Estimation of the Bit Error Rate for Direct-Detected OFDM Signals With Optically Preamplified Receivers

Wei-Ren Peng, Kai-Ming Feng, Alan E. Willner, *Fellow, IEEE, Fellow, OSA*, and Sien Chi, *Fellow, OSA*

***Abstract*—In this paper, we provide a numerical bit error rate (BER) estimation approach for direct-detected orthogonal frequency division multiplexing (OFDM) signals in the presence of optical preamplified receivers. The individual BER of each subcarrier is first computed by considering their electrical signal-to-noise ratio (SNR), and then the ensemble BER is derived simply by taking the average of all the subcarriers' BERs. The calculated BER is verified by the conventional error-counting approach with high precision and is still accurate with higher quadratic-amplitude modulation (QAM) formats, even under the influences of the optical filtering and polarization mode dispersion (PMD) effects. Based on our simulation approach, the required extra power budget for 16- and 64-QAM relative to 4-QAM format are found to be $\sim$3.8 and 8.2 dB, respectively, at a BER of 10^{-9}. Furthermore, we use this approach to compare the receiving sensitivities and PMD tolerances for the previous proposed gapped and interleaved radio-frequency (RF)-tone-assisted OFDM systems. The results show that the gapped OFDM has a better sensitivity while the interleaved OFDM has a more PMD-tolerable capability.**

***Index Terms*—Bit error rate (BER), direct detection, optical fiber communication, orthogonal frequency division multiplexing (OFDM), optical modulation.**

I. Introduction

OPTICAL orthogonal frequency division multiplexing (OFDM) has recently been proposed as a promising format for optical long-haul transmission since the fiber chromatic dispersion (CD) could be electrically compensated for by digital signal process (DSP) procedures at the receiver [1]–[3]. The direct-detected OFDM, which uses only one photodiode and thus is very simple to be implemented, with eight-channel wavelength division multiplexing (WDM) system has been successfully transmitted through 1000 km of uncompensated standard single-mode fiber (SSMF) [4], and with single channel using virtual single sideband OFDM (VSSB-OFDM) format also has reached 1600 km of SSMF with only $\sim$3 dB power penalty [5]. Moreover, by utilizing the polarization division multiplexing (PDM) and the self-polarization diversity receiving, the capacity of a direct-detected OFDM system can be further doubled with a moderate receiver complexity [6].

To explore in detail how the signal performance would change with various system parameters, one direct and simple approach is to evaluate system performances in terms of the bit error rate (BER). Conventional BER calculation is evaluated by the error-counting approach, which typically is time and memory consuming, because to get a certain degree of statistic confidence at low BER, numerous OFDM symbols have to be generated for counting the errors bit by bit. Thus, an accurate and highly efficient BER estimation is crucially desired for understanding, designing, and optimizing an OFDM system.

A Q-factor approach, which assumes a Gaussian distribution for the electrical beat noise and estimates the average electrical signal-to-noise ratio (ESNR) from the received constellations, is suggested approximating the BER of a 4-quadratic-amplitude modulation (QAM), direct-detected OFDM system [7], [8]. However, there is no evidence in [7] and [8] to justify how well this Q-factor approach can match the practical error-counting method in an OFDM system. Thus, both the appropriateness of the Q-factor approach, which is simply extracted from the received constellations without considering the individual ESNR of each subcarrier, and the suitability of the Gaussian assumption to the electrical beat noise are still unsolved issues.

In this paper, we show that the exact BER of an optically preamplified direct-detected OFDM signal can be accurately estimated by first evaluating the individual ESNR of each subcarrier and then averaging the BERs of all subcarriers [9]. The ESNR of each subcarrier can be easily obtained by numerically computing the power spectral density (PSD) of both the signal and the beat noises. We found that both the PSD of the received data subcarrier and the beat noises are colored due to the filtering effect, which also has been described in [10], thus yielding a nonuniform ESNR distribution over the subcarriers. The numerical results, supported by the error-counting method, show our scheme can well predict the exact BER even under tight optical filtering or severe polarization mode dispersion (PMD) conditions. Moreover, although a larger QAM size is more spectrally efficient, the required extra power at a BER of 10^{-9} for the 16- and 64-QAM are found to be $\sim$3.8 and 8.2 dB, respectively, relative to the 4-QAM format based on our BER evaluation technique. In addition, by using our approach, we compare the sensitivities and PMD tolerances at $\mathrm{BER} = 10^{-9}$ for the previous gapped and interleaved radio-frequency (RF)-tone-assisted OFDM systems. The results show that the gapped OFDM exhibits a $\sim$2.3 dB better sensitivity

