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Risk measurement and risk identification for BOT projects: A multi-attribute utility approach

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ABSTRACT

This paper aims to identify and assess the potential risks faced by private sectors in holding BOT projects through the risk assessment model developed herein. The multi-attribute utility function and aggregation utility are established using the multi-attribute utility theory to evaluate the risk state of each uncertainty, and in turn to determine whether such an uncertainty is a risk factor or not from the negotiator group's viewpoint. This model shows that the uncertainty is regarded as a risk factor only when the aggregation utility value is less than the average aggregation utility value when the outcome, attribute, and states of a factor as well as its occurrence probability are all independent. A numerical example is also utilized to demonstrate the application of the developed risk assessment model. Results of the numerical example reveal that the concession period of a BOT project is the primary risk factor whereas the foreign exchange ratio is the secondary risk factor. Accordingly, the concession period dominates the negotiation results of BOT projects.

1. Introduction

Build, Operate and Transfer (BOT) is an approach the private sector utilizes to obtain a granted concession for completing a specific project independently. However, the ownership of the project has to be returned to the public sector once it is entirely completed [1,2]. To carry out a BOT project, both sectors take an advantage of risk sharing from each other. For the public sector, it is already known that its inherent risks in financial, technological as well as managerial problems have been greatly reduced. For the private sector, by contrast, what is known is the magnitude of concession contract negotiation and what is unknown is the critical risk among many uncertain factors [2]. The issues related to the risks for the BOT project, however, still remain unclear. Therefore, this study tries to identify what the risks are and assess those the private sector faces in holding a BOT project.

Risk assessment predominates the success in investment and contract negotiation [2–6]. Specifically, Tiong [4], Levitt et al. [5], and Jaselskis and Russell [6] found that risk assessment serves as a significant incentive in dealing with engineering contracts whereas Tiong [4] and Sidney [7] also showed its importance in bidding activity and contract negotiation for the BOT projects.

Risk analysis studies in the current literature can be classified into two broad categories, namely qualitative and quantitative analysis [2]. For instance, Tiong [3] and Walker and Smith [2] identified various risk factors such as political risk, commercial risk, legislative risk, operational risk and risk of construction completion for BOT projects through qualitative analysis. However, such an approach can hardly explain measures of effectiveness (MOE), the level of risk and the probability of risk occurrence. Quantitative analysis, including statistical analysis [6,8], financial analysis [9], engineering economic

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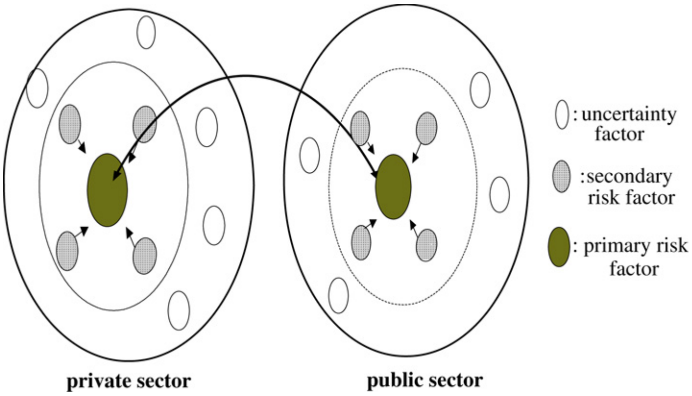

Fig. 1. Problems of risk importance measure.

analysis [10], and the weighted method [11], on the other hand, has been broadly applied to risk evaluation of project investment. For example, Hwang [1] investigated the relationship between the level of risk and the investment return rate on the BOT project using property rights and transaction cost theories assuming the probability of risk to be of normal distribution. David [12] further considered the uncertainty of electrical energy supplies in determining the price of electricity and penalty charge between investor and host utility. Additionally, Hwee and Tiong [13] presented a cash flow forecasting model to analyze the impact risk factors. Ye and Tiong [14] have also presented appropriate combination mechanisms to manage key risks of BOT projects through a simulation method to analyze the risk of determining tariff magnitude, the choice of tariff structure, and the design of adjustment mechanisms of minimum operation level for BOT projects during the operation period.

From the above, the financial risk evaluation of BOT projects and risk factors identification on BOT contract can be observed as two major research streams. Some widely used methods such as Net Present Value (NPV), Benefit–Cost (B/C ratio) and Internal Rate of Return (IRR) can be employed to measure financial risk and evaluate competitive tender concession of BOT projects [15]. However, these methods all have their own inherent weakness in the estimation of future cash flow and are improper for extremely long-term BOT project assessment [2,7,15]. The utility theory is another approach for assessing risk or evaluating tender concession contract for BOT projects [16,15]. Kang et al. [17] developed a multi-objection dynamic programming model to imitate negotiators' behaviors using utility function and dynamic programming approaches; and the model was employed to assess the risk in concession contract for BOT projects. Therefore, this study adopted the utility approach and the multi-attribute utility model [18], with some modifications to the risk identification and assessment for BOT projects.

The remainder of the paper is structured as follows. Section 2 outlines the research problems in risk importance measure on BOT projects. Section 3 describes the procedure of model construction and solution algorithm. Section 4 presents a numerical example. Finally, a discussion is presented and conclusions are drawn in Section 5.

2. Problems in risk important measure

The host utility and private sectors will face many uncertain factors associated with BOT projects during the planning, construction, and operation periods [2,3]. As to the varied uncertainty factors such as land acquisition delay, completion delay in construction, concession period change, interest rate, toll regulation, political change, some may become risk factors and some will remain uncertain when BOT projects are being undertaken [2]. The aim of a BOT concession agreement therefore is to reduce those mentioned risks produced through contract negotiation. However, it is a critical step for two parties to assess risks among many uncertain factors, and then to determine the primary and secondary risky factors of concern to individual parties before the concession negotiation.

Usually, the primary risk factors of BOT concession contract that the host utility or private sector is concerned about are the first items to be negotiated items in concession negotiation and in turn the secondary risk factors are the items negotiated next. Negotiators of the private sector or host utility have to first evaluate risks for many uncertain factors that they are concerned about before determining primary and secondary factors.

The way to achieve risk sharing is to transfer risks to the party with a good financial condition. However, things will not always happen as expected. If any side rejects or disputes with what has been negotiated, the negotiation fails. Accordingly, the first problem concerns the "negotiator controllability", while the second issue is associated with "risk importance measure". Fig. 1 presents the concept mentioned above.

3. Model construction

As seen in Fig. 1, primary and secondary risk factors of BOT concession are determined through a negotiators' evaluation made by either the private or public party. Thus, the preference of negotiators and cost of agents will affect the risk level

Table 1
Relationship between state, attribute outcome, and assessment value.

Event f	State (S)					
	s_1,	s_2,	…,	s_j,	…,	s_n
Outcome of attribute x	x_1,	x_2,	…,	x_j,	…,	x_n
Probability p	p_1,	p_2,	…,	p_j,	…,	p_n
Assessment value υ	$\upsilon_1(x_1)$,	$\upsilon_2(x_2)$,	…,	$\upsilon_j(x_j)$,	…,	$\upsilon_n(x_n)$

for uncertainty factors. Therefore, the assumptions made by the proposed model are as follows. (1) The cost of agents is independent of the utility of negotiators. (2) The negotiator makes decisions rationally, i.e. she/he optimizes her/his expected utility in a risky environment. Assumption (1) indicates that the negotiator is authorized by a specific organization; and if the agency cost is not zero, adverse-selection might occur in the negotiation process. Assumption (2) satisfies the principle of maximizing utility for a negotiator and negotiator group. It implies that the utility function satisfies the Von Neumann–Morgenstern (V–M) axioms [19].

3.1. Definition of risk state

To determinate risks and to identify primary or secondary risk factors for BOT projects, we first define the risk state and then construct the risk assessment model for negotiation.

(1) Definition of risk state

Many different definitions of risk have been made by previous studies. For instance, Rowe [20] defines risk as "The potential for unwanted negative consequences of an event or activity". Meanwhile, Rescher [21] suggests that "Risk is the chance of a negative outcome", and Lowrance [22] defines risk as "A measure of the probability and severity of adverse effects". Cooper and Chapman [10] define risk as "Exposure to the possibility of economic or financial loss or gain, physical damage or injury, or delay, as a consequence of the uncertainty associated with pursuing a particular course of action". Keeney and Raiffa [18] apply the expected utility value to assess the risk shelter, while Jia and Dyer [23] develop a risk assessment model from the utility theory, and define risk as negative expected utility in preference, which implies the concept of risk loss.

