Optics Communications 282 (2009) 1976–1983

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Equiangular-spiral bent lightpipes with arbitrary bent angle

Shu-Chun Chu [a,*], Yi-Kai Cheng [b], Jyh-Long Chern [b]

[a] Department of Physics, National Cheng Kung University, No. 1, University Road, Tainan City 701, Taiwan
[b] Department of Photonics, Institute of Electro-Optical Engineering, National Chiao Tung University, Hsinchu 300, Taiwan

ARTICLE INFO

Article history:
Received 12 August 2008
Received in revised form 17 November 2008
Accepted 5 February 2009

PACS:
42.15.–i
42.15.Eq

Keywords:
Lightpipe
Light pipe
Illumination
Optical design

ABSTRACT

A recent study proposed the concept of a leakage-free bent lightpipe with an equiangular-spiral shape [S.-C. Chu, J.-L. Chern, Opt. Lett. 30 (2005) 3006]. This paper extends the design of a leakage-free equiangular-spiral bent lightpipe with arbitrary-bend-angle and addresses in detail the mathematical formalism required to build such an equiangular-spiral bent lightpipe. Furthermore, a comparison of the proposed equiangular-spiral bent lightpipe and a conventional circular bent lightpipe is provided. Numerical verifications are discussed, and experimental explorations with different bent shapes and angles are carried out for comparison. Results show that equiangular-spiral bent lightpipes with different bent angles exhibit a theoretical 100% transmission with more than 76% efficiency when practically propagating, which is much better than conventional circular bent lightpipes. The output irradiance distributions of bent lightpipes with different bent shapes are also investigated.

1. Introduction

Lightpipes have the unique ability to transport light energy and change its direction of propagation with a high degree of efficiency. Lightpipe design can increase the illumination efficiency of an entire system, which is very critical for practical applications, and has become a useful design tool. Lightpipes are widely used in electro-optical applications, including projector engine illumination [1], liquid crystal panel backlight systems [2], automobile dashboards [3], headlights [4], etc. Lightpipe shapes vary depending on designer needs, and it is often necessary to bend them. Hence, lightpipe design is important for maintaining efficiency and the longstanding issue of avoiding light-leakage caused by bending a lightpipe.

Previous studies discussing the flux transmittance of lightpipes are available, but they focus primarily on conventional bent shapes [5–8]. Derlofske and Hough developed a flux confinement diagram model to discuss the flux propagation of square light pipes [5]. Gupta et al. developed an approach to determine light-leakage in circular bent lightpipes by analyzing rays in the principle section of the lightpipe [6]. Gupta et al. discussed the ratio of the ray acceptance angle to the bend ratio and the refractive index of a conventional circular bent lightpipe that bending once. They found that reducing the lightpipe bend ratio or increasing the refractive index causes an increase in the acceptance angle, i.e. reduces light-leakage. Our recent study [9] reports an equiangular-spiral bent lightpipe with a bend angle of *only 90°* that is capable of multiple bends without light-leakage or acceptance angle limitations. Practical applications for bent lightpipes often require that a lightpipe be bent in an arbitrarily *specified bend* angle without light-leakage.

This paper extends the bending scheme of a leakage-free bent lightpipe to *an arbitrary-bend-angle*. This paper also addresses in detail the mathematical formalism required to build such an equiangular-spiral bent lightpipe. To improve the implementation of the proposed equiangular-spiral bent lightpipe in common applications, this study also compares the transfer efficiency of the proposed equiangular-spiral bent lightpipe with a conventional circular bent lightpipe. Detailed simulations help illustrate and compare the light-leakage from both equiangular-spiral bent lightpipes and circular bent lightpipes. In addition, this study compares the transfer efficiencies of both kinds of lightpipes using numerical simulation and experimental data.

The paper is organized as follows: Section 2.1 summarizes the ray guidance concept of a leakage-free equiangular-spiral bent lightpipe. Section 2.2 provides detailed mathematical formulas for building a leakage-free equiangular-spiral bent lightpipe. Section 3.1 compares the simulations of an equiangular-spiral bent lightpipe and a circular bent lightpipe. Section 3.2 investigates the influence of multiple factors on equiangular-spiral bent lightpipe performance. Section 4 shows the experimental transfer efficiency measurements for both kinds of bent lightpipe. Section

* Corresponding author. Tel.: +886 6 275 7575; fax: +886 6 274 7995.
E-mail address: scchu@mail.ncku.edu.tw (S.-C. Chu).

0030-4018/$ - see front matter Crown Copyright © 2009 Published by Elsevier B.V. All rights reserved.
doi:10.1016/j.optcom.2009.02.012

5 discusses the irradiance distribution of equiangular-spiral and circular bent lightpipes. The final section presents the study's conclusions.

2. Formation of the leakage-free equiangular-spiral bent lightpipe

2.1. The basic concept of ray guiding

This section summarizes the basic requirements for guiding rays without light-leakage and the geometric form considerations of a leakage-free equiangular-spiral bent lightpipe. To avoid light-leakage caused by bending a lightpipe, recall that a lightpipe guides light by means of total internal reflection (TIR). Thus, to avoid any light-leakage, the guided ray incident angle at the lightpipe guiding surface should be greater than or equal to the total internal reflection angle, θ_c. Hence, the guided ray angular distribution inside the lightpipe is between $-\theta_c$ to θ_c. This condition focuses discussion on identifying the basic leakage-free bent lightpipe geometric unit, which connects to the end of the lightpipe with an angular distribution between $-\theta_c$ to θ_c and transfers the light to the other end of the lightpipe without light leakage. To avoid light-leakage, the bent lightpipe must have a geometric form in which all the guiding rays have incident angles on the bent surface greater than or equal to θ_c. This is the basic requirement of a leakage-free bent lightpipe.

The "principle section" concept of Gupta et al. [6] is used to derive the geometric form of a leakage-free bent lightpipe. Rays in the principal section determine the smallest incident ray angle at the bent surface [6]. The rays that strike the bent surface with smaller incident angle escape the lightpipe more easily. Rays that strike the outer bend surface from the inner bend point of the principle section are most critical to light-leakage [6]. These rays are also the most crucial rays to be guided in a bent lightpipe. To avoid critical ray leakage, the basic requirement of the outer surface of the a leakage-free bent lightpipe is that *the angle between the critical ray incident ray direction and the outer surface tangent should be greater than or equal to the value* $\pi/2 + \theta_c$. In addition to confining the outer surface of leakage-free bent lightpipe, the geometric form inner surface should not only guide all rays reflected from the bent lightpipe outer surface, but should also behave in such a way that rays reflected from the inner surface can be further guided by the outer surface. To allow all rays to be guided in a straight lightpipe infinitely without any light-leakage, the inner surface of the leakage-free bent lightpipe should behave similarly to the outer surface. Hence, all rays can be guided between the inner and outer surfaces without any light-leakage just as in a straight lightpipe.