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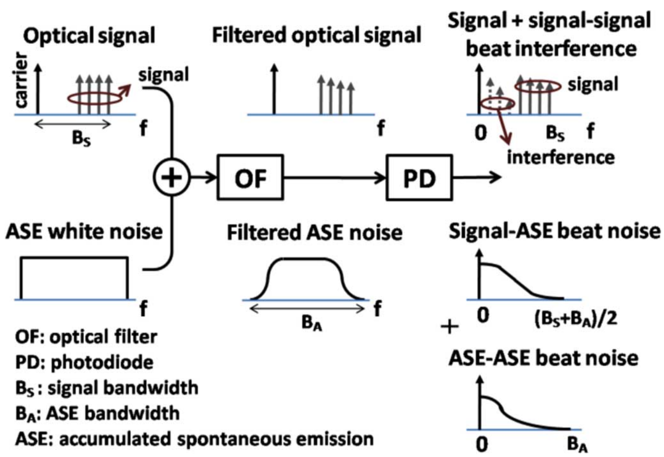

Fig. 1. Progress of the power spectra of the signal and the ASE noise for SSB-OFDM systems.

while the interleaved OFDM is less sensitive to the PMD effects.

This paper is organized as follows. In Section II, the semi-analytical BER estimation is detailed for the direct-detected OFDM system. In Section III, the simulation results for 10-Gb/s direct-detected OFDM system, validated by the conventional error counting approach, are presented under the conditions of strong optical filtering and severe PMD effects. Then, Section IV compares the receiving sensitivities and PMD tolerances of the previous two direct-detected OFDM systems (i.e., the gapped OFDM and the interleaved OFDM). Finally, in Section V, we conclude this paper.

II. Bit Error Rate Calculation

Fig. 1 depicts a typical receiver model for an optical direct-detected OFDM system in the presence of an optical preamplifier. After the signal passing through the optical amplifier, the amplified spontaneous emission (ASE) noise that typically can be assumed as the additive white Gaussian noise (AWGN) is added to the signal. An optical filter following the optical amplifier is used to reduce the out-of-band ASE noise. This optical filter will not only unequalize the signal power among the multiple data subcarriers but also will shape the PSD of the ASE noise. The signal and ASE noise could be treated independently in the case of a linear transmission before the beating at the photodiode. After the square-law detection in the photodiode, the desired data subcarriers, resulted from the beating between the carrier and the optical data subcarriers, can be obtained directly without being interfered by the signal–signal beat interference (SSBI). On the other hand, the electrical beat noises can be mainly categorized as the signal-ASE beat noise (SABN) and the ASE-ASE beat noise (AABN), which will be detailed later in this section.

In the following subsections, we will first describe how to derive the electrical PSD of the signal and the beat noises, and use the obtained ESNR of each individual subcarrier to calculate the ensemble BER of the whole system.

A. Signal Power

To compute the electrical power of each data subcarrier, we need only one OFDM symbol with each subcarrier modulated by the root mean square (RMS) amplitude of the data symbol's amplitude. The discrete time model for this optical OFDM symbol can be written as

$$E(n) = A\exp\left(\frac{-j2\pi(N/2)n}{N}\right) + \sum_{k=0}^{N_d-1} d_{\mathrm{rms}}\exp\left(\frac{j2\pi kn}{N}\right) \tag{1}$$

where A is the amplitude of the optical carrier [1] or the inserted RF tone [11] at the $(-N/2)$th subcarrier, k is the subcarrier index ranging from $(-N/2)$ to $(N/2-1)$, N_d and N are the numbers of the data subcarrier and the size of discrete Fourier transform (DFT), respectively, with $N = 2N_d$, and d_{rms} stands for the RMS amplitude of all the data symbol's amplitude, i.e., $d_{\mathrm{rms}} = \sqrt{\langle|d_k|^2\rangle}$ where d_k is the data symbol on the kth subcarrier. Note that the total optical power is $P_{\mathrm{opt}} = \varepsilon[|E(n)|^2] = |A|^2 + N_d d_{\mathrm{rms}}^2$, where $\varepsilon[x]$ is the expectation of x.

