國立交通大學

電信工程研究所

碩 士 論 文

論在固定能量及異質鏈路狀態下之中繼站選取

Relay Selection with Fixed-Energy Relays and Heterogeneous Link Conditions

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摘要

在傳統的中繼站輔助通訊的系統中，一般討論課題不外是如何提高系統的效率(throughput)、有待傳序列的中繼站(Relays with queues)或用戶的公平性(Fairness)等等。然而對於有多中繼站、感測器的各種感測及無線通訊網路之節能傳輸措施則較少著墨。這類網路或供緊急救難時使用，或設置於土質敏感區，於溪流沿岸、橋邊或環境保護區內以偵測水位、風速、溫度、氣壓或異常氣體等都是位在比較偏遠，電力不便或無法到達之處，中繼站或感測器之電源只能仰賴電池供應，其電能的使用效率遂格外重要。

本論文便是考慮這類有固定能量中繼站(Fixed-Energy Relays)的無線網路環境下的中繼站選擇。我們提出有效率運用這些固定能量中繼站以傳送最大量資訊的中繼站選擇方案。由於各中繼站離傳輸端與目的端之距離不一，選擇通道狀況最好的鏈路—此即許多文獻使用或提及的BRS (Best Relay Selection) 選擇法—通常無法達到此目標。我們選擇依據統計的相對鏈路增益(relative equivalent end-to-end link gain)最好的中繼站反而有較BRS為佳的總傳輸量。為防止因要等待較佳的鏈路（通道）狀況而延宕中繼站的選擇（因統計上未來一定有更佳的鏈路狀況出現）並考量每次傳輸前的通道估計與回報所耗損的能量，我們因此設定

了最低的單次傳輸量，只要所選擇的中繼站其等效鏈路增益足夠支持此最低傳輸量即必須進行傳輸。

我們針對上述選擇策略詳細分析了其統計行為，以估測相關的網路生命期與總傳輸量。這個部份在中繼點的研究中非常少見，但如此分析可以避免耗時的電腦模擬式預測，也方便瞭解各種系統及通道參數如何影響系統性能表現。事實上我們的電腦模擬也證明我們的分析的確可以提供非常準確的性能預測。

Relay Selection with Fixed-Energy Relays and Heterogeneous Link Conditions

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Abstract

Relay assisted network has been a very popular and important issue recently, and various selection methods have been put forth by other fellow researchers. The main topic discussed includes the throughput improvement, the fairness awareness, even MIMO can sometimes be involved. Recently, there are a group of researchers start to consider a new and more practical scenario, the energies on the relays are limited.

The energy-limited relay scheme can be adopted in the area where it is hard to provide a constant power supply, for example, in the dessert, in the middle of Pacific ocean, or in the mountains. It is very difficult and not economical profitable to assume the relays to have constant power supply in the above mentioned areas, and thus energy-limited relay schemes must be considered. In addition, energy-limited relay scheme is also suitable to provide essential emergency communications when either a natural or a human-made disaster strikes.

In this study, we propose and analyze a highly-efficient and excellent-performance relay selection method. We compare our proposed algorithm to both the well-known conventional BRS (Best Relay Selection) algortihm and a rather trivial Max-Life algorithm, and we can see in the simulation part that our proposed algorithm is better. Moreover, since the total energy is fixed for the network and our scheme uses this energy more efficiently than BRS, we will call ourself a greener technology.

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最後我更要感謝在身邊關心我的家人、朋友以及曾經關心過我的人，若不是你們在背後無條件的支持，這漫漫的研究長路我將無走得如此自如，也因為在挫折時有他們的笑臉與鼓勵，每每賜予了我再往前進、再精進研究的動力，僅獻上此論文來表達我最深的感謝。

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Chapter 1

Introduction

The communication requirement grows surprisingly fast in the past decade, and the coverage of the communication network is crucial to guarantee a complete and continuous communication service. However, it will not be practical to assume there is always a base station to serve all the users who are trying to establish communication links. The most common and effective solution to this problem is to use relays to assist the transmission, and there are tons of research already done toward the relay assisted network.

There are already a lot of researches about relay networks, and they may consider from relay selection methods[1],[2],[3], fairness[4], QoS requirement[5],[6], relays with queues[7],[8],[9], outage probabilities[10],[11] to even MIMO [12]can be taken into consideration. What's more, some of the papers even consider multistage relaying[12],[13], two-way transmission[9][11].

Although there are so many efforts done as above mentioned, there is a corner of the relay network that is forgotten, energy-constrained relays. That is, we want to explore what will be different when relays are energy-limited and thus have limited transmitting opportunities to help the source to transmit the packet to the destination.

This is not, however, a hypothetical issue. We do need to establish the communication network to cover the desert or to reach the bottom of the oceans one day, and we think relays are the only way to achieve this goal since you will not be able to find a constant power supply to build a base station in the desert or on the floor of the ocean nor is it economical profitable.

Another practical scene to apply our research is that when a major natural disaster strikes, we need to establish an emergency communication network. For example, a major earthquake is most likely to destroy the electric power network, and thus the power supply to the base stations near the origin of the earthquake will be cut off, and the base station will not be able to work properly. However, this area is usually the area that takes the most severe hit from the earthquake, and the recovery of the communication network will be essential and vital since effective communications from the victims trapped under the debris to the rescuers could save lives. This is when we can deploy several temporary relay stations running on the batteries to establish the important life-saving communication network.

In this research, we will propose a very effective relay selection algorithm to give better performance when the relays are energy-limited. We will compare our proposed algorithm to the well-known conventional BRS algorithm [8],[14],[15]with slight modification, and a Max Life selection algorithm serves as an intuitive CSI-free algorithm.

If the relays are not energy constrained, the conventional BRS algorithm will give us the best performance as far as the throughput is concerned. Nevertheless, if we consider the relays to be energy constrained, it will not be the case. We will give further details and the simulation results in later chapters.

Besides proposing a more efficient algorithm to use when relays are energy constrained, we also do the mathematical analysis for our proposed algorithm. This is something really hard to do and rarely seen in any other relay-related researches; however, we manage to overcome the difficulties and finally obtain a very accurate mathematical analysis and performance prediction for our proposed algorithm.

Chapter 2

System Model

We present in this chapter the system model and the environment setting for our energy-limited network. We also describe a criterion for evaluating a relay-selection algorithm as an optimization framework at the end of this section.

2.1 System Model

We consider a one Source (S), N relays ($R_i, i = 1, 2, ..., N$), one destination (D) network, with only one relay chosen to assist the source to transmit the desired packet to the destination. Decode-and-Forward (DF) is assumed. We also make the assumption that the distance between the source and the destination is too great to have a significant direct link, and thus it is ignored. An illustration of a relay network can be found in Fig 2.1. For simplicity, we assume a fixed-power transmission policy for both the source and the relays across all time slots represented by P_S and P_{R_i}, respectively. Note that every relay is going to use the same transmitting power P_R no matter what the channel condition is. That is, P_{R_i}=P_R, $\forall\ i = 1, 2, 3, \cdots, N$.

We denote the distance between the source and relay i by $d_{S,i}$ and that between relay i and destination by $d_{i,D}$. With these, we introduce the path loss model

$$L_{ab} = n \times 10 \log_{10} (d_{ab}), \tag{2.1}$$

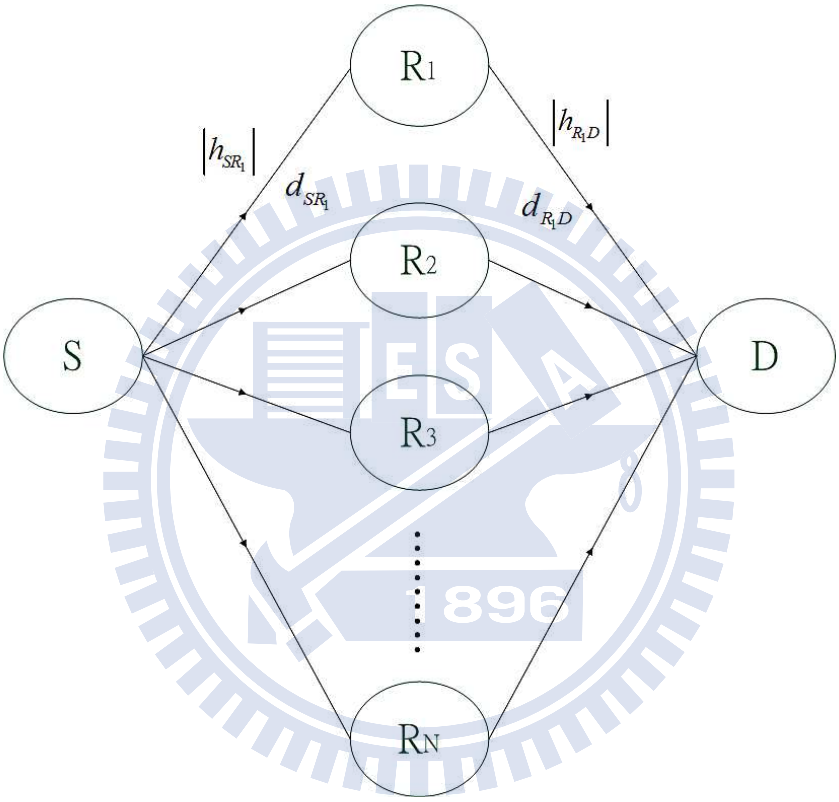

Figure 2.1: System model of a relay network.

where L_{ab} is the path loss from node a to node b in dB, d_{ab} is the distance from node a to node b. n is the path loss exponent.

Single carrier scheme is adopted, and the energy required for the relays to decode and process the packet is neglected. We assume that it is much smaller than the energy used for transmission. Perfect CSI is assumed unless otherwise specified.

The relay network adopts a two-stage transmission scheme[8]. In the first time slot, the source will first do a CSI acquisition and decide the selected relay according to different algorithms. Then it broadcasts the data to all relays, and every relay tries to decode the message transmitted from the source. In the second time slot, the selected relay transmit the received packet to the destination. Such transmission process is demonstrated in Table 2.1.

We would like to mention here that Table 2.1 is not representing the real time spending ratio between the CSI acquisition and the actual transmission. The former takes much less time than the later.

Table 2.1: An example of how a two-stage transmission takes place.