2.2. Detailed formalization on equiangular-spiral bent lightpipe

This section presents in detail the geometric form of a proposed leakage-free equiangular-spiral bent lightpipe which can redirect light flux to an arbitrary direction. Fig. 1a illustrates the geometric form of an equiangular spiral. An equiangular spiral has the unique property that the included angle between the tangent $\vec{T}$ and radial line $\vec{r}$ is a fixed angle ϕ. This special geometric form could be a suitable solution for the geometric form of a no-loss lightpipe as illustrated in the previous paragraph. Fig. 1b illustrates the cross-section of the proposed leakage-free equiangular-spiral bent lightpipe. As Fig. 1b shows, except for the input and output lightpipe ports, a leakage-free bent lightpipe can be treated as a three part combination: straight surface *AB*, outer bent surface *BD*, and inner bent surface *OE*. The following section illustrates this geometric form part by part:

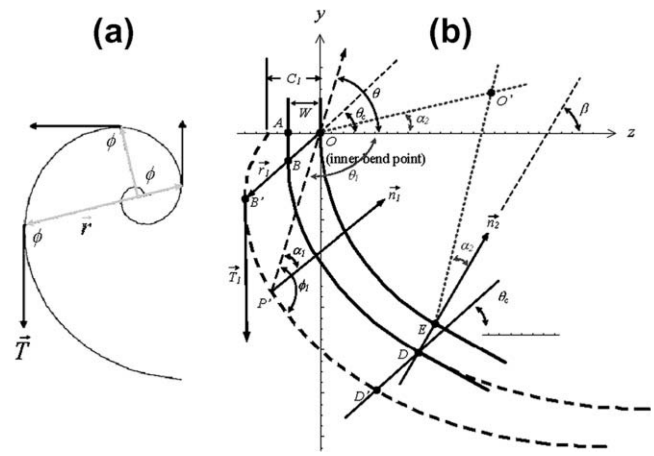

Fig. 1. Diagram of bending light in an equiangular-spiral shape. (a) Equiangular spiral; (b) cross-section of the proposed leakage-free equiangular-spiral bent lightpipe with an arbitrary-bend-angle β.

2.2.1. The geometric form of the outer surface

The outer surface contains two parts, straight surface *AB*, and outer bent surface *BD*. First, the surface *AB* is a straight surface where the included angle of $\overline{AO}$ and $\overline{BO}$ is the total internal reflection angle θ_c because all rays striking the straight surface from the inner bend point cannot escape in this portion as they are guided in a parallel lightpipe. Second, the outer bent surface of the leakage-free bent lightpipe is set as a shifted equiangular-spiral, the original equiangular-spiral surface $B'D'$ before shift is

$$r_1(\theta_1) = -C_1 e^{-\theta_1 \cot\phi_1}, \tag{1}$$

in which the polar origin position is situated at the origin of the Cartesian coordinate. The actual outer surface of equiangular-spiral bent lightpipe is BD, which is obtained by shifting the original equiangular-spiral surface B'D' by the vector $\mathbf{B'B}$, with the length $B'B = |r_1(\theta_c)| - W\sec\theta_c$, where W is the width of the lightpipe and $|r_1(\theta_c)|$ is OB'. The fixed tangential angle of equiangular-spiral, the included angle between the tangent $\vec{T_1}$ and radial line $\vec{r_1}$ of a equiangular spiral, is $\phi_1 = \pi/2 + \alpha_1$, and the angle α_1 is chosen as θ_c. Comparing the general mathematical form of the equiangular spiral $r_1(\theta) = C_1 e^{-\theta\cot\phi_1}$, the minus sign in Eq. (1) is introduced by a simple variable change $\theta_1 = \theta - \pi$, and the minus sign of Eq. (1) indicates that the counting of variable θ_1 starts from the third quadrant. C_1 is the length parameter, which is reformulated as $C_1 = mC_0$, where m is the multiple factor and C_0 is the minimum length parameter such that the original equiangular-spiral surface before shift can be connected and tangential to the straight surface *AB*, while no shifting is needed. In other words, when the length parameter is chosen as C_0, the length of $r(\theta_c)$ equals the hypotenuse *BO*. This means that C_0 satisfies the following condition:

$$C_0 e^{-\theta_c \cot\phi} = W\sec(\theta_c). \tag{2}$$

Note that the *multiple factor, m*, influences the size of the equiangular-spiral bent lightpipe. A larger equiangular-spiral bent lightpipe will have a larger multiple factor. The two reasons for choosing the outer surface as a *shifted* equiangular spiral of larger length parameter are:

(1) All critical rays are guided by this outer bent surface because all incident angles are equal to or greater than the original incident angle on an un-shifted equiangular-spiral surface (total internal reflection angle) to fulfill the basic requirement for the outer surface of a leakage-free bent lightpipe, and

(2) The incident angle of the ray reflected from the inner bent surface increases because choosing a shifted equiangular spiral eases restrictions in choosing the inner surface shape.

2.2.2. The geometric form of the inner surface

There are two considerations of geometric form for the inner bent surface. The first is the basic requirement of a leakage-free bent lightpipe that the inner surface geometric form should be similar to the outer surface so that all rays can be guided as in a parallel lightpipe. The other consideration is that the inner surface form should be chosen in such a way that the cross-section of the lightpipe end surface after bending will maintain the same size as before bending. To fulfill these two considerations, the inner bent surface *OE* is also chosen as a shifted equiangular-spiral surface which tangents to the input and output port of the bent lightpipe:

$$r_2(\theta_2) = -C_2 e^{-\theta_2 \cot\phi_2}. \tag{3}$$

Constructing a leakage-free bent lightpipe capable of bending at an arbitrary-bend-angle β requires four parameters of the inner bent surface of leakage-frees bent lightpipe which fulfill the above two considerations: the polar origin position of the inner equiangular spiral in the Cartesian coordinate $O'(Z_0, Y_0)$, the fixed tangential angle $\phi_2 = \pi/2 + \alpha_2$, and the length parameter C_2.

