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Physica C 275 (1997) 135–140

PHYSICA C

Interaction between a cylindrical superconducting impurity and a vortex in a type-II superconductor

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Received 9 October 1996; revised manuscript received 4 December 1996

Abstract

We examined the effectiveness of an infinite cylindrical superconducting impurity of radius a in an infinite type-II superconductor as a pinning center. The impurity and the superconducting background have the penetration depth λ_i and λ_s, respectively. With regard to a single-quantum vortex, the free energy of the system and the pinning potential for a vortex were calculated, based upon the London model of the vortex line. Here, the pinning potential U_{pin} is defined by the difference between the free energy of a vortex being at the center of the impurity and that being infinitely distant from the impurity. U_{pin} is negative for $\lambda_i > \lambda_s$. On the other hand, U_{pin} is positive for $\lambda_i < \lambda_s$. In the limit of $a \gg \lambda_i$ and λ_s, the pinning potential is found to be $U_{pin} = \phi_0^2(1/\lambda_i^2 - 1/\lambda_s^2) \ln \kappa/4\pi\mu_0$. In the limit of $a \ll \lambda_i$ and λ_s, the pinning potential is found to be $U_{pin} = \phi_0^2[(1/\lambda_i^2 - 1/\lambda_{ss}^2) \ln \kappa + (1/\lambda_{ss}^2) \ln(\lambda_s/a) - (1/\lambda_i^2) \ln(\lambda_i/a)]/4\pi\mu_0$. There is a clear correlation between the intrinsic property of these two superconducting materials and the pinning potential.

1. Introduction

For technical applications of superconductors, investigating the pinning properties for vortices is one of the most important problems. The properties of pinning centers strongly affect a number of electrical and magnetic behaviors of the superconductors. Many experimental trials have been done to introduce the pinning centers into high-T_c superconductors [1–6]. These pinning materials were modeled in analogy to defects or insulating precipitates. These nonsuperconducting defects which interact with vortices would induce the screening currents and lead to the so-called electromagnetic pinning [7].

For the purpose of understanding theoretically the pinning nature for bulk type-II superconductors, the simple case of a single straight vortex in the presence of a cylindrical defect has been calculated [8] in the London limit $\kappa \gg 1$. The theory of Ref. [8] has been generalized to a periodic structure of columnar defects [9]. The numerical results of the Ginzburg-Landau equations for various values of κ in type-II superconductors with a columnar defect have been obtained by Takezawa and Fukushima [10]. Buzdin et al. [7,11] have analyzed the interaction of a vortex with a cylindrical defect of radius a by employing the image method. They have derived the interaction

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PII S0921-4534(96)00717-4

energy $U_{int}(r) \propto \ln(1 - a^2/r^2)$ at short distances, both in superconducting thin film and bulk material. Chen [12] has obtained a general expression of the interaction energy and the pinning potential for a vortex interacting with a circular defect in a thin film. His results do not only verify the results of Buzdin et al., obtained by considering the interaction at short distances but also predict that the vortex-defect interaction energy decrease with r^{-4} at large distances; this is in contrast to the exponential decrease in the case of a vortex interacting with a cylindrical defect in a bulk superconductor. If, however, the pinning centers are not defects but instead superconducting materials, the pinning of the vortex could be an issue of concern.

In this paper we consider an infinite cylindrical type-II superconducting impurity of radius a in an infinite type-II isotropic superconductor, and a vortex parallel to this cylindrical impurity. We investigate the interaction between a vortex and a superconducting impurity inside a superconductor based on the London theory. We calculate the free energy of this system and the dependence of the pinning potential on the size of the impurity pinning center. In general we shall examine certain limiting cases in which progress can be made analytically.

