}$ and $(e, f) \in R$, $P_{(e,f)(i,j)} = 0$, before (O2). Condition (b) prevents the creation of new recurrent classes in $S \backslash R \cup \{(1, 1)\}$ (see the proof of Proposition 3.4 below). Note that, since P is stochastic, if in (O2) (i), $R \stackrel{(+)}{\to} (1, 1)$ is performed by $(k, t) \stackrel{(+)}{\to} (1, 1)$, with $(k, t) \in R$, we need to adjust some or all of the probabilities $P_{(k,t)(e,f)}, (e, f) \in R$ by decreasing them (see Remark 3.1). Condition (c) is necessary to achieve the objective of Operation (O2) (ii). Operation (O2) (i) makes the focal profile $(1, 1)$ communicate with the profiles of the recurrent class R, while (O2) (ii) is necessary to make $R \cup \{(1, 1)\}$ a recurrent class. These two operations integrate the focal profile $(1, 1)$ with R as a recurrent class.

Remark 3.4 In terms of habitual domain theory, when the focal profile $(1, 1)$ is transient, at least one player would perceive that the outcome of $(1, 1)$ is not good enough. The charge level created by this profile is high and can be decreased by taking action. The process may visit this profile for a finite number of times, then move to other profiles until it enters a recurrent class, where it will remain. Thus, players have no (or weak) incentive to reach the focal profile $(1, 1)$ or to remain in it once it is reached. In order to integrate the focal profile $(1, 1)$ with a recurrent class, the players have to use Operation (O2) supported by the techniques of reframing games mentioned in Sect. 3.1.

Operation 3 Making the Focal Profile $(1, 1)$ Absorbing and Becoming Win-Win.

(O3) For any profile $(k, t) \neq (1, 1)$ with $P_{(1,1)(k,t)} > 0$, perform $(1, 1) \stackrel{(-)}{\to} (k, t)$. This operation should be performed while keeping the transition probabilities within $S \backslash \{(1, 1)\}$ and from $S \backslash \{(1, 1)\}$ to $\{(1, 1)\}$ constant. In addition, if $P_{(1,1)(k,t)} = 0$ for some $(k, t) \neq (1, 1)$, this probability should be kept equal to zero.

It is easy to see that by performing Operation (O3) the focal profile $(1, 1)$ becomes absorbing, i.e. a win-win profile (see Definition 2.2). This operation makes use of disconnection defined in Definition 3.1(iii). Operation (O3) has the following consequence.

Proposition 3.3 *Performing* (O3) *does not create new recurrent classes not containing the focal profile* $(1,1)$. *Moreover, in the case that* $(1,1)$ *is recurrent, the recurrent class containing* $(1,1)$ *will be dissolved, i.e. its profiles other than* $(1,1)$ *become transient.*

Proof See Appendix A.2. □

According to Hypothesis H1 of HD theory, the stronger the circuit patterns in the brain corresponding to an idea, concept or behavior, the higher the probability that this idea, concept or behavior can be activated. In the Toolbox 3 of Appendix A.1, Yu [4, 5] provides a step-by-step method based on the eight basic hypotheses H1–H8 of the HD theory to make an idea, concept or behavior part of the core of the HD of a decision maker with respect to this idea, concept or behavior. Once an idea, concept or behavior is integrated with the core of existing HD, the activation probability of this idea, concept or behavior becomes very high, as to close to 1. This means that whenever attention is paid to the game, the idea, concept or behavior is almost surely activated.

If the profile $(1,1)$ is integrated with the core of the HDs of both players, then it could quickly become a win-win profile if their corresponding charge levels are at their lowest levels in all possible profiles relevant to the game. Note that this method may require long time when the HDs of the players are not ready to accept a new idea. The effectiveness of implementing the method depends on that of restructuring the game by changing the relevant parameters including the rules of the game and HDs of the players. See [4, 5] for further discussion.

3.3 Reaching the Focal Profile $(1,1)$: Restructuring Procedures

Depending on the game, usually there are many ways to reach the focal profile $(1,1)$. In this section, for illustration, we present two procedures for reaching the focal profile $(1,1)$ based on Operations (O1)–(O3). One is straightforward, the other is progressive. Let R^0 be any recurrent class containing the focal profile $(1,1)$.

Procedure 3.1 Identify the recurrent classes of the game, by transforming the transition probability matrix P into the canonical form (3) of [1]. If the focal profile $(1,1)$ is in a recurrent class R^0 go to Step 1. Otherwise, go to Step 2.

Step 1. If the process starts in the recurrent class R^0, consider only the Markov sub chain consisting of the profiles of the recurrent class R^0. Perform Operation (O3), go to Step 3. Otherwise, go to Step 2.

Step 2. Perform Operation (O3). If the process is in the focal profile $(1,1)$ go to Step 3. Otherwise, perform $R \overset{(+)}{\to} \{(1,1)\}$ or Operation (O1) for all recurrent classes R such that $(1,1) \notin R$.