To determine the inner bent surface of a leakage-free bent lightpipe, first derive the output inner bent point *E* position that the inner bent surface is tangent to. Because point *E* is derived from two steps of position-shift from point D', determine the correspondence output outer bent point D' on the equiangular-spiral surface $B'D'$ before shifting to connect to the straight surface *AB*. The radial line $\vec{r_1}$ of any point on the outer equiangular-spiral surface $B'D'$ is represented in Cartesian coordinate as $\vec{r_1} = r_1(\theta_1)\cos\theta_1\hat{z} + r_1(\theta_1)\sin\theta_1\hat{y}$, and its tangent $\vec{T}_1$ is $\vec{T}_1 = d\vec{r_1}/d\theta_1 = [r_1'(\theta_1)\cos\theta_1 - r_1(\theta_1)\sin\theta_1]\hat{z} + [r_1'(\theta_1)\sin\theta_1 + r_1(\theta_1)\cos\theta_1]\hat{y}$, where $\hat{z}$ and $\hat{y}$ are the Cartesian coordinate unit vector. For a leakage-free bent lightpipe with a bend angle β, the included angle between the tangent $\vec{T}_1$ and the $\hat{z}$ at the output outer bend point D' will be $\pi/2 - \beta$, such that

$$\tan(\pi/2 - \beta) = -\left[\frac{r_1'(\theta_{D'})\sin\theta_{D'} + r_1(\theta_{D'})\cos\theta_{D'}}{r_1\prime(\theta_{D'})\cos\theta_{D'} - r_1(\theta_{D'})\sin\theta_{D'}}\right]. \tag{4}$$

Solving Eq. (4), the polar angle of the output outer bent point D' is

$$\theta_{D'} = \tan^{-1}\left[\frac{1 + \tan\alpha_1\cot\beta}{\cot\beta - \tan\alpha_1}\right], \tag{5}$$

such that the position of the output outer bent point D' is $(Z_{D'}, Y_{D'}) = [r_1(\theta_{D'})\cos\theta_{D'}, r_1(\theta_{D'})\sin\theta_{D'}]$. After determining the output position of the outer bend point D', the output position of the inner bend point *E* can be derived from two steps of position-shift from point D'. The first step shifts along an angle of elevation θ_c for a distance $l_1 = |r_1(\theta_c)| - W\sec(\theta_c)$, which provides the position of *D*. The second step shifts *D* along the normal surface direction $\vec{n_2}$ of the outer bent surface for a lightpipe width *W* distance to produce the output position of the inner bend point *E*. The two position-shift steps are presented as

$$\vec{r_E} = \vec{r_{D'}} + \Delta\vec{r}, \tag{6}$$

where $\Delta\vec{r} = (l\cos\theta_c + W\cos\beta, l\sin\theta_c + W\sin\beta)$. In other words, the position of point *E* is $(Z_{D'} + l\cos\theta_c + W\cos\beta, Y_{D'} + l\sin\theta_c + W\sin\beta)$.

After determining the output position of the inner bend point *E*, it is now possible to derive the exact inner bent surface form for a leakage-free bent lightpipe, which is a shifted equiangular-spiral surface that is tangent to both the input inner bend point *O* and the output inner bend point *E*. The polar position of the inner equiangular spiral O' is specified as (Z_0', Y_0') in the Cartesian coordinate, which can be easily derived because it is the intersection point of the two lines $\overline{OO'}$ and $\overline{EO'}$:

$$\begin{cases} Z_0' = [Y_E - Z_E\tan(\alpha_2 + \beta)]/[\tan\alpha_2 - \tan(\alpha_2 + \beta)] \\ Y_0' = Z_0\tan\alpha_2 \end{cases} \tag{7}$$

Applying the properties of the inner equiangular spiral

$$\begin{cases} \overline{OO'} = |r_2(\alpha_2)| \\ \overline{EO'} = |r_2(\beta + \alpha_2)| \end{cases} \tag{8}$$

and substituting Eq. (7) into Eq. (8) produces the following two equations,

$$2\beta\tan\alpha_2 = \ln[\overline{EO'}/\overline{OO'}], \tag{9}$$

and

$$C_2 = \overline{OO'}e^{-\alpha_2\tan\alpha_2}. \tag{10}$$

From three derived equations, it is possible to obtain the exact geometric form of the inner equiangular-spiral surface of a leakage-free bent lightpipe. Solve the combinations of Eqs. (7) and (9) to obtain the first three parameters of the inner equiangular spiral, the inner equiangular-spiral polar origin in the Cartesian coordinate $O'(Z_0', Y_0')$ and the fixed tangential angle $\phi_2 = \pi/2 + \alpha_2$; substituting the value α_2 into Eq. (10) produces the value C_2. With the origin position of the inner equiangular-spiral surface $O'(Z_0', Y_0')$, the fixed tangent angle ϕ_2, and the length parameter C_2, the explicit geometrical form of the inner bent surface is derived, creating a leakage-free equiangular-spiral bent lightpipe.

3. Simulation verification

3.1. Simulation model of arbitrary-bend-angle lightpipes

This section provides numerical verification of the leakage-free characteristic of an equiangular-spiral bent lightpipe with an *arbitrary-bend-angle*. These simulations compare the transfer efficiency of a leakage-free equiangular-spiral bent lightpipe with a conventional circular bent lightpipe in concerning practical applications. All the simulations in this paper were modeled by the commercial ray-tracing simulation package, TracePro (version 4.1.5) [12]. The geometrical profiles of equiangular-spiral bent lightpipes were created by Macro Scheme, a Macro commend language in TracePro.

