

A scenario-based stochastic programming model for the control or dummy wafers downgrading problem

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SUMMARY

The subject of this paper is to study a realistic planning environment in wafer fabrication for the control or dummy (C/D) wafers problem with uncertain demand. The demand of each product is assumed with a geometric Brownian motion and approximated by a finite discrete set of scenarios. A two-stage stochastic programming model is developed based on scenarios and solved by a deterministic equivalent large linear programming model. The model explicitly considers the objective to minimize the total cost of C/D wafers. A real-world example is given to illustrate the practicality of a stochastic approach. The results are better in comparison with deterministic linear programming by using expectation instead of stochastic demands. The model improved the performance of control and dummy wafers management and the flexibility of determining the downgrading policy. Copyright © 2008 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Control or dummy (C/D) wafers are indispensable to manufacturing processes in semiconductor wafer fabrication. Control wafers are used to test the quality of equipment and monitor the process prior to risking the real product wafer. This ensures process stability and normal equipment operation. Control wafers may also be used with products together as proof of product quality in the process. Dummy wafers are used to distribute heat uniformly inside the furnaces. Any lack or surplus of C/D wafers may cause the loss of equipment capacity and production movement because they occupy equipment capacity of wafer fabrication, not only affecting production planning but also decreasing process yield.

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Control and dummy wafers become a critical issue since their cost is expensive. The benchmark of C/D wafers to product ratio for fabs is 0.93, courtesy of SEMATECH. A large number of C/D wafers must be prepared and stored in order to avert shortages. The cost of a 200 mm C/D wafer varies between \$35 and \$120 according to quality and characteristics. For a wafer fab that yields 3000 pieces of product wafers per week, the WIP level of C/D wafers may reach as many as 3000 pieces or more. A great portion of investment in a wafer fab, more than about 20 millions a year, is occupied by C/D wafers, which are non-production but necessary in the manufacturing process.

Cost is only a part of the C/D wafer issue because they also occupy the capacity of equipment, which is capital intensive. Good management can reduce the number of C/D wafers brought in to the manufacturing process and improve the efficiency of C/D wafer usage since the major characteristics of C/D wafers is that they can repeat the same functional test several times until they fail to conform to quality specifications related to requirements for cleanliness or thickness. The reuse statuses consist of the pre-disposition, in-use, and recycle stages, termed the PUR process, illustrated in Figure 1 with dotted arrows. We called it internal downgrading or recycling. Once a C/D wafer no longer conforms to the pre-defined specifications, it will be scraped or downgraded to next lower grade of which the quality specifications are not so high. Hence, such a kind of downgrading is referred to here as external downgrading due to wafer quality, indicated by a dotted line and bold arrows in Figure 1. Furthermore, releasing new raw wafers as any grade of C/D wafers or downgraded C/D wafers that directly bypass the PUR process to lower grades where there is a deficit of C/D wafers are regarded as external downgrading due to demand, indicated by a solid line and bold arrows. Wong and Hood [1] used discrete event simulation to run a hypothetical fab model with an industry-standard CMOS base process. They did not provide a method for efficiently managing test wafers but only examined the impact of test wafers on process cycle time, wafer throughput, and fab line equipment capacity requirements. Popovich *et al.* [2] mentioned that Motorola MOS12 designed a re-use matrix to determine the possible uses for C/D wafers. However, it is manual and thus has limitations due to the complexity of identifying downgrading paths and controlling the inventory of C/D wafers.

A great amount of research has been conducted on C/D wafer management but most was based on the assumption of known or expected demand. Chung *et al.* [3] used a nonlinear program to set a safe inventory level for control wafers. Although they assumed that demand follows an approximately normal distribution, the optimal solutions were based on deterministic expected values (EVs) to simplify stochastic events and dynamics that might reach misleading solutions. Some researchers focused on downgrading rules. Foster *et al.* [4] studied test wafer consumption by simulation. Although simulation can realize stochastic events and observe the effects by the current state of the system during a specific simulation run, it needs more time to produce results, and the randomness does not guarantee the same results between different runs. Chung *et al.* [5] proposed a linear programming (LP) model for the C/D wafers downgrading problem to minimize the total cost of C/D wafers by using expected demand in the photolithography area of a wafer fab. Wu *et al.* [6] aimed to minimize the long-term daily use of brand-new C/D wafers in a fab by an LP model. Özelkan and Çakanyıldırım [7] represented a resource downgrading problem as a network model with side constraints, which results in an integer programming formulation. Of the above, little work has been done to include the uncertainty of demands so as to meet the rapidly changing demands of the future. Liou *et al.* [8] established a capacity forecast model for C/D wafers for decision support instead of basing it on personal experience or the historical reservation data in practice.