Timeslot	1		2	3	
Source	Getting CSI	Broadcast	X	Getting CSI	Broadcast
Selected Relay	X		Send to Destination	X	

The received signal at the relay R_i and the destination D can be written as

$$\begin{aligned} r_{R_i} &= \sqrt{\frac{P_s}{10^{\frac{L_{Si}}{10}}}} h_{SR_i} x + v_i, \\ r_D &= \sqrt{\frac{P_R}{10^{\frac{L_{i^*D}}{10}}}} h_{R_{i^*}D} x + v_D \end{aligned} \quad (2.2)$$

The x in 2.2 is the transmitted symbol, and v_i and v_D are Additive White Gaussian Noise (AWGN) at the relay i and the destination D, respectively. h_{SR_i} and $h_{R_{i*}D}$ are zero-mean circular symmetric complex Gaussian variables. i^* denotes the selected relay for the second-stage transmission.

We realize that v_i and v_D should be independent random variables. However, for the simplicity sake, we will use σ^2 as the power of the thermal noise for both v_i and v_D in the following text.

The received SNR at the selected relay i^* and the destination is given by

$$\gamma_{i^*} = \frac{P_s |h_{SR_{i^*}}|^2}{10^{\frac{L_{Si^*}}{10}} \sigma^2} \tag{2.3}$$

$$\gamma_D = \frac{P_R |h_{R_{i^*}D}|^2}{10^{\frac{L_{i^*D}}{10}} \sigma^2} \tag{2.4}$$

σ^2 is the power for the AWGN noise.

Then, we can easily find out the equivalent received SNR at the destination for the signals through the selected relay i^* is [8]

$$\Gamma_{i^*} = \min\{\gamma_{i^*}, \gamma_D\} \tag{2.5}$$

Let's further let $\overline{\gamma}_{i^*} = E[\gamma_{i^*}]$, $\overline{\gamma}_D = E[\gamma_D]$. Since we have a Rayleigh fading channel, the γ_{i^*} and γ_D are exponentially distributed with parameter $\lambda_1 = \frac{1}{\overline{\gamma}_{i^*}}$, and $\lambda_2 = \frac{1}{\overline{\gamma}_D}$. Actually, the Γ_{i^*} is also exponentially distributed with parameter $\lambda = \lambda_1 + \lambda_2 = \frac{\overline{\gamma}_{i^*} + \overline{\gamma}_D}{\overline{\gamma}_{i^*} \times \overline{\gamma}_D}$. [8]

We will give a brief proof to illustrate that (2.5) is still exponentially distributed with coefficient $\lambda = \lambda_1 + \lambda_2$

Proof. For any chosen relay i^*, we denote the CDF of Γ_{i^*} to be $M_{i^*}(x)$:

$$\begin{aligned}
M_{i^*}(x) &= \mathrm{P}[\gamma_{i^*} \leq x, \gamma_D > x] + \mathrm{P}[\gamma_D \leq x, \gamma_{i^*} > x] \\
&= \int_0^x \lambda_1 e^{-\lambda_1 t} e^{-\lambda_2 t} \mathrm{d}t + \int_0^x \lambda_2 e^{-\lambda_2 t} e^{-\lambda_1 t} \mathrm{d}t \\
&= \left(\frac{\lambda_1}{\lambda_1 + \lambda_2} - \frac{\lambda_1}{\lambda_1 + \lambda_2} e^{-(\lambda 1 + \lambda 2)x} \right) + \left(\frac{\lambda_2}{\lambda_1 + \lambda_2} - \frac{\lambda_2}{\lambda_1 + \lambda_2} e^{-(\lambda 1 + \lambda 2)x} \right) \\
&= 1 - e^{-(\lambda 1 + \lambda 2)x}
\end{aligned} \tag{2.6}$$

□

As we can see from (2.6), the CDF for the minimum of two exponentially distributed random variables still takes the exponential random variable's CDF form. That is, the minimum of two exponentially distributed random variable is still exponentially distributed, with the parameter $\lambda = \lambda_1 + \lambda_2$.

There is also a fixed rate constraint, C(bits/sec/Hz), caused by the service requirement or the requirement to make the transmission more power efficient. This rate constraint can also be represented by an SNR constraint, γ_{cst} on the equivalent receiving SNR of the network. The relationship between the rate constraint, C and the SNR constraint, γ_{cst} is

$$C = 0.5 \times \log_2 (1 + \gamma_{cst}) \tag{2.7}$$

2.2 Energy Limited Relays

The most innovative and challenging part of our work is to set energy limits on the relays so that each of them has only limited transmitting opportunities. Let $E_i(t)$ be the remaining energy for the relay i at the beginning of the time slot t.

We set the initial energy of each relay to be the same. That is, $E_i(t = 0) = E,$ for $i = 1, 2, \cdots, N$. The source is, however, energy unlimited. We also assume that the energy consumed for decoding and processing the packet at the selected relay i^* is very small compared to the one used to retransmit the packet to the destination, and thus it is ignored.

Combining the limitation on the energy and the fixed power transmission, we can easily see that the possible transmission attempts will be finite and the same for each relay. In the beginning of the first time slot, the possible transmission attempts for each relay is $M = E/P_R$, where we normalize the transmission time in each time slot to 1.

Table 2.2: An example of how an idle time slot is dealt with.

Timeslot	1	2	3
Source (S)	Broadcast	X	Broadcast
Selected Relay (R_{i^*})	X	Send to Destination	X

Timeslot	4	5	6
Source (S)	X	Idle	X
Selected Relay (R_{i^*})	Send to Destination	X	X

2.3 Network Lifetime and Idle Time Slots

For each time slot, the source will choose the chosen relay according to different algorithms, and check if the instantaneous channel status of the chosen relay supports a rate that is greater than the rate constraint. If it comes back positive, the transmission will take place. However, if the chosen relay cannot support a rate that is greater than the rate constraint, that time slot will be abandoned.

A possible scenario is presented in Tab 2.2 where at time slot 5, the source finds out that the chosen relay does not possess a quality link to support a rate greater than the rate constraint, so the source decides not to transmit.

We omitted the getting CSI session in the beginning of the first stage to make Tab 2.2 more readable, and caution readers to deny any possible confusions.

We define the Network lifetime to be the total count of the time slots from the beginning to the last relay depletes its energy. This indicator, however, is not the longer the better as most other paper does.

This is due to the fact that the longer network lifetime represents that larger amount of idle time slots are generated. As a result, we will want this indicator to be as small as possible to reduce the total number of idle time slots. We will see more about how longer lifetime (more idle time slots) hurts our optimization goal in the next section.

2.4 Optimization Goal

In this research, we want to maximize the throughput of the network before all relays are running out of energies. The definition of the throughput η is

$$\eta = \frac{\sum_{t=1}^{\infty} \sum_{i=1}^{N} \frac{1}{2} \rho_i(t) \log_2 (1 + \Gamma_i(t))}{\arg\min_t \left(\sum_{i=1}^{N} E_i(t) = 0 \right)} \tag{2.8}$$

$$\rho_i(t) = \begin{cases} 1, & \text{if } i = i^* \\ 0, & \text{otherwise} \end{cases}$$

The $\rho_i(t)$ in (2.8) indicates whether the relay i is selected or not for the time slot t, and $E_i(t)$ is the remaining energy for the relay i at the beginning of the time slot t. In addition, $\rho_i(t)$ will be different due to different relay choosing algorithms.

The numerator of (2.8) is the total transmitted bits until all relays are out of energies, and the denominator is the network lifetime. The goal for each algorithm is to maximize the throughput η before all energy in the relays are consumed.

We can also acknowledge from the denominator of (2.8) that the idle slots should be avoided and the network lifetime should be as less as possible in order to maximize η.

That is, although we do not enforce a hard constraint on the total sum of the generated idle time slots in this study, the penalty is already on the optimization goal η here.

Chapter 3

Conventional Relay Selection Algorithms

In this chapter, we introduce two relay selection algorithms, namely, mBRS (modified Best Relay Selection) and Max-Life algorithms. Performance comparisons with the proposed algorithm are given in the ensuing chapters. BRS itself is a well-known algorithm and we develop an improved version for fair comparison purpose.

On the other hand, Max-Life algorithm serves as a reference algorithm we put forth to show what would happen when we are completely blind to the CSI information. As a result, the Max Life algorithm's performance should serve as a lower-bound for comparison with any reasonable CSI-aided relay selection algorithm.

3.1 Modified Best Relay Selection

Conventional BRS (Best Relay Selection) is a very intuitive algorithm. By selecting the relay to obtain a Source-Relay-Destination link with the best instantaneous equivalent SNR, we should be able to achieve the best possible performance for each time slot.

It is trivial and true that the BRS is indeed the best algorithm in maximizing the network throughput when the relays are not energy-limited and there is no delay bound constraint. However, when the relays are energy-limited, it has only finite transmission opportunities and we are not so sure that the BRS remains being the best solution. In fact, as we explain later in this section, it may yield unsatisfactory performance in the presence of energy limitation.

First, we would like to explain how we modify the conventional BRS algorithm and why. We can first see how the conventional BRS different from our modified BRS algorithm in Fig.3.1.

We can see that the conventional BRS algorithm will definitely transmit in every time slot. That is, after finding out the selected relay i^*, the source will start the transmission even if the channel status of the selected relay will cause an outage. This