The first simulations shows that an equiangular-spiral bent lightpipe with *any bend angle* β can guide light without any light-leakage. Fig. 2 demonstrates the light-flux of a point-like Lambertian source transmitting through leakage-free bent lightpipes with three typical bend angles of 30°, 60°, and 90°, where the multiple factor *m* of all equiangular-bent lightpipes shown in this section are all 2.0. Without loss of generality, all the simulations in this paper use a square size, point-like Lambertian source of 0.01×0.01 mm that emits 500 thousands rays with an angular distribution between −90° and 90°. The source was put near the lightpipe input surface with an air gap of 1 μm. The lightpipe examples used are made of acrylic material with an index of 1.49378 for the simulated wavelength 532 nm. A detector situated near the end of the bent lightpipe covered all of the exiting light-flux. Since the goal of this simulation is to illustrate the leakage-free properties of the equiangular-spiral bent lightpipe, the entrance and exit surfaces are set as a perfect transmitter and perfect absorber, respectively. In other words, the simulations in this study ignore Fresnel losses at both ends of the equiangular-spiral bent lightpipe. The crucial loss, or light-leakage, during light transference in the bent lightpipe is the only major concern here. Simulation results show that 500 thousand rays of a point-like Lambertian

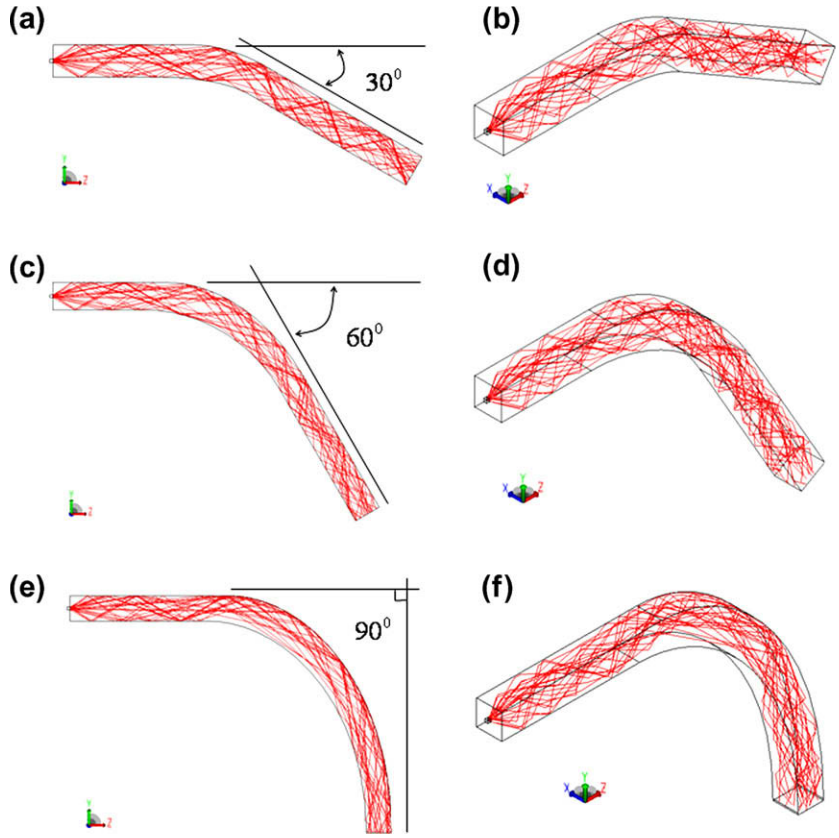

Fig. 2. The leakage-free equiangular-spiral bent lightpipe with a bend angle β of: (a) and (b) 30°; (c) and (d) 60°; (e) and (f) 90°.

source can all be guided in the equiangular-spiral bent lightpipe without any light-leakage. In Fig. 2, only 25 rays are arbitrarily selected to illustrate the guided ray path and leakage-free properties of the equiangular-spiral bent lightpipe. Note that there is no constraint on the bend angle β in showing the leakage-free properties of the equiangular-spiral bent lightpipe with any other bend angle β different from the bend angle shown in Fig. 2.

The second simulations models both an equiangular-spiral bent lightpipe and a conventional circular bent lightpipe to compare their practical transfer efficiencies. Fig. 3 illustrates the simulated models. Concerning practical transfer efficiency, both kinds of bent lightpipes are bent at the same bend angle β and use the same relative entrance and exit in simulations and experiments. As the inset formula of Fig. 3 shows, a conventional circular bent lightpipe is characterized by a bend ratio b that is defined as the ratio of the outer bend surface radius, R_{out}, to the inner bend surface radius, R_{in} [5]. In simulations, the cross-section of both the entrance and exit for both lightpipes share the same lateral square size of 9 mm × 9 mm. An acrylic lightpipe material with an index of 1.49378 is used for a simulated wavelength 532 nm. Fig. 3 shows that the side length L, which is the length from one lightpipe's end to the intersection point of the extension lines of the center of two bent lightpipe's ends, is 80 mm.

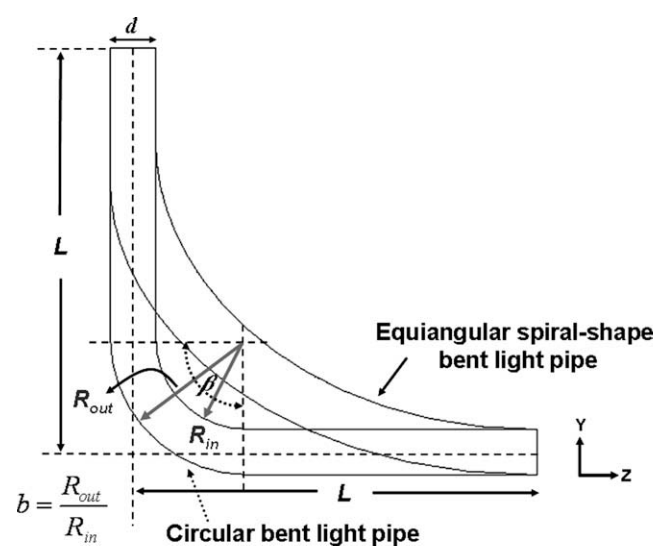

Fig. 3. Perspective diagram of the equiangular-spiral and circular bent lightpipe adopted in simulations and experiments.

The simulations in this study consider two categories of lightpipes bent at 45° and 90°. Table 1 shows the simulation transfer efficiencies for equiangular-spiral bent lightpipes and conventional circular bent lightpipes at three bend ratios, b = 1.2, 1.5, and 1.8. The inner overall sizes, $(\Delta Z, \Delta Y)$, of lightpipes that bent in 45° and 90° could be found in Tables 2 and 3 respectively, where the inner overall size of a lightpipe is described by the displacement from the inner bend point (O') to the inner output point (E). Two situations are considered: including and excluding Fresnel losses at both ends of the lightpipe. The first two rows of Table 1 show the transfer efficiencies of a bent lightpipe without considering Fresnel losses at the entrance or exit of the bent lightpipe. These results show the transfer efficiency of the lightpipes while serving as bent unit in an optical lightpipe system, i.e. the materials before and after the bent unit are the same as the lightpipe itself. These simulation results show that *the proposed equiangular-spiral bent lightpipe can arbitrarily redirect all rays without causing any light-leakage*, and further demonstrate that the transfer efficiency of a

Table 1
Simulation results for bent lightpipe transfer efficiency.