After passing through an optical filter with a discrete frequency response of $H_o(k)$, the filtered optical signal $E_o(n)$ becomes

$$E_o(n) = AH_o(-N/2)\exp(-j\pi n) + \sum_{k=0}^{N_d-1} d_{\mathrm{rms}}H_o(k)\exp\left(\frac{j2\pi kn}{N}\right). \tag{2}$$

If the first-order PMD effect is further introduced into the system with the discrete frequency transfer function of

$$H_x(k) = \exp\left(\frac{-j\pi k\gamma_d}{N}\right)$$

and

$$H_y(k) = \exp\left(\frac{j\pi k\gamma_d}{N}\right) \tag{3}$$

where $H_x(k)$ and $H_y(k)$ stand for the frequency responses of PMD in x- and y-polarizations of the fiber, and $\gamma_d = T_d/T_s$ in which T_d is the differential group delay (DGD) and T_s is the time-domain sampling duration. Then, the PMD-affected and optically filtered signal in the x-polarization $E_{xo}(n)$ and y -polarization $E_{yo}(n)$ are

$$E_{xo}(n) = \sigma\begin{bmatrix} AH_o(-N/2)H_x(-N/2)\exp(-j\pi n)+ \\ \sum_{k=0}^{N_d-1} d_{\mathrm{rms}}H_o(k)H_x(k)\exp\left(\frac{j2\pi kn}{N}\right)\end{bmatrix} \tag{4}$$

$$E_{yo}(n) = \sqrt{1-\sigma^2}\begin{bmatrix} AH_o(-N/2)H_y(-N/2)\exp(-j\pi n)+ \\ \sum_{k=0}^{N_d-1} d_{\mathrm{rms}}H_o(k)H_y(k)\exp\left(\frac{j2\pi kn}{N}\right)\end{bmatrix} \tag{5}$$

where σ^2 is the power ratio of the optical signal that is transmitted along the x-polarization.

After the photodiode, the photocurrent contains both the desired electrical signal and the SSBI I_{SSBI} which is

$$\begin{aligned} I(n) &= |E_{xo}(n)|^2 + |E_{yo}(n)|^2 \\ &= \left\{ d_{\mathrm{rms}} A^* H_o^* \left(\frac{-N}{2}\right) \sum_{k=0}^{N_d-1} G(k) H_o(k) \right. \\ &\quad \left. \times \exp\left(\frac{j2\pi\left(k+\frac{N}{2}\right)n}{N}\right) + I_{\mathrm{SSBI}} \right\} + c.c. \end{aligned} \quad (6)$$

where

$$G(k) = \left[\sigma^2 \exp\left(-j\frac{\pi\left(k+\frac{N}{2}\right)\gamma_d}{N}\right) + (1-\sigma^2)\exp\left(j\frac{\pi\left(k+\frac{N}{2}\right)\gamma_d}{N}\right)\right] \quad (7)$$

and $c.c.$ stands for the complex conjugation term.

The electrical power of each subcarrier $P_{\mathrm{el}}(k)$ can be written as

$$P_{\mathrm{el}}(k) = \left| d_{\mathrm{rms}} A H_o\left(\frac{-N}{2}\right) H_o(k)\right|^2 |G(k)|^2. \quad (8)$$

For the given values of σ^2 and γ_d, we found the electrical power $P_{\mathrm{el}}(k)$ of some of the subcarriers would be strongly attenuated via $|G(k)|^2$, and this phenomenon is referred to as the "PMD induced power fading." If the optical signal is equally split into the two orthogonal x- and y-polarizations, i.e., $\sigma^2 = 1/2$, the most severely PMD-induced power fading can be observed directly via

$$P_{\mathrm{el}}(k) = \left| d_{\mathrm{rms}} A H_o\left(\frac{-N}{2}\right) H_o(k)\right|^2 \cos^2\left(\frac{\pi(k+N/2)\gamma_d}{N}\right) \quad (9)$$

of which the fading is a function of the subcarrier index k.