2. Calculation of the free energy

2.1. Vortex located outside the superconducting impurity

We choose cylindrical coordinates (ρ, θ, z) with the z-axis parallel to the central axis of the cylindrical superconducting impurity, and consider a vortex parallel to the z-axis and located at a distance r from the center of the impurity, where $r > a$, as shown in Fig. 1. The London equation can be expressed as

$$B^2 + \lambda_s^2 \nabla \times \nabla \times B = \phi_0 \delta(\rho - r) \hat{e}_z, \quad \rho > a, \quad (1)$$

$$B^2 + \lambda_i^2 \nabla \times \nabla \times B = 0, \quad \rho < a. \quad (2)$$

The effective depth to which the applied field penetrates into the ideally isotropic superconductor is λ_s and the superconducting impurity is λ_i. ϕ_0 is the flux quantum and $\delta(\rho - r)$ is the two-dimensional delta function.

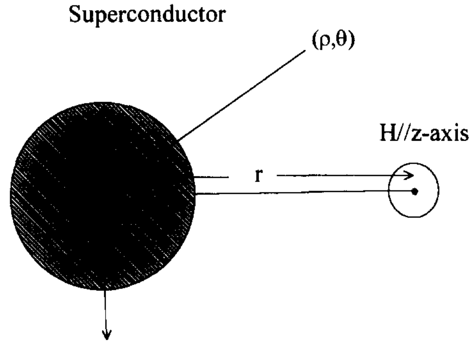

Fig. 1. A cylindrical superconducting impurity with radius a and infinite length embedded in an isotropic superconductor. We choose a cylindrical coordinate system whose z-axis is parallel to the central axis of the impurity. A vortex line exists parallel to this impurity and is located at a distance r from it. These two superconductors have penetration depths λ_i and λ_s.

The magnetic field $B = B\hat{e}_z$ can be written in the form

$$B = \begin{cases} B_s + B_{out}, & \rho > a, \\ B_{in}, & \rho < a, \end{cases} \quad (3)$$

where B_s is the particular solution of Eq. (1) (i.e., directly contributed from the vortex line), B_{out} and B_{in} satisfy, respectively, the homogeneous solutions of Eq. (1) and Eq. (2). The solutions of Eqs. (1) and (2) are

$$B_s = \left(\frac{\phi_0}{2\pi\lambda_s^2}\right) K_0(|\rho - r|/\lambda_s),$$
$$= \left(\frac{\phi_0}{2\pi\lambda_s^2}\right) \sum_{m=-\infty}^{\infty} I_m(r_</\lambda_s) K_m(r_>/\lambda_s) e^{im\theta}, \quad (4)$$

where $r_> = \max(\rho, r)$, $r_< = \min(\rho, r)$, meaning, respectively, the smaller or larger ρ and r,

$$B_{in} = \left(\frac{\phi_0}{2\pi\lambda_i^2}\right) \sum_{m=-\infty}^{\infty} A_m(r) I_m(\rho/\lambda_i) e^{im\theta}, \quad (5)$$

$$B_{out} = \left(\frac{\phi_0}{2\pi\lambda_s^2}\right) \sum_{m=-\infty}^{\infty} B_m(r) K_m(\rho/\lambda_s) e^{im\theta}, \quad (6)$$

where both I_m and K_m are called the modified Bessel functions of order m. To determine the coefficients $A_m(r)$ and $B_m(r)$, the following boundary conditions should be met: (i) the magnetic field $\boldsymbol{B}$ is continuous at $\rho = a$, (ii) the tangential component of $\lambda^2(\nabla\times \boldsymbol{B})$ is continuous at $\rho = a$, i.e., ensuring the continuity of supercurrents across the interface between these two superconductors. From Eqs. (4)–(6) and the boundary conditions, we have

$$\begin{aligned} I_m(a/\lambda_i)A_m &= I_m(a/\lambda_s)K_m(r/\lambda_s) + K_m(a/\lambda_s)B_m, \\ \frac{\lambda_i}{\lambda_s}I'_m(a/\lambda_i)A_m &= I'_m(a/\lambda_s)K_m(r/\lambda_s) + K'_m(a/\lambda_s)B_m, \end{aligned} \tag{7}$$