Bend angle	E-SBL (%)	CBL (%)		
		Bend ratio $b = 1.2$	Bend ratio $b = 1.5$	Bend ratio $b = 1.8$
45° (exclude F-L)	100	98.95	91.95	85.97
90° (exclude F-L)	100	99.99	90.13	74.76
45° (include F-L)	80.18	80.74	69.98	61.38
90° (include F-L)	81.41	81.79	74.08	63.47

F-L, E-SBL and CBL denote Fresnel loss, equiangular-spiral and circular bent lightpipe separately.

Table 2
Simulation results for bent lightpipe light-leakage (bend angle = 45°).

	E-SBL	CBL		
		Bend ratio $b = 1.2$	Bend ratio $b = 1.5$	Bend ratio $b = 1.8$
Overall size in mm ($\Delta Z, \Delta Y$)	(12.81,34.63)	(13.18,31.82)	(5.72,12.73)	(3.29,7.95)
Fresnel loss (%)	9.32	9.32	9.32	9.32
1st light-leakages (%)	0	0.95	7.30	12.72
2nd light-leakages (%)	10.50	8.99	13.40	16.57

E-SBL and CBL denote equiangular-spiral and circular bent lightpipe separately.

Table 3
Simulation results for bent lightpipe light-leakage (bend angle = 90°).

	E-SBL	CBL		
		Bend ratio $b = 1.2$	Bend ratio $b = 1.5$	Bend ratio $b = 1.8$
Overall size in mm ($\Delta Z, \Delta Y$)	(74.98,56.53)	(45.00,45.00)	(18.00,18.00)	(11.25,11.25)
Fresnel loss (%)	9.32	9.32	9.32	9.32
1st light-leakages (%)	0	0.01	8.95	22.89
2nd light-leakages (%)	9.27	8.88	7.65	4.32

E-SBL and CBL denote equiangular-spiral and circular bent lightpipe separately.

bent circular lightpipe depends on its bend ratio. A circular bent lightpipe with a higher bend ratio leads to more light-leakage.

The last two rows of Table 1 reflect the transfer efficiencies of lightpipe bent units being used alone, i.e. the material before and after the lightpipe bent unit is simply air. To compare the simulation results with experimental results, this step considers the Fresnel losses at all surfaces. Surprisingly, *the transfer efficiency of a bent lightpipe with a larger bend angle (90°) is better than a bent lightpipe with a smaller bend angle (45°)*. However, the area of lightpipe bending surface leading to light-leakage is larger in lightpipes with a larger bend angle. To find out why, we checked all leakage rays in detail. Tables 2 and 3 show the light-leakage levels for lightpipes with bend angles of 45° and 90°, respectively. In both tables, the Fresnel loss item represents the light-flux loss at the entrance of the bent lightpipe, and it is the same for all bent lightpipes. The "1st light-leakage" in Tables 2 and 3 means the light-leakage occurs while light transfers from the entrance to the exit. Simulation results show that the equiangular-spiral bent lightpipe design actually redirects light to an arbitrary direction without causing any light-leakage. The index change at the end lightpipe surface leads to light-reflection at the end surface of a bent lightpipe. This reflection results in secondary light-leakages, which are indicated by "2nd light-leakages" in Tables 2 and 3. The secondary *light leakage of lightpipes bent at 45° is larger than for lightpipes bent at 90°*.

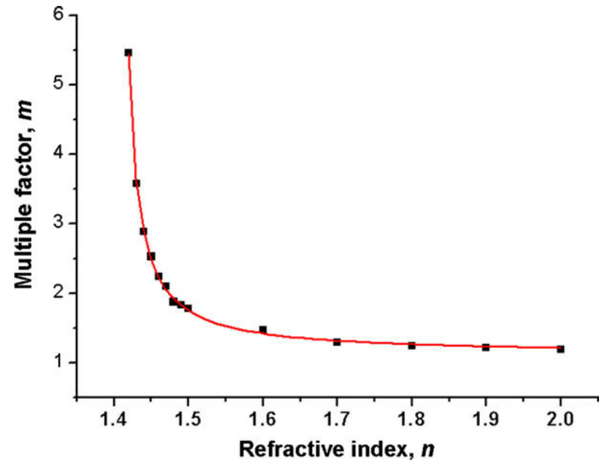

Fig. 4. The relationship curve of the smallest multiple factor m to the refractive index of the equiangular-spiral bent lightpipe.

This result suggests that using a lightpipe with a greater bend angle is a better choice for redirecting light-flux in air, and is applicable to applications in transferring solar energy and solar indoor lighting systems. It is also suggests that anti-reflection coatings on the input/output surfaces are necessary for increasing the transfer efficiency.

3.2. Influence of multiple factors on equiangular-spiral bent lightpipes

The value of the least *multiple factor m* of a leakage-free equiangular-spiral bent lightpipe is relevant to the lightpipe index. This paper uses simulations to determine the minimum *multiple factor m* of a leakage-free equiangular-spiral bent lightpipe at different refractive indices. Fig. 4 plots the relationship between the lightpipe index n and the *multiple factor m* of a leakage-free equiangular-spiral bent lightpipe. This curve shows that a leakage-free equiangular-spiral bent lightpipe with a higher refractive index can be constructed with a smaller *multiple factor m*. This is because a lightpipe with a higher refractive index has a smaller total internal reflection angle and can be constructed more compactly. The relationship curve of the least *multiple factor* m to the index of a leakage-free equiangular-spiral bent lightpipe is a hyperbola. Fig. 5 gives an example of an equiangular-spiral bent lightpipe with