Step 3. The obtained Markov chain has a unique recurrent class which consists of the focal profile $(1,1)$ only. Transform this matrix into the form (3) of [1], if necessary. Then, compute the expected number of steps required to reach the focal profile $(1,1)$ by (5)–(7) of [1]. It is not necessary to compute this number if the process is in $(1,1)$, and $(1,1)$ is a win-win profile.

Remark 3.5 In Step 1 of Procedure 3.1, when the process starts in the recurrent class R^0, we know that it will never leave it. Then, we can restrict the study of the process to the profiles of the recurrent class R^0 only.

Remark 3.6 Procedure 3.1 can be performed by changing the parameters of the game such as introducing a penalty via a contract (see Sect. 3.1 for more details). If the players have no adequate perception about the focal profile $(1,1)$ i.e. the transition probabilities from the other profiles to the focal profile $(1,1)$ could be very low and difficult to change, we may have to work hard on reframing the game to make Procedure 3.1 effective. Recall that the changes in transition probabilities correspond to changes of the actual domains of players. When the change of these probabilities requires high cost, Procedure 3.1 may take time to finish. Procedure 3.1 can be more easily implemented in practice if there is an authority (government, arbitrage...) that can enforce the changes in the game.

Procedure 3.2

Step 1. Identify the recurrent classes of the game, by transforming the transition probability matrix P into the canonical form (3) of [1]. There are two possible cases: *Case* (i): there are more than one recurrent class; if the focal profile $(1,1)$ is recurrent, go to Step 2; otherwise, go to Step 3. *Case* (ii): there is only one recurrent class; in the case the focal profile $(1,1)$ is recurrent, go to Step 5; otherwise, go to Step 4.

Step 2. Identify the recurrent class R^0 that contains the focal profile $(1,1)$. If the process is in R^0 at the present step, consider the recurrent Markov subchain R^0 only; go to Step 5. Otherwise, perform Operation (O1) on all recurrent classes not containing $(1,1)$; go to Step 5.

Step 3. Perform Operation (O3) or integrate the focal profile $(1,1)$ with a recurrent class by Operation (O2). Go to Step 2.

Step 4. Perform Operation (O2) or (O3). If (O3) is performed, go to Step 2.

Step 5. Transform the transition probability matrix into the form (3) of [1] if necessary. Then compute the expected number of steps required to reach the focal profile by (5)–(7) of [1]. Note that it is not necessary to compute this number if the process is in the focal profile $(1,1)$ and $(1,1)$ is a win-win profile.

The main steps of the Procedure 3.2 are described in Fig. 1.

Proposition 3.4 *Procedures* 3.1 *and* 3.2 *terminate in a finite number of steps.*

Proof See Appendix A.3. □

Remark 3.7 Note that we have assumed that the party (players, third party, government, arbitrage...) implementing Procedure 3.1 or 3.2 is able to satisfy the conditions imposed on Operations (O1)–(O3). This is to ensure that, at each step, no new recurrent class emerges other than those generated by Operations (O1)–(O3). This can be