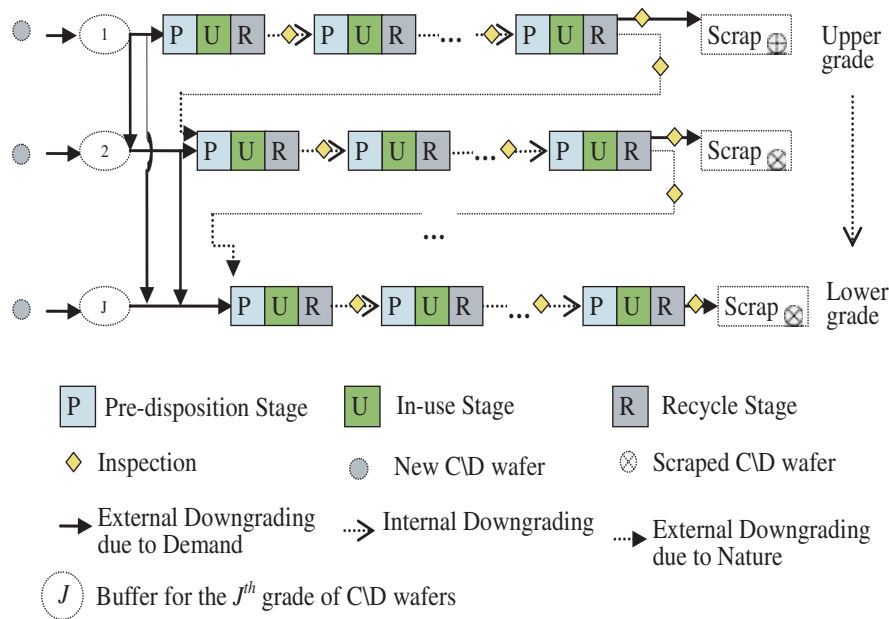


Figure 1. General multi-level downgrading diagram.

To attain the mission of C/D wafers in a fab, we define the C/D wafer problem (C/DWP) to minimize the total cost of C/D wafers while simultaneously determining their inventory policies, downgrading policies, and release rules for new wafers. We consider that the uncertainty of demands will result in more realistic planning decisions to meet rapidly changing future demands. Therefore, the purpose of this paper is to develop a two-stage stochastic programming (SP) model for the control or dummy wafers problem (SC/DWP) to minimize the total cost of C/D wafers and to set the amount of new C/D wafers released and C/D wafers recycled or downgraded to meet the stochastic demands of each grade. The proposed stochastic model, which is balanced and hedges against various scenarios, can describe the real-world production setting more realistically than the static approach can. Furthermore, a discrete approximation of stochastic demand gives advantages in terms of retaining a linear model and easier solutions by utilizing a single large equivalent LP model. It is more useful and efficient than a simulation approach.

In the remainder of this paper, we first introduce SP and related works in Section 2. A two-stage stochastic model is formulated for SC/DWP considering uncertain demands in Section 3. In Section 4, a numerical example shows the effectiveness and implications of the proposed model. Parameter sensitivity analysis was also conducted. Finally, concluding remarks are made.

2. TWO-STAGE SP APPROACH

Uncertainty is one of the main characteristics of semiconductor manufacturing systems. To handle uncertainty, it is appropriate to use a two-stage SP with recourse, which was first independently presented by Dantzig [9] and Beale [10]. It is a dynamic LP model characterized by uncertain

future outcomes for some parameters, as follows:

$$\begin{aligned}
 z = \min \quad & cx + E_{\omega}[Q(x, \omega)] \\
 \text{s.t.} \quad & Ax = b \\
 & x \geq 0 \quad \text{where} \\
 & Q(x, \omega) = \min \quad f(\omega)y \\
 \text{s.t.} \quad & D(\omega)y = d(\omega) + B(\omega)x \\
 & y \geq 0 \\
 & \omega \in \Omega
 \end{aligned}$$