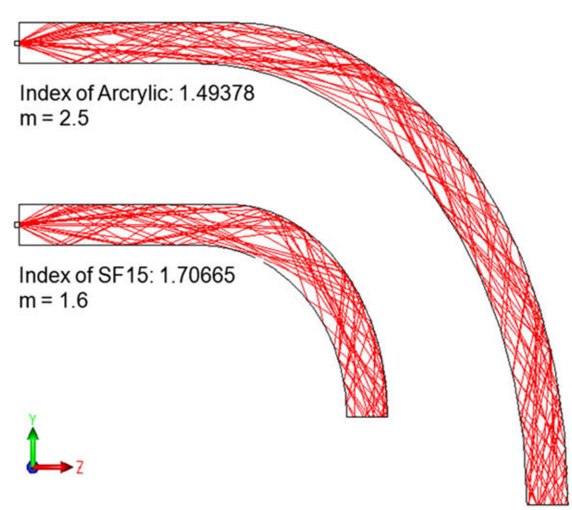

Fig. 5. Two leakage-free equiangular-spiral bent lightpipes made of different materials: acrylic and SF15.

a higher refractive index but smaller geometric size. A lightpipe made of acrylic, where the index for the simulated wavelength 532 nm is 1.49378, enables a leakage-free bent lightpipe with a larger geometric size and a larger *multiple factor* of $m = 2.5$. A lightpipe made of SF15 glass material, where the index for the simulated wavelength 532 nm is 1.70665, provides a leakage-free bent lightpipe with a smaller geometric size. Its *multiple factor* $m = 1.6$ is also smaller, as Fig. 5 demonstrates.

4. Experimental explorations

In this section, experiments are implemented to measure the performance of two kinds of bent lightpipes in practical situations. This study created equiangular-spiral bent lightpipes with a multiple factor m of 2.0 and circular bent lightpipes of rectangular cross-section with bend ratios of $b = 1.2, 1.5$, and 1.8 bent at 45° and 90° using computer numerical control (CNC) machining. Fig. 4 shows the experimental setup for measuring the lightpipe flux-transferring efficiency of the bent lightpipe. As Fig. 6 shows, a point-like Lambertian source was constructed by passing a collimated green laser beam though a diffuser. The laser beam has a 532 nm wavelength and a beam spot size of 1 mm. The inset figure in the lower left-hand corner of the Fig. 6 depicts an equiangular-spiral bent lightpipe in a darkroom with a point-like Lambertian source passing through it. While measuring the light-flux passing through the bent lightpipe, the entrance end of the bent lightpipe nearly touches the diffuser. The end surface of the bent lightpipe simultaneously connects to one port of the integral sphere to collect all exiting light-flux from the bent lightpipe. The other port of the integral sphere connects to the detector to measure the exiting light-flux. When measuring the entering light-flux of the point source, the bent lightpipe was removed from the integral sphere port and the port was directly connected to the diffuser. The transfer efficiency of the bent lightpipe is defined as the ratio of exiting light-flux to entering light-flux. Table 4 shows the experimental light-transferring efficiencies for both kinds of bent lightpipes bent at 45° and 90°. The

Table 4
Experimental results for bent lightpipe transfer efficiency.

Bend angle	E-SBL (%)	CBL (%)		
		Bend ratio b = 1.2	Bend ratio b = 1.5	Bend ratio b = 1.8
45°	77.55	75.63	66.45	54.25
90°	76.24	73.42	67.93	57.17

E-SBL and CBL denote Fresnel loss, equiangular-spiral and circular bent lightpipe separately.

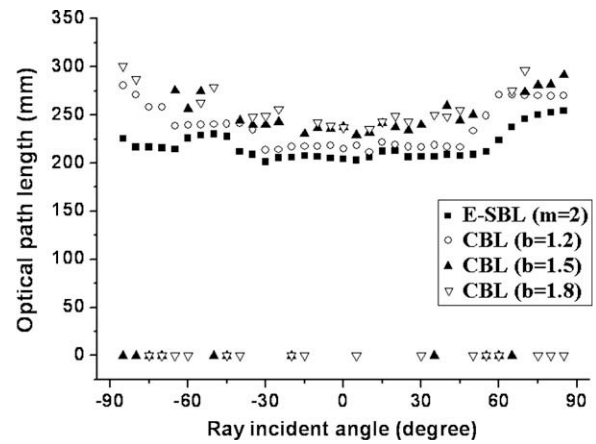

Fig. 7. The relation of the optical path length to the rays in the principle plane (Z–Y plane) of the equiangular-spiral bent lightpipe (E-SBL) and circular bent lightpipe (CBL) with bend angle of 90°, with the ray incident angles ranges from −85° to 85° in 35 steps, where the OPL of leakage ray in the figure is defined as zero.

experiments confirm that *the transfer efficiencies of equiangular-spiral bent lightpipes are all better than conventional circular bent lightpipes.* However, the experimental results show that for lightpipes with a larger bending surface (the equiangular-spiral bent lightpipe

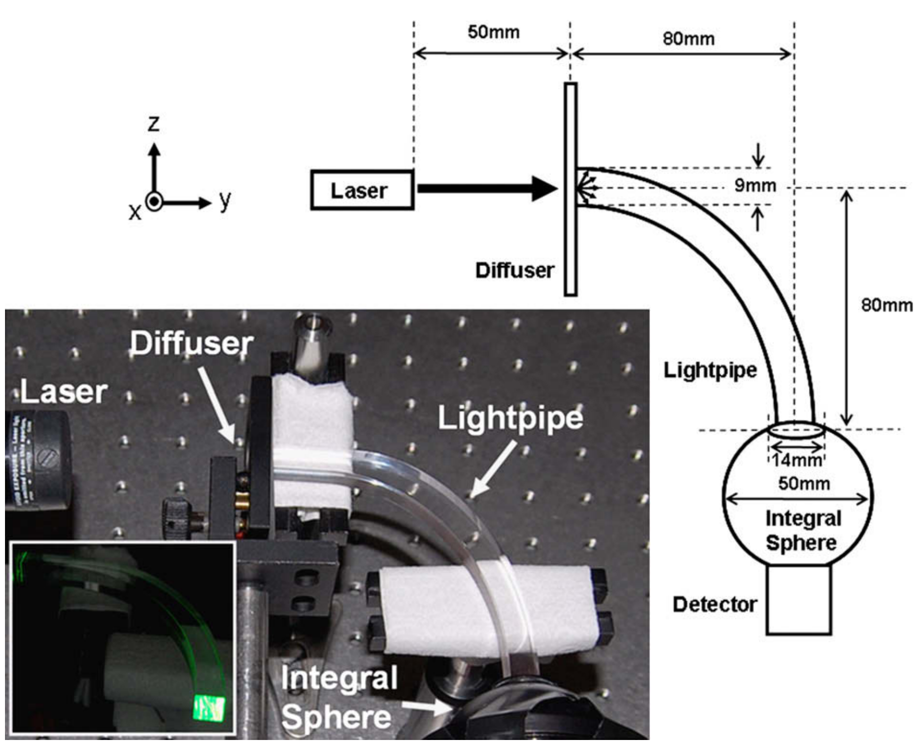

Fig. 6. Experimental setup for measuring the transfer efficiencies of lightpipe bent units.