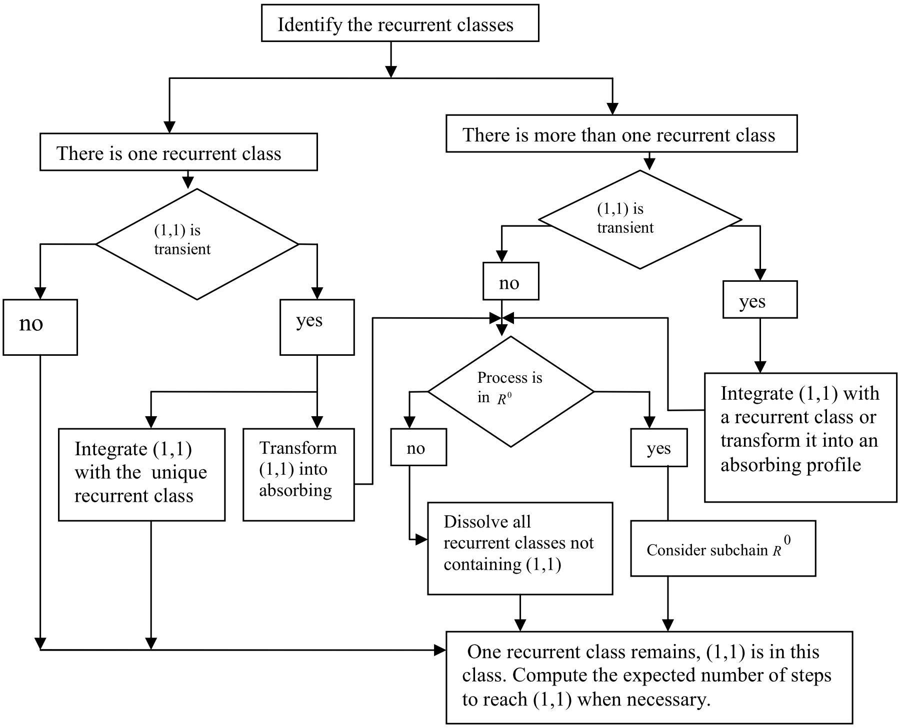

Fig. 1 Procedure 3.2

achieved by the techniques provided in the Toolbox 3 of Appendix A.1. In the case where some unexpected event occurs, which makes the implementing party unable to prevent the emergence of new recurrent classes beside the ones created by (O1)–(O3), then the implemented procedure has to be restarted from Step 1. The Procedure 3.2 seems lengthy. However, from a practical standpoint, it is suitable to use when there is a mediator or when the game necessitates a progressive (or gradual) treatment due to the difficulty in changing the habitual domains of the players quickly and effectively.

4 Reaching a Win-Win Profile

In this section we will show how to make the focal profile $(1,1)$ a win-win profile when it is possible to reach it. Finally, we present the main result of this paper, the possibility theorem.

In terms of Markov chain theory, the problem of transforming the focal profile $(1,1)$ into a win-win profile reduces to that of making $(1,1)$ an absorbing state i.e. $P_{(1,1)(1,1)}=1$ (see Definition 2.2). In Sect. 3.2, we saw that, by performing Operation (O3), the focal profile $(1,1)$ becomes an absorbing profile. By performing the elementary operation $(1,1)\overset{(-)}{\rightarrow}(k,t)$ of Operation (O3), we increase the transition probability $P_{(1,1)(1,1)}$ from the focal profile $(1,1)$ to itself. According to Bath and

Miller [7],

$$\alpha_{(1,1)} = P_{(1,1)(1,1)}/(1 - P_{(1,1)(1,1)})$$

provides the average number of steps the process stays in the focal profile (1, 1) once it is reached, known as the *sojourn time*. Thus, the larger $P_{(1,1)(1,1)}$, the better the stability of the focal profile (1, 1). When $P_{(1,1)(1,1)} \to 1$, $\alpha_{(1,1)} \to +\infty$. Precisely, once Operation (O3) is completed, we get $P_{(1,1)(1,1)} = 1$, and the focal profile (1, 1) becomes a win-win profile. The following procedure shows how to make the focal profile (1, 1) a win-win profile.

Procedure 4.1 Use Procedure 3.1 or Procedure 3.2 to make the focal profile (1, 1) reachable. In the case the focal profile (1, 1) is not made a win-win profile when Procedure 3.2 is implemented, perform Operation (O3) to transform (1, 1) into a win-win profile.

Remark 4.1 To implement Operations (O1)–(O3) and Procedure 4.1 effectively, one could apply the framing techniques mentioned in Sect. 3.

We summarize the results of Sects. 3 and 4 by the following theorem.

Theorem 4.1 (Possibility Theorem) *Suppose that both players are aware of Hypotheses H*1*–H*8 *and use them fully so that the Operations* (*O*1)–(*O*3) *can be performed effectively. Then, it is always possible to reach a win-win profile in the game* (1) *by implementing Procedure* 4.1 *effectively.*

As an application, we show now how the possibility theorem can be applied to solve the games of Examples 2.1–2.3 of [1].

Example 4.1 (Prisoner Dilemma) In Examples 2.1 and 3.1 of [1], we have analyzed the game by HD theory; we have found that the game has no stable solution and (C, C) is a focal point of the game. Let us analyze the transition probability matrix P of the game given in Example 3.1 of [1]. Notice that all the entries of P are positive. According to Definition 4.2 of [1], the Markov chain of the game is irreducible. The Markov chain of the game has only one recurrent class and the focal profile (C, C) is recurrent. Then, Proposition 4.1 of [1] implies that (C, C) can be reached. We can estimate the expected number of steps required to reach the focal profile (C, C) by Proposition 4.3 of [1]. For this purpose, we have to introduce a matrix of the form (4) of [1] with respect to (C, C). Proceeding as in Example 4.1 of [1], we get the average number of steps needed to reach the focal profile (C, C) from different profiles: from (C, N),

$$\Delta_{(\mathrm{C,N})(\mathrm{C,C})} = 1.78 + 0.78 + 2.63 = 4.19,$$

from (N, C),

$$\Delta_{(\mathrm{N,C})(\mathrm{C,C})} = 0.8 + 1.82 + 2.8 = 5.42,$$

and from (N, N),

$$\Delta_{(\mathrm{N,N})(\mathrm{C,C})} = 0.87 + 0.87 + 3.48 = 5.22.$$

As we see, the average number of steps required to reach the focal profile (C, C) is quite small. However, the average number of steps the game will stay in the focal profile (C, C) or sojourn time is

$$\alpha_{(\mathrm{C},\mathrm{C})} = P_{(\mathrm{C},\mathrm{C})(\mathrm{C},\mathrm{C})}/(1 - P_{(\mathrm{C},\mathrm{C})(\mathrm{C},\mathrm{C})}) = 0.1/0.9 = 0.11.$$