According to these definitions, the concept of risk includes two basic elements, one is "the possibility of an event", and the other is "the potential consequences". We can clearly find that risk can be measured via probability, expected value, variance and so on [8,10,20,21]. Schmeidler and Wakker [19] have pointed out the expected utility theory can be employed to analyze choice under uncertain environment and to measure risk for decision-making. Hence, Kang et al. [17] also used the expected utility to simulate risk assessment performed by negotiators, and they defined the "risk-state" of a factor as follows: "For the decision-maker, the actual utility of a specific factor under a certain state is less than the averaged utility for all the states".

(2) The assessment model

Appendix A shows the symbols and notations of variables for measuring the risk-state for a factor. Assume that an event f has n uncertain states, $s_1, s_2, \ldots, s_j, \ldots, s_n$, and an outcome of attribute x. Every state corresponds to $x, x_1, x_2, \ldots, x_j, \ldots, x_n$, and every x_j corresponds to $\upsilon_j(x_j)$, where $\upsilon_j(x_j)$ is the value assessed by negotiators corresponding to the outcome of an attribute. In addition, p_j indicates the occurrence probability of state j. Let $p_j \times \upsilon_j(x_j)$ be the utility value for s_j and x_j, $j = 1, 2, \ldots, n$.

Table 1 shows the relationship between event state, attribute outcome, and assessment value. Before exploring the risk, this study first defines the risk-state according to the concept of Table 1. The definition of risk-state proposed by Kang et al. [17] is adopted as defined in Eq. (1).

$$u^f(s_j) = p_j \upsilon_j(x_j) < \bar{u}(s), \quad \forall j = 1, 2, \ldots, n \tag{1}$$

where $u^f(s_j)$ is the utility value of the negotiator regarding state j for a specific event. Since $\upsilon_j(x_j) \in [0, 1]$ and $p_j \in [0, 1]$, hence $u^f(s_j) \in [0, 1]$. In addition, $\bar{u}(s)$ represents the averaged utility value of $u^f(s_j)$, where $\bar{u}(s) = \frac{1}{n}\sum_j^n u(s_j)$. Eq. (1) shows a risk-state under state s_j through a negotiator's evaluation of a specific event if $u^f(s_j) < \bar{u}(s)$. Consider a factor f with m outcomes for multiple attribute, x_i, $i = 1, 2, \ldots, m$ and n states, s_j, $j = 1, 2, \ldots, n$. Table 2 shows the relationship between states, outcome of attribute, and utility for a factor f.

In Table 2, for each state and outcome of attribute, $u(x_i, s_j)$ is referred to as the level evaluated by each negotiator. As for factor f, let $\upsilon^f(x_i, s_j)$ represent the assessment value of a negotiator regarding outcome of attribute x_1 at state s_j for factor f. Moreover, we assume that the relationship between x_i, s_j, and p_{ij} is mutually independent, and we compute the averaged utility value for all states using $\bar{u}^f(x_i, \bar{s}) = \frac{1}{n}\sum_{j=1}^n p_{ij} \times \upsilon(x_i, s_j)$. Furthermore, the averaged utility value of all outcomes of attribute and for all states of factor f, $\bar{u}^f(\bar{x}, \bar{s})$, can also be obtained using $\bar{u}^f(\bar{x}, \bar{s}) = \frac{1}{mn}\sum_{i=1}^m \sum_{j=1}^n p_{ij} \times \upsilon(x_i, s_j)$. Similarly, according to Eq. (1), the risk-state for multiple attributes can be defined as Eq. (2).

$$u^f(x_i, s_j) = p_{ij} \times \upsilon(x_i, s_j) < \bar{u}^f(\bar{x}, \bar{s}), \quad \forall i, j \tag{2}$$

Table 2
Relationship between state, outcome of attribute, and utility.

Event f	States S
	$s_1, s_2, \ldots, s_j, \ldots, s_n$
Outcome of Attribute X	$x_{11}, x_{12}, \ldots, x_{1j}, \ldots, x_{1n}$ $(p_{11}), (p_{12}), \ldots, (p_{1j}), \ldots, (p_{1n})$ $\vdots \quad \vdots \quad \vdots \quad \vdots$ $x_{i1}, x_{i2}, \ldots, x_{ij}, \ldots, x_{in}$ $(p_{i1}), (p_{i2}), \ldots, (p_{ij}), \ldots, (p_{in})$ $\vdots \quad \vdots \quad \vdots \quad \vdots$ $x_{m1}, x_{m2}, \ldots, x_{mj}, \ldots, x_{mn}$ $(p_{m1}), (p_{m2}), \ldots, (p_{mj}), \ldots, (p_{mn})$
Assessment value $v(x_i, s_j)$	$v(x_1, s_1), \ldots, v(x_1, s_j), \ldots, v(x_1, s_n)$ $\vdots \quad \vdots \quad \vdots$ $v(x_i, s_1), \ldots, v(x_i, s_j), \ldots, v(x_i, s_n)$ $\vdots \quad \vdots \quad \vdots$ $u(x_m, s_1), \ldots, u(x_m, s_j), \ldots, u(x_m, s_n)$

$p_{ij} = \text{Prob}(x_i, s_j)$ denotes the probability corresponding to each state and outcome for multi-attribute, where $0 \le p_{ij} \le 1$.

where $u^f(x_i, s_j)$ is risk-state of outcome of attribute i at state j for factor f. Eq. (2) presents the utility assessment of a negotiator for outcome of attribute x_i at state j of factor f. If $u^f(x_i, s_j) < \bar{u}^f(\bar{x}, \bar{s})$, it indicates that the negotiator believes that this situation represents a risk-state for the outcome of attribute corresponding to state s_j for factor f. Otherwise, this situation represents the non-risk-state.

If $x_i = 1$ in Eq. (2), i.e., $\bar{u}^f(\bar{x}, \bar{s}) = \bar{u}^f(1, \bar{s})$, the multi-attribute case for the risk-state assessment model of Eq. (2) will become the single-attribute model, as illustrated in Eq. (1). Eq. (1) shows a risk-state under state s_j through a negotiator's evaluation of a specific event if $u^f(x_j) < \bar{u}(s)$. According to Eq. (1), although $0 \le v_j(x_j) \le 1$, $0 \le p_j \le 1$, the value of $\bar{u}(s)$ may be greater than 1. To simplify comparison, we utilize the concept of transformation utility proposed by Keeney and Raiffa to modify Eq. (1) as follows:

$$u_q^f(s_j) = \begin{cases} \dfrac{p_j \times v(s_j) - \min\limits_j\{u^f(s_j)\}}{\max\limits_j\{u^f(s_j)\} - \min\limits_j\{u^f(s_j)\}}, & \text{if } \max\limits_j\{u^f(s_j)\} \ne \min\limits_j\{u^f(s_j)\}, \quad \forall j \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where $u_q^f(s_j)$ is the utility value of the qth negotiator for attribute-outcome x_j of event f and is the normalized utility value. q is the number of negotiators, $q = 1, 2, \ldots, Q$. If $\max_j\{u^f(s_j)\} = \min_j\{u^f(s_j)\}$, then let $u_q^f(s_j) = 0$ which is the non-risk state for attribute-outcome x_j. $u_q^f(s_j) = 1$ as $\min_j\{u^f(s_j)\} = 0$ and $\max_j\{u^f(s_j)\} = p_j v(s_j)$; therefore $u_q^f(x_j) \in [0, 1]$. Eq. (3) describes that the outcome of attribute x_j for the specific event under state s_j is a risk-state according to the evaluation of the qth negotiator. Similarly, we can define the normalized utility for the multi-attribute case as follows:

$$u_q^f(x_i, s_j) = \begin{cases} \dfrac{p_{ij} \times v(x_i, s_j) - \min\limits_j\{u^f(x_i, s_j)\}}{\max\limits_j\{u^f(x_i, s_j)\} - \min\limits_j\{u^f(x_i, s_j)\}}, & \text{if } \max\limits_j\{u^f(x_i, s_j)\} \ne \min\limits_j\{u^f(x_i, s_j)\}, \quad \forall i, j \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

where $u_q^f(x_i, s_j)$ is a normalized utility value ranging between 0 and 1.

Take the land acquisition event of the High Speed Rail (HSR) BOT project as an example. If the government cannot acquire the land for the route and stations in time, the company cannot start construction on schedule, and delays on both construction and operation will then occur. Let the delay time be $0, 1, 2, \ldots, 7$ years, and the increased construction cost, x, be the outcome of the attribute due to a year-long delay. Given the assessment value $v(x_j)$ of each negotiator, and the probability value p of event occurrence, the data are shown in Table 3.