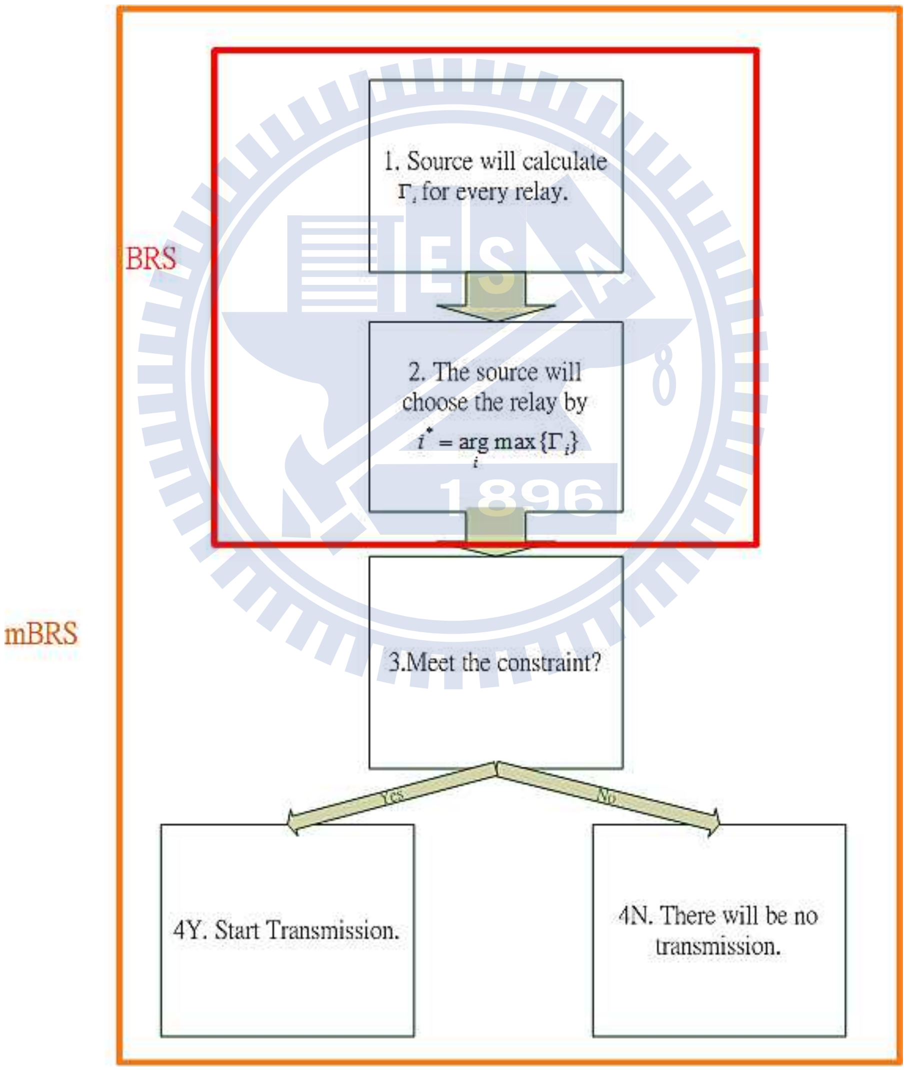

Figure 3.1: mBRS vs. BRS algorithm.

is something we would like to change in the conventional BRS algorithm, since it isn't allowed to transmit if the transmission does not fulfill the rate constraint C assuming having knowledge of the complete CSI. Therefore, we change the conventional BRS algorithm to make the source to stop the transmission if the selected relay i^* can not support a link with equivalent rate larger than the rate constraint. We call this revised version of the conventional BRS algorithm the modified BRS algorithm (mBRS). An illustration of the modified BRS algorithm can be found in Fig.3.1

The reason why the conventional BRS works fine in tons of other researches is the fact that they do not consider the energy consumption. As a result, there is no benefits to stop the transmission.

3.1.1 Analysis of Modified BRS algorithm

To further understand exactly why modified BRS algorithm is not the best choice when it comes to energy-limited relays, we give some mathematical facts in this section.

First, we claim that the mBRS tends to exhaust all the energy of the relay with better $\overline{\Gamma}_i$ first, and then continue on accordingly. This fact is directly from the theorem below.

Lemma 3.1.1. *For two relays m, n, if $\overline{\Gamma}_m \gg \overline{\Gamma}_n$, then mBRS will deplete all the energy in relay m first before any energy being consumed from relay n.*

Proof.

$$
\begin{aligned}
&P[\text{relay } m \text{ to transmit} \mid \text{relay } m, n \text{ still have energy}] \\
&= \int_0^\infty P[\Gamma_m = x, \Gamma_n < x]\,\mathrm{d}x = \int_0^\infty \frac{1}{\overline{\Gamma}_m} \mathrm{e}^{\frac{-x}{\overline{\Gamma}_m}} \left(1 - \mathrm{e}^{\frac{-x}{\overline{\Gamma}_n}}\right) \mathrm{d}x \\
&= \frac{\overline{\Gamma}_m}{\overline{\Gamma}_m + \overline{\Gamma}_n} = 1 \\
&P[\text{mBRS choose relay } n \text{ to transmit} \mid \text{relay } m, n \text{ still have energy}] = 0
\end{aligned}
$$

□

From the above lemma, we can derive the following theorem:

Theorem 3.1.1. *Modified BRS algorithm tends to empty the energy of the relay i with the best $\overline{\Gamma}_i$ first and then accordingly.*

Proof.

Let's suppose there are M relays with $\overline{\Gamma}_1 \gg \overline{\Gamma}_2 \gg \cdots \overline{\Gamma}_M$,

then let's assume that for any $m < n$, relay n transmits when relay m still has energy, that is :

P[relay n transmits | relay m, n still have energy]$\neq$0

But we know from the assumption that $\overline{\Gamma}_m \gg \overline{\Gamma}_n$,

which is a direct violation to the lemma 3.1.1. As a result,

P[relay n transmits | relay m, n still have energy]=0

□

3.1.2 Conclusion on Modified BRS

Therefore, we can see that mBRS tends to deplete the energy of the relays with better average SNR first, and save the worst ones for the last. This behavior immediately gives mBRS a huge disadvantage since it will certainly suffer a lot of idle time slots after the relays with good link qualities are all firstly consumed.

Let's check again our optimization goal in (2.8). It is not hard to find out that the excess idle time slots will hurt the throughput and thus decrease the total performance. As a result, it is both intuitive and mathematically proven that mBRS will not perform well when the relays are energy constrained.

Yet another disadvantage this property brings is the fact that even for those relays with good average link qualities (high average SNR), mBRS tends to use all their energies consecutively without any diversity.

In other words, the relay with high average SNR can have a bad instantaneous channel gain compared to its own statistical distribution, but the instantaneous channel gain still wins over the other remaining relays. On the contrary, the other relay may have a little worse instantaneous channel gain, but compared to its own distribution, it may be in its lucky days. However, mBRS will choose the former to transmit since it only takes instantaneous channel SNR into account.

To sum up, mBRS not only generates excess idle time slots by leaving those relay with disadvantage links to the last, but also uses the energy of the relays with better average SNRs unwisely.

3.2 Max Life algorithm

What we would like to do in this section is to introduce a CSI -free relay selection algorithm, and thus should serve as a lower bound toward all robust CSI related algorithm when relays are energy-constrained.

We will introduce a Max Life algorithm, which uses no CSI information :

$$\text{At time slot } t:$$

$$i^*_{ML} = \arg\max_i E_i(t)$$

We can see that the Max Life algorithm choose the relay with the most energy left, so that it can make the rate of the energy consumption of each relay to be very close. The most special property of this selection algorithm will be the fact that the number of alive relays will be almost the same from the beginning to the end of the network lifetime. This property also allows us to reset the relay network easily, since you can replace all the batteries in relays in one trip.

Max Life algorithm should set a lower-bound on all reasonable CSI aware relay selection algorithms, since it does not use any form of CSIs to do the relay selection. If an algorithm uses CSI and still loses to the Max Life algorithm, it is definitely not a reasonable one. We will see both our proposed algorithm and mBRS outperform the Max Life algorithm by much in the simulation part.

Chapter 4

A New Relay Selection Algorithm

We have shown why mBRS and Max Life algorithms perform poorly when the relays have energy constrains in previous chapter. We will now propose a new relay selection algorithm in this chapter and try to analyze and predict its performance.

Unlike mBRS and the Max Life algorithm in the previous chapter, our proposed algorithm uses a more reasonable way to choose the selected relay i^* by considering both the instantaneous channel status and the statistical SNR mean of each relay.

As the result, the proposed algorithm utilizes the energy of the relays more efficiently and reasonably than the other two algorithms, and thus we call it a greener technology.

4.1 Introduction to the proposed algorithm

In this section, we explain how the proposed algorithm works and give better performance when relays are energy-constrained.

We can see that mBRS tends to choose the relay based on the instantaneous channel gains only. On the other hand, Max Life algorithm chooses the selected relay by how much energy is in each relay without any CSI information. Both of them have a disadvantage as we either stated or proved in previous chapter.

The proposed algorithm, on the contrary, chooses the selected relay i^* by both the instantaneous SNR performance and the statistical SNR mean. By comparing the instantaneous SNR performance to its statistical mean for each relay, we can select a desirable relay so that its performance will be the best with the statistical mean concerned.

Actually, our proposed algorithm uses the CDF (Cumulative Density Function) to do the selection to ensure that a fair comparison is done among all relays. In each time slot, the source will calculate the CDF values $D_i(\Gamma_i(t))$ of the current instantaneous channel gains for every relay i, and the greatest one will then be chosen as the selected relay i^*. The transmission will take place if the selected relay i^* supports a rate greater than the rate constraint C.

The following is the selection algorithm of the proposed algorithm

$$\begin{aligned} &\text{At time slot } t: \\ &i^* = \arg\max_i D_i\left(\Gamma_i(t)\right) \\ &D_i\left(\Gamma_i(t)\right) = 1 - \mathrm{e}^{\frac{-\Gamma_i(t)}{\bar{\Gamma}_i}} \end{aligned}$$

Let's make clear here that our proposed algorithm uses no more information than the mBRS algorithm mentioned in the previous chapter but statistical means, which are usually assumed known in related researches and easy to obtain.

By converting the instantaneous channel gains to the CDF values, we can now eliminate the chance of blindly choosing the relay with a better instantaneous SNR to transmit which is a major defect for mBRS stated and proved in 3.1.1.

More importantly, every relay has a better chance to be selected when its instantaneous channel equivalent SNR is performing better with respect to its own statistic distribution. This gives a fair chance for the relays with weaker average SNR to be chosen as a transmitted relay if they are performing well according to their own statistics.

What's more, for those relays with high average SNR, this selection mechanism prevents them from transmitting when their instantaneous CSI is the best among all relays but weak with respect to their own statistic distribution.

The above facts result in that the energy of the relays are used much more efficiently than either mBRS or Max Life algorithm.

4.2 Markovian Formulation of the Relay Energy Consumption Process

There are many relay-related researches, but few of them try to analyze the performance of their proposed algorithms. However, in this research, we will try to give some mathematical analysis about the algorithm we proposed. It is already very difficult to analyze the performance of a relay system and merely impossible when the system has a limited lifetime. In this study, the system lifetime comes to an end when all relays depleted their energies.

This section is composed of four subsections. In the first subsection, we analyze the total transmittable bits under no rate constraint for our algorithm during the network lifetime. For the second subsection, we analyze the network lifetime for our algorithm. As for the third part, we analyze the total transmittable bits for our algorithm under rate constraints.

For the first and third parts, we use the Laplace transform to estimate the distribution of total transmittable bits and then calculate their means. However, in the forth part, we use Gauss approximation by directly estimating the mean and variance of the distribution of total transmittable bits.

We would like to mention that in the first and third subsection, the numerical Laplace transform performed contains extreme high computational complexity, and it will be practically impossible to commence if the total number of the relays or the transmission opportunity of each relay becomes large.