where I'_m (K'_m) is the first derivative of I_m (K_m) with respect to its own argument. The coefficients are obtained immediately:

$$A_m(r) = \frac{(\lambda_s/a)K_m(r/\lambda_s)}{(\lambda_i/\lambda_s)I'_m(a/\lambda_i)K_m(a/\lambda_s) - I_m(a/\lambda_i)K'_m(a/\lambda_s)}, \tag{8}$$

$$B_m(r) = \frac{I_m(a/\lambda_i)I'_m(a/\lambda_s) - (\lambda_i/\lambda_s)I'_m(a/\lambda_i)I_m(a/\lambda_s)}{(\lambda_i/\lambda_s)I'_m(a/\lambda_i)K_m(a/\lambda_s) - I_m(a/\lambda_i)K'_m(a/\lambda_s)} \times K_m(r/\lambda_s). \tag{9}$$

Once the fields are determined, then the free energy is given by the formula

$$F(r) = \frac{1}{2\mu_0}\int (B^2 + \lambda^2|\nabla\times\boldsymbol{B}|^2)\,dv. \tag{10}$$

The integral is taken over the sample volume except for the core region of the vortex line, which is excluded. The free energy per unit length of the vortex located outside the impurity, $F_{out}(r)$, is expressed as

$$F_{out}(r) = \frac{1}{4\pi\mu_0}\left(\frac{\phi_0}{\lambda_s}\right)^2 \left(K_0(\xi_s/\lambda_s) + \sum_{m=-\infty}^{\infty} B_m K_m(r/\lambda_s)\right), \quad r > a, \tag{11}$$

where ξ_s is the effective coherence length of the isotropic superconductor. The first term in Eq. (11) represents the self-energy of the single vortex in this isotropic superconductor. The second term in Eq. (11) represents the interaction energy between the vortex and the superconducting impurity.

2.2. Vortex located inside the superconducting impurity

Now we deal with the vortex located inside the superconducting impurity, i.e., $r < a$. The London equation of the system in question can be expressed as

$$\boldsymbol{B} + \lambda_s^2\nabla\times\nabla\times\boldsymbol{B} = 0, \quad \rho > a, \tag{12}$$

$$\boldsymbol{B} + \lambda_i^2\nabla\times\nabla\times\boldsymbol{B} = \phi_0\delta(\boldsymbol{\rho} - \boldsymbol{r})\hat{e}_z, \quad \rho < a. \tag{13}$$

We follow the same procedures as we did in the previous case. The solution can be written as

$$B = \begin{cases} B_{out}, & \rho > a, \\ B_s + B_{in}, & \rho < a. \end{cases} \tag{14}$$

The solutions of the equations are

$$B_s = \left(\frac{\phi_0}{2\pi\lambda_i^2}\right)\sum_{m=-\infty}^{\infty} I_m(r_</\lambda_i)K_m(r_>/\lambda_i)e^{im\theta}, \tag{15}$$

$$B_{in} = \left(\frac{\phi_0}{2\pi\lambda_i^2}\right)\sum_{m=-\infty}^{\infty} C_m(r)I_m(\rho/\lambda_i)e^{im\theta}, \tag{16}$$

$$B_{out} = \left(\frac{\phi_0}{2\pi\lambda_s^2}\right)\sum_{m=-\infty}^{\infty} D_m(r)K_m(\rho/\lambda_s)e^{im\theta}. \tag{17}$$

In the present case the boundary conditions imposed to Eqs. (15)–(17) yield

$$C_m(r) = \frac{(\lambda_s/\lambda_i)K_m(a/\lambda_i)K'_m(a/\lambda_s) - K'_m(a/\lambda_i)K_m(a/\lambda_s)}{K_m(a/\lambda_s)I'_m(a/\lambda_i) - (\lambda_s/\lambda_i)K'_m(a/\lambda_s)I_m(a/\lambda_i)} \times I_m(r/\lambda_i), \tag{18}$$

$$D_m(r) = \frac{(\lambda_i/a)I_m(r/\lambda_s)}{K_m(a/\lambda_s)I'_m(a/\lambda_i) - (\lambda_s/\lambda_i)K'_m(a/\lambda_s)I_m(a/\lambda_i)}. \tag{19}$$