We can obtain the utility values using $p \times v(x_j)$ as 0.2009, 0.0241, . . ., and 0.0004, respectively. As seen in the table, the maximum and minimum value of $p \times v(x_j)$ are 0.2009 and 0.0004, respectively. Substituting the data of Table 2 into Eq. (2), we can get $u_1^f(s_1) = \frac{0.2009-0.0004}{0.2009-0.0004} = 1$, $u_1^f(s_2) = \frac{0.0241-0.0004}{0.2009-0.0004} = 0.1182, \ldots, u_1^f(s_6) = 0.0262$, $u_1^f(s_7) = 0$ and the mean utility value $\bar{u}(s) = 0.21855$. This shows that state 0, that is no delay, is a non-risk state because $u_1^f(s_1) > \bar{u}(s)$; while the other states are risk-states.

Table 3
Data for the example of land acquisition event of the HSR BOT project.

Land acquisition event	State (delay in number of years)							
1st negotiator	0	1	2	3	4	5	6	7
Outcome of attribute: Increased construction cost (NT$: 100 million dollars)	0	2.98	4.45	6.89	9.68	11.98	15.45	20.36
Assessment value $v(x_j)$	0.95	0.82	0.64	0.55	0.32	0.11	0.06	0.005
probability p	0.2115	0.0294	0.0602	0.0762	0.0995	0.097	0.09448	0.0806
$p \times v(x_j)$	0.2009	0.0241	0.0385	0.0419	0.0318	0.0107	0.0057	0.0004
$u_1^f(s_j)$	1.0000	0.1182	0.1898	0.2066	0.1565	0.0511	0.0262	0.0000

3.2. Group aggregation utility function

As described in Section 2, the risk-state assessment for each state and outcome of attribute factor f have been evaluated by each negotiator. However, the risk-state assessment factor f should be evaluated through some negotiators from the BOT sector or host utility. Thus, we must develop a further group utility value to assess the risk-state for factor f from the viewpoint of group sectors or the host utility.

We assume that there are q negotiators for the private group or host utility group. According to Eq. (4), let $u_q^f(x_i, s_j)$ be a normalized utility value evaluated by the qth negotiator. Thus, it can be employed to evaluate risk-state or non-risk-state for factor f through the qth negotiator. Additionally, to determine both primary and secondary risk factors as described in Fig. 1, the process of concession contract negotiation involves group participation, one is the private group and the other is the host utility. Both negotiation groups can develop the group utility function to represent the private group or host utility in order to measure the risks that occur in BOT projects.

According to the concept of multi-attribute utility theory (MAU) [18] and multi-attribute risk utility function [24], the MAU theory can be employed to assess risk and reflect the risk preference of negotiator for a construction project [16]. This model can be divided into the additive and multiplicative models. The multiplicative model can be expressed as Eq. (5).

$$U(x) = \sum_{i=1}^{n} k_i U_i(x_i) + k \sum_{\substack{i=1 \\ j>i}}^{n} k_i k_j U_i(x_i) U_j(x_j) + \cdots + k^{n-1} k_1 k_2 \cdots k_n U_1(x_1) U_2(x_2) \cdots U_n(x_n). \tag{5}$$

Eq. (5) is a generalized representation of the MAU model, the MAU is an additive model as $\sum_i k_i = 1$; otherwise, as $\sum_i k_i \neq 1$, the MAU model is a multiplicative model and there are q negotiators in the BOT company. According to the concept of Eq. (5), the multi-attribute utility function of negotiators can then be expressed as follows.

$$\begin{aligned} GU_q^f(x_i, s_j) = {} & \sum_{q=1}^{Q} k_q (u_q^f(x_i, s_j)) + k \sum_{\substack{q=1 \\ a>q}}^{Q} k_q k_a u_q^f(x_i, s_x j) u_a^f(x_i, s_j) + \cdots \\ & + k^{Q-1} k_1 k_2 \cdots k_Q u_1^f(x_i, s_j) u_2^f(x_i, s_j) \cdots u_Q^f(x_i, s_j) \end{aligned} \tag{6}$$

where $GU_q^f(x_i, s_j)$ represents the mutual assessment result of the negotiation team at a specific state for factor f in Eq. (6). Since $0 \le u_q^f(x_i, s_j) \le 1$ and $0 \le k_q \le 1$, then $0 \le GU_q^f(x_i, s_j) \le 1$. Using the definition of risk-state mentioned previously, we compute $\bar{u}_q^f(\bar{x}, \bar{s})$ where $\bar{u}_q^f(\bar{x}, \bar{s})$ is the averaged utility value for all (x_i, s_j) of factor f. Moreover, we can obtain $GU_q^f(x_i, s_j)$ using Eq. (6). It indicates that the negotiation group believes that a risk-state exists at state s_j and outcome of attribute x_i for factor f if $GU_q^f(x_i, s_j) < \bar{u}_q^f(\bar{x}, \bar{s})$; otherwise, the non-risk-state exists at state s_j and outcome of attribute x_i for factor f.

To solve the problem in Eq. (6), we assume that the negotiator's utility, attribute of factor and state are independent. Thus, Eq. (6) should become the additive model of MAU and k_q can be solved by the weighted method, which was developed by Tzeng et al. [25] and is shown in Eq. (7).

$$\begin{cases} k_q u_q^f(x_i, s_j)^U - k_q u_q^f(x_i, s_j)^L = k_{q+1} u_{q+1}^f(x_i, s_j)^U - k_{q+1} u_{q+1}^f(x_i, s_j)^L \\ \sum_{q=1}^{Q} k_q = 1. \end{cases} \tag{7}$$

The variables in Eq. (7) are defined in Appendix A. The relative weighted value k_q can be obtained using Eq. (7) and the value of k_q can then be substituted into Eq. (6) to compute $GU_q^f(x_i, s_j)$. The $GU_q^f(x_i, s_j)$ can determine the risk-state and non-risk-state for factor f through the evaluation of q negotiators. Fig. 2 depicts the process for determining the risk-state and non-risk-state for factor f through q negotiators.

As seen in Fig. 2, since $0 \le GU_q^f(x_i, s_j) \le 1$, it can be ranked from 0 to 1 according to the variable of state s_j. Therefore, let s_j be the horizontal axis and let $GU_q^f(x_i, s_j)$ be the variable of the vertical axis for factor f. We distinguish state s_j through group

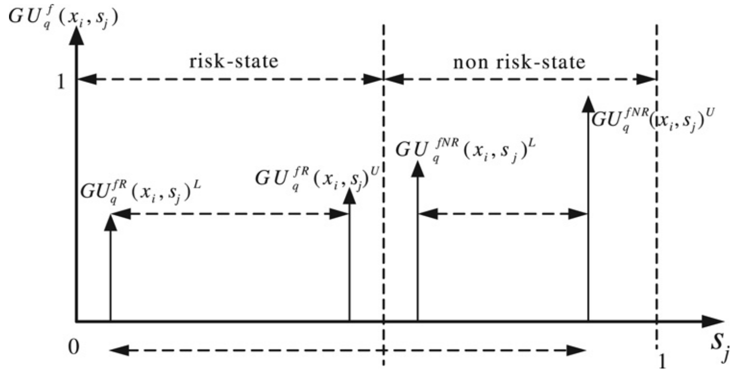

Fig. 2. Aggregating utility in risk-state and non risk-state.

assessment into risk-state and non-risk-state by $GU_q^f(x_i, s_j)$. According to the definition of risk-state mentioned above, the utility assessment value of the non-risk-state exceeds that of the risk-state. Therefore, we can rank the state by $GU_q^f(x_i, s_j)$. In turn, we can get the maximum value and minimum value of the negotiation group for factor f in the non-risk-state and the risk-state, respectively. Multiplying values k_q also have taken into the aggregation utility function account, and yields the utility value of both states; GU_q^f, which is the aggregation utility function of the negotiation group for factor f can be obtained by Eq. (8).

$$GU_q^f = \sum_{q=1}^{Q} k_q \times \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \left| \frac{(GU_q^{fNR}(x_i, s_j)^U - GU_q^{fNR}(x_i, s_j)^L) - (GU_q^{fR}(x_i, s_j)^U - GU_q^{fR}(x_i, s_j)^L)}{GU_q^{fNR}(x_i, s_j)^U - GU_q^{fR}(x_i, s_j)^L} \right| \right). \tag{8}$$

The denominator of Eq. (8) is divided into two parts, namely non-risk-state and risk-state. Let $GU_q^{fNR}(x_i, s_j)^U - GU_q^{fNR}(x_i, s_j)^L$ be the utility difference of the negotiation group of factor f in the non-risk-state, and let $GU_q^{fR}(x_i, s_j)^U - GU_q^{fR}(x_i, s_j)^L$ be the utility difference of the negotiation group of factor f in the risk-state. The numerator of Eq. (8) is the distance differential between the non-risk-state and risk-state, which shows the magnitude of the difference between the two different states and falls between 0 and 1. This process aims to integrate the assessment results of the negotiator toward utility at various states. Eq. (8) enables us to obtain GU_q^f of the negotiation group, which is the consensus of the negotiation group on the risk-state or non-risk-state for factor f. The value of GU_q^f should also range between 0 and 1 since $GU_q^{fNR}(x_i, s_j)^U$, $GU_q^{fNR}(x_i, s_j)^L$, $GU_q^{fR}(x_i, s_j)^U$, and $GU_q^{fR}(x_i, s_j)^L$ range between 0 and 1. Furthermore, we can obtain the averaged utility value of the aggregation utility function of negotiation group for factor f by $\bar{U}^* = \frac{1}{Q} \sum_{q=1}^{Q} GU_q^f$ for $q = 1, 2, \ldots, Q$. It can be seen that the factor f is a risk factor if $GU_q^f < \bar{U}^*$; meaning that the negotiation group believes that f is a risk factor. Meanwhile, when $GU_q^f \geq \bar{U}^*$, f is a non-risk factor.