This is in fact the motivation for the forth part, since the Gaussian approximation method can give us an approximation with much less complexity.

We will define here the state $A_m = (T_{N-1}, T_{N-2}, \cdots, T_i, \cdots, T_0)$. i is the relay index, and the state index m is determined by $m = \sum_{i=0}^{N-1} T_i \times (M+1)^i$,where $T_i = 0, 1, \cdots, M$ is the remaining transmission opportunity of relay i, and M is the initial transmitting opportunity for each relay as defined in Sec 2.2. In fact, the state index m is the decimal value of $(M+1)$-ary tuple $(T_{N-1}, T_{N-2}, \cdots, T_0)$.

It is not always possible for a state A_m to transit to another state A_n in one transition, or in other words, in one transmission, since we only select one relay to assist the transmission, and we should only consume one transmission opportunity.

That is, the only possible transition for one transmission from the state A_m to state A_n is $\log_{M+1}(m-n) \in \{0, 1, 2, \cdots, N-1\}$, which represents that exactly one transmission opportunity is consumed. In fact, the selected relay i^* is also determined by $i^* = \log_{M+1}(m-n)$.

Also, by the definition of our states, we are facing a absorbing Markov chain problem[16].

For example, the state transition diagram of a case with a 2 relay system with each relay has 2 transmission opportunities in the beginning $(N = 2, M = 2)$ is presented below.

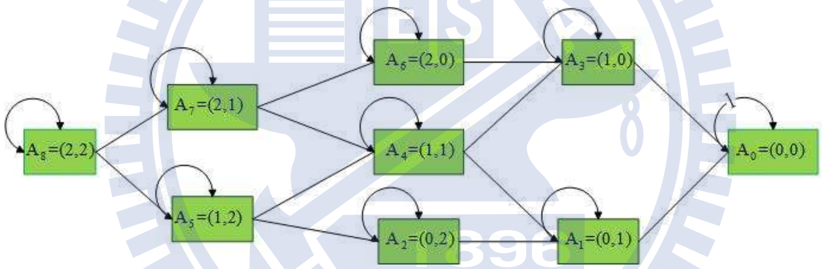

Figure 4.1: A state transition diagram with 2 relays and 2 initial transmission opportunities each.

We note that all possible transitions from state A_m to A_n in one transmission follow the rule $\log_{2+1}(m-n) \in \{0, 1\}$ mentioned before and the selected relay i^* for the transmission is exactly $i^* = \log_{2+1}(m-n)$. Any transition that does not fit $\log_{2+1}(m-n) \in \{0, 1\}$ is not possible as Fig 4.1 shows.

We also define a function υ_m to be the Hamming weight of the state A_m, and $\delta_m = \{i | T_i \in A_m, T_i \neq 0\}$ to be a set of relay indexes of the nonzero entries in the state A_m.

4.3 Semi-Markov Processes

We base our performance analysis on the concept of Semi-Markov processes (also known as Markov renewal processes). A finite-state Semi-Markov process is a finite Markov process for which the dwell time and state transition time is a random variable whose statistic depends on the state of concern or the starting and the destination states when a sate transition is involved. Formly, we have

Definition 1. *A random process $S(t)$ is called a finite-state Semi-Markov process if and only if it takes value from a finite set of states. Let T_n be the time when the n^{th} state transition occurs with the corresponding state X_n, and $\tau_n = T_n - T_n - 1$ is the inter-transition time. We have*

$$\begin{aligned} &P\left[\tau_n \leq t, X_n = j \mid (X_0, T_0), (X_1, T_1), \cdots, (X_{n-1} = i, T_{n-1})\right] \\ &= P\left[\tau_n \leq t, X_n = j \mid (X_{n-1} = i, T_{n-1})\right] \end{aligned}$$

In our case, our rate constraint and the corresponding idle time slots have randomized the transition time between any two different states.

We then derive the state transition probability matrix $\boldsymbol{U}$ of the corresponding state transition diagram (suppose there are total L states), and use the property that $\boldsymbol{U}^n$ will find all possible trellis and their possibilities after n state transitions to continue our analysis on transmittable bits and network lifetime.

$$\boldsymbol{U} = \begin{pmatrix} p_{12} & \cdots & p_{1L} \\ \vdots & \ddots & \vdots \\ p_{L1} & \cdots & p_{LL} \end{pmatrix}$$

p_{ij} is the transition probability from the state i to state j.

$$\boldsymbol{U}^n = \begin{pmatrix} p_{12} & \cdots & p_{1L} \\ \vdots & \ddots & \vdots \\ p_{L1} & \cdots & p_{LL} \end{pmatrix}^n$$

$\boldsymbol{U}^n(i, j)$ represents the probability for the transition from state i to state j in exact n transitions.

If we take a closer look at the $\boldsymbol{U}^n(i,j)$, we can find out that it actually finds out all possible route from state i to state j in n transitions and sums up the probability of each route. For our total transmittable bits analysis in later sections, we use this special property and multiply the distributions of the transmittable bits of the transition from state i to state j to the corresponding entries in $\boldsymbol{U}$.

$$\boldsymbol{O} = \begin{pmatrix} p_{12}b_{12}(s) & \cdots & p_{1L}b_{1L}(s) \\ \vdots & \ddots & \vdots \\ p_{L1}b_{L1}(s) & \cdots & p_{LL}b_{LL}(s) \end{pmatrix}$$

$b_{ij}(s)$ is the distribution of the transmittable bits for state i to transit to state j in Laplace domain.

Eventually, $\boldsymbol{O}^n(i,j)$ ($\boldsymbol{O}^n(i,j)$denotes the (i,j) entry of matrix $\boldsymbol{O}^n$) helps us to find the distribution of transmittable bits from state i to state j in n transitions. However, we are only interested in the distribution of the transmittable bits from the initial state to the absorbing state since it represents the distribution of the total transmittable bits of the network.

There is, nevertheless, yet another twist in deriving $\boldsymbol{O}$. Since the addition of random variables (in this case, the distribution of the transmittable bits from state changes) results in a convolution of the distributions in time domain and it is very difficult to perform. We thus transfer all distributions to the Laplace domain to make all further analysis possible.

As for our analysis on the network lifetime, we use the property that $\boldsymbol{U}^n(i,j)$ gives us the possibility of the transition from state i to state j in exact n transitions, and by choosing the state i to be the initial state and j to be the absorbing state, the network lifetime is then $\sum_{p=0}^{\infty} p \times \boldsymbol{U}^p(i,j)$.

4.4 Unconstrained Transmitted Bits (Rates) Analysis

In this section, we want to derive the distribution about how many bits can be transmitted without the rate constraint until all relays are out of energy. We will find the distribution to each state transition first, and derive the state transition probability matrix. By using the Laplace transform and inverse Laplace transform, we can obtain the total distribution of how many bits our proposed algorithm can transmit during its lifetime. Therefore, we can get a estimated total transmittable bits as the mean of this distribution.

For a relay to be selected as the selected relay i^*, it must have the following property:

$$\begin{aligned}
&D_{i^*}\left(\Gamma_{i^*}(t)\right) > D_i\left(\Gamma_i(t)\right), \forall i \neq i^* \Rightarrow \\
&\left(1 - \mathrm{e}^{\frac{-\Gamma_{i^*}(t)}{\overline{\Gamma}_{i^*}}}\right) > \left(1 - \mathrm{e}^{\frac{-\Gamma_i(t)}{\overline{\Gamma}_i}}\right), \forall i \neq i^* \Rightarrow \\
&\frac{\Gamma_{i^*}(t)}{\overline{\Gamma}_{i^*}} > \frac{\Gamma_i(t)}{\overline{\Gamma}_i}, \forall i \neq i^*
\end{aligned} \tag{4.1}$$

Since $\Gamma_i(t)$ is a exponential distributed random variable with mean $\overline{\Gamma}_i$, we can observe that both $\frac{\Gamma_{i^*}(t)}{\overline{\Gamma}_{i^*}}$ and $\frac{\Gamma_i(t)}{\overline{\Gamma}_i}$ shares the same exponential distribution with mean equal to 1. Therefore, all relays are equally likely to be chosen as the transmitting relay

$$\mathrm{P}(i = i^*) = \frac{1}{N}, \forall i = 1, 2, \cdots, N \tag{4.2}$$

Next, we want to derive the rate distribution of one-time transmission. We are now assuming that all N relays are alive (with transmission opportunities left) at this moment.

We can derive the CDF for the selected relay i^* below:

$$\begin{aligned}
B_{i^*}(\Gamma_{i^*}(t)) &= \left(1 - \mathrm{e}^{\frac{-\Gamma_{i^*}(t)}{\overline{\Gamma}_{i^*}}}\right) \prod_{i=0, i\neq i^*}^{N-1} \left(1 - \mathrm{e}^{\frac{\frac{\Gamma_{i^*}(t)\overline{\Gamma}_i}{\overline{\Gamma}_{i^*}}}{\overline{\Gamma}_i}}\right) \\
&= \left(1 - \mathrm{e}^{\frac{-\Gamma_{i^*}(t)}{\overline{\Gamma}_{i^*}}}\right)^N
\end{aligned} \tag{4.3}$$

Next, let's consider a more realistic case, the distribution of the transmittable bits when the state transits from state A_m to A_n. There can be less than N alive relays in the state A_m, so we will have to modify (4.3) by replacing N with υ_m.

We also use the well known Shannon capacity bound to establish the relationship between the average SNR, $\Gamma_{i^*}(t)$, with the rate, c.

$$c = 0.5 \log_2 (1 + \Gamma_{i^*}(t)) \tag{4.4}$$

where the coefficient 0.5 is due to the two-stage transmission as introduced in chapter 2.

With the helps above, we can now rewrite (4.3) as

$$B_{mn}(c) = \left(1 - \mathrm{e}^{\frac{-(2^{2c}-1)}{\Gamma_{i^*}}}\right)^{\upsilon_m} \tag{4.5}$$

By Taking derivative of (4.5) against c, we can derive the PDF of the transmittable bits for state m to state n to be

$$b_{mn}(c) = \upsilon_m \frac{1}{\Gamma_{i^*}} \left(1 - \mathrm{e}^{-\frac{2^{2c}-1}{\Gamma_{i^*}}}\right)^{\upsilon_m - 1} \mathrm{e}^{-\frac{2^{2c}-1}{\Gamma_{i^*}}} 2^{2c+1} \log(2) \tag{4.6}$$

Since we have multiple transmissions, we can see that this question involves a convolution of the distribution in the time domain. The convolution itself, however, is really difficult to perform, so we will deal with this problem in the Laplace domain where the convolution will become simple multiplication.