The model is separated into two stages. At the first stage, the decision variables are chosen to minimize the direct cost and expected recourse cost that faces the recourse action taken. At the second stage, the decision variables are chosen due to the future uncertainty defined by probability space (Ω, P) . Matrix A , vector b , and vector c are known with certainty. The function $Q(x, \omega)$ is referred as the recourse function. The technology matrix $D(\omega)$, the right-hand side $d(\omega)$, the inter-stage link matrix $B(\omega)$, and the objective function coefficients $f(\omega)$ may be random. For a realization ω , the corresponding recourse action y is determined by $Q(x, \omega)$. Therefore, the optimal solution of the objective function hedges against all possible events $\omega \in \Omega$ that might occur in the future. Kall [11] suggests that ‘here and now’ (HN) and ‘wait and see’ (WS) are two different solution approaches to the SP. The WS approach assumes that the decision maker would not make the optimal decision until the outcome of a random variable can be observed. It is clear that such a solution is not implemented. The HN approach represents the true stochastic optimization solution without knowledge of the realization of random variables. Wets [12] surveyed the use of large-scale LP techniques and, using mathematical programming techniques, pointed out what seemed to be one of the most promising approaches to solve stochastic problems in some special cases.

3. A TWO-STAGE SP MODEL FOR C/DWP

In the following, an SP model for C/DWP is developed to minimize the total cost of C/D wafers and to determine the number of new wafers, recycled, and downgraded C/D wafers for each grade. The indices, parameters, and variables used in SC/DWP are listed in Table I.

3.1. Overview of stochastic management system of C/D wafers

The SC/DWP is depicted on a simplified representation of network system, as shown in Figure 2. It should be emphasized that the stages of a two-stage stochastic model refer to time periods. While $t = 1$ represents the time period called ‘HN’, $t = 2$ is the next time period to ‘wait and see’, and $t = 0$ is the previous time period. In Figure 2, each node represents the random demand for each grade of C/D wafers in each period. The solid arrows refer to external downgrading action due to demand while the segmented arrows refer to external downgrading action due to nature. The recycle or inventory is presented by dotted arrows. Therefore, in a stabilized system, the arrivals of C/D wafers at each node are equal to the departures. The inventory at the end of period

Table I. Indices, parameters, and variables used in SC/DWP.

Index	Definition	Sets
m	Product	$\mathbf{M}=\{1, 2, \dots, M\}$
j	Sequencing grade of C/D wafer	$\mathbf{J}=\{1, 2, \dots, J\}$
k	Number of grades changed	$\mathbf{K}=\{1, 2, \dots, J-1\}$
s	Demand scenario	$\xi=\{1, 2, \dots, S\}$
t	Time period	$\mathbf{T}=\{0, 1, 2\}$
c_0	Cost of the new C/D wafer/per unit	
c_{jj}	Total recycling cost in the j th grade/per unit	
$c_{ij}^{(n)}$ $c_{ij}^{(d)}$	Natural or demanding downgrading cost from the i th grade to the j th grade/per unit	
h_j	Cost of holding the C/D wafer of the j th grade	
I_j^0	Amount of inventory for the j th grade at time $t=0$	
r_j	Number of times of recycling for the C/D wafer in the j th grade	
u	Minimum level of inventory of the C/D wafers at each grade	
$R_r R_s$	Maximum rate of recycled and minimum rate of scraped C/D wafers at each grade	
$\mu_m \sigma_m^2$	Expected demand instantaneous drift rate and instantaneous variance of product m	
D_m^t	Demand of the product m at time t	
f_{jm}	Frequency of using the j th grade C/D wafers in the process for product m	
$d_j^t(s)$	Total demand for the j th grade of C/D wafers at time t (scenario s)	
$I_j^t(s)$	Amount of inventory for the j th grade of C/D wafers at time t (scenario s)	
$x_{0j}^t(s)$	Amount of new C/D wafers released to the j th grade at time t (scenario s)	
$x_{jj}^t(s)$	Amount of the j th grade of the C/D wafers for recycling or external downgrading at time t (scenario s)	
$x_{j(j+k)}^t(s)$	Amount of the j th grade of the C/D wafers downgraded to the $(j+k)$ th grade due to demand at time t (scenario s)	

t is available for withdrawal in the next period, and is also as the transit matrix that provides the linkage between the periods of the model. To avoid affecting the processes, backlogging is not allowed. In the next section, we will focus on the stochastic nature of demand, which is the essential contribution of this study.