The free energy (per unit length) of the single vortex located inside the superconducting impurity, $F_{in}(r)$, is expressed as

$$F_{in}(r) = \frac{1}{4\pi\mu_0}(\phi_0/\lambda_i)^2 \times \left(K_0(\xi_i/\lambda_i) + \sum_{m=-\infty}^{\infty} C_m I_m(r/\lambda_i) \right), \quad r < a, \tag{20}$$

where ξ_s is the effective coherence length of the impurity. The first term in Eq. (20) represents the self-energy of the single vortex in this superconducting impurity.

3. Calculation of the pinning potential

The pinning potential per unit length for a vortex, $U_{pin}(a)$, is defined by

$$U_{pin}(a) = F_{in}(0) - F_{out}(\infty), \tag{21}$$

where $F_{in}(0)$ is the free energy per unit length of the vortex at the center of the superconducting impurity, and $F_{out}(\infty)$ is that of the vortex infinitely distant from the impurity. Substituting Eqs. (11) and (20) into Eq. (21) leads to the following form for $U_{pin}(a)$,

$$U_{pin}(a) = \frac{\phi_0^2}{4\pi\mu_0}\left[\left(\frac{1}{\lambda_i^2} - \frac{1}{gl_s^2}\right)\ln\kappa + \frac{1}{\lambda_i^2} \times \frac{K_1(a/\lambda_i)K_0(a/\lambda_s) - (\lambda_s/\lambda_i)K_0(a/\lambda_i)K_1(a/\lambda_s)}{K_0(a/\lambda_s)I_1(a/\lambda_i) + (\lambda_s/\lambda_i)K_1(a/\lambda_s)I_0(a/\lambda_i)}\right]. \tag{22}$$

For simplicity of calculation and analysis, we have set $K_0(\xi_i/\lambda_i) = K_0(\xi_s/\lambda_s) \propto \ln\kappa$. The motivation for this assumption comes from the study of the phase diagram of the YBCO system. For instance, $YBa_2Cu_4O_8$(124) has been identified as intergrowths in $YBa_2Cu_3O_7$(123), produced in thin films [13,14], and subsequently synthesized in bulk as a majority phase [15]. From the experimental measurements of the in-plane London penetration depth [16–18] $\lambda_{ab}(0)$ and the coherence length [16,19] $\xi_{ab}(0)$ of Y-123 and Y-124, the ratio of $\lambda_{ab}(0)/\xi_{ab}(0) \approx 100$ was obtained for both superconductors.

We consider first the case, a/λ_s, $a/\lambda_i \gg 1$, then Eq. (22) can be reduced to

$$U_{pin}(a) \cong \frac{\phi_0^2}{4\pi\mu_0}\left(\frac{1}{\lambda_i^2} \frac{-1}{\lambda_s^2}\right)\ln\kappa. \tag{23}$$

In another limiting case, a/λ_s, $a/\lambda_i \ll 1$, Eq. (22) can be reduced to

$$U_{pin}(a) \cong \frac{\phi_0^2}{4\pi\mu_0}\left[\left(\frac{1}{\lambda_i^2} - \frac{1}{\lambda_s^2}\right)\ln\kappa + \frac{1}{\lambda_s^2}\ln(\lambda_s/a) - \frac{1}{\lambda_i^2}\ln(\lambda_s/a)\right]. \tag{24}$$