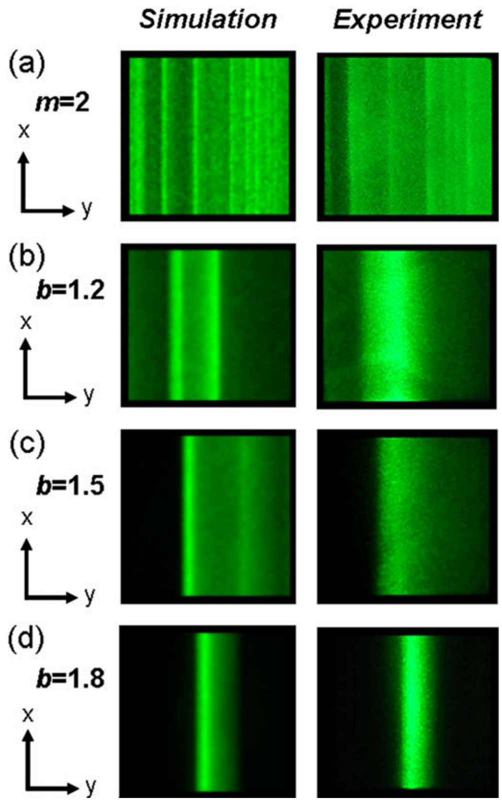

Fig. 8. Simulation and experimental results of irradiance distributions for 90° bent light pipes with different bending methods: (a) equiangular-spiral multiple factor $m = 2$; (b) circular bend ratio $b = 1.2$; (c) $b = 1.5$; (d) $b = 1.8$.

and circular bent lightpipe with a bend ration of $b = 1.2$), the transfer efficiencies of 45° bent lightpipes are higher than 90° bent lightpipes. These experimental results conflict with the previous simulation results, and are affected by light-scattering loss from CNC manufacturing roughness at the bend surface. Thus, in the two kinds of bent lightpipes (an equiangular-spiral bent lightpipe and a circular bent lightpipe with a bend ratio of $b = 1.2$), the bent lightpipe with a larger bend angle (90°) and larger bending surface had a higher light-flux loss in these experiments. Note that all experimental transfer efficiency values are 5–8% smaller than the simulated transfer efficiency values. This is because the simulations do not consider unknown material absorption, and absorption levels vary depending on the composite in use. However, comparing Table 4 to the last two rows of Table 1 reveals that an equiangular-spiral bent lightpipe exhibits a smaller difference between the experimental and simulation results. This suggests that an equiangular-spiral bent lightpipe guides rays through a shorter optical path than a circular bent lightpipe, which in turns leads to lower absorption loss in the bent unit. To confirm this supposition, we check the optical path length (OPL) of the rays for both kinds of lightpipes. Fig. 7 shows the relation of the OPL to the rays in the principle plane (Z–Y plane) of the four lightpipes with bend angle of 90°, with the ray incident angles ranges from −85° to 85° in 35 steps. Note that the OPL of leakage ray in the figure is defined as zero. Fig. 7 shows that except leakage rays, an equiangular-spiral bent lightpipe actually guides the rays through a shorter optical path than that of a circular bent lightpipe did, which in turns leads to lower absorption loss in the bent unit. Note that another simulation result of bent lightpipes with bend angle of 45° also shows the equiangular-spiral bent lightpipe actually guides the rays through a shorter optical path than that of circular bent lightpipes did, which in turns leads to lower absorption loss in the bent unit.

5. Irradiance distribution

The irradiance distributions of straight light pipes with square apertures are uniform, regardless of whether they use hollow or dielectric-filled material [10,11]. This section investigates the irradiance distributions of equiangular-spiral and circular bent light pipes. Previously, Gupta et al. analyzed the angular and spatial distributions for square and circular cross-section light pipes, including a 90° circular bending [6]. Note that the bending method of Gupta et al. differs from the equiangular-spiral bent light pipe in this study; hence, the analysis shown here offers an alternative illustration. Square aperture light pipes with 90° bend angles, an equiangular-spiral multiple factor of $m = 2$, and different circular bend ratios of $b = 1.2, 1.5$, and 1.8 serve as illustrations. An acrylic

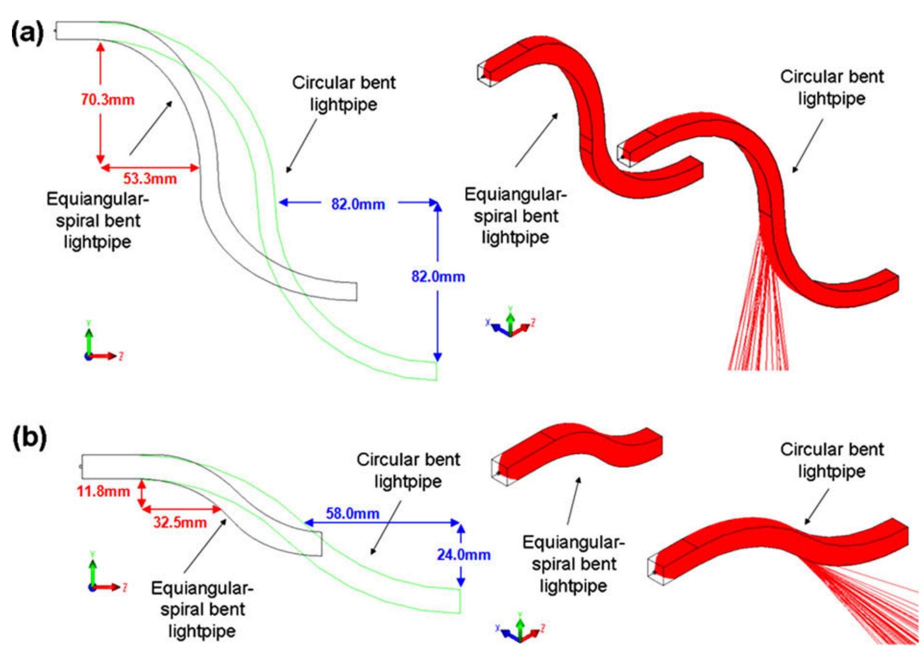

Fig. 9. Simulation results for a twice-bent equiangular-spiral bent lightpipe and circular bent lightpipe with a bend angle of: (a) 45° and (b) 90°.