3.2. Estimating demand model and constructing scenarios

One of the purposes of this paper is to study the economic effect due to volatility of demand for C/D wafers. Benavides *et al.* [13] first applied geometric Brownian motion as the demand model because the historical demand data from Semiconductor Industry Association were consistent with it. Later, Lin *et al.* [14] proposed using a geometric Brownian motion process to calibrate the degree of demand variation. Based on the above, we modeled the demands of products as a geometric Brownian motion process rather than conventional normal distribution. According to Dixit and Pindyck [15], and letting D_m^t be the demand of product m at time period t for SC/DWP, the rate of change of this demand is assumed to be governed by

$$dD_m^t = \mu_m D_m^t dt + \sigma_m D_m^t dz \quad (1)$$

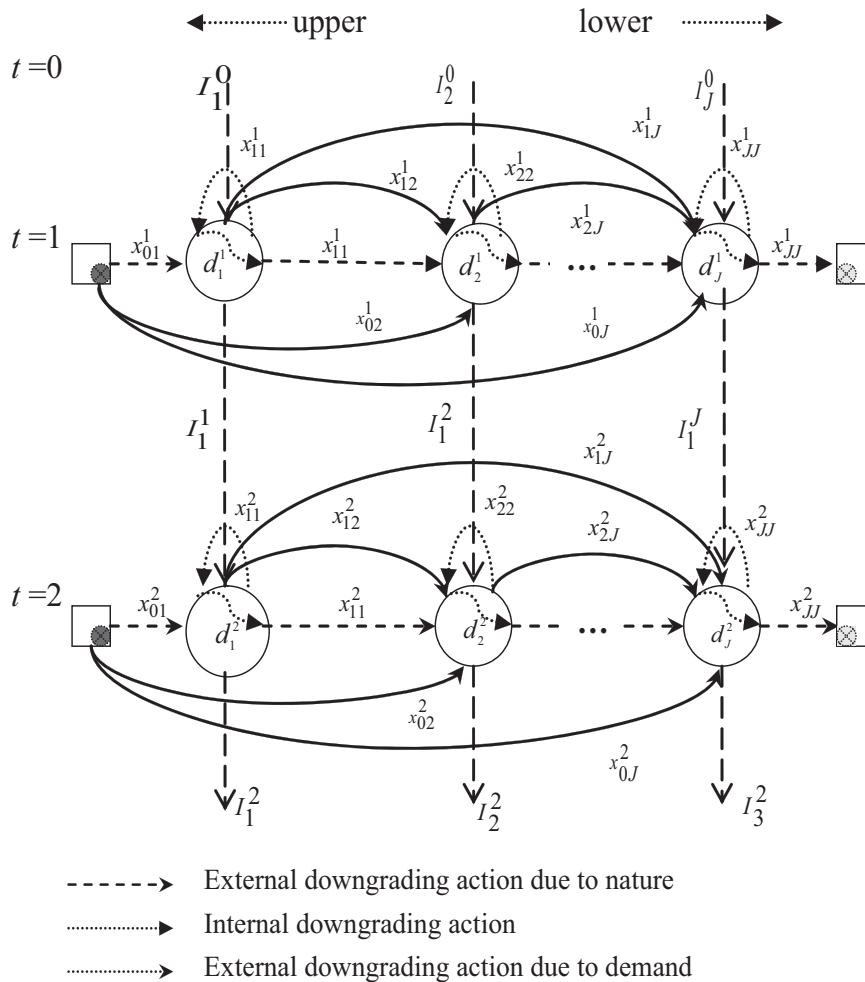


Figure 2. Schematic of the SC/DWP system.

where $dz = \varepsilon_t \sqrt{dt}$ and ε_t is assumed as a standard normal random variable with respect to the time interval t . This model of demand implies that the variability of demand increases linearly with the length of the demand forecast horizon so that over a finite time interval t , the change in the logarithm of demand is distributed as follows:

$$\ln(D_m^t) - \ln(D_m^0) = \ln\left(\frac{D_m^t}{D_m^0}\right) \sim N\left(\left[\mu_m - \frac{\sigma_m^2}{2}\right]t, \sigma_m^2 t\right), \quad m = 1, 2, \dots, M, \quad t = 0, 1, \dots, T \quad (2)$$