3.3. Analysis of risk factor

Assume that there are t factors associated with the BOT projects, $t = 1, 2, \ldots, T$ and all the t factors of the BOT projects are independent of each other. The aggregation utility values for these t factors are obtained through Eq. (8) to analyze whether there is risk factor for these t events through the group risk assessment process. Although we can obtain the risk factors for t events via Eq. (8) step by step, this approach cannot distinguish between the primary and secondary risk factors. Therefore, the critical risk level for these risk factors must be further determined in order to reveal the primary and secondary risk factors. The critical risk level of risk factors are shown in Appendix B.

As seen in Appendix B, the critical risk level is the averaged aggregation utility value of the negotiation group toward all the risk factors. $GU_q^r < (\overline{GU}^r)$ shows that the aggregation value of the negotiation group for a given risk factor is less than the critical risk level, and that the given factor is a primary risk factor, while $GU_q^r \geq (\overline{GU}^r)$ shows that the given risk factor is a secondary risk factor. According to concept of the utility preference theory, the lower the utility level, the lower the preference, and thus the higher the risk level will be [18]. Thus, the main bargaining items for the concession company during contract negotiations are the primary risk factors, followed by the secondary risk factors.

4. Numerical example

A numerical example using the data from Kang [26] is given in this section to illustrate the application of the risk assessment model developed herein.

4.1. *Description of events*

In Taiwan, the government used the BOT approach to carry out the High Speed Rail BOT project (HSRBOT). During the negotiation, the two parties are concerned about what the primary and secondary risk factors are.

Assume that the rights for contract negotiation regarding a HSRBOT project are granted to a BOT concession company. This BOT company will face numerous uncertainty factors during the franchise term, such as land acquisition, loan credit ratio, discount ratio, concession period, price regulation, and foreign exchange ratio. For illustration, we assume that the concession company's negotiation team comprises six negotiators and measures six uncertainty factors. In addition, the utility values and probability for attribute outcome are given herein, and each factor is described in detail below.

a. Land acquisition (L)

If the government cannot acquire the land for the route and stations in time, the concession company will face the problem that the construction of stations and truck-line of HSR cannot start on schedule, delaying both completion and operation. Assume that the delay can be 0, 1, 2, . . . , 10 years, corresponding to 10 states, denoted as d_y. Meanwhile, let the increased construction cost, C_b, be the outcome of the attribute resulting from a yearlong delay. The probability $p(C_b, d_y)$ and assessment value $v(C_b, d_y)$ correspond to the attribute outcome and states, respectively, and have 10 values each.

b. Discount ratio (D)

Let the discount ratio be 7%, 8%, 9%, 10%, 11%, 12%, 13%, 14%, 15%, 16%, 17% and 18%, so a total of 12 states exist, where d_r is the state of discount ratio. The interest cost is the outcome of attribute, denoted as C_i. Meanwhile, the value assessed by the negotiator regarding attribute is $v(C_i, d_r)$, while the probability value is $p(C_i, d_r)$. Furthermore, $v(C_i, d_r)$ and $p(C_i, d_r)$ each correspond to different d_r, and each has 12 values.

c. Loan credit ratio (C)

Assume that there are seven loan credit ratios, namely 6.5%, 7%, 7.5%, 8%, 8.5%, 9%, and 10%. A total of seven states exist where L_{cr} represents the level of the loan credit ratio. The outcome of attribute for this event is interest cost, denoted as C_L. Meanwhile, $v(C_L, L_{cr})$ is the value assessed by the negotiator regarding the outcome of attribute and for each state. The probability for each state is $p(C_L, L_{cr})$, $v(C_L, L_{cr})$, L_{cr}, and $p(C_L, L_{cr})$ each corresponding to one state.

d. Price regulation (P)

Assume that the government regulates the fares and their bi-annual changes. The original fare is to be NT\$ 180. If no price regulation occurs, the fare will be adjusted according to the fluctuation of the price index. Suppose that the price inflection level is 3%, the future fare will become NT\$ 185, 191, 197, . . ., and 298. Instead, with price regulation, the fare will be NT\$ 180, 180, 180, 191, 191, . . . , 215, 215, and 215. Let P_f be the change in fare. The outcome of attribute is the revenue loss, denoted as R_L. Let $v(R_L, P_f)$ be the assessment value of the negotiator at the outcome of attribute and the state probability, and let $p(R_L, P_f)$ be the probability value at the outcome of attribute R_L and at the states P_f. R_L, $v(R_L, P_f)$ and $p(R_L, P_f)$ correspond to each of the 18 states, so each of them has 18 values.

e. Concession period (T)

The concession period is set according to the characteristics of the BOT project, and any change in the concession period will significantly impact the concession company. Assume that the state of this event is the number of years of the concession period; hence, for a concession period of 27 to 35 years, nine states exist, denoted as T_c, while T_c is the concession period of BOT project. The attribute outcome is the operational revenue, denoted as R_O. The value assessed by the negotiators regarding attribute outcome and state is $v(R_O, T_c)$, while the probability is $p(R_O, T_c)$. R_O, T_c, $v(R_O, T_c)$ and $p(R_O, T_c)$ each correspond to each of the nine states, and each has nine values.

f. Foreign exchange ratio (E)

The foreign exchange ratio is defined as New Taiwan Dollars versus the US Dollar, which is $e_r = NT\$/US\$$. Assume that a purchasing plan for the concession company during the concession period will be carried out, and that this plan is priced in US dollars, then this purchasing plan faces fluctuations in foreign exchange. If the magnitude of exchange rate fluctuations is too dramatic, the cost will increase.

Assume that the purchasing plan of the concession company includes buying 24 vehicles at $e_r = 31.0$ in the second year of the operational period, 24 vehicles at $e_r = 30.5$ in the seventh year, and 24 vehicles at $e_e = 31.8$ in the twelfth year. The predicted actual exchange rate ranges from 29.8 to 32.5; therefore, there are 17 states, denoted as e_r. The attribute outcome is the purchasing cost, denoted as C_p. The value assessed by the negotiator regarding attribute outcome and state is $v(C_p, e_r)$, while the probability of the exchange ratio state at the outcome of attribute and state is $p(C_p, e_r)$.

4.2. *Risk assessment of a factor*

According to the definition for these factors as mentioned above, the data for outcome of attribute, probability and assessment value are given as the land acquisition factor. These values of $v_1^L(1, d_y) \times p(1, d_y)$ are 0.2009, 0.0241, . . . , 0 for each state as obtained using Eq. (1), respectively. They are then incorporated into Eq. (3) to compute $u_1^L(1, d_y)$ for each state, giving $u_1^f(1, d_y = 0) = \frac{0.2009-0.0000}{0.2009-0.0000} = 1$, $u_1^f(1, d_y = 1) = \frac{0.0241-0.0000}{0.2009-0.0000} = 0.1199$, $u_1^f(1, d_y = 2) = \frac{0.0385-0.0000}{0.2009-0.0000} = 0.1918, \ldots, u_1^f(1, d_y = 9) = 0.0000$, $u_1^f(1, d_y = 10) = 0.0000$ and the averaged utility value under normalized utility. Results of the risk-state and non-risk-state evaluation of the 1st negotiator for the land acquisition factor are shown in Table 4. As can be seen, $d_y = 0$, it indicates that it is a risk-state by the 1st negotiator's evaluation, since $u_1^L(1, 0) = 1$ is

Table 4
Results of state-state and non-risk-state for the 1st negotiator for land acquisition.