4. Discussion and summary

We have derived the free energy of a vortex in an isotropic superconductor interacting with a cylindrical superconducting impurity of radius a, its axis parallel to the vortex, as expressed in Eq. (11) and Eq. (20). The second term in parentheses on the right hand side of both Eq. (11) and Eq. (20) represents the interaction energy between the vortex and the superconducting pinning center. By definition, $K_m(x)$ tends to zero as x becomes large. Thus for the interaction energy to be small and negligible, the vortex must be located at a large distance from the pinning center. In this situation, the vortex may be considered as an isolated vortex; so only the self-energy of the single vortex remains in Eq. (11). Moreover, $I_m(x)$ is bounded as x when small in any bounded domain of x. In Eq. (20), the behavior of the interaction energy depends on the radius of the impurity and the location of the vortex inside the impurity. For certain limiting cases, one can get the interaction behavior in an analytical form. Consider the limiting case of a/λ_s and $a/\lambda_i \gg 1$. Then the asymptotic forms of the sum term (or interaction energy) in Eqs. (11) and (20) can be reduced to

$$\sum_{m=-\infty}^{\infty} B_m K_m(r/\lambda_s) \cong \left(\frac{\lambda_s - \lambda_i}{\lambda_s + \lambda_i}\right)\sqrt{\frac{a}{r}}\, e^{-(r-a)/\lambda_s} K_0\left(\frac{r-a}{\lambda_s}\right), \quad r \gg \lambda_s, \tag{25}$$

$$\sum_{m=-\infty}^{\infty} C_m I_m(r/\lambda_i) \cong \left(\frac{\lambda_s - \lambda_i}{\lambda_s + \lambda_i}\right)\sqrt{\frac{a}{r}}\, e^{-(r-a)/\lambda_i} K_0\left(\frac{r-a}{\lambda_i}\right), \quad r \gg \lambda_i. \tag{26}$$

The exponential part controls the main behavior of the interaction between the vortex and the impurity, except for near the interface between the impurity and the superconducting background. It means that the interaction energy is small when this vortex is far from the interface for a large radius of the impurity. In another extreme case of a/λ_s and $a/\lambda_i \ll 1$, it is easy to show that

$$\sum_{m=-\infty}^{\infty} B_m K_m(r/\lambda_s) \cong \frac{\lambda_s(\lambda_s-\lambda_i)}{\lambda_s^2+\lambda_i^2}\ln\left(1-\frac{a^2}{r^2}\right) + \frac{a^4}{16\lambda_s^2}\left(\frac{1}{\lambda_i^2}-\frac{1}{\lambda_s^2}\right)K_0^2(r/\lambda_s),$$

$$r \ll \lambda_s, \tag{27}$$

$$\sum_{m=-\infty}^{\infty} C_m I_m(r/\lambda_i) \cong \frac{\lambda_s^2-\lambda_i^2}{\lambda_s^2+\lambda_i^2}\ln\left(1-\frac{r^2}{a^2}\right) + \frac{\lambda_i^2}{\lambda_s^2}\ln\left(\frac{\lambda_i}{a}\right) - \ln\left(\frac{\lambda_s}{a}\right),$$

$$r \ll \lambda_i. \tag{28}$$

Both show that the interaction energy between the superconducting impurity and the vortex line behave logarithmically at short distances for a small radius of the impurity. The interaction energy between a vortex and a columnar defect at short distances has been calculated in Refs. [8,11]. Formulas (27) and (28) coincide with the results of these papers with logarithmic behavior, if the radius a of the impurity is much smaller than the quantities λ_i and λ_s. Figs. 2 and 3 show the free energy as a function of the distance r from the center of impurity for an impurity radius $a = \lambda_i$, $\kappa = 10^2$, and $\lambda_i/\lambda_s = 0.5$ and 2, respectively. It seems from the figures that there is a potential barrier opposing the entry of the vortex line for $\lambda_i/\lambda_s = 0.5$, in which case the vortex located outside the impurity is energetically favorable. But it seems there is a potential well, which always attracts the vortex line to itself, for $\lambda_i/\lambda_s = 2$, in which case the vortex located inside the impurity is energetically favorable.