B. Noise Power

Now we derive the electrical power distribution for the beat noises which can be categorized as the SABN and the AABN as shown in Fig. 1. The baseband PSD of the filtered ASE noise $N_A(f)$ is basically the same as the optical filter, i.e., $N_A(f) = \mathcal{N}_o H_o(f)$, where $\mathcal{N}_o$ is a constant representing the spectral density of the unfiltered ASE and $H_o(f)$ is the baseband transfer function of the optical filter. After the squared-law photodiode, we can write the converted continuous-time photocurrent $I(t)$ as

$$\begin{aligned} I(t) &= |E_s(t) + N_A(t)|^2 \\ &= |E_s(t)|^2 + 2\mathrm{Re}[E_s(t)N_A^*(t)] + |N_A(t)|^2 \end{aligned} \quad (10)$$

where $E_s(t)$ is the continuous-time filtered optical OFDM symbol including both the optical carrier and data subcarriers, and $N_A(t)$ is the continuous-time filtered ASE noise. The first term in (10), $|E_s(t)|^2$, contains the direct current (dc), the desired data subcarriers, and SSBI which has been discussed in the previous subsection. The second term in (10), $2\,\mathrm{Re}[E_s(t)N_A^*(t)]$, is the SABN and has a PSD expressed as

$$N_{SA}(f) = N_s(f) \otimes N_A(-f) + N_s(-f) \otimes N_A(f) \quad (11)$$

where $N_s(f)$ is baseband PSD of both the filtered optical carrier and signal, $N_A(f)$ is the baseband PSD of the filtered ASE noise, and $\otimes$ is the convolution operator. This implies that the SABN is colored if ASE is bandlimited by an optical filter. The complete derivation of the PSD of $N_{SA}(f)$ is described in the Appendix. The third term in (10), $|N_A(t)|^2$, is the AABN and has a PSD of $N_{AA}(f) = N_A(f) \otimes N_A(-f) + N_{AA}(0)$ [12], where $N_{AA}(0)$ is simply a dc. Similarly, the bandlimited ASE yields a colored AABN although its effect is insignificant compared with SABN due to its relatively low power. If we assume that the SABN and the AABN are independently Gaussian distributed, the total PSD of the beat noise $N_{\mathrm{tol}}(f)$ can be simply expressed as a sum of the PSD of SABN and AABN, i.e., $N_{\mathrm{tol}}(f) = N_{SA}(f) + N_{AA}(f)$.

C. Bit Error Rate

For computing the BER, we define the ESNR for the kth subcarrier as $\mathrm{ESNR}(k) = P_{\mathrm{el}}(k)/[N_{\mathrm{tol}}((k+N/2)\Delta f) \times \Delta f]]$, where Δf is the frequency spacing between the data subcarriers. Note that we use the index $(k+N/2)$ for computing the noise power since based on (6) the subcarriers have been shifted by $N/2$ units after beating. Since the beat noise is assumed as Gaussian distributed, we can relate $\mathrm{ESNR}(k)$ to $\mathrm{BER}(k)$ of each subcarrier as [13]

$$\mathrm{BER}(k) = \left(1 - \frac{1}{L}\right) \frac{\mathrm{erfc}\left(\sqrt{\frac{3\mathrm{ESNR}(k)\log_2 L}{(L^2-1)\log_2 M}}\right)}{\log_2 L} \quad (12)$$

where L is the number of levels in each dimension of the M-ary modulation system for each subcarrier [13] and M is the QAM format used on each data subcarrier. Then the ensemble BER will be the mean of the error rate of each subcarrier

$$\mathrm{BER} = \frac{1}{N_d} \sum_{k=0}^{N_d-1} \mathrm{BER}(k) \quad (13)$$

where N_d is the number of the data subcarriers.

III. Simulation Result

In simulations, the ASE noise is considered for both the x- and y-polarizations. The transfer function of the optical modulator is assumed liner which would be reasonable when a low optical modulation index (OMI) is adopted. The simulated data rate is fixed at 10 Gb/s with a 4-QAM format unless mentioned otherwise. The optical carrier to signal power ratio (CSPR), defined as $\mathrm{CSPR} = |A|^2/(N_d d_{\mathrm{rms}}^2)$, is set at 0 dB, which has been proved as the optimum value both numerically [7] and experimentally [11]. For Figs. 2–4, the number of data subcarrier is 72 and the total subcarrier number (i.e., the size of the discrete Fourier transform) is 512. Thus, the oversampling rate is $512/72 \approx 7.1$ which should be high enough for modeling a real system. The inserted cyclic prefix (CP) occupies