Attribute	State (d_y)										
	0	1	2	3	4	5	6	7	8	9	10
Increment cost (NT$ 100 million)	0	2.98	4.45	6.89	9.68	11.98	15.45	20.36	24.23	30.45	35.558
$v_1^L(1, d_y)$	0.95	0.82	0.64	0.55	0.32	0.11	0.06	0.005	0.0004	0.0003	0.00005
$p(1, d_y)$	0.2115	0.0294	0.0602	0.0762	0.0995	0.0970	0.0945	0.0806	0.0624	0.0465	0.03017
$v_1^L(1, d_y) * p(1, d_y)$	0.2009	0.0241	0.0385	0.0419	0.0318	0.0107	0.0057	0.0004	0.0000	0.0000	0.0000
$u_1^L(1, d_y)$	1.0000	0.1199	0.1918	0.2084	0.1584	0.0531	0.0282	0.0020	0.0000	0.0000	0.0000

larger than the averaged utility value; the other states are not risk-states. Results of risk-state and non-risk-state for the land acquisition factor are shown in Tables B.1 and B.2.

Similarly, we can calculate the utility value for other negotiators and other factors, and the results of the risk analysis are obtained and summarized in Tables B.1 and B.2. Table B.1 shows the non-risk-state and risk-state of each negotiator's assessment for these six factors. Take land acquisition for example, negotiators #2, #3 and #4 believe that if the delay is less than a year, the delay year state is non-risky, while negotiators #1 and #5 believe if there is no delay, i.e., delay year equals zero, the state is non-risky; while all other states are risky. Negotiator #6 believes that if the delay is less than two years, the state of delay year is non-risky while all other states are risky. Similarly, for other factors such as discount ratio, loan credit ratio, foreign exchange ratio, different negotiators hold different assessment standards, resulting in different assessment results, as presented in Tables B.1 and B.2 regarding outcome of attribute and state.

Since we assume that the utility of negotiators and the state of factor are independent, we use Eqs. (6) and (7) to obtain the multi-attribute utility of the negotiation group. As mentioned in Section 4.1, there are 10 states and a single attribute outcome for the land acquisition factor. Adopting the concept of Eq. (7) proposed by Tzeng et al. [25] gives the solution process as Eq. (9).

$$\begin{cases} k_1u_1^L(1,0) - k_1u_1^L(1,10) = k_2u_2^L(1,0) - k_2u_2^L(1,10) \\ k_2u_2^L(1,0) - k_2u_2^L(1,10) = k_3u_3^L(1,0) - k_3u_3^L(1,9) \\ k_3u_3^L(1,0) - k_3u_3^L(1,9) = k_4u_4^L(1,3) - k_4u_4^L(1,10) \\ k_4u_4^L(1,3) - k_4u_4^L(1,10) = k_5u_5^L(1,0) - k_5u_5^L(1,10) \\ k_5u_5^L(1,0) - k_5u_5^L(1,10) = k_6u_6^L(1,2) - k_6u_6^L(1,10) \\ k_1 + k_2 + k_3 + k_4 + k_5 + k_6 = 1. \end{cases} \tag{9}$$

The notations of variables of Eq. (9) are shown in Appendix A and the data are displayed in Table B.2. As seen in the table, these values are the maximum and minimum values for each negotiator regarding each state. For instance, $u_1^L(1,0) = 0.2009$, $u_1^L(1,10) = 0$, $u_2^L(1,0) = 0.8964$, $u_2^L(1,10) = 0$, $u_3^L(1,0) = 0.5870$, $u_3^L(1,9) = 0$, $u_4^L(1,3) = 0.0097$, $u_4^L(1,10) = 0$, $u_5^L(1,0) = 0.0460$, $u_5^L(1,10) = 0$, $u_6^L(1,2) = 0.2958$, and $u_6^L(1,10) = 0$, respectively. Substituting these values into Eq. (9) gives

$$\begin{cases} k_1 \times 0.2009 - k_1 \times 0 = k_2 \times 0.8964 - k_2 \times 0 \\ k_2 \times 0.8964 - k_2 \times 0 = k_3 \times 0.5870 - k_3 \times 0 \\ k_3 \times 0.5870 - k_3 \times 0 = k_4 \times 0.0097 - k_4 \times 0 \\ k_4 \times 0.0097 - k_4 \times 0 = k_5 \times 0.0460 - k_5 \times 0 \\ k_5 \times 0.0460 - k_5 \times 0 = k_6 \times 0.2958 - k_6 \times 0 \\ k_1 + k_2 + k_3 + k_4 + k_5 + k_6 = 1. \end{cases} \tag{10}$$

Solving Eq. (10) and the relative weight values of the negotiator of land acquisition yields $k_1 = 0.0366$, $k_2 = 0.0082$, $k_3 = 0.0125$, $k_4 = 0.7580$, $k_5 = 0.1599$, and $k_6 = 0.0249$, respectively, which are shown in Table B.2. Similarly, we can also compute the k_q values for the other factors using Eq. (9) since we assume that these factors are independent of each other.

The results of the relative weighted values of the negotiator are shown in Table B.2. The weighted value indicates the relative weighted value for a negotiator regarding the attribute outcome of the factor [18]). For the land acquisition factor, $k_4 = 0.7580$ means that compared with other negotiators, negotiator #4 has a higher utility regarding the attribute outcome of the event. For the foreign exchange ratio factor, the weighted values for negotiators #2 and #3 are 0.0041 and 0.0038, respectively, indicating that both individuals have a lower utility regarding the attribute outcome of exchange ratio. The weighted value of the remaining events are similar among the negotiators, meaning no significant difference in recognition among the six negotiators regarding the attribute outcome of the factors.

Using the values of $k_1 = 0.0366$, $k_2 = 0.0082$, $k_3 = 0.0125$, $k_4 = 0.7580$, $k_5 = 0.1599$, and $k_6 = 0.0240$, which are coefficients of Eq. (6), we can then get the multi-attribute utility function for the land acquisition factor as

$$\begin{aligned} GU_q^L(C_b, d_y) = {} & 0.0366u_1^{L*}(C_b, d_y) + 0.0082u_2^{L*}(C_b, d_y) + 0.0125u_3^{L*}(C_b, d_y) \\ & + 0.7580u_4^{L*}(C_b, d_y) + 0.1599u_5^{L*}(C_b, d_y) + 0.0249u_6^{L*}(C_b, d_y). \end{aligned} \tag{11}$$

Similarly, we can calculate the multi-attribute utility function for the other factors, and the results are shown in Table 5.

Table 5
Results of the multi-attribute utility function for events.

Factor	Multi-attribute utility function
Land Acquisition (L)	$GU_q^L(C_b, d_y) = 0.0366u_1^{L*}(C_b, d_y) + 0.0082u_2^{L*}(C_b, d_y) + 0.0125u_3^{L*}(C_b, d_y) + 0.7580u_4^{L*}(C_b, d_y) + 0.1599u_5^{L*}(C_b, d_y) + 0.0249u_6^{L*}(C_b, d_y)$
Discount Ratio (D)	$GU_q^D(C_i, d_r) = 0.2643u_1^{D*}(C_i, d_r) + 0.1386u_2^{D*}(C_i, d_r) + 0.1323u_3^{D*}(C_i, d_r) + 0.1584u_4^{D*}(C_i, d_r) + 0.1296u_5^{D*}(C_i, d_r) + 0.1768u_6^{D*}(C_i, d_r)$
Concession Period (T)	$GU_q^T(R_O, T_C) = 0.1545u_1^{T*}(R_O, T_C) + 0.1548u_2^{T*}(R_O, T_C) + 0.1694u_3^{T*}(R_O, T_C) + 0.1931u_4^{T*}(R_O, T_C) + 0.1931u_5^{T*}(R_O, T_C) + 0.1734u_6^{T*}(R_O, T_C)$
Loan Credit Ratio (C)	$GU_q^C(C_L, L_{cr}) = 0.1478u_1^{C*}(C_L, L_{cr}) + 0.1196u_2^{C*}(C_L, L_{cr}) + 0.2494u_3^{D*}(C_L, L_{cr}) + 0.1881u_4^{D*}(C_L, L_{cr}) + 0.1364u_5^{D*}(C_L, L_{cr}) + 0.1573u_6^{D*}(C_L, L_{cr})$
Price Regulation (P)	$GU_q^P(R_L, P_f) = 0.1668u_1^{P*}(R_L, P_f) + 0.1651u_2^{P*}(R_L, P_f) + 0.1668u_3^{P*}(R_L, P_f) + 0.1651u_4^{P*}(R_L, P_f) + 0.1713u_5^{P*}(R_L, P_f) + 0.1651u_6^{P*}(R_L, P_f)$
Foreign exchange ratio (E)	$GU_q^E(C_p, e_r) = 0.1848u_1^{E*}(C_p, e_r) + 0.0041u_2^{E*}(C_p, e_r) + 0.0038_3^{E*}(C_p, e_r) + 0.2652u_4^{E*}(C_p, e_r) + 0.2514u_5^{E*}(C_p, e_r) + 0.2906u_6^{E*}(C_p, e_r)$

Table 6
Results of risk and non-risk factors.