This means that the focal profile (C, C) is very unstable. Thus, at the time the focal profile (C, C) is reached, the players have great incentive to leave it. In order to convert (C, C) into a win-win profile, we need to apply Operation (O3). To achieve this, the charge levels of both players in the focal profile (C, C) should be reduced to a minimum level compared to the other profiles of the game. For this purpose the players have to use the techniques of reframing games that we have mentioned in Sect. 3. We have seen in Sect. 3.1 how the players achieved this by reframing payoffs via a contract with the following new payoff matrix:

$$\begin{array}{cc} & \begin{array}{cc} \mathrm{RP} & \mathrm{SP} \end{array} \\ \begin{array}{c} \mathrm{RP} \\ \mathrm{SP} \end{array} & \begin{pmatrix} (11,11) & (10,6) \\ (6,10) & (5,5) \end{pmatrix} \end{array}.$$

In the new payoff matrix, the focal profile (C, C) is very attractive for both players. Whatever profile they are in, they will shift to the focal profile (C, C) in one step. Moreover, if they have reached (C, C), they will never leave it, because at (C, C) their charge levels are at their minimum levels. Thus, (C, C) is a win-win profile. The new transition probability matrix of the game is of the form

$$P' = \begin{array}{c} \\ (\mathrm{C},\mathrm{C}) \\ (\mathrm{C},\mathrm{N}) \\ (\mathrm{N},\mathrm{C}) \\ (\mathrm{N},\mathrm{N}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathrm{C},\mathrm{C}) & (\mathrm{C},\mathrm{N}) & (\mathrm{N},\mathrm{C}) & (\mathrm{N},\mathrm{N}) \end{array} \\ \begin{pmatrix} 1 & 0.0 & 0.0 & 0.0 \\ 1 & 0.0 & 0.0 & 0.0 \\ 1 & 0.0 & 0.0 & 0.0 \\ 1 & 0.0 & 0.0 & 0.0 \end{pmatrix} \end{array}$$

where there is only one recurrent class consisting of a single profile, the win-win profile (C, C) with $P_{(\mathrm{C},\mathrm{C})(\mathrm{C},\mathrm{C})} = 1$. Thus, by using the mentioned penalty technique of reframing the payoffs, the players could reach a win-win profile.

Example 4.2 (Game of Silence) The psychological atmosphere of the game can be seen on the transition probability matrix P of the game given in Example 3.2 of [1]. By rearranging the profiles, we get the canonical form (3) of [1]

$$P = \begin{array}{c} \\ (\mathrm{N},\mathrm{N}) \\ (\mathrm{C},\mathrm{N}) \\ (\mathrm{N},\mathrm{C}) \\ (\mathrm{C},\mathrm{C}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathrm{N},\mathrm{N}) & (\mathrm{C},\mathrm{N}) & (\mathrm{N},\mathrm{C}) & (\mathrm{C},\mathrm{C}) \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.8 & 0.1 & 0.1 & 0 \\ 0.8 & 0.1 & 0.1 & 0 \\ 0.9 & 0 & 0.1 & 0 \end{pmatrix} \end{array}.$$

It is easy to see that $\{(\mathrm{N},\mathrm{N})\}$ is the only recurrent class and the profile (N, N) is absorbing. Since in this phase (the first two days) the players are in the profile (N, N), both players are trapped in this recurrent class that does not contain the focal profile (C, C). They will not leave (N, N) unless some event occurs to restructure the game. In this phase both players are not satisfied. However, their charge levels increase with time. At the beginning of the third day both want a change, but, it is difficult because of being seized by self-importance feeling. Since the focal profile (C, C) is transient, it is necessary to implement Operation (O2) or (O3). The transition probability of this phase can be assumingly estimated as

$$P' = \begin{array}{c} \\ (\mathrm{C,C}) \\ (\mathrm{C,N}) \\ (\mathrm{N,C}) \\ (\mathrm{N,N}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathrm{C,C}) & (\mathrm{C,N}) & (\mathrm{N,C}) & (\mathrm{N,N}) \end{array} \\ \begin{pmatrix} 0.4 & 0.3 & 0.3 & 0 \\ 0.3 & 0.3 & 0.3 & 0.1 \\ 0.8 & 0.1 & 0.1 & 0 \\ 0.2 & 0.3 & 0.3 & 0.2 \end{pmatrix} \end{array}.$$

As we have seen in Sect. 3.1, the game was solved by the wife, who came out with the idea of opening and closing the drawers. Let us describe how the game is solved by HD theory. By Hypothesis H5 (goal setting and state evaluation), the wife evaluated the situation and felt that there was a serious deviation from the ideal of the most important family goals, the stability and peaceful life. This created a high level of charge (charge structure and attention allocation Hypothesis H6). The charge level became a drive for change in the mind of the wife, which made her look for a solution or way out of this dangerous and stressful situation as to reduce the charge level (discharge Hypothesis H7). Looking deeply into her potential domain and knowing well her husband's habitual domain, she came out with the idea (strategy) of showing love without hurting her self-importance feeling. She started opening and closing the drawers "looking for his voice". This event triggered the cooperation spirit of her husband (efficient restructuring Hypothesis H3). The charge was completely released and their life became normal (discharge Hypothesis H7).