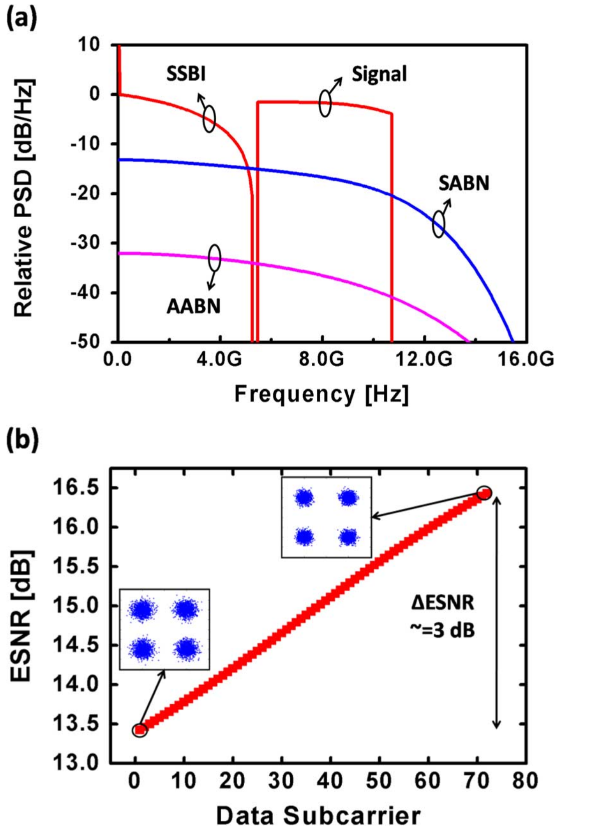

Fig. 2. (a) Simulated electrical power spectra of the desired signal, the SSBI, the colored signal-ASE, and ASE–ASE beat noise (SABN and AABN). (b) The corresponding ESNR as a function of the data subcarriers.

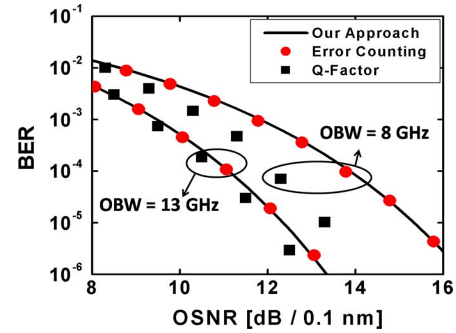

Fig. 3. BER versus OSNR with different OBW for the SSB-OFDM systems. The data rate is 10 Gb/s with 4-QAM. The OFDM bandwidth is ∼10.63 GHz. The Q-factor is extracted from all the received constellation points [7], [8].

$\sim 1/16$ of the duration of one OFDM symbol. The BER is calculated using (13) and the zero-forcing equalization [14] is utilized throughout this paper. The OSNR is defined as $\mathrm{OSNR} = P_{\mathrm{opt}}/(\mathcal{N}_0 \cdot \mathrm{BW})$ and the noise bandwidth (BW) is set at 0.1 nm in this paper.

Fig. 2 depicts the electrical PSD of both the signal and the beat noises, and the corresponding ESNR with an OSNR of ∼16 dB. The optical bandwidth, including both the carrier and the signal, of the OFDM signal is ∼10.63 GHz after the insertion of the CP. The optical filter is modeled as a second-order Gaussian type filter with a 3-dB bandwidth of ∼13 GHz. From Fig. 2(a), we found that the power of the desired data subcarriers, SABN, and AABN are all nonuniformly distributed over the subcarriers which is attributed to the limited bandwidth of the optical filter. The derived ESNR as a function of the data subcarrier from 1 to 72 has been shown in Fig. 2(b). The colored beat noise results in a worse ESNR for the low-indexed subcarriers, and gives a better ESNR for the high-indexed subcarriers. The maximum ESNR difference reaches ∼3 dB between the first and last data subcarriers. The constellations of the first and the last data subcarriers are shown in the insets.