Factor	Aggregation utility value	$\bar{U}^*$	Risk/Non-risk
Land Acquisition (L)	0.5128	0.0655	Non-risk
Discount Ratio (D)	0.6620	0.2317	Non-risk
Loan Credit Ratio (I)	0.4001	0.2877	Non-risk
Price Regulation (P)	0.4386	0.2154	Non-risk
Concession Period (T)	0.2113	0.2972	Risk
Foreign Exchange Ratio (E)	0.2282	0.2565	Risk

Incorporating these values of $u_q^L(C_b, d_y), q = 1, 2, \ldots, 6$ into Eq. (10), we compute $GU_q^L(C_b, d_y)$ for each state. As for land acquisition, $GU_q^L(C_b, d_y = 0) = 0.0366 \times 0.2009 + 0.0082 \times 0.8964 + 0.0125 \times 0.5870 + 0.7580 \times 0.0012 + 0.1599 \times 0.0460 + 0.0249 \times 0.2279 = 0.03598$. Likewise, we can compute $GU_q^L(C_b, d_y)$ for all states of land acquisition using Eq. (10) and the values obtained are 0.03598, 0.0261, 0.0175, 0.0178, 0.0006, 0.0025, 0.0008, 0.00009, 0, 0, and 0, respectively. In turn, we get the averaged utility value of $GU_q^L(C_b, d_y)$ for all states, being 0.0097. Thus, we find that states with delay less than zero, one year, two years, and three years are not risky from the negotiator group's viewpoint. It was denoted as "U' and the other states are risky, denoted as "R". The results are shown in Table B.2. Similarly, we can analyze the risk-state and non-risk-state for the other factors using the multi-attribute utility function. These results are shown in Table B.2, Continuity 1 and Continuity 2.

Table B.2 shows the outcomes of the common concerns among all negotiators. As for discount ratio, all the negotiators believe that the states with discount ratio of 12%, 13%, 14%, 15% and 16% are non-risky; while the others are risky. For the concession period, all the negotiators believe that the concession years at 30, 31 and 32 are non-risky, while the others are risky. As for credit ratio, all the negotiators believe that the states with credit ratio of 8.5%, 9% and 10% are risky but the states with credit ratio of 6.5%, 7.0%, 7.5% and 8% are non-risky.

Results of Tables B.1 and B.2 only reveal risky state and non-risky state for all these factors with the values of negotiators' utility, but cannot identify the risk factors or non-risk factors. To identify such, we use Eq. (8) to compute the aggregate utility value for each factor with the data of Table B.2.

The data of Table B.2 can be divided into non-risky state and risky state for the six factors. As for land acquisition, the maximum value and minimum value of the non-risky state from the 1st negotiator are 0.2009 and 0.241, respectively; while the maximum value and minimum value of the risky state are 0.0318 and 0, respectively. Moreover, we can find that these maximum value and minimum value correspond to risky state and non-risky state for the other negotiators. Furthermore, substituting these maximum value, minimum value and k_q into Eq. (7) gives the aggregation utility function, and $GU_q^L = 0.5128$. Similarly, we can calculate the aggregation utility values using Eq. (7) for the other factors. The aggregation utility values are $GU_q^D = 0.6620$, $GU_q^C = 0.4001$, $GU_q^P = 0.4386$, $GU_q^T = 0.2113$, and $GU_q^E = 0.2282$, respectively. The values of $\bar{U}^*$ for all factors are 0.0655, 0.2317, 0.2877, 0.2154, 0.2972, and 0.2565, respectively. Results are shown in Table 6.

We assume that these factors, namely Land acquisition, Discount Ratio, Loan Credit Ratio, Price Regulation, Concession Period, and Foreign Exchange Ration, are independent of each other. In Table 3, we calculate the averaged utility value for all factors, and get 0.2257. Since $\bar{U}^* = 0.2972$ of concession period and $\bar{U}^* = 0.2565$ of foreign exchange ratio are larger than 0.2257, we can thus find out that both of them are risk factors, while the others are non-risk factors. In addition, we use the critical risk level, which is shown in Appendix B, for these risk factors to find the primary and secondary risky factors. The critical risk level is 0.2769 and we can easily find that the concession period is a primary risk factor because the aggregation utility value of 0.2113 is less than 0.2769. While the aggregation utility value of the foreign exchange ratio is larger than the averaged aggregation utility value, the foreign exchange ratio is the secondary risk factor. Therefore, during concession contract negotiation, the private sector or host utility should take the factor of concession period as the main negotiation concern, followed by the foreign exchange ratio.

4.3. Discussion

As reported in the appendix Tables, we can realize that there are different risk-states and non-risk states for each uncertain factor by each negotiator's evaluation. Additionally, we can distinguish between risky and uncertain factors; finally, the primary and secondary risk events, concession period and foreign exchange ration, are determined based on these models.

From the literature reviews, Tiong and Alum [27] pointed out that there were 13 items, including the initial level of toll, future tariff increase, length of concession period, fixed interest rate for loads profits and revenue sharing with government and so on, that are regarded as both important and difficult to negotiate during the negotiation. In comparison with the results of Tiong and Alum [27], the concession period is a risk factor in numerical example as the meaning which the authors have pointed out. However, beside the mention-above factors in Section 4.2, many uncertain factors associated with BOT project, such as revenue-sharing, level of passenger traffic volume, fixed construction schedule, rate of return on investment and so on, could be incorporated into this model and then identify risk.

As noted, the data of the probability and utility value for assessing risk are from Kang [27] in the numerical example. However, to identify and to evaluate risk for uncertain factors in a real BOT case, it is needed to investigate data of the negotiator's evaluation value and the occurrence probability of a factor by questionnaire approach from the negotiation group during the negotiation phase. In application, it requires to relax the assumptions of the development model so as to identify the primary and secondary risks in a real BOT case.

5. Conclusion

The purpose of this study is to identify which uncertainty factors are risk factors, and which are non-risk factors, and then to distinguish the primary risk factors from the secondary through the private sector's viewpoint.

According to the concept of risk assessment and multi-attribute utility theory, this study develops a risk assessment model for the negotiation group and then to assess the utility of an uncertain factor in relation to the concession contract negotiation of a BOT project. Meanwhile, a numerical example is presented to demonstrate the application of the risk assessment utility model developed herein. The results have the following implications. (1) Assuming that the outcome of an attribute and the state of a factor are independent, the definition of risk-state for multiple attributes associated with a factor is a generalized equation of the single attribute. (2) By applying the MAU concept and ranking the utility, the aggregation utility function can be developed. Furthermore, (3) the critical risk level of the risk factors can also be developed here, and this critical risk level is an average of aggregation utility value for all risk factors. Therefore, the primary and secondary risk factors can be identified using the critical risk level. (4) According to the example presented herein, the aggregation utility value for the six factors, namely land acquisition, discount ratio, loan credit ratio, price regulation, concession period, and foreign exchange ratio, are 0.5128, 0.6620, 0.4001, 0.4386, 0.2113 and 0.2282, respectively. Hence, the concession period and foreign exchange ratio are risk factors, while the others are not. Among the risk factors, the concession period is the primary risk factor, while the foreign exchange ratio is the secondary risk factor.

As shown in the numerical example, the developed model could be used to apply risk identification and risk assessment for a BOT project from the perspective of private sector or the host utility. In addition, the results herein are achieved under the assumptions that the outcome, attribute, and state of a factor as well as the utility among negotiators are independent. However, factors vary through interactions among negotiators. For future studies, the above assumptions can be relaxed, the negotiators' attitude toward risk and the other factors of BOT projects can be further explored. The models in this study are developed from the perspective of the BOT concession company, but a risk assessment model can also be developed from the perspective of the government. Above all, a real case study of a BOT project implemented in Taiwan is presented to verify the application of the model developed herein.