The idea of the wife restructured the game, which affected also the transition probability matrix, to get out of the class (decision trap) {(N, N)} and make the focal profile (C, C) reachable. The effect of this idea on the transition probability matrix is exactly Operation (O3). We can assumingly estimate the new transition probability matrix as

$$P'' = \begin{array}{c} \\ (\mathrm{C,C}) \\ (\mathrm{C,N}) \\ (\mathrm{N,C}) \\ (\mathrm{N,N}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathrm{C,C}) & (\mathrm{C,N}) & (\mathrm{N,C}) & (\mathrm{N,N}) \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.9 & 0 & 0.1 & 0 \\ 0.9 & 0.1 & 0 & 0 \\ 0.8 & 0.1 & 0.1 & 0 \end{pmatrix} \end{array}.$$

It is clear that (C, C) is now a win-win profile. Note that in the first two days, we have

$$\alpha_{(\mathrm{C,C})} = P_{(\mathrm{C,C})(\mathrm{C,C})}/(1 - P_{(\mathrm{C,C})(\mathrm{C,C})}) = 0/(1-0) = 0.$$

That is, the process will not stay in the focal profile (C, C) if it is reached. In the third day before the wife's idea is implemented, we have

$$\alpha_{(\mathrm{C,C})} = 0.4/(1-0.4) = 0.66.$$

The stability of the focal profile (C, C) has increased, but it is far from being stable. After the wife's idea is implemented, we have $P_{(\mathrm{C,C})(\mathrm{C,C})} = 1$, that is, (C, C) is a win-win profile.

Example 4.3 (Alynsky's Strategy) The psychological atmosphere of the game in Phase 2, is described by the transition probability matrix P' given in Example 3.3 of [1]. From the form of P', it is clear that $\{(\mathrm{C,N}), (\mathrm{N,C}), (\mathrm{N,N})\}$ is the only recurrent class and it does not contain the focal profile (C, C). Moreover, the players are trapped in this class. Thus, if no new important event or idea could restructure the game, the focal profile (C, C) will never be reached. In terms of Operations (O1)–(O3), the Operation (O2) or (O3) has to be implemented because the profile (C, C) is transient.

As we have seen in Sect. 3.1, the game was solved by consulting with Alinsky, who came out with the idea of legally occupying all toilets of Chicago airport. Let us describe the solution by HD theory. Thus, by active solving problem principle and H8 (information input hypothesis), the organization could come out with the idea of consulting with Alinsky who proposed a solution to the problem. When the authorities heard about the action that the organization intended to take (information input H8), by analyzing the possible impacts of this action (goal setting and state evaluation H5), their charge level increased sharply to a high level (charge structure and attention allocation H6). Then, by efficient restructuring hypothesis H3, their mind changed from non cooperation state to a cooperation state. At the same time the charges of the organization and the authorities were also released (discharge hypothesis H7). Thus, by consulting with Alinsky the organization could come out with an idea that makes it possible to apply Operation (O3) effectively. Just by leaking the idea to the authorities, the game was restructured so that the focal profile (C, C) became a win-win profile. We can estimate the new transition probability matrix of the game as

$$P'' = \begin{array}{c} \\ (\mathrm{C,C}) \\ (\mathrm{C,N}) \\ (\mathrm{N,C}) \\ (\mathrm{N,N}) \end{array} \begin{array}{c} \begin{array}{cccc} (\mathrm{C,C}) & (\mathrm{C,N}) & (\mathrm{N,C}) & (\mathrm{N,N}) \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \end{array}.$$

We have $P''_{(\mathrm{C,C})(\mathrm{C,C})} = 1$, that is, the focal profile (C, C) is now a win-win profile. Moreover, in whatever state the players are, they will shift to the profile (C, C) in one step.

Remark 4.2 Different reframing techniques were used to reach a focal profile or transform it into a win-win profile in the examples we have treated in this paper. In Example 4.1 (prisoner dilemma) a new rule of the game has been introduced to reframe the payoffs (see Sect. 3.1); in Example 4.2 (game of silence), the general techniques of understanding the game situation, identifying solvable solutions and creating charge for these solutions has been used (see Sect. 3.1); in Example 4.3 (Alinsky strategy), the expansion of the potential domain by consulting with experts has been used (or using external information input Hypothesis H8). Note that one restructuring technique may perform many operations at the same time. For example, in the Alinsky strategy example, the Alinsky strategy alone was sufficient for transforming the profile (C, C) into a win-win profile and dissolving all the other recurrent classes (Procedure 3.1).