Let's denote $b_{mn}(s)$ to be the Laplace transform of the $b_{mn}(c)$,

$$b_{mn}(s) = \mathcal{L}\{b_{mn}(c)\} \tag{4.7}$$

With the help of (4.2), the state transition probability for state A_m to transit to state A_n is

$$p_{mn} = \begin{cases} \frac{1}{\upsilon_m}, \text{ if } \log_{M+1} m - n \in \{0, 1, 2, \cdots, N-1\} \\ 0, \text{ otherwise} \end{cases} \tag{4.8}$$

As we stated and explained before, it is not always possible for any state m to transit to any other state n as shown in (4.8), since the transition of the state represents a one-time transmission. Any transition between two states m, n which do not satisfy $\log_{M+1} m - n \in \{0, 1, 2, \cdots, N-1\}$ is not possible.

The state transition matrix can then be expressed as

$$\boldsymbol{F} = \begin{pmatrix} p_{(M^N-1)(M^N-1)} & \cdots & p_{(M^N-1)0} \\ \vdots & \ddots & \vdots \\ p_{0(M^N-1)} & \cdots & p_{00} \end{pmatrix}$$

We should note that the transition matrix starts with the initial state A_{M^N-1}, and the absorbing state is A_0.

The distribution of total transmittable bits after all relays are out of energy in the Laplace domain is:

$$\boldsymbol{X}^{NM}(s) = \begin{pmatrix} p_{(M^N-1)(M^N-1)}b_{(M^N-1)(M^N-1)}(s) & \cdots & p_{(M^N-1)0}b_{(M^N-1)0}(s) \\ \vdots & \ddots & \vdots \\ p_{0(M^N-1)}b_{0(M^N-1)}(s) & \cdots & p_{00}b_{00}(s) \end{pmatrix}^{NM}$$

We want to know the distribution of the total transmittable bits from the initial state $M^N - 1$ to the absorbing state 0, so we take the inverse Laplace transform to get the PDF of the total transmittable bits distribution:

$$b(c) = \mathcal{L}^{-1}\{\boldsymbol{X}^{NM}(M^N - 1, 0)\} \tag{4.9}$$

The above analysis shows that to find the resulting pdf of the total transmitted rate we need to be able to derive Laplace transform of the related state transition PDF and and the inverse Laplace transform of the moment-generating function of the overall sum rate, which seems to be a formidable challenge. We avoid this difficulty by resorting to numerical methods to perform these transformations.

However, since we sample the PDF to do the numerical Laplace transform, the reconstructed distribution of total transmittable bits will be PMF (Possibility Mass Function) rather than PDF. This is not a serious issue since we can always do a simple integration to derive the CDF and derive the PDF if it exists.

4.5 Network Lifetime Analysis

In this section, we analyze the network lifetime of our proposed algorithm.

Since the rate constraint is no longer zero, we will need to add the requirement that the selected relay i^* must have an instantaneous link quality that is greater than the rate constraint. Otherwise, the transmission will not occur and the state will not change.

The transition probability p^c_{mn} will be different from the previous section and it is found as

$$p^c_{mn} = \begin{cases} \frac{1}{\upsilon_m}\left\{1 - \left[1 - \mathrm{e}^{-\left(\frac{2^{2C}-1}{\overline{\Gamma}_{\log_{N+1} m-n}}\right)}\right]^{\upsilon_m}\right\}, \text{ if } \log_{M+1} m - n \in \{0, 1, 2, \cdots, N-1\} \\ \sum_{i\in\delta_m} \frac{1}{\upsilon_m}\left(1 - \mathrm{e}^{-\left(\frac{2^{2C}-1}{\overline{\Gamma}_i}\right)}\right)^{\upsilon_m}, \text{ if } m = n \neq 0 \\ 0, \text{ otherwise} \end{cases} \tag{4.10}$$

The requirement for meeting the rate constraint contributes to $1-\left(1 - \mathrm{e}^{-\left(\frac{2^{2C}-1}{\overline{\Gamma}_{\log_{N+1} m-n}}\right)}\right)^{\upsilon_m}$ in p^c_{mn} to prevent the case when the selected relay does not fulfill the rate constraint. C is the rate constraint as introduced in chapter 2.

As we stated before, any state transition from state A_m to A_n is not possible unless $\log_{M+1} m - n \in \{0, 1, 2, \cdots, N-1\}$. However, with the rate constraint presents, we do allow the state m to transit back to itself (idle time slot), which results in $m = n \neq 0$. Any other transition is still impossible.

The state transition matrix $\boldsymbol{G}$ will be a $M^N \times M^N$ square matrix, which is defined to be:

$$\boldsymbol{G} = \begin{pmatrix} p^c_{(M^N-1)(M^N-1)} & \cdots & p^c_{(M^N-1)0} \\ \vdots & \ddots & \vdots \\ p^c_{0(M^N-1)} & \cdots & p^c_{00} \end{pmatrix},$$

$$\boldsymbol{G}^i = \begin{pmatrix} p^c_{(M^N-1)(M^N-1)} & \cdots & p^c_{(M^N-1)0} \\ \vdots & \ddots & \vdots \\ p^c_{0(M^N-1)} & \cdots & p^c_{00} \end{pmatrix}^i$$

We know from the definition of the transition probability matrix that $\boldsymbol{G}^i(m, n)$ denotes the possibility to transit from state m to state n in exactly i transmissions, and thus we are only interested in the $\boldsymbol{G}^i(M^N-1,0)$, since it represents the possibility to transit from the initial state to the absorbing state after i time slots.

As a result, we can derive the estimated network lifetime T by multiply the used time slots i with the probability to reach the absorbing state A_0 from the initial state A_{M^N-1} in exact i time slots and add them together

$$T = \sum_{i=1}^{\infty} i \times \boldsymbol{G}^i(M^N-1,0) \tag{4.11}$$

4.6 Constrained Transmittable Bits (Rates) Analysis

In this section, we will introduce how to analysis the total transmittable bits of our algorithm under rate constrained scenario.

We can, of course, still use (4.10) in this chapter for the state transition probability, but we would like to make some adjustments since the transition from any state m to itself does not accumulate any transmitted bits.

As a result, we modified the transition probability to exclude the case when the state transits to itself. Basically, we average out the probability of idling and recalculate the possibility of the scene when the transmission does occur.

$$p_{mn}^{cr} = \begin{cases} \dfrac{\frac{1}{v_m}\left(1-\left(1-\mathrm{e}^{-\left(\frac{2^{2C}-1}{\overline{\Gamma}_{\log_{N+1} m-n}}\right)}\right)^{v_m}\right)}{\sum_{i\in\delta_m}\frac{1}{v_m}\left(1-\left(1-\mathrm{e}^{-\left(\frac{2^{2C}-1}{\overline{\Gamma}_i}\right)}\right)^{v_m}\right)}, \text{ if } \log_{M+1} m-n \in \{0,1,2,\cdots,N-1\} \\ 0, \text{otherwise} \end{cases} \tag{4.12}$$

The denominator of (4.12) is the sum of the probabilities from all possible transmitting scenario (chosen relay i^* is going to fit the rate constraint C) in state A_m. Therefore, the division will tell us how much chance it is for a particular relay to be chosen as the transmitting relay i^* and exceed the rate constraint.

We next derive the state transition matrix as we always do:

$$\boldsymbol{H} = \begin{pmatrix} p^{cr}_{(M^N-1)(M^N-1)} & \cdots & p^{cr}_{(M^N-1)0} \\ \vdots & \ddots & \vdots \\ p^{cr}_{0(M^N-1)} & \cdots & p^{cr}_{00} \end{pmatrix} \tag{4.13}$$

In this new modified state transition probability, we exclude the probability of a state m to transit to itself and recalculate the possibility for each state transition under the condition that the state does change to another state other than itself.

The benefit of this modification is that we are going to be able to calculate the total transmittable bits in finite multiplications free from the unknown idle slot loops in each state.

Next, we will have to modify the distribution of total transmittable bits. First, we introduce a general fact about the distribution of a random variable.

If $f_X(x)$ is the PDF of the random variable X, then a truncated distribution where $x \in [a, b]$ is derived as $\frac{f_X(x)}{F_X(b)-F_X(a)}$. $F_X(x)$ is the CDF of the random variable X.

Since we introduce the rate constraint, the PDF of each one-time transmittable bits is a truncated version of (4.6) with $a = \left(1 - \mathrm{e}^{\frac{-(2^{2C}-1)}{\Gamma_{i^*}}}\right)^{\upsilon_m}$, $b = \infty$.

$$b^c_{mn}(c) = \frac{\frac{\upsilon_m}{\Gamma_{i^*}}\left(1 - \mathrm{e}^{-\frac{2^{2c}-1}{\Gamma_{i^*}}}\right)^{\upsilon_m - 1} \mathrm{e}^{-\frac{2^{2c}-1}{\Gamma_{i^*}}} 2^{2c+1}\log(2)}{1 - \left(1 - \mathrm{e}^{\frac{-(2^{2C}-1)}{\Gamma_{i^*}}}\right)^{\upsilon_m}} \tag{4.14}$$

After we obtain the transition probability and the distribution to each one-time transmission, we can apply similar method we used in 4.3.1.

First, we obtain the Laplace transform of $b^c_{mn}(c)$ to be

$$b^c_{mn}(s) = \mathcal{L}\{b^c_{mn}(c)\} \tag{4.15}$$

Then we can obtain the one-time transmittable rate distribution matrix by

$$\boldsymbol{Y}(s) = \begin{pmatrix} p^{cr}_{(M^N-1)(M^N-1)} b^c_{(M^N-1)(M^N-1)}(s) & \cdots & p^{cr}_{(M^N-1)0} b^c_{(M^N-1)0}(s) \\ \vdots & \ddots & \vdots \\ p^{cr}_{0(M^N-1)} b^c_{0(M^N-1)}(s) & \cdots & p^{cr}_{00} b^c_{00}(s) \end{pmatrix}$$

After NM transmissions, we get the distribution of the total transmittable rate matrix to be $\boldsymbol{Y}^{NM}$.

$$\boldsymbol{Y}^{NM}(s) = \begin{pmatrix} p^{cr}_{(M^N-1)(M^N-1)} b^{c}_{(M^N-1)(M^N-1)}(s) & \cdots & p^{cr}_{(M^N-1)0} b^{c}_{(M^N-1)0}(s) \\ \vdots & \ddots & \vdots \\ p^{cr}_{0(M^N-1)} b^{c}_{0(M^N-1)}(s) & \cdots & p^{cr}_{00} b^{c}_{00}(s) \end{pmatrix}^{NM} \tag{4.16}$$

Since we already handle and prevent the state from going to itself, we can enjoy a finite power NM instead of ∞ in (4.17). The distribution of total transmittable bits is:

$$b^c(c) = \mathcal{L}^{-1}\{\boldsymbol{Y}^{NM}(M^N - 1, 0)\} \tag{4.17}$$

4.7 Performance Analysis using Gaussian Approximation

In the previous section, we use the numerical integration methods to compute the Laplace transforms for (4.6) and (4.14). In this section, we adopt a Gaussian approximation approach on analysis of the total transmitted bits (rates) by assuming the corresponding random variables are Gaussian distributed. With this Gaussian assumption, we to evaluate the mean and variance for the transmitted bits associated with a state transition by using the PDF given in (4.14).