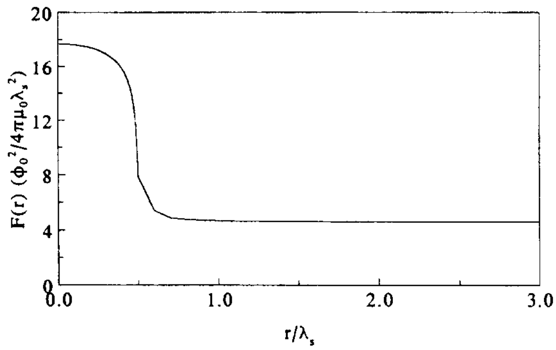

Fig. 2. The free energy as a function of the distance r from the center of the impurity of radius $a = \lambda_i$, $\kappa = 10^2$, and $\lambda_i/\lambda_s = 0.5$.

For simplicity, the pinning potential is defined by the difference between the free energy of a vortex at the center of the impurity and of a vortex infinitely distant from the impurity. Then we obtain a dependence of the pinning potential on the impurity radius a, as shown in Eq. (22). We also consider two extremely limiting cases. They show a simple and clear expression for the pinning potential for a single vortex. The pinning potential explicitly correlates with the difference between the intrinsic property of the penetration depth of the superconducting impurity and that of the superconducting background. If, for example, both superconductors are of the same material, the pinning potential will be reduced to zero. As seen in Eqs. (23) and (24), the pinning potential U_{pin} is negative if $\lambda_i > \lambda_s$. The interaction between the vortex and the impurity pinning center is attractive. Therefore, the capture of a vortex by the superconducting impurity is favored. On the other hand, if $\lambda_i < \lambda_s$, the pinning potential U_{pin} is posi-

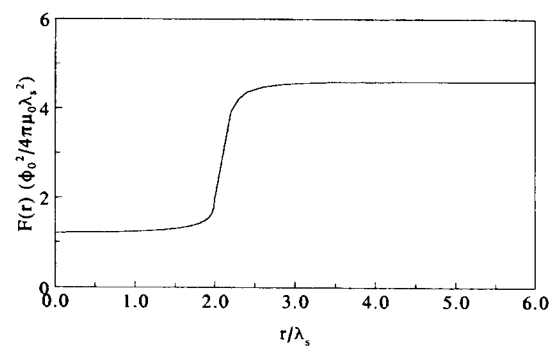

Fig. 3. The free energy as a function of the distance r from the center of the impurity of radius $a = \lambda_i$, $\kappa = 10^2$, and $\lambda_i/\lambda_s = 2$.

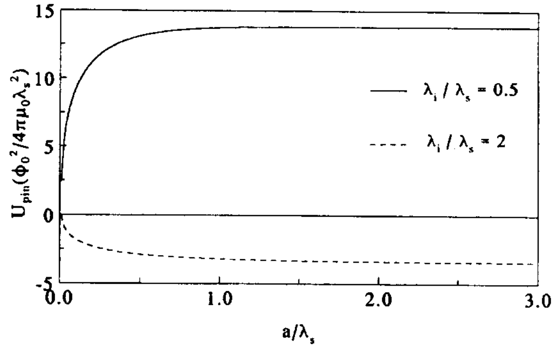

Fig. 4. The pinning energy for a vortex as a function of the radius a of the impurity for two different ratios of $\lambda_i/\lambda_s = 0.5$ and 2, and $\kappa = 10^2$.

tive. An energy barrier opposing the vortex seems to enter in the impurity pinning center.

Generally, the weaker the superconductivity of a superconducting material, the larger its the penetration depth. Hence one of these two superconductors with the weaker superconductivity always attracts the vortex line. Fig. 4 shows the pinning energy for a vortex as a function of the radius a of the impurity for two different ratios of the effective penetration depth λ_i of the impurity to the λ_s of the superconducting background, with $\kappa = 10^2$. The depth of U_{pin} increases rapidly with increasing radius a for $a \lesssim \lambda_s$ and is saturated when the radius a is much greater than the effective penetration depth λ_s. For a single vortex, the superconducting impurity pinning center with radius $a \gtrsim \lambda_s$ gives a stronger interaction with the vortex. These results are to be contrasted with the dependence of the minimum pinning potential U_p on the radius a of a cylindrical insulating inclusion by solving the G-L equations numerically [10]. These calculations led to the conclusion that, as regards the single-quantum vortex, U_p is negative for all a and is very shallow for a smaller than the coherence length ξ. And the depth of U_p increases rapidly with increasing a up to the penetration depth λ, and there U_p is found to saturate.