In Fig. 3, we evaluate the system BER versus the OSNR using our method with two different optical bandwidths (OBW): 8 and 13 GHz. For comparisons, we also show the results obtained by the conventional error counting and the previously suggested Q-factor approaches. The Q-factor approach has been provided recently for computing the BER using the averaged signal and noise power estimated from the constellations of all the received symbols. For both tested OBWs, the results of our approach are well validated by the error-counting method. The BER results calculated from the previous Q-factor approach slightly deviate from the error-counting approach when a wider optical filter is used (i.e., when OBW = 13 GHz), and fail to predict the exact BER when a tight optical filter is employed (i.e., when OBW = 8 GHz). Since the Q-factor approach considers only the averaged ESNR instead of the individual ESNR of each subcarrier, it will fail to predict an exact BER when the difference among the subcarrier's ESNR is large, as in a system when a tight optical filter is utilized.

Shown in Fig. 4 is the simulated BER versus the DGD with our approach and the Q-factor method. For PMD simulations, we consider only the first-order PMD effect and use the signal model given in Section II. The optical power is assumed equally split into the two orthogonal polarizations, which is typically considered as the worst case. The results of error-counting approach are also shown as a reference for the exact BER. When there is no PMD effect (DGD = 0), the results of all the three methods are very similar except a little deviation by the Q-factor approach. When a strong PMD effect kicks in with a 40-ps DGD, our approach still keeps a good approximation while the Q-factor approach starts to underestimate the BER. The Q-factor approach fails to predict the BER of a PMD-affected signal because of the strong power variation among the subcarrier (PMD-induced power fading) which in turn will result in a strong ESNR variation among the data subcarriers. Since the Q-factor approach calculates the BER without considering the individual subcarrier's ESNR, it could not well predict the BER performance under a strong influence of the PMD.

To verify that our approach can still work for high QAM formats, Fig. 5 depicts the BER versus the OSNR for 10-Gb/s date rate but different QAM sizes of 4-, 16-, and 64-QAM formats. The numbers of the data subcarrier are 72, 36, and 24, respectively, for $\mathrm{M} = 4, 16$, and 64, and the number of the total subcarriers is fixed at 512. The optimum optical bandwidths of ∼13, 6.7 and 4.6 GHz for $\mathrm{M} = 4, 16$, and 64 are used in simulation. Each QAM format is verified by the error-counting method for proving that our approach can still function well with formats from $\mathrm{M} = 4$ to 64. We first show that at $\mathrm{BER} = 10^{-9}$, the extra power of ~ 3.8 or 8.2 dB is required if we extend the QAM size

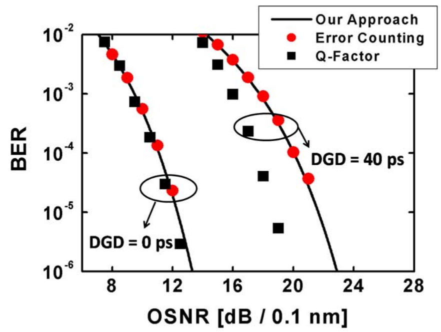

Fig. 4. BER versus OSNR calculated by our approach, error counting, and the previous Q-factor method for the SSB-OFDM systems.

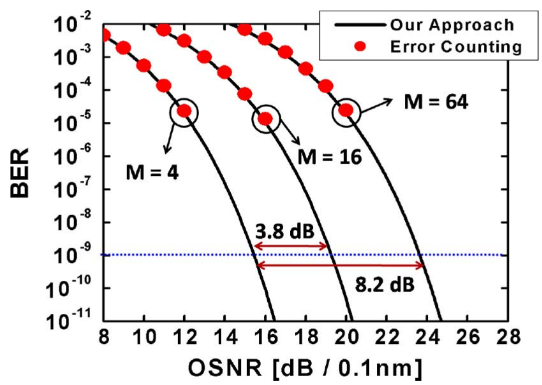

Fig. 5. BER versus OSNR with the same 10-Gb/s data rate but different OFDM QAM formats for the SSB-OFDM systems. The optimum optical bandwidths for 4-, 16-, and 64-QAM 10-Gb/s OFDM signals are 13, 6.7, and 4.6 GHz.