In making decisions under uncertainty, it is essential to represent uncertainties in a form suitable for quantitative models. The most popular method for SP is to generate a limited number of discrete scenarios that satisfy the specified random distribution. However, the number of scenarios increases exponentially while the number of random variables considered increases. This will lead

to SC/DWP intractably solved by an equivalent LP. To generate as small a number of scenarios as possible, Jarrow and Rudd [16] proposed a binary tree with equal probability method and proved it was a reasonably good approximation of the distribution of the specified random variable. Hence, we apply this method to generate the scenario tree of demands. There are two possibilities of demands for product m at time period $(1+t)$ with probability 0.5 as follows:

$$D_m^{t\pm} = D_m^1 \exp\left(\left[\mu_m - \frac{\sigma_m^2}{2}\right]t \pm \sigma_m \sqrt{t}\right) \quad (3)$$

3.3. Formulating the stochastic LP model

A two-stage SP model for SC/DWP was constructed for a theoretical manufacturing system based on the following assumptions:

- The product mix is given at time period $t = 1$, which represents ‘now’.
- The multi-level downgrading rule is applied.
- Engineering lots are not considered.
- A shortage of C/D wafers is not allowed.
- The C/D wafers are classified into J grades.
- The maximum recycle ratio of C/D wafers for each grade is determined to allow for the occurrence of unexpected breakages. (The ratio is the number of recycled C/D wafers to that of available C/D wafers.)
- The minimum scrap ratio of C/D wafers for each grade is determined to avoid waste due to excess of inventory. (The ratio is the number of scrapped C/D wafers to that of available C/D wafers.)

The integrated demand of the j th grade C/D wafers at each time period is calculated, respectively by

$$d_j^1 = \sum_{m=1}^M f_{jm} \times D_m^1, \quad d_{j(s)}^2 = \sum_{m=1}^M f_{jm} \times D_{m(s)}^2, \quad m = 1, 2, \dots, M, \quad s = 1, 2, \dots, 32 \quad (4)$$

The production planner’s objective is to minimize the total cost of the C/D wafers and to determine the supply amount of C/D wafers at each grade for each period and inventory amount at each grade for the next period. Equation (5) is the objective function that includes the cost of new C/D wafers, recycling cost, downgrading cost due to natural, downgrading cost due to demand, holding cost, and the expected cost of the second stage. The operative constraints, of which the first stage and the second stage given a specific scenario are formulated on the right-hand and left-hand side, respectively, are as follows. Equation (7) presents that the recycling capacity of the C/D wafers must meet the integrated demand of the j th grade. Equation (8) consists of balance constraints representing that the arrivals are equal to the departures at each grade. The recycle ratio is not more than a positive percentage given by Equation (9). The scrap rate is not less than a positive percentage given by Equation (10). The inventory of each grade is kept at a minimum level by Equation (11). Equation (12) represents that all variables are non-negative integers. In particular, the inventory variables for each grade at the end of the first stage are available for withdrawal during the second stage, and they also provide the linkage between the two stages. Finally, the first and second stage can be summed up in a single large LP model. Therefore, we determine all x ’s and I ’s to be optimal over all the scenarios because we solve the large LP for

all decision variables simultaneously.

$$\begin{aligned} \min \quad Z = & \sum_{j=1}^J c_0 x_{0j}^1 + \sum_{j=1}^J c_{jj} x_{jj}^1 + \sum_{j=1}^{J-1} c_{(j+1)(j+1)}^{(n)} x_{jj}^1 + \sum_{j=1}^J \sum_{k=1}^{J-j} c_{j(j+k)}^{(d)} x_{j(j+k)}^1 \\ & + \sum_{j=1}^J h_j I_j^1 + E_{\xi}[Q(X, I, \xi^S)] \end{aligned} \tag{5}$$

s.t.