In summary, we have modeled the superconducting impurity in a type-II superconductor as a pinning center and investigated the dependence of the pinning potential per unit length, U_{pin}, on the radius a of a cylindrical superconducting impurity when a single-quantum vortex line is situated parallel to the central axis of the impurity. Our study presents an analytical method to understand the variety of flux pinning mechanisms in superconductors.

Acknowledgment

This research was supported by the National Science Council of Taiwan through grant number NSC 85-2112-M009-029-PH and NSC 86-2112-M009-007.

References

[1] P.J. Kung, M.P. Maley, M.E. McHenry, J.O. Willis, M. Murakami and S. Tanaka, Phys. Rev. B 48 (1993) 13922.
[2] A. Legris, F. Rullier-Albenque and P. Lejay, Phys. Rev. B 48 (1993) 10634.
[3] J.E. Tkaczyk, J.A. DeLuca, P.L. Karas, P.J. Bednarczyk, D.K. Christen, C.E. Klabunde and H.R. Kerchner, Appl. Phys. Lett. 62 (1993) 3031.
[4] L. Klein, E.R. Yacoby, Y. Wolfus, Y. Yeshurun, L. Burlachkov, B.Ya. Shapiro, M. Konczykowski and F. Holtzberg, Phys. Rev. B 47 (1993) 12349.
[5] F.M. Sauerzopf, H.P. Wiesinger, W. Kritscha, H.W. Webber, G.W. Crabtree and J.Z. Liu, Phys. Rev. B 43 (1991) 3091.
[6] L.W. Lombardo, D.B. Mitzi, A. Kapitulnik and A. Leone, Phys. Rev. B 46 (1992) 5615.
[7] A. Buzdin and D. Feinberg, Physica C 235–240 (1994) 2755.
[8] G.S. Mkrtchyan and V.V. Shmidt, Sov. Phys. – JETP 34 (1972) 195.
[9] I.B. Khalfin and B.Ya. Shapiro, Physica C 207 (1993) 357.
[10] N. Takezawa and K. Fukushima, Physica C 228 (1994) 149.
[11] A. Buzdin and D. Feinberg, Physica C 256 (1996) 303.
[12] J.L. Chen, in: Proc. Taiwan Int. Conf. on Superconductivity (Hualien, Taiwan, 1995); Phys. Rev. B 54 (1996) 502.
[13] A.F. Marshall et al., Phys. Rev. B 37 (1988) 1353.
[14] P. Marsh, R.M. Fleming, M.L. Mandich, A.M. De Santolo, J. Kwo, M. Hong and L.J. Martinez-Miranda, Nature 334 (1988) 141.
[15] D.E. Morris, A.G. Markelz, B. Fayn and J.H. Nickel, Physica C 169 (1990) 53.
[16] D.R. Harshman and A.P. Mills Jr, Phys. Rev. B 45 (1992) 10684.
[17] A. Schilling, F. Hulliger and H.R. Ott, Physica C 168 (1990) 272.
[18] D.R. Harshman, L.S. Schneemeyer, J.V. Waszczak, G. Aeppli, R.J. Cava, B. Batlogg and L.W. Rupp, Phys. Rev. B 39 (1989) 851.
[19] B. Oh, K. Char, A.D. Kent, M. Naito, M.R. Beasley, T.H. Geballe, R.H. Hammond, A. Kapitulnik and J.M. Graybeal, Phys. Rev. B 37 (1988) 7861.