First of all, we would like to say that (4.14) is more general than (4.6), and thus we will only deal with (4.14) in this section. For the rate constraint free case, simply set the rate constraint C in (4.14) to zero to derive the (4.6).

For any two independent uncorrelated random variables X, Y, and let $Z = X + Y$, we can have the following property.

$$\overline{Z} = \overline{X} + \overline{Y} \tag{4.18}$$

$$\sigma^2(Z) = \sigma^2(X) + \sigma^2(Y) \tag{4.19}$$

$\overline{A}$ denotes the mean of the random variable A, and $\sigma(A)$ is the variance of the random variable A. It is also well known that for random variable A, we have the following property.

$$\sigma^2(A) = E[A^2] - \overline{A}^2 \tag{4.20}$$

By using the relationship $c = 0.5\log_2(1+x)$, $C = 0.5\log_2(1+\gamma_{cst})$, we can rewrite (4.14) as

$$b_{mn}^c(x) = \frac{\frac{\upsilon_m}{\overline{\Gamma}_{i^*}}\left(1-\mathrm{e}^{\frac{-x}{\overline{\Gamma}_{i^*}}}\right)^{\upsilon_m-1}\mathrm{e}^{\frac{-x}{\overline{\Gamma}_{i^*}}}}{1-\left(1-\mathrm{e}^{\frac{-(\gamma_{cst})}{\overline{\Gamma}_{i^*}}}\right)^{\upsilon_m}} \tag{4.21}$$

(4.21) is the PDF of the equivalent channel SNR for the selected relay i^*.

Let b_{mn}^c denotes the random variable of the total transmittable bits from state A_m to A_n, and b^c denotes the random variable of the transmittable bits for the entire network lifetime. We can find the mean of b_{mn}^c by applying integration by parts to be

$$\begin{aligned}
\overline{b_{mn}^c} &= \int_{\gamma_{cst}}^{\infty} 0.5\log_2(1+x)b_{mn}^c(x)\mathrm{d}x \\
&= \int_{\gamma_{cst}}^{\infty} \frac{0.5\log_2(1+x)\frac{\upsilon_m}{\overline{\Gamma}_{i^*}}\left(1-\mathrm{e}^{\frac{-x}{\overline{\Gamma}_{i^*}}}\right)^{\upsilon_m-1}\mathrm{e}^{\frac{-x}{\overline{\Gamma}_{i^*}}}}{1-\left(1-\mathrm{e}^{\frac{-(\gamma_{cst})}{\overline{\Gamma}_{i^*}}}\right)^{\upsilon_m}}\mathrm{d}x \\
&= \int_{\gamma_{cst}}^{\infty} \frac{0.5\log_2(1+x)\frac{\upsilon_m}{\overline{\Gamma}_{i^*}}\left(\sum_{k=0}^{\upsilon_m-1}\binom{\upsilon_m-1}{k}(-1)^k\mathrm{e}^{\frac{-kx}{\overline{\Gamma}_{i^*}}}\right)\mathrm{e}^{\frac{-x}{\overline{\Gamma}_{i^*}}}}{1-\left(1-\mathrm{e}^{\frac{-(\gamma_{cst})}{\overline{\Gamma}_{i^*}}}\right)^{\upsilon_m}}\mathrm{d}x \\
&= \frac{0.5}{1-\left(1-\mathrm{e}^{\frac{-(\gamma_{cst})}{\overline{\Gamma}_{i^*}}}\right)^{\upsilon_m}}\frac{\upsilon_m}{\overline{\Gamma}_{i^*}}\sum_{k=0}^{\upsilon_m-1}\binom{\upsilon_m-1}{k}\frac{(-1)^k}{\log 2} \\
&\left(\frac{k+1}{\overline{\Gamma}_{i^*}}\log(1+\gamma_{cst})\mathrm{e}^{-\frac{(k+1)\gamma_{cst}}{\overline{\Gamma}_{i^*}}} + \frac{\overline{\Gamma}_{i^*}\mathrm{e}^{\frac{k+1}{\overline{\Gamma}_{i^*}}}}{k+1}\mathbf{\Gamma}\left(0, \frac{(k+1)(1+\gamma_{cst})}{\overline{\Gamma}_{i^*}}\right)\right)
\end{aligned} \tag{4.22}$$

where γ_{cst} is the equivalent SNR constraint, $C = 0.5\log_2(1+\gamma_{cst})$.

We can also find the variance in a similar way. To find the variance, we want to derive $E[{b^c_{mn}}^2]$ first.

$$
\begin{aligned}
E[{b^c_{mn}}^2] &= \int_{\gamma_{cst}}^{\infty} \left(0.5\log_2(1+x)\right)^2 b^c_{mn}(x)\mathrm{d}x \\
&= \int_{\gamma_{cst}}^{\infty} \frac{\left(0.5\log_2(1+x)\right)^2 \frac{\upsilon_m}{\overline{\Gamma}_{i^*}} \left(1-\mathrm{e}^{\frac{-x}{\overline{\Gamma}_{i^*}}}\right)^{\upsilon_m-1} \mathrm{e}^{\frac{-x}{\overline{\Gamma}_{i^*}}}}{1-\left(1-\mathrm{e}^{\frac{-(\gamma_{cst})}{\overline{\Gamma}_{i^*}}}\right)^{\upsilon_m}}\mathrm{d}x \\
&= \int_{\gamma_{cst}}^{\infty} \frac{\left(0.5\log_2(1+x)\right)^2 \frac{\upsilon_m}{\overline{\Gamma}_{i^*}} \left(\sum_{k=0}^{\upsilon_m-1}\binom{\upsilon_m-1}{k}(-1)^k \mathrm{e}^{\frac{-kx}{\overline{\Gamma}_{i^*}}}\right) \mathrm{e}^{\frac{-x}{\overline{\Gamma}_{i^*}}}}{1-\left(1-\mathrm{e}^{\frac{-(\gamma_{cst})}{\overline{\Gamma}_{i^*}}}\right)^{\upsilon_m}}\mathrm{d}x
\end{aligned}
$$

Let $t = x - \gamma_{cst}$:

$$
= \frac{0.5\upsilon_m}{\overline{\Gamma}_{i^*}\left(1-\left(1-\mathrm{e}^{\frac{-(\gamma_{cst})}{\overline{\Gamma}_{i^*}}}\right)^{\upsilon_m}\right)} \sum_{k=0}^{\upsilon_m-1}\binom{\upsilon_m-1}{k}(-1)^k \int_0^{\infty} \left(\log_2(1+\gamma_{cst}+t)\right)^2 \mathrm{e}^{\frac{-(k+1)(t+\gamma_{cst})}{\overline{\Gamma}_{i^*}}}\mathrm{d}t \tag{4.23}
$$

We then use the Gauss Laguerre quadrature rule to calculate (4.23). The Gauss Laguerre quadrature rule is a numerical integration method for computing an integral with a semi-infinity integration interval. This numerical rule is given by

$$
\int_0^{\infty} f(x)\mathrm{e}^{-x}\mathrm{d}x \approx \sum_{i=1}^{n} w_i f(x_i)
$$

where $w_i = \frac{x_i}{(n+1)^2[L_{n+1}(x_1)]^2}$ and x_i is the i^{th} root of the Laguerre polynomial $L_n(x)$.

Using (4.20), we compute the variance of the one-time transmittable bits from state A_m to A_n by

$$
\sigma^2(b^c_{mn}) = E[{b^c_{mn}}^2] - \overline{b^c_{mn}}^2 \tag{4.24}
$$

After we find the variance for the transmittable bits for each state transition, we use Gaussian distributions to approximate every state transition, and transfer them to frequency domain using Fourier transform.

For state m to transit to state n, we denote the corresponding distribution in frequency domain as $\mathcal{W}_{mn}$

$$
\mathcal{W}_{mn} = \mathcal{F}\left\{\mathcal{N}\left(\overline{b^c_{mn}}, \sigma^2\left(b^c_{mn}\right)\right)\right\} \tag{4.25}
$$

$\mathcal{N}(\mu, \sigma^2)$ is a gaussian distribution with mean μ, and variance σ^2. $\mathcal{F}\{\}$ denotes Fourier transform.

We then multiply the acquired gaussian distribution in frequency domain into the state transition matrix.

$$\boldsymbol{Z} = \begin{pmatrix} p^{cr}_{(M^N-1)(M^N-1)}\mathcal{W}_{(M^N-1)(M^N-1)} & \cdots & p^{cr}_{(M^N-1)0}\mathcal{W}_{(M^N-1)0} \\ \vdots & \ddots & \vdots \\ p^{cr}_{0(M^N-1)}\mathcal{W}_{0(M^N-1)} & \cdots & p^{cr}_{00}\mathcal{W}_{00} \end{pmatrix}$$

At last, we get the reconstructed gaussian distribution by inverse Fourier transforming $\boldsymbol{Z}^{NM}((M^N-1)0)$, and the reconstructed Gaussian distribution I is found as

$$I = \mathcal{F}^{-1}\left\{\boldsymbol{Z}^{NM}(M^N-1, 0)\right\} \tag{4.26}$$

Chapter 5

Simulation Results and Discussions

We have shown in previous chapters that in theory, our relay selection algorithm should perform better than the mBRS and the Max-Life algorithm. In this chapter, we validate our claims by showing some numerical results obtained by computer simulations.

Before we start describing the simulation settings, we would like to emphasize that the transmitting power of the relays in the ensuing discourse has been normalized such that noise variance is always 1.

We assume that there are $N = 5$ relays and the average received SNR at each relay is 30 dB above the noise floor. The distance between the relay and the destination is Rayleigh distributed with a mean of 355 meters.