$$\min \quad Q(s) = \sum_{j=1}^J c_0 x_{0j(s)}^2 + \sum_{j=1}^J c_{jj} x_{jj(s)}^2 + \sum_{j=1}^J \sum_{k=1}^{J-j} c_{j(j+k)} x_{j(j+k)(s)}^2 + \sum_{j=1}^J h_j I_{j(s)}^2 \tag{6}$$

$$r_j x_{jj}^1 \geq d_j^1, \quad r_j x_{jj(s)}^2 \geq d_{j(s)}^2 \tag{7}$$

$$I_j^0 + x_{(j-1)(j-1)}^1 + \sum_{k=1}^j x_{(j-k)j}^1 = I_j^1 + \sum_{k=0}^{J-j} x_{j(j+k)}^1$$

$$I_j^0 + x_{(j-1)(j-1)(s)}^1 + \sum_{k=1}^j x_{(j-k)j(s)}^1 = I_{j(s)}^1 + \sum_{k=0}^{J-j} x_{j(j+k)(s)}^1 \tag{8}$$

$$x_{jj}^1 / I_j^0 + x_{(j-1)(j-1)}^1 + \sum_{k=1}^j x_{(j-k)j}^1 \leq R_r$$

$$x_{jj(s)}^2 / I_{j(s)}^1 + x_{(j-1)(j-1)(s)}^2 + \sum_{k=1}^j x_{(j-k)j(s)}^2 \leq R_r \tag{9}$$

$$x_{jJ}^1 / \left(I_j^1 + \sum_{k=1}^{J-(j+1)} x_{j(j+k)j}^1 \right) \geq R_s, \quad x_{jJ(s)}^2 / \left(I_{j(s)}^2 + \sum_{k=1}^{J-(j+1)} x_{j(j+k)j(s)}^2 \right) \geq R_s \tag{10}$$

$$I_j^1 \geq u, \quad I_{j(s)}^2 \geq u \tag{11}$$

$$x_{ij}^1 \text{ are non-negative integers, } \quad i=0, 1, \dots, J-1, \quad j=1, 2, \dots, J \tag{12}$$

4. NUMERICAL EXAMPLE

To investigate the effect of the stochastic management system on the planning, real-world data are taken from a wafer fabrication factory located in the Science-Based Industrial Park in Hsin-Chu, Taiwan.

4.1. Data for SC/DWP

In this production system, there are five products A, B, C, D, and E with product mix 5:7:3:4:1 at the first stage. Based on historical data, we applied a geometric Brownian motion model and

Table II. The parameters of demands for each product.

Parameters	Product				
	A	B	C	D	E
μ	0.14	0.18	0.09	0.06	0.07
σ^2	0.22	0.19	0.14	0.14	0.13

Table III. The number of times for C/D wafer consumed at each grade.

Product	A	B	C	D	E
Grade 1	6	4	6	7	5
Grade 2	5	6	5	6	9
Grade 3	9	7	8	6	5

Table IV. The unit cost for (holding, recycling/natural downgrading, demand downgrading).

To	From			
	New	Grade 1	Grade 2	Grade 3
Grade 1	(-, 0,100)	(6, 80, -)		
Grade 2	(-, 0,100)	(-, 70, 80)	(6, 70, -)	
Grade 3	(-, 0,100)	(-, -, 80)	(-, 60, 70)	(6, 60, -)
Scrap		(-, -, 5)	(-, -, 5)	(-, -, 5)

estimated the drift and variance parameters of the demands for each product, as given in Table II. The monthly throughput target is 640 lots and the planning period is 28 days. C/D wafers can be categorized into three levels according to their conditions suitable for use in process. At the end of time period $t=0$, the inventory amount is 30 for grade 1, 40 for grade 2, and 50 for grade 3. The maximum times of recycling a C/D wafer at each grade is 4, 5, and 6 for grade 1, 2, and 3, respectively. Table III gives the frequencies of using the j th grade C/D wafers for each product and the unit cost for each kind is given in Table IV. The multilevel downgrading rule is implemented to minimize the total cost for SC/DWP. Finally, the large LP model is solved by using LINDO 6.01.

4.2. Economic benefit analysis and experimental study

The solution procedure includes the 'HN', 'wait and see' (WS), and 'EV' approaches. To assess the benefit of the SC/DWP model, the EV of perfect information (EVPI) and optimality index are investigated. EVPI measures the value of knowing the future with certainty. Optimality is defined by the ratio of EVPI to the WS optimal solution. It indicates how costly the incomplete information about the future is. To assess the value of knowing and using distributions on future outcomes, the value of the stochastic solution (VSS) and benefit are computed. Since benefit is the ratio of VSS to the HN optimal solution, the larger the benefit of the stochastic solution, the more the implemental stochastic optimization.