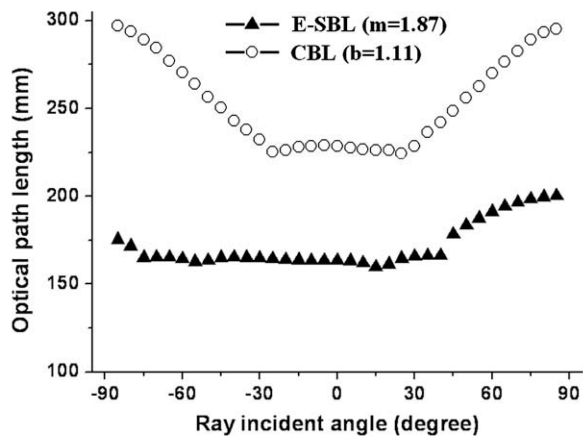

Fig. 10. The ray optical path length in the principle plane (*Z–Y* plane) of leakage-free equiangular-spiral bent lightpipe (E-SBL) and leakage-free circular bent lightpipe (CBL).

material and a wavelength of 532 nm were selected. The point-like Lambertian source was placed in front of the lightpipe entrance and the extension on both ends of bent light pipe as Fig. 3 shows was retracted. Fig. 8 shows the simulation result of irradiance distributions for the light pipe output. These results show that uniform characteristics appear in the vertical *x*-direction, like the straight light pipes in Refs. [10,11] . In the horizontal *y*-direction, however, bending the lightpipe brings on flux localizations. This study also provides experimental comparisons for further investigation. Fig. 6 indicates that the integral sphere and detector in the light pipe output were replaced by a diffusive sheet that served as an image plane. All measurements were taken in a dark room to prevent ambient light interference. The output image from the diffusive sheet was captured by a commercial digital camera, Nikon model D40 [13]. Fig. 8 shows these images, and the flux localization characteristics that appeared in the horizontal *y*-direction. The experiment and numerical exploration are clearly in good agreement. Note that though bending lightpipes brings on flux localizations in the horizontal *y*-direction, better horizontal irradiance distribution uniformity in the exiting surface is provided by equiangular-spiral bent lightpipe.

6. Conclusions

In conclusion, this paper shows how to construct a novel leakage-free equiangular-spiral bent lightpipe with an arbitrary-bend-angle. Such a lightpipe bent unit is capable of arbitrary-bend-angles and can be adopted in many optical applications, including light-splitting elements, light-mixing elements, solar-light redirection systems, and several other designs.

As seen in Table 1, a circular bent lightpipe with a smaller bend ratio produces less light-leakage. It is worth noting that one could use larger circular bent lightpipes with smaller bent ratios to achieve leakage-free circular bent lightpipes. Therefore, it would be meaningful to compare both the overall size and performance of two kinds of leakage-free bent lightpipes. The maximum bend ratio of a leakage-free circular bent lightpipe can be solved following Gupta et al. [6]. Fig. 9 shows the inner overall size and compare the simulation results of two kinds of smallest leakage-free bent lightpipes that bend twice, with bend angles of 45° and 90° in (a) and (b), respectively, where the multiple factors *m* of equiangular-spiral bent lightpipes are 1.87. The proposed leakage-free equiangular-spiral bent lightpipes are better than leakage-free circular bent lightpipes in three ways. First, when two kinds of leakage-free bent lightpipes bent in same angle, the proposed equiangular-spiral is smaller than that of circular bent lightpipe. Secondly, the proposed leakage-free equiangular-spiral bent lightpipe has the capability to bend multiple times with arbitrary bent angle without causing light-leakage. Besides, we also have checked the optical path lengths of both kinds of leakage-free bent lightpipes, as shown in Fig. 10. Leakage-free equiangular-spiral bent lightpipes guides the rays through a shorter optical path than what Leakage-free circular bent lightpipes did, which in turns leads to lower absorption loss in the bent unit.

The current bending scheme does have some limitations, which are outlined here. First, this study chose a rectangular cross-section for the proposed leakage-free bent lightpipe. This choice greatly simplified analysis. The refractive index of the lightpipe materials must be larger than $\sqrt{2}$ (which is true for most transparent materials), ensuring that rays distributed between $-\theta_c$ and θ_c will be guided in two parallel planes without light-leakage when the ray incident angles are greater than the critical angles at the parallel surface. On the other hand, a rectangular cross-section is easier to connect in a perpendicular direction. A different form of cross-section form, such as a polygon, requires significantly more effort to establish bending rules without light-leakage.

Acknowledgements

This work was supported in part by a grant from the National Science Council of Taiwan, R.O.C., under Contract No. NSC 96-2112-M-006-019-MY3. We also thank Lambda Research Corporation for their support in providing the simulation package TracePro.

References

[1] K.K. Li, Illumination engine for a projection display using a tapered light pipe, US Patent 67,39,726, May 25, 2005.
[2] N. Takahashi, S. Umemoto, Liquid crystal display apparatus having light pipe with reflective polarizer, US Patent 67,78,235, August 17, 2004.
[3] J. Lee, J.E. Greivenkamp, Opt. Eng. 43 (2004) 1537.
[4] H. Delattre, Motor vehicle headlight with light pipe, US Patent 65,47,428, April 15, 2003.
[5] J.F.V. Derlofske, T.A. Hough, Opt. Eng. 43 (7) (2004) 1503.
[6] A. Gupta, J. Lee, R.J. Koshel, Appl. Opt. 40 (2001) 3640.
[7] D.G. Hawthorn, T. Timusk, Appl. Opt. 36 (1999) 2787.
[8] E. Fu, J. Opt. Soc. Am. B 13 (1996) 702.
[9] S.-C. Chu, J.-L. Chern, Opt. Lett. 30 (2005) 3006.
[10] Y.-K. Cheng, J.-L. Chern, J. Opt. Soc. Am. A 23 (2006) 427.
[11] Y.-K. Cheng, M.-H. Wang, J.-L. Chern, J. Opt. Soc. Am. A 24 (2007) 2748.
[12] see http://lambdares.com for technical information.
[13] see http://imaging.nikon.com/products/imaging/lineup/digitalcamera/slr/d40/index.htm.