Remark 4.3 Because of the many restrictive assumptions mentioned in Sect. 3.1 of [1], the framework of normal form games does not allow players to make use of all possible psychological aspects to solve games like those of Examples 2.2–2.3 of [1] or to reach a stable solution in the prisoner dilemma game. We have shown that, within the framework of habitual domain theory it is possible to solve these games. By habitual domains theory, we could analyze these games, describe their evolution and explain how a solution (win-win profile) can be reached.

5 Conclusions

In [1] and the present Part 2 of the paper, a HD theory based model for two person games has been formulated and presented. In the proposed model, sets of strategies and utility functions are not involved; it focuses on the states of mind of players and their charge level. Thus, the model could significantly enlarge the scope of applications of games in strategic (normal) form. The assumptions of our model allowed us to use the theory of Markov chains to analyze the game. Some techniques on how to direct the game towards a focal profile and how to make it a win-win profile are presented. We have justified the possibility theorem that claims that, under some reasonable assumptions, and when the players use effectively Hypotheses H1–H8 of HD theory, it is always possible to reach a win-win state. The possibility theorem shows the *feasibility* of the HD approach for solving two person games. As Operations (O1)–(O3) can be performed by many ways, a very challenging problem remains open, the problem of *optimality*. That is, how to find optimal ways of restructuring the game for reaching a win-win profile when time, cost and other criteria are considered to evaluate changes in transition probabilities or equivalently changes in the minds of players? Finally we hope that second order games will be a useful and effective tool for solving real social, political and economic conflicts in order to avoid or minimize losses in lives and property.

Appendix A

A.1 Expanding HD and Integrating New Ideas with the HD

Toolbox 1. *Basic Methods for Expanding the HD*. (i) Active learning, (ii) projecting from higher position, (iii) active association, (iv) changing the relevant parameters, (v) changing the environment, (vi) brain storming, (vii) retreating, and (viii) meditation.

Toolbox 2. *Ideas that Capture our Attention*. (i) Ideas that change the charge level, (ii) ideas that could trigger echoing in memory, (iii) ideas that arrive at the right time, (iv) ideas presented in the right ways, and (v) credible ideas.

Toolbox 3. *Method for Integrating an Idea or Concept with the Core of a Decision Maker's HD.* (i) Suggesting the idea, (ii) implanting the idea, (iii) nurturing the idea, (iv) habituating and repeating the idea and (v) integrating the idea with the core of existing HD.

We refer the reader to [4, 5] for more details on these three toolboxes.

A.2 Proofs of Propositions 3.1, 3.2 and 3.3

Definition A.2.1 (See [8]) A subset R of the set of states of a finite Markov chain is said to be closed if

$$\sum_{j\in R} P_{ij}^{(n)} = 1, \quad \text{for all } i \in R \text{ and } n \geq 0.$$

Proposition A.2.1 (See [8]) *A recurrent class is closed.*

Proof of Proposition 3.1 According to Proposition A.2.1, a subset of S which is not closed cannot be a recurrent class. Before implementing Operation (O1), we had

$$\sum_{(e,f)\in R} P_{(k,t)(e,f)}^{(n)} = 1, \quad \text{for all } (k,t) \in R \text{ and } n \geq 0.$$

Then, assume that $R \overset{(+)}{\to} R'$ is done by $(i, j) \overset{(+)}{\to} (u, v)$, with $(i, j) \in R$ and $(u, v) \in R'$. After Operation (O1), we have