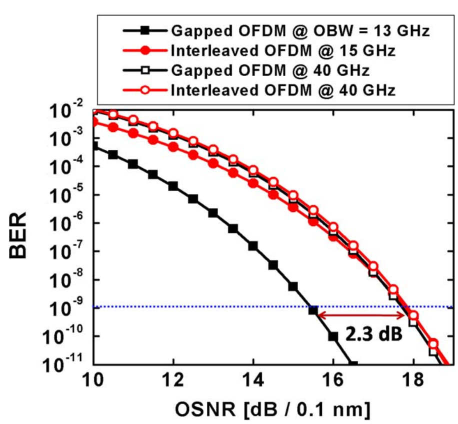

Fig. 6. Sensitivity comparisons for the gapped and interleaved OFDM systems.

from $\mathrm{M} = 4$ to $\mathrm{M} = 16$ or 64 to reach a higher spectral efficiency (SE).

IV. Performance Comparisons

In this section, we use our approach to compare the BER performance for the previous proposed systems: the gapped and the interleaved RF-tone assisted OFDM systems [11]. For Figs. 6 and 7, the data rate is 10 Gb/s with 4 QAM. The number of data and the number of total subcarriers are 72 and 512, respectively, and the CP is 1/16 of one OFDM symbol duration. The signal bandwidths of the two systems are equal to $\sim$10.63 GHz.

Shown in Fig. 6 are the back to back sensitivities of the two systems with their optimum optical bandwidths of 13 and 15 GHz, respectively. Under the optimum optical bandwidth, the gapped OFDM has an $\sim$2.3 dB better sensitivity at $\mathrm{BER} = 10^{-9}$ compared with the interleaved OFDM. This $\sim$2.3-dB difference mostly comes from different allocations of the data subcarriers. Depending on the results shown in Fig. 2, the noise PSD is higher for subcarriers closer to the dc value. For the gapped OFDM, because all the data subcarriers are located clusterly at the other side far from the dc and would not fall into the "deep noise" region, the signal averagely has a higher ESNR and thus has a better sensitivity. The interleaved OFDM has all the data subcarriers uniformly distributed over the signal bandwidth and therefore half the data subcarriers will fall into the "deep noise" regime, thus averagely degrading the receiving sensitivity. This effect would be mitigated when we use a larger optical bandwidth, i.e., $\mathrm{OBW} = 40$ GHz in Fig. 6. The performances of the two systems become very similar when the optical bandwidth extends to 40 GHz. This can be explained as follows. When a broader optical bandwidth is utilized, the power of the signal and noise becomes more uniformly distributed over the signal bandwidth. Under such a condition, the allocation of the data subcarriers becomes less important and thus the two OFDM systems have a comparable performance. Also from Fig. 6, we observe that the gapped OFDM has a $\sim$2.3-dB difference when the optical bandwidth is changed from 13 to 40 GHz and thus is more sensitive to the optical bandwidth compared with the interleaved OFDM, which has almost no difference when the filter bandwidth has been changed. Note that their similar performance under the broader optical bandwidth matches the previous measured results in [11], in which a 0.3-nm (37.5-GHz) optical filer is used for both the gapped and interleaved systems.

Fig. 7 draws the optical penalties at a BER of 10^{-9} as a function of the DGD for both systems. Again we consider only the first-order PMD effect and assume equal power distribution in x- and y-polarizations. The tolerable DGD for the gapped and interleaved OFDM are 22.5 and 31 ps, respectively, with 1-dB power penalty. The penalty curve of the interleaved OFDM does not even start to rise up until the DGD exceeds 25 ps. The better PMD tolerance of the interleaved OFDM is attributed to the frequency-dependent power fading. From (9), the PMD-induced power fading is more serious for those high-indexed subcarriers, i.e., subcarriers far away from the RF tone. Because the gapped OFDM has all the data subcarriers located clusterly far from the RF tone, the averagely suffered PMD fading would be more severe than that of interleaved OFDM, which puts the subcarriers more uniformly on the signal bandwidth and thus averagely suffer less PMD impairment. Note that in addition to the better PMD tolerance and the insensitivity to the optical bandwidth, the interleaved OFDM also shows a better tolerance to the fiber nonlinearities [11] and behaves more robust to the I/Q imbalances of the optical modulator when combined with a 2×2 matrix equalizer [15].