To make the comparison fair, we assume that the number of transmit opportunities for each relay to be inversely proportional to the relay transmitting power. The transmit opportunities for each relay ranges from 50 to 600 incremented by 50. Furthermore, the normalized transmitting powers are $90 - 10 \times \log_{10}(1, 2, \cdots, 12)$ dB watt, respectively. The rate constraint is set to be 0.7 bits/sec/Hz.

We can see that the same transmitting power for each relay is inversely proportional to the transmitting opportunities. This will in turn gives us a fixed total energy charged for each relay to form a fair comparison.

We adopt the log-distance path loss model for the transmission from the relay to the destination as shown in (2.1), and the path loss exponent is set to be 3. The distance

Table 5.1: Simulation Parameters.

Average received SNR from the source (at Relays)	30dB
Normalized Transmitting power of each relay	$90 - 10 \times \log_{10}(1, 2, \cdots, 12)$ dBw
Transmitting Opportunities of each relay	$50 - 600$ times with 50-time increment
Number of relays	5
Path loss exponent	3
Rate Constraint	0.7 bits/sec/Hz
Distance between Relays to Destination	Rayleigh distributed (mean 355 meters)

between each relay to destination is a Rayleigh distributed random variable with mean 355 meters. On the other hand, we assume the distance between the source and all the relays are the same, and produces a 30 dB average received SNR at all relays for simplicity.

The details of the simulation parameters are listed in the Tab 5.1

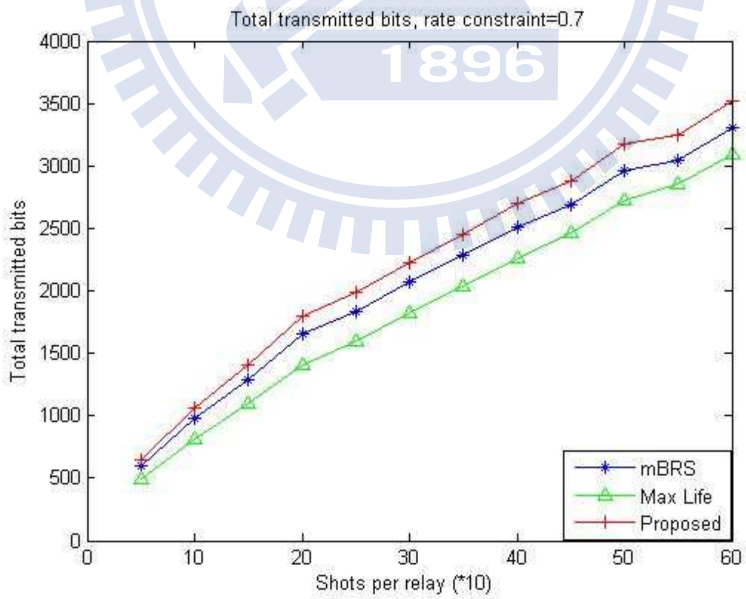

Figure 5.1: Transmitted bits.

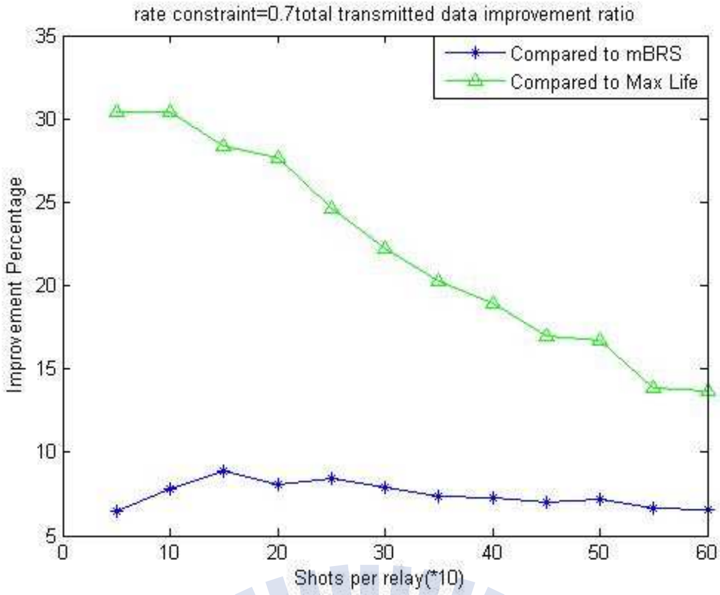

Figure 5.2: Improvement ratio of transmitted bits.

In Fig. 5.1, and Fig. 5.2, we can see that our proposed algorithm outperform the modified conventional BRS algorithm and the Max Life algorithm in the total transmitted bits. This result comes with no surprise since our proposed algorithm utilize the relay's energy when it is at its best moments with respect to its statistical distribution. mBRS comes second, since it uses only instantaneous channel status to do the decision which is not really a good strategy. The Max Life approach has the poorest performance as it uses no channel information at all.

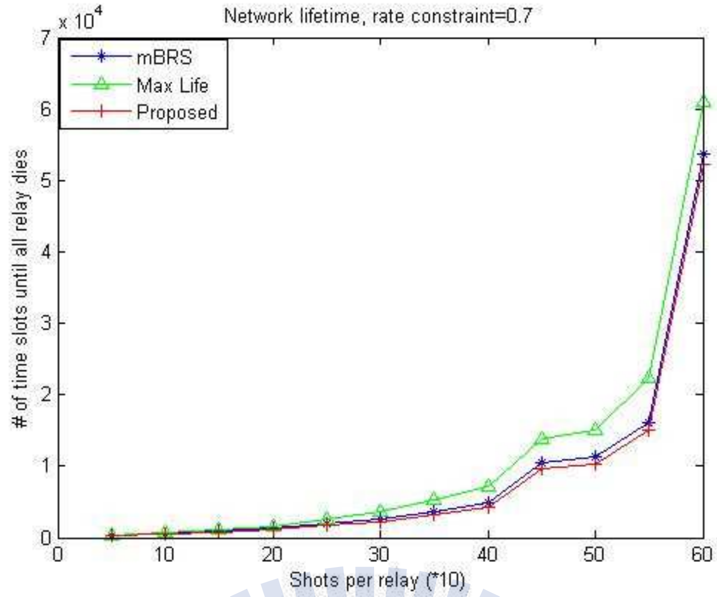

Figure 5.3: Network Lifetime.

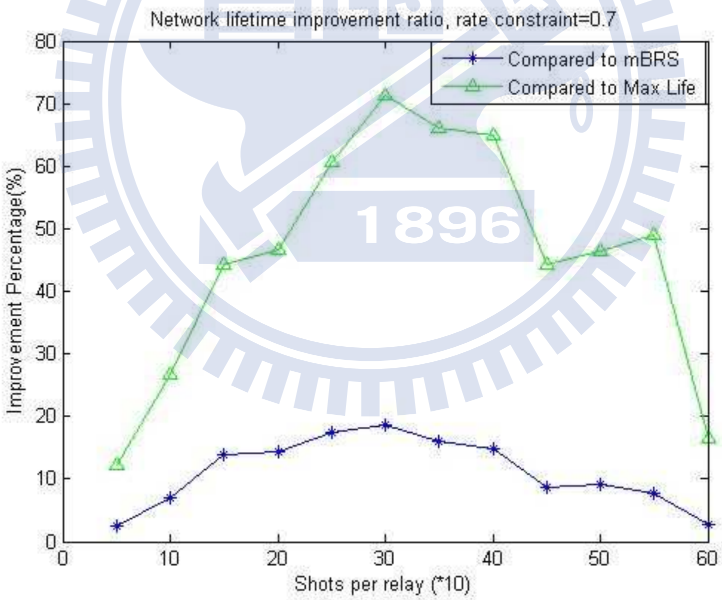

Figure 5.4: Improvement ratio of Network Lifetime.

We then examine the network lifetime of each of the three algorithm. Note that although in many cases one wants to maximize the network lifetime, a longer lifetime is often preferred, in this case, it is not. Longer network lifetime in our studies implies more idle time slots which is not desired. We can see in Fig. 5.3 and Fig. 5.4 that the proposed algorithm has the shortest network lifetime and therefore has the least idle time slots among the three.

Although we do not enforce a hard delay constraint in this research, it is very vital to keep the count of the idle time slots low for practical situations. For example, the request for the realtime traffic. In Fig. 5.4, we see the consequence of mBRS leaving tons of ill-performance relays to the end of the network lifetime and causing more excessive idle time slots compared to our proposed algorithm.

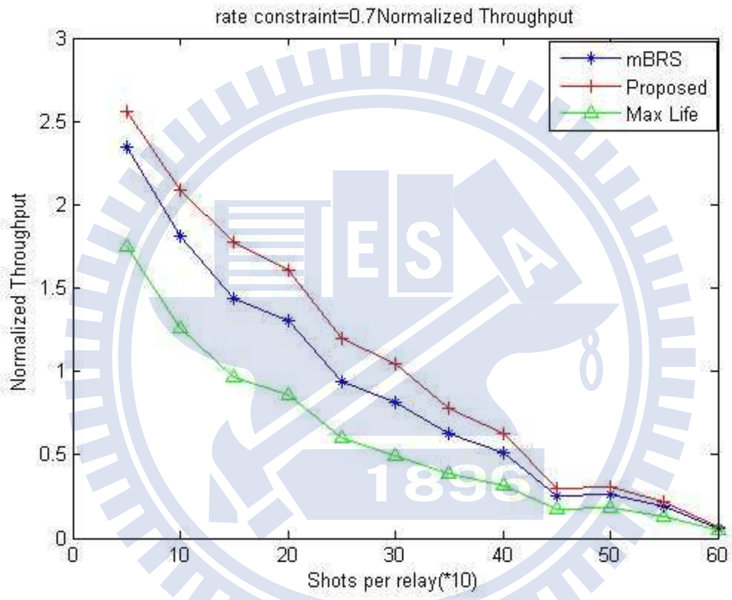

Figure 5.5: Throughput (Optimization Goal).