$$\sum_{(e,f)\in R} P_{(i,j)(e,f)} = 1 - P_{(i,j)(u,v)},$$

with $P_{(i,j)(u,v)} > 0$, and $(u, v) \notin R$. This means that the subset R is no more closed. Thus R itself cannot be a recurrent class. Let us now prove that no profile of R can be recurrent in any other recurrent class after Operation (O1). Let R'' be a recurrent class after Operation (O1). We need to prove that $R'' \cap R = \emptyset$. First assume that $R'' = R'$. Let us first show that R' is actually a recurrent class after Operation (O1). Operation (O1) does not affect the transition probabilities within $S\backslash R$ and from $S\backslash R$ to R. Then it does not affect the transition probabilities within R' and from R' to outside R', because $R' \subset S\backslash R$. Hence, R' being a recurrent class before Operation (O1), it is a recurrent class after Operation (O1) as well. We had $R' \cap R = \emptyset$ before Operation (O1), then $R' \cap R = \emptyset$ after Operation (O1) as well. Indeed, since the transition probabilities from R' to outside R' remain the same after Operation (O1), no profile outside R' can communicate with any profile of R', hence no new profile can join the recurrent class R'. Thus, no profile of R can be a recurrent profile of R'. Next, assume that $R'' \neq R'$. Since both R' and R'' are recurrent classes, we have $R' \cap R'' = \emptyset$. Assume now, by contradiction, that $R'' \cap R \neq \emptyset$. Let $(k, t) \in R'' \cap R$. Recall that $R \overset{(+)}{\to} R'$ is done by $(i, j) \overset{(+)}{\to} (u, v)$, with $(i, j) \in R$ and $(u, v) \in R'$. Since $(k, t) \in R$ and $(i, j) \in R$, then $(i, j) \leftrightarrow (k, t)$ before Operation (O1). Since the positive transition probabilities within R remain positive after Operation (O1), we have also $(i,j) \leftrightarrow (k,t)$ after Operation (O1). Then, using the transitivity of accessibility and the fact that $R' \cap R'' = \emptyset$, we obtain $(k,t) \to (u,v)$, with $(k,t) \in R''$ and $(u,v) \notin R''$. This means that R'' is not closed. Thus, by Proposition A.2.1, it cannot be a recurrent class. This contradiction implies that $R'' \cap R = \emptyset$. □

Proposition A.2.2 *Let D be a subset of the set of states of a finite Markov chain. If D is closed and all its states communicate between each other, it is a recurrent class.*

Proof Since D is closed, we have

$$\sum_{j \in D} P_{ij}^{(n)} = 1, \quad \text{for all } i \in D \text{ and } n \geq 0.$$

Hence,

$$\lim_{n \to \infty} \sum_{j \in D} P_{ij}^{(n)} = 1, \quad \text{for all } i \in D. \tag{3}$$

Since the states of D communicate between each other, according to Theorem 4.1 of [1], they are all recurrent or all transient. Now, assume that the states of D are transient. Then, by Theorem 4.2 of [1], we have $\lim_{n\to\infty} P_{ij}^{(n)} = 0$, for all $i, j \in D$, hence $\lim_{n\to\infty} \sum_{j\in D} P_{ij}^{(n)} = 0$, for all $i \in D$, which leads to a contradiction with (3). Hence, the states of D are recurrent. Next, by assumption, the states of D communicate between each other and D is closed, then no state outside D can communicate with a state in D. Hence, D is a class. Thus, D is a recurrent class. □

Proof of Proposition 3.2 We need to prove that the set of profiles $R \cup \{(1,1)\}$ obtained after performing Operation (O2) is a recurrent class. We will use Proposition A.2.2. First we prove that all the profiles of $R \cup \{(1,1)\}$ communicate between each other. As the positive transition probabilities between the profiles of R are not transformed into zero by Operation (O2) and before this operation R was a recurrent class, the profiles of R communicate with each other after Operation (O2) is performed. Hence it remains to prove that the focal profile $(1,1)$ communicates with all the profiles of R. The Operation (O2) makes the focal profile $(1,1)$ accessible from some profile (i,j) in R and some profile (k,t) in R accessible from the focal profile $(1,1)$. That is, $(i,j) \to (1,1)$ and $(1,1) \to (k,t)$. Since both (i,j) and (k,t) are in R, before Operation (O2), we had $(i,j) \leftrightarrow (k,t)$, which remains true after Operation (O2). Then by transitivity $(i,j) \leftrightarrow (1,1)$. Again by transitivity, we have $(e,f) \leftrightarrow (1,1)$, for all $(e,f) \in R$. Thus, the focal profile $(1,1)$ communicates with all the profiles of R. Let us now prove that $R \cup \{(1,1)\}$ is closed. By Operation (O2) (ii), we get $P_{(1,1)(u,v)} = 0$ for all (u,v) not in $R \cup \{(1,1)\}$. We had $P_{(e,f)(u,v)} = 0$ for all (e,f) in R and (u,v) not in $R \cup \{(1,1)\}$ before Operation (O2), because R was a recurrent class. As (O2) affects only the transition probabilities within $R \cup \{(1,1)\}$ and from the focal profile $(1,1)$ to other profiles, $P_{(e,f)(u,v)} = 0$ for all (e,f) in R and (u,v) not in $R \cup \{(1,1)\}$ will remains true after Operation (O2). Hence,

$$\sum_{(i,j)\in R\cup\{(1,1)\}} P_{(e,f)(i,j)} = 1, \quad \text{for all } (e,f) \in R \cup \{(1,1)\}.$$

Consequently, from [7] we get

$$\sum_{(u,v)\in R\cup\{(1,1)\}} P^{(n)}_{(e,f)(u,v)} = 1, \quad \text{for all } (e,f)\in R\cup\{(1,1)\} \text{ and } n\geq 1.$$