The simulated results in this paper are under the assumption of equal transmission power among the data subcarriers. Adaptive power control for each subcarrier [16], i.e., the power can be judiciously allocated to each subcarrier depending on the known

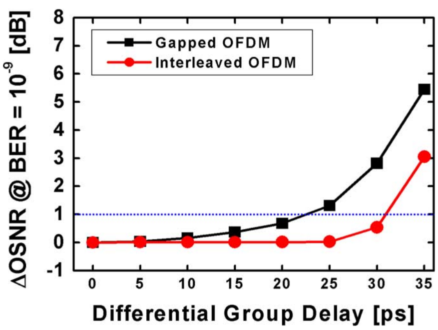

Fig. 7. Simulated power penalties versus the first-order PMD DGD for both the gapped and interleaved OFDM systems.

channel conditions, would possibly yield an even better performance.

V. Conclusion

We provide the first numerical BER calculation approach for direct detection OFDM systems in the presence of optically preamplified receivers. Our approach considers the PSD of both the electrical signal and noise, and then further uses the obtained ESNR to derive the BER directly. With our approach, accurate estimation of the BER can be obtained even under a strong optical filtering, serious PMD impairment, and different QAM sizes from 4 to 64. All the simulated results are verified by the conventional error-counting approach. Moreover, we compare the performance for the previous two RF-tone-assisted OFDM systems in terms of our calculated BER. The gapped OFDM outperforms the interleaved OFDM in the receiving sensitivity by ∼2.3 dB, while the interleaved OFDM has a better PMD tolerance compared with the gapped OFDM.

Appendix

The PSD of the signal-ASE beat noise (SABN) $N_{SA}(f)$ can be written as

$$N_{SA}(f) = \mathcal{F}\{R_{SA}(\tau)\} \tag{A-1}$$

where $\mathcal{F}\{\cdot\}$ is the Fourier transform operation and $R_{SA}(\tau)$ is the autocorrelation function of the SABN $N_{SA}(t)$, that is

$$R_{SA}(\tau) = \varepsilon[N_{SA}^*(t)N_{SA}(t+\tau)] \tag{A-2}$$

where $\varepsilon[x]$ is the expectation of x. With the formula of $N_{SA}(t) = 2\mathrm{Re}[E_s(t)N_A^*(t)]$ for SABN, $R_{SA}(\tau)$ can be further manipulated as follows:

$$\begin{aligned} R_{SA}(\tau) &= \varepsilon[\{E_S(t)N_A^*(t) + E_S^*(t)N_A(t)\}^* \\ &\quad \times \{E_S(t+\tau)N_A^*(t+\tau) + E_S^*(t+\tau)N_A(t+\tau)\}] \\ &= \varepsilon[E_S^*(t)E_S(t+\tau)]\cdot\varepsilon[N_A(t)N_A^*(t+\tau)] \\ &\quad + \varepsilon[E_S(t)E_S^*(t+\tau)]\cdot\varepsilon[N_A^*(t)N_A(t+\tau)] \\ &\quad + \varepsilon[E_S(t)E_S(t+\tau)]\cdot\varepsilon[N_A^*(t)N_A^*(t+\tau)] \\ &\quad + \varepsilon[E_S^*(t)E_S^*(t+\tau)]\cdot\varepsilon[N_A(t)N_A(t+\tau)]. \end{aligned} \tag{A-3}$$

Note that the third and fourth terms in (A-3) are equal to zero so that the autocorrelation function of $R_{SA}(\tau)$ can be simply written as a function of $R_S(\tau)$ and $R_A(\tau)$, which are the autocorrelation functions of $E_S(t)$ and $N_A(t)$, respectively, as follows:

$$R_{SA}(\tau) = R_S(\tau)R_A^*(\tau) + R_S^*(\tau)R_A(\tau) \tag{A-4}$$

where

$$R_S(\tau) = \varepsilon[E_S^*(t)E_S(t+\tau)]$$

and

$$R_A(\tau) = \varepsilon[N_A^*(t)N_A(t+\tau)].$$

After taking the Fourier transform as in (A-1), we obtain

$$\begin{aligned} N_{SA}(f) &= \mathcal{F}\{R_{SA}(\tau)\} \\ &= \mathcal{F}\{R_S(\tau)R_A^*(\tau) + R_S^*(\tau)R_A(\tau)\} \\ &= N_S(f)\otimes N_A(-f) + N_S(-f)\otimes N_A(f) \end{aligned} \tag{A-5}$$

where $\otimes$ is convolution operator.

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