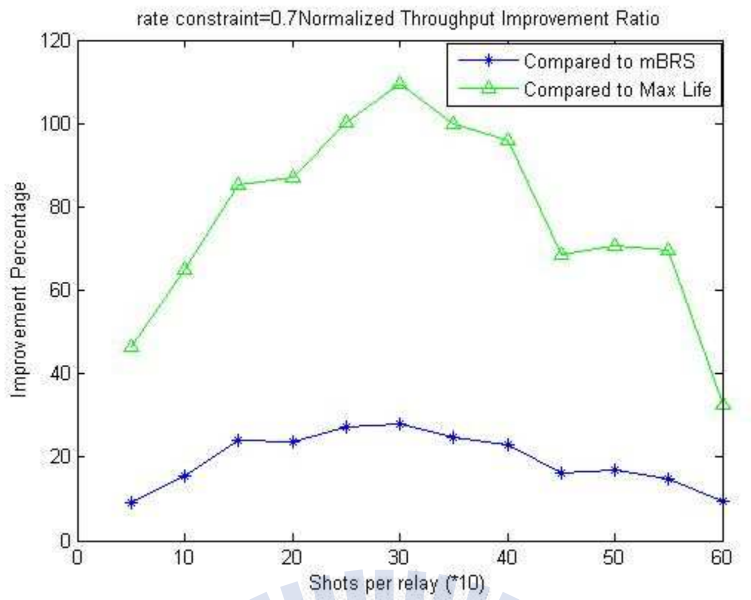

Figure 5.6: Improvement ratio of Throughput (Optimization Goal).

Next, we want to observe how the three algorithm scores our optimization goal defined in equation (2.8). In Fig. 5.5 and Fig. 5.6, we can see that our proposed algorithm still stands out from the rest two algorithm, and achieve better results towards our optimization goal, which again proves our proposed algorithm is a better option to choose when the relays are energy constrained.

We can see in Fig. 5.6, our proposed algorithm has the best advantage against the other two algorithm when the transmission probability for each relay is medium. For larger transmission opportunities, the transmitting power of each relay is reduced, and thus causing difficulties to meet the rate constraint. In the cases when the transmission power is so low that meeting the rate constraint is highly unlikely, there will be almost equal performance for any algorithms, since there will be most likely only one eligible relay or none for selection.

We also compare the theoretical results we derived in the previous chapter to the simulation results under the the assumption that there is 2 relays and each relay has 2 transmitting opportunities.

First, we use (4.9) to obtain the theoretical total transmittable bits for the constraint-free case, and (4.17) to obtain the theoretical value for the constrained one. The result is in Tab. 5.2.

We also use (4.11) to estimate the network lifetime and compare the results to the simulated value. The estimation of the network lifetime is presented in Tab. 5.3.

Table 5.2: Transmittable bits: Theoretical value using (4.9), (4.17) vs. Simulation.

Rate constraint(bits/sec/Hz)	0	0.3	0.6	0.9
Theoretical mean (bits)	12.3878	12.4138	12.4502	12.5013
Simulated results (bits)	12.2875	12.3022	12.5163	12.5414
Error (%)	0.8	0.9	-0.5	-0.3

Table 5.3: Network Lifetime: Theoretical value using (4.11) vs. Simulation.

Rate constraint(bits/sec/Hz)	0	0.7	1	2
Theoretical iteration (times)	4	4.03	4.0628	4.4073
Simulated results (times)	4	4.01	4.08	4.45
Error (%)	0	0.4	-0.4	-0.9

In both Tab. 5.2 and Tab. 5.3, we can see that our theoretical value is very close to the actual simulation results. This again proves the accuracy of the numerical Laplace transform is acceptable.

In the last section of the chapter 4, we mentioned that it is also possible to estimate the total transmittable bits by using Gaussian approximation. The estimated total transmittable rate using (4.26) is provided in the following table.

Table 5.4: Transmittable bits: Theoretical value using (4.26) vs. Simulation.

Rate constraint(bits/sec/Hz)	0	0.3	0.6	0.9
Theoretical mean (bits)	12.3878	12.4139	12.4502	12.5013
Simulated results (bits)	12.2875	12.3022	12.5163	12.5414
Error (%)	0.8	0.9	-0.5	-0.3

It is clear that the Gaussian approximation also gives us a very precise prediction.

The reconstructed CDF using 4.26 versus the simulated CDF is presented in the following figure.

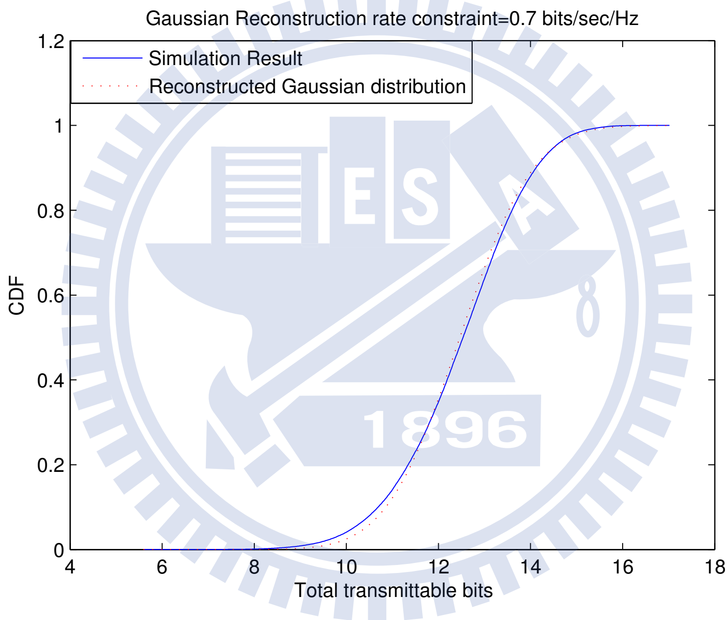

Figure 5.7: The CDF of the reconstructed total transmittable bits distribution.

We can see that our reconstructed CDF matches the CDF from the simulation very well, and this again proves the Gaussian approximation is a very good approximation with little error for our scheme.

We have already given a way to reconstruct the total transmittable bits distribution with excellent precision. With this powerful tool in hand, we can derive every property we would like to explore.

Chapter 6

Conclusions and Future Works

As our demands toward a more complete and continuous communication service increases, we proposed a very useful and robust algorithm to use when the relays are energy limited in this research.

When a severe natural disaster strikes, by using portable battery-powered relays, we can recover the basic communication network and establish a lifeline for the victims to have a fighting chance. On the other hand, we can also establish communications deep in the desert, mountains, or even ocean floors to provide all kinds of services imaginable. For example, we can allow the people who are lost in the desert to use their cell phone to directly talk to the rescuer, and maybe we can even triangulate their positions to save their lives.

Our proposed algorithm gives us a more power-efficient choice when we have to deploy relays with energy constraints, and achieve a higher throughput. We overcome the defects that conventional BRS brings from choosing blindly the best instantaneous channel SNR, and outperform it under all circumstances in the simulation.

We also manage to analyze our proposed algorithm, and predict its performance in both the total transmittable bits and the network lifetime. The precision of our prediction is incredible as shown in the bottom of the previous chapter.

All in all, we are really confident that our proposed algorithm is the promising candidate when we deploy the energy constrained relays into modern communication system.

For our future work, we will try to find the true optimal selection algorithm for the model with energy-limited relays. This requires a tons of optimization skills, mathematical techniques, and, perhaps, some good lucks. However, we can anticipate that the optimal selection algorithm may be very complex, and our proposed algorithm may well still be a more practical solution with an acceptable performance loss.

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程式語言	Matlab 、 C、Verilog
相關證照	TOEIC 945 分金色證書

四、已修畢之相關課程

專業課程類別	課程列表		
數理基礎課程	微積分	微分方程	線性代數
	機率	普通物理	普通化學
	偏微分方程與複變函數		
基礎專業課程(I)	訊號與系統	電磁學	電路學
	電子學	數位電路分析與設計	邏輯設計
基礎專業課程(II)	通訊概論	數位訊號處理概論	經濟學原理
	通訊系統		
電腦&實驗課程	計算機網路概論	計算機程式設計	計算機結構
	電路學實驗	普通化學實驗	通訊系統實驗
	光電實驗	邏輯設計實驗	普通物理實驗

研究所專業課程	排隊理論	數位通訊	數位訊號處理
	隨機過程	適應性訊號處理	計算機網路
	無線通訊	檢測與估計理論	
旁聽過課程	消息理論		

五、研究所個人經歷

1987 年，12 月，生於台灣台南。

從小就生長在以熱情著名的南台灣，養成了我熱心助人的性格，也對人文有著另一個比一般人更深的關懷，這份情感直到日後到北部讀書都一直沒有改變。

令我印像深刻的求學心路大概得從國中說起，那時我就讀的是以升學為導向的台南市立復興國中，從國中開始就發現週遭的同學都為了考高中而付出自己所有的心血和努力，在這種競爭的環境裡，我也不願意服輸，憑著自己的努力一步一步的往高中邁進，雖然辛苦，但是充實。

在台南一中的求學階段，突然從男女合班的環境轉換到了一個全男生的學校，一開始還真的不太適應，隨後我便發現，在台南一中的求學期間尤於是全男生的環境，也讓我更能和同學培養出更深厚的友情，雖然如此環境也造就了我高中的感情生活留下了空白，但確實也讓我最後考上了不錯的大學。但全盤加減後，我還是覺得應該要男女合校才是正確的選擇，有很多和異性相處的能力是不可以不學習的，也不應該被犧牲的。

大學時，我選擇了清大電機系，一開始只是因為它的分數和我的學測分數最接近，但進去以後發現了清大電機系的廣大，從大的都市配電系統，到小的積體電路，清大電機系都有完整的課程可以供我選擇，最後尤於上了蔡育仁老師一堂通訊系統的課，讓我下定決心我這輩子就是想走通訊相關的領域了。

我是用推甄上交大電信所的，當時就聽說蘇育德博士的實驗室是非常精實的實驗室，也承蒙蘇老師不嫌棄，因此在研究所這兩年就在蘇老師的指導下進行學術上的研究，這兩年讓我看到了蘇老師在學術上的認真及專業，老師展現的是一種精益求精的精神，成功的路上總是崎嶇，甚至有時我們無法到達成功，但其實走在路上的態度才是最重要的，如何保持著一種樂觀進取的精神，不因為一兩次的失敗就挫敗而放棄，這是蘇老師教給我的人生知識，也是我覺得除了學術知識以外我從蘇老師手上學到的最寶貴的人生資產。

展望我的未來，我相信前面是一片的光明，就算有時太陽不露臉，那也沒關系，我會拿出我的手電筒勇往直前，我真的要感謝很多人，無論是親人、好朋友、抑或是在我學生生涯最後兩年讓我獲益良多的蘇育德博士，如果將來我有那麼一點點的成就，金錢上也好，非金錢能衡量的也好，你們都絕對是那個幫我打下堅固基石的貴人，且長存於我心中！