That is, $R\cup\{(1,1)\}$ is closed. According to Proposition A.2.2, $R\cup\{(1,1)\}$ is a recurrent class. □

Proof of Proposition 3.3 Let R be a recurrent class after completion of (O3). We will prove that it was a recurrent class before (O3) is performed. As both R and $\{(1,1)\}$ are recurrent classes, $R\cap\{(1,1)\}=\varnothing$, which implies that $R\subset S\backslash\{(1,1)\}$. Then, the transition probabilities within R and from R to outside R are the same before and after Operation (O3) is performed. Thus, if R is a recurrent class after (O3) is performed, then R was a recurrent class before (O3). Now assume that (1, 1) belongs to a recurrent class R^0 and $\{(1,1)\}\neq R^0$ before Operation (O3). As after completion of Operation (O3), $\{(1,1)\}$ becomes a recurrent class, R^0 cannot be a recurrent class for $\{(1,1)\}\subset R^0$ and $\{(1,1)\}\neq R^0$. Now we prove that no profile of $R^0\backslash\{(1,1)\}$ can be recurrent in any other recurrent class after Operation (O3). Let R be a recurrent class after Operation (O3) is performed. We need to prove that $R\cap(R^0\backslash\{(1,1)\})=\varnothing$. In the case $R=\{(1,1)\}$ the result is obvious. Assume that $R\neq\{(1,1)\}$. Since R and $\{(1,1)\}$ are recurrent classes after (O3) is performed, we have $(1,1)\notin R$. Hence, $R\subset S\backslash\{(1,1)\}$. Consequently, Operation (O3) does not affect the transition probabilities within R and from R to outside R. Hence, R being a recurrent class after Operation (O3), it was a recurrent class before this operation. This implies $R\cap R^0=\varnothing$. Then, we have

$$R\cap(R^0\backslash\{(1,1)\})\subset R\cap R^0=\varnothing \quad\Rightarrow\quad R\cap(R^0\backslash\{(1,1)\})=\varnothing. \qquad \square$$

A.3 Proof of Proposition 3.4

(i) In Procedures 3.1 or 3.2 when Operation (O1) is performed on a recurrent class R, there is no creation of new recurrent classes. Recall that Operation (O1) does not affect the transition probabilities within $S\backslash R$ and from $S\backslash R$ to R. Let R'' be a recurrent class after Operation (O1), in the proof of Proposition 3.1 (see Appendix A.2) we have proved that $R''\cap R=\emptyset$. Hence $R''\subset S\backslash R$. This inclusion implies that the transition probabilities within R'' and from R'' to outside R'' are the same before and after Operation (O1). Thus, R'' being a recurrent class after Operation (O1), it was also a recurrent class before this operation. Moreover, in the Step 2 of Procedure 3.1, an operation of the form $R\overset{(+)}{\to}\{(1,1)\}$ implemented on a class not containing (1, 1) does not create new recurrent classes, because it is a particular case of Operation (O1).

(ii) In Steps 3–4 of Procedure 3.2, by integrating the focal profile (1, 1) with a recurrent class of the Markov chain of the game (Operation (O2)), there is no creation of new recurrent classes apart from the recurrent class $R\cup\{(1,1)\}$, where R is the recurrent class of the Markov Chain of the game with which the focal profile (1, 1) is integrated. Assume that A (such that $A\neq R\cup\{(1,1)\}$) is a recurrent class after Operation (O2) is performed. Note that we have $A\cap\{R\cup\{(1,1)\}\}=\emptyset$. Since a recurrent class is closed (Proposition A.2.1), this means that for any profile $(i, j) \notin A$ we have $P_{(k,t)(i,j)} = 0$, for all $(k,t) \in A$. According to the conditions imposed on (O2), Operation (O2) does not affect the transition probabilities within $S \backslash R \cup \{(1,1)\}$ and from $S \backslash R \cup \{(1,1)\}$ to $R \cup \{(1,1)\}$. Since $A \cap \{R \cup \{(1,1)\}\} = \emptyset$, then $A \subset S \backslash R \cup \{(1,1)\}$. Hence, the transition probabilities within A and from A to outside A are the same before and after Operation (O2) is performed. Thus, A being a recurrent class after Operation (O2) is performed, it was a recurrent class before this operation as well.

(iii) In Steps 1–2 of Procedure 3.1 or Steps 3–4 of Procedure 3.2, by performing Operation (O3) there is no creation of new recurrent classes. We have proved this in Proposition 3.3. Procedures 3.1 and 3.2 terminate when only one recurrent class, containing the focal profile $(1,1)$, remains in the Markov Chain of the game. According to Proposition 3.1, when Operation (O1) is performed on a recurrent class it dissolves it, i.e. the number of recurrent classes decreases by one. Since the number of recurrent classes is finite in a finite Markov chain, (i)–(iii) and Proposition 3.1 imply that Procedures 3.1–3.2 terminate in a finite number of steps.

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