Table V. Economic benefit analysis for SC/DWP.

Benefit (%)	Optimality (%)	VSS	EVPI	EV	HN	WS
0.78	0.02	1139	29	146 414	145 275	145 246

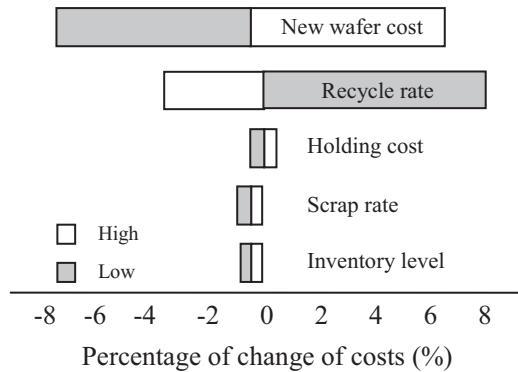


Figure 3. Sensitivity of value of optimal alternatives.

The results for SC/DWP are shown in Table V. With perfect information, the minimized total cost of C/D wafers is 145 246 dollars. With a 'HN' decision, we would make a minimized cost of 145 275 dollars. Note that the optimality index is 0.02%, which means the stochastic solution is nearly optimal. In other words, the EVPI is worthless. On the part of the value of the stochastic solution, SP is superior to the expected approach by 0.78%, as shown in Table V. This implies that, considering demand uncertainty, the accumulated capital could be saved up to 0.3 million U.S. dollars per year for wafer fabrication yielding 30 000 pieces of product wafers a month, since the WIP level of C/D wafers may be as many as 30 000 pieces priced at USD 100 each.

4.3. Sensitivity analysis

Here sensitivity analysis was conducted to determine how the results of the base case analysis reported above vary with changes in the principal parameters of the model. Given a product mix of 5:7:3:4:1, we considered the following as the principal parameters of the model: cost of new wafers, cost of holding, maximum recycle rate, minimum rate of scrap, and inventory level.

The results of the sensitivity analysis are summarized in the 'tornado' diagram of Figure 3. The figure shows how the value of the optimal minimized total cost changes as the individual parameters are changed to the high and low values of the parameter ranges shown in Table VI. The dramatic impact of new wafer cost and maximum recycle rate illustrates the central roles in SC/DWP management given demand uncertainties. The total cost of C/D wafers varies linearly by 7% with the cost of new wafers. Since the price of new wafers is market driven, it should be considered as a random variable in future research. Similarly, the impact of the maximum recycle rate on the results reflects the importance of the implemented downgrading rules. Nevertheless, the impact due to a 10% decrease in the recycle rate is twice as great as an increase in the same amount. One of the thumbs-up rules in C/D wafer management is to keep the recycle rate as high

Table VI. Parameter values for analysis.

Parameter	Low	Base	High
New wafer cost (\$)	80	100	120
Recycle rate (%)	70	80	90
Holding cost (\$)	4	6	8
Scrap rate (%)	5	10	15
Inventory (\$)	30	40	50

as possible. In contrast, the results show that holding cost, minimum scrap rate, and inventory level restrict the impact that the volatility of demands has on total cost.

5. CONCLUSION

This paper has proposed a two-stage stochastic model for C/DWP to minimize the total cost of control and dummy wafers when demands are uncertain. By explicitly considering a number of demand scenarios, a stochastic model can determine quantities of new wafer supply, recycling, and downgrading for each C/D wafer grade. We verified that this type of model can provide different insights than can the deterministic optimization model, in essence, which assumes that future demands are known with certainty. Given the substantial uncertainties of the semiconductor manufacturing business environment, the ability of a stochastic model to deliver the leading performance across a wide range of sensitivity analysis is impressive and valuable. In the presence of uncertainty, it is believed that implementing the multilevel downgrading rule will result in relatively significant savings and increase utilization efficiency by prolonging the life cycle of C/D wafers.

However, it is impossible to find a solution that is ideal under all circumstances; even decisions in stochastic models are balanced, or hedged against various scenarios. Therefore, care must be taken not to overstate the benefits of stochastic models. For future study, we can focus on estimating a demand model and generating economic scenarios to improve the discrete approximation of the probability distribution. In addition, establishing a multi-objective stochastic model for SC/DWP to minimize total cost can achieve multiple planning targets at one time.

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