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Appendix A. The notations of variables

$U_i(x_i)$: The utility value of attribute x_i, $0 \leq U_i(x_i) \leq 1$.
$U(x)$: The multi-attribute utility function.
k_i: The relative weighted value of attribute x_i, $0 \leq \lambda_i \leq 1$; and k is the scale constant.
$GU_q^f(x_i, s_j)$: The utility value of the group negotiators for outcome of attribute x_i at state s_j for factor f.
k_q: The relative weighting value of a negotiator and k_q is the scale constant.
$u_q^f(x_i, s_j)^U$: The maximum utility value of the qth negotiator for factor f at outcome of attribute x_i and state s_j.

$u_q^f(x_i, s_j)^L$: The minimum value of the qth negotiator's utility for factor f at outcome of attribute x_i and state s_j.

$u_{q+1}^f(x_i, s_j)^U$: The maximum value of the $(q+1)$th negotiator's utility for factor f at outcome of attribute x_i and state s_j.

$u_{q+1}^f(x_i, s_j)^L$: The minimum value of the $(q+1)$th negotiator's utility for factor f at outcome of attribute x_i and state s_j.

$GU_q^{fNR}(x_i, s_j)^U$: The maximum utility value of the negotiation group for factor f in the non-risk-state.

$GU_q^{fNR}(x_i, s_j)^L$: The minimum utility value of the negotiation group for factor f in the non-risk-state.

$GU_q^{fR}(x_i, s_j)^U$: The maximum utility value of the negotiation group for factor f in the risk-state.

$GU_q^{fR}(x_i, s_j)^L$: The minimum utility value of the negotiation group for factor f in the risk-state.

$u_1^L(1, 0)$: The minimum utility value at the single attribute outcome and the 0-delay-year state of negotiator #1 for land acquisition.

$u_1^L(1, 10)$: The maximum utility value at the single attribute outcome and the 10-delay-year state of negotiator #1 for land acquisition.

$u_2^L(1, 0)$: The minimum utility value at the single attribute outcome and the 0-delay-year state of negotiator #2 for land acquisition.

$u_2^L(1, 10)$: The maximum utility value at the single attribute outcome and the 10-delay-year state of negotiator #2 for land acquisition.

$u_3^L(1, 0)$: The minimum utility value at the single attribute outcome and the 0-delay-year state of negotiator #3 for land acquisition.

$u_3^L(1, 9)$: The maximum utility value at the single attribute outcome and the 9-delay-year state of negotiator #3 for land acquisition.

$u_4^L(1, 3)$: The minimum utility value at the single attribute outcome and the 3-delay-year state of negotiator #4 for land acquisition.

$u_4^L(1, 10)$: The maximum utility value at the single attribute outcome and the 10-delay-year state of negotiator #4 for land acquisition

$u_5^L(1, 0)$: The minimum utility value at the single attribute outcome and the 0-delay-year state of negotiator #5 for land acquisition

$u_5^L(1, 10)$: The maximum utility value at the single attribute outcome and the 10-delay-year state of negotiator #5 for land acquisition.

$u_6^L(1, 2)$: The minimum utility value at the single attribute outcome and the 2-delay-year state of negotiator #6 for land acquisition

$u_6^L(1, 10)$: The maximum utility value at the single attribute outcome and the 10-delay-year state of negotiator #6 for land acquisition.

Appendix B. The critical risk level for risk factors

According to the assumptions made in Sections 3 and 4, Q negotiators exist for the concession company, for $q = 1, 2, \ldots, Q$. And assume that there are r risk factors, $r = 1, 2, \ldots, N$; and let $f(r)$ be the occurrence probability of factor r. Moreover, all r factors are assumed herein to be independent of each other. The decision-making behavior of the negotiation group is also assumed to be rational. Then the negotiation group would pursue the maximizing expected utility value for the negotiation group. Therefore, the critical risk level for the risk factor can be solved by first-order differentiation of the expected utility value $E(U^r)$. The process of determining the critical risk level is as follows.

Table B.1
Analysis results of a negotiator regarding risk-state and non risk-state for factor.

Event state	Land acquisition, d_y		Discount ratio, d_r		Concession period, T_c	
Negotiator	Non risk-state (year)	Risk-state (year)	Non risk-state (%)	Risk-state	Non risk-state	Risk-state
1st	0	>=1	13	Others	30	Others
2nd	< 1	>=2	13	Others	30 and 31	Others
3rd	<1	>=2	15	Others	30	Others
4th	<1	>=2	14, 15	Others	30	Others
5th	0	>=1	14, 15, 16	Others	30 and 31	Others
6th	< 2	>=3	14	Others	30	Others
Event state	Loan credit ratio, L_{cr} (%)		Price regulation, P_f		Foreign exchange ratio, e_r	
1st	7.5	Others	\$ NT 180, \$ NT 191	Others	30.1, 29.8, 30.4, 30.9	Others
2nd	6.5	Others	\$ NT 180	Others	29.8, 30.1, 30.9,	Others
3rd	6.5	Others	\$ NT 180	Others	29.8, 30.4, 30.9	Others
4th	7.5	Others	\$ NT 180	Others	29.8, 30.1, 30.4, 30.9	Others
5th	6.5	Others	\$ NT 180	Others	30.9	Others
6th	7.5	Others	\$ NT 180	Others	29.8, 30.1, 30.4, 30.9	Others

Table B.2
Analysis results of the risk-state and non risk-state for factor.

Factor state	Land acquisition, d_y																		
Utility	0	1	2	3	4	5	6	7	8	9	10	k_q	–	–	–	–	–	–	–
1st	0.2009	0.0241	0.0385	0.0419	0.0318	0.0107	0.0057	0.0004	0.0000	0.0000	0.0000	0.0366	–	–	–	–	–	–	–
2nd	0.8964	0.7704	0.0615	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0082	–	–	–	–	–	–	–
3rd	0.5870	0.5137	0.0615	0.0008	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0125	–	–	–	–	–	–	–
4th	0.0012	0.0048	0.0066	0.0097	0.0034	0.0020	0.0008	0.0001	0.0000	0.0000	0.0000	0.7580	–	–	–	–	–	–	–
5th	0.0460	0.0200	0.0156	0.0319	0.0144	0.0025	0.0000	0.0000	0.0000	0.0000	0.0000	0.1599	–	–	–	–	–	–	–
6th	0.2279	0.2275	0.2958	0.1505	0.0033	0.0099	0.0000	0.0000	0.0000	0.0000	0.0000	0.0249	–	–	–	–	–	–	–
$u^{L*}(1, d_y)$	0.0360	0.0261	0.0175	0.0177	0.0061	0.0026	0.0008	0.0001	0.0000	0.0000	0.0000	–	–	–	–	–	–	–	–
Result	*U*	*U*	*U*	*U*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	–	–	–	–	–	–	–	–
Factor state	Discount ratio, d_r																		
Utility	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	k_q	–	–	–	–	–	–
1st	0.0015	0.0223	0.0376	0.0850	0.2209	0.2866	0.4001	0.3760	0.1769	0.0887	0.0113	0.0091	0.2643	–	–	–	–	–	–
2nd	0.0009	0.0148	0.1093	0.1440	0.2305	0.2907	0.7609	0.3751	0.2043	0.1789	0.0746	0.0110	0.1386	–	–	–	–	–	–
3rd	0.0014	0.0133	0.0599	0.0768	0.1169	0.1472	0.5870	0.6378	0.7973	0.3930	0.2357	0.0598	0.1323	–	–	–	–	–	–
4th	0.0157	0.0369	0.0431	0.0657	0.1301	0.2187	0.3856	0.6647	0.6808	0.5006	0.1070	0.0359	0.1584	–	–	–	–	–	–
5th	0.0000	0.0157	0.0401	0.0797	0.1242	0.3573	0.7569	0.8127	0.7802	0.7880	0.0977	0.0905	0.1296	–	–	–	–	–	–
6th	0.0043	0.0223	0.0440	0.0710	0.1070	0.2552	0.4449	0.6000	0.5584	0.3796	0.1127	0.0183	0.1768	–	–	–	–	–	–
$u^{D*}(1, d_r)$	0.0039	0.0215	0.0528	0.0859	0.1614	0.2616	0.5267	0.5524	0.4882	0.3487	0.0941	0.0325	–	–	–	–	–	–	–
Result	*R*	*R*	*R*	*R*	*R*	*U*	*U*	*U*	*U*	*U*	*R*	*R*	–	–	–	–	–	–	–
Factor state	Concession period, T_c																		
Utility	27	28	29	30	31	32	33	34	35	k_q	–	–	–	–	–	–	–	–	–
1st	0.0000	0.0000	0.0002	1.0000	0.8910	0.6980	0.5095	0.2100	0.1100	0.1545	–	–	–	–	–	–	–	–	–
2nd	0.0000	0.0000	0.0000	0.9980	0.9500	0.8500	0.4500	0.3000	0.1000	0.1548	–	–	–	–	–	–	–	–	–
3rd	0.0000	0.0000	0.0015	0.8910	0.9120	0.4950	0.1000	0.0010	0.0001	0.1694	–	–	–	–	–	–	–	–	–
4th	0.0000	0.0000	0.0200	0.8000	0.4750	0.4950	0.1100	0.0010	0.0001	0.1931	–	–	–	–	–	–	–	–	–
5th	0.0000	0.0000	0.0110	0.9980	0.9000	0.6000	0.3500	0.2500	0.0100	0.1548	–	–	–	–	–	–	–	–	–
6th	0.0000	0.0000	0.0002	0.7984	0.4275	0.2970	0.0385	0.0003	0.0000	0.1734	–	–	–	–	–	–	–	–	–
$u^{T*}(1, T_c)$	0.0000	0.0000	0.0059	0.9073	0.7444	0.5632	0.2474	0.1180	0.0341	–	–	–	–	–	–	–	–	–	–
Result	*R*	*R*	*R*	*U*	*U*	*U*	*R*	*R*	*R*	–	–	–	–	–	–	–	–	–	–
Factor state	Loan credit ratio, L_{cr}																		
Utility	6.5%	7.0%	7.5%	8%	8.5%	9%	10%	k_q	–	–	–	–	–	–	–	–	–	–	–
1st	0.1990	0.4701	0.8737	0.4020	0.0613	0.0121	0.0089	0.1478	–	–	–	–	–	–	–	–	–	–	–
2nd	0.9589	0.8693	0.4330	0.2432	0.0992	0.0449	0.0020	0.1196	–	–	–	–	–	–	–	–	–	–	–
3rd	0.4680	0.3306	0.2906	0.0546	0.0660	0.0171	0.0090	0.2494	–	–	–	–	–	–	–	–	–	–	–
4th	0.1157	0.5478	0.6089	0.4990	0.0024	0.0002	0.0090	0.1818	–	–	–	–	–	–	–	–	–	–	–
5th	0.8404	0.6843	0.4235	0.2908	0.0261	0.0018	0.0011	0.1364	–	–	–	–	–	–	–	–	–	–	–
6th	0.2016	0.7225	0.7280	0.3570	0.0990	0.0121	0.0001	0.1573	–	–	–	–	–	–	–	–	–	–	–
$u^{C*}(1, L_{cr})$	0.4282	0.5625	0.5364	0.2887	0.0570	0.0136	0.0056	–	–	–	–	–	–	–	–	–	–	–	–
Result	*U*	*U*	*U*	*U*	*R*	*R*	*R*	–	–	–	–	–	–	–	–	–	–	–	–

Table B.2 (*continued*)

Factor state	Foreign exchange ratio, e_r																	
Utility	32.5	31.8	32.3	31.1	30.9	30.10	29.80	30.40	30.90	32.56	32.98	33.10	32.35	31.65	30.81	30.56	30.11	29.91	k_q
1st	0.0008	0.0069	0.0009	0.0185	0.7125	0.7546	0.8019	0.7380	0.7225	0.0004	0.0000	0.0000	0.0007	0.0005	0.1904	0.5394	0.5005	0.4760	0.1848
2nd	0.0095	0.0008	0.0000	0.5002	0.8448	0.8372	0.8508	0.7553	0.8280	0.0000	0.0000	0.0000	0.0000	0.0000	0.4270	0.3120	0.4148	0.4550	0.0041
3rd	0.1220	0.0550	0.0047	0.2800	0.5600	0.7371	0.9120	0.8649	0.8554	0.0001	0.0005	0.0000	0.0005	0.0004	0.1600	0.2250	0.2860	0.3621	0.0038
4th	0.0048	0.0052	0.0000	0.3360	0.5346	0.5525	0.5510	0.5124	0.5589	0.0000	0.0000	0.0000	0.0000	0.0000	0.1218	0.2040	0.2240	0.2025	0.2652
5th	0.0065	0.0000	0.0000	0.3060	0.4836	0.4930	0.5280	0.5580	0.5896	0.0350	0.0000	0.0000	0.0000	0.0044	0.1870	0.2240	0.2565	0.2378	0.2514
6th	0.0000	0.0000	0.0000	0.0580	0.3250	0.5100	0.4368	0.4840	0.5015	0.0000	0.0000	0.0000	0.0000	0.0049	0.0650	0.0915	0.2640	0.1225	0.2906
$u^{E*}(1, e_r)$	0.0036	0.0029	0.0002	0.1894	0.4951	0.5644	0.5609	0.5596	0.5823	0.0089	0.0000	0.0000	0.0001	0.0026	0.1357	0.2388	0.2959	0.2403	–
Result	*R*	*R*	*R*	*R*	*U*	*U*	*U*	*U*	*U*	*R*	*R*	*R*	*R*	*R*	*R*	*U*	*U*	*U*	–
Factor Utility	Price regulation State P_f (original price, regulation price)																		Unit: $ NT
Original	180	185	191	197	203	209	215	221	228	235	242	249	257	264	272	280	289	298	k_q
Regulate	180	180	180	191	191	191	197	197	197	203	203	203	209	209	209	215	215	215	
1st	0.9900	0.8100	0.2000	0.6790	0.2500	0.1000	0.0095	0.0100	0.0010	0.0009	0.0010	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.1668
2nd	1.0000	0.5000	0.2500	0.6675	0.5100	0.2100	0.2275	0.1200	0.0100	0.1440	0.0030	0.0010	0.0010	0.0001	0.0001	0.0000	0.0000	0.0000	0.1651
3rd	0.9900	0.8010	0.5000	0.7425	0.4850	0.3200	0.2450	0.1500	0.0900	0.1843	0.0100	0.0000	0.0010	0.0001	0.0001	0.0000	0.0000	0.0000	0.1668
4th	1.0000	0.9100	0.6100	0.6684	0.5600	0.2410	0.2410	0.1210	0.0110	0.1010	0.0100	0.0000	0.0010	0.0001	0.0001	0.0000	0.0000	0.0000	0.1651
5th	0.9640	0.8300	0.4610	0.6426	0.3640	0.1910	0.2385	0.1110	0.1100	0.1000	0.0600	0.0120	0.0090	0.0001	0.0001	0.0000	0.0000	0.0000	0.1713
6th	1.0000	0.5900	0.2200	0.6831	0.3210	0.1210	0.1269	0.1100	0.0910	0.1010	0.0220	0.0012	0.0011	0.0001	0.0001	0.0000	0.0000	0.0000	0.1651
$u^{P*}(1, P_f)$	0.9907	0.7411	0.3740	0.6805	0.4146	0.1972	0.1816	0.1037	0.0525	0.1051	0.0179	0.0024	0.0022	0.0001	0.0001	0.0000	0.0000	0.0000	–
Result	*U*	*U*	*U*	*U*	*U*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	–

“U” denotes the non-risk ; the “R” denotes the risk.

$Max(E(U^r)) = Max\left(\sum_{r=1}^{N} f(\mathrm{r}) GU_q^r\right)$, the first-order differentiation of the expected utility value is

$$\frac{\partial(E(U^r))}{\partial(GU_q^r)} = \frac{\partial(\sum\limits_{r=1}^{N} f(r) GU_q^r)}{\partial(GU_q^r)} = \frac{\partial(E(U^r - \overline{GU}^r + \overline{GU}^r))}{\partial(GU_q^r)}.$$

Since $\overline{GU}^r$ is constant, then

$$\frac{\partial(E(U^r - \overline{GU}^r + \overline{GU}^r))}{\partial(GU_q^r)} = \frac{\partial(E(U^r - \overline{GU}^r))}{\partial(GU_q^r)} + \frac{\partial(E\overline{GU}^r)}{\partial(GU_q^r)}.$$

Let the first-order differentiation of the expected utility value be equal to zero, giving

$$\frac{\partial(E(U^r - \overline{GU}^r))}{\partial(GU_q^r)} + \frac{\partial(E(\overline{GU}^r))}{\partial(GU_q^r)} = 0.$$

Since $E(U^r)$; is constant, we get

$$\frac{\partial(E(U^r - \overline{GU}^r))}{\partial(GU_q^r)} = 0, \quad \text{while } \frac{\partial(E(U^r))}{\partial(GU_q^r)} = 0.$$

Thus $E(U^r) - (\overline{GU}^r) = 0$; hence $E(U^r) = (\overline{GU}^r)$, where GU^r is the aggregation utility function for the negotiation group toward risk factor r; and $\overline{GU}^r$ is the averaged utility for risk factor r.

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