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Fuzzy-identification-based adaptive backstepping control using a self-organizing fuzzy system

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Abstract In this paper, a fuzzy-identification-based adaptive backstepping control (FABC) scheme is proposed. The FABC system is composed of a backstepping controller and a robust controller. The backstepping controller, which uses a self-organizing fuzzy system (SFS) with the structure and parameter learning phases to on-line estimate the controlled system dynamics, is the principal controller, and the robust controller is designed to dispel the effect of approximation error introduced by the SFS. The developed SFS automatically generates and prunes the fuzzy rules by the proposed structure adaptation algorithm and the parameters of the fuzzy rules and membership functions tunes on-line in the Lyapunov sense. Thus, the overall closed-loop FABC system can guarantee that the tracking error and parameter estimation error are uniformly ultimately bounded; and the tracking error converges to a desired small neighborhood around zero. Finally, the proposed FABC system is applied to a chaotic dynamic system to show its effectiveness. The simulation results verify that the proposed FABC system can achieve favorable tracking performance even with unknown controlled system dynamics.

Keywords Adaptive control · Backstepping control · Chaotic dynamic system · Self-organizing fuzzy system · Structure adaptation

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1 Introduction

Fuzzy control (FC) has achieved many practical successes; however, it has not been viewed as rigorous. Because FC lacks a systematic design procedure to determine proper membership functions and fuzzy rules, and the way to guarantee the global stability still needs to be explored (Timothy 1995; Wang 1997). To tackle this problem, some researchers have focused on the use of the Lyapunov synthesis approach to construct a stable adaptive fuzzy control (AFC) scheme (Leu et al. 2005; Lin 2002; Lin and Hsu 2002; Wang 1994; Wang and Lin 2000). Based on the universal approximation property of fuzzy systems, the AFC design methods can provide stabilizing controller in the Lyapunov sense for nonlinear systems with dominant uncertain nonlinearities by using sufficiently complex approximation functions (Wang 1994). With these approaches, the fuzzy rules and membership function can be automatically adjusted by some adaptation laws to achieve satisfactory system response. If an adaptive fuzzy controller uses fuzzy systems as a model of the plant, it is referred to as a indirect adaptive fuzzy controller. If an adaptive fuzzy controller uses fuzzy systems as controllers, it is referred to as a direct adaptive fuzzy controller (Wang 1994).

Though the performances of AFC are acceptable in Leu et al. (2005), Lin (2002), Lin and Hsu (2002), Wang (1994) and Wang and Lin (2000), the learning algorithms only take care of parameter learning but neglect structure learning of fuzzy system. Time-consuming trial-and-error process or experts' knowledge is still needed to determine the structure's size of fuzzy system. However, if much time or experienced experts are unavailable, an ill-structure of fuzzy system may be constructed and causes serious problems. In general, more fuzzy rules are needed as more favorable performance is required. On the other hand, an overly-small fuzzy rule base may not be robust enough to achieve favorable control performance. To solve the problem of structure determination in fuzzy system, much interest has been focused on the adaptive structure learning approach for self-organizing fuzzy system (SFS) or self-organizing fuzzy neural network (SFNN) (Leng et al. 2005; Lin et al. 2005; Pal and Pal 1999). Lin et al. proposed a self-constructing fuzzy neural network; unfortunately, the proposed approach cannot prevent the structure of the network from growing unboundly (Lin et al. 2005). Pal and Pal proposed a rule pruning algorithm to prune the fuzzy rules; however, the searching space for the connection weights is restricted to R^+ (Pal and Pal 1999). This may harm the capability of the proposed network to lower the value of residual square error. In (Leng et al. 2005), both rule adding and pruning are performed in the structure learning of the proposed self-organizing fuzzy neural network. It has been demonstrated that by the proposed adaptive structure learning approach the fuzzy rules can be automatically generated or pruned. The adding approach is derived form geometric growing criterion and satisfies ε-completeness of fuzzy rules; on the other hand, second derivative information is used to find the unimportant rule which should be pruned. However, it can be imagined that such complicated structuring learning may lead to computational load so that they are not suitable for online practical applications. Recently, some intelligent control schemes utilize the SFS approach proposed in Gao and Er (2003), Hsu (2007), Lin et al. (2001) and Park et al. (2003, 2005). However, some of them use the gradient descent method to derive the parameter learning algorithms which cannot guarantee the system stability (Lin et al. 2001). Some of them derive the parameter learning algorithms in the Lyapunov sense to guarantee system stability, but the structure learning algorithm is too complex (Gao and Er 2003; Hsu 2007; Park et al. 2003, 2005). In Hsu (2007), Lin et al. (2001) and Park et al. (2005), a self-constructing fuzzy neural network control is proposed to avoid the newly generated membership function being too similar to the existing ones. However, the structure would grow large as the input data has large variations. Gao and Er use an error reduction ratio with QR decomposition to prune the rules; however, the design procedure is overly complex (Gao and Er 2003). In Park et al. (2003), the developed adaptation law does not consider the tuning of membership functions, and the structure learning does not consider the pruning algorithm of fuzzy rules. This maybe causes slow convergence of the parameters and unbound structure size.

In this paper, a fuzzy-identification-based adaptive backstepping control (FABC) system for an unknown nonlinear system is proposed. The FABC system is composed of a backstepping controller and a robust controller. The backstepping controller containing a SFS is designed in the sense of the adaptive backstepping control technique. The SFS with simultaneous structure and parameter learning is used to on-line estimate the controlled system dynamics. The structure learning considers both growing and pruning of fuzzy rules. A new membership function is generated (resulting in new generated rules) when a new incoming input signal lies far away from the input range, i.e., the membership degrees in all its fuzzy sets are quite small. A rule is considered redundant and pruned when it has little contribution to the output of the SFS for a period of time. The robust controller is designed to dispel the effect of approximation error between SFS approximation and controlled system dynamics. The developed structure learning algorithm empowers the SFS to automatically generate and prune the fuzzy rules during the learning process. All the control parameters of FABC can be on-line tuned by the adaptive laws derived in the Lyapunov sense; thus the uniformly ultimately bounded stability of the closed-loop control system can be guaranteed. Finally, the FABC system using the developed SFS is applied to a chaotic dynamic system. Simulation results verify that the proposed control scheme can achieve favorable tracking performance.

2 Problem formulation

Chaotic dynamic systems have been studied and known to exhibit complex dynamical behavior. The interest in chaotic dynamic systems lies mostly upon their complex, unpredictable behavior, and extreme sensitivity to initial conditions as well as parameter variations. Consider a second-order chaotic dynamic system, the well known Duffing's equation, which describes a special nonlinear circuit or a pendulum moving in a viscous medium under control. The dynamics of Duffing's equation is described as (Chen and Dong 1993; Loria et al. 1998)

$$\ddot{x} = -p\dot{x} - p_1 x - p_2 x^3 + q\cos(wt) + u = f + u \qquad (1)$$

where t is the time variable; w is the frequency, $f = -p\dot{x} - p_1 x - p_2 x^3 + q\cos(wt)$ is the system dynamic function, u is the control effort, and p, p_1, p_2 and q are real constants. Depending on the choices of these constants, the solutions of system (Eq. 1) may display complex phenomena, including various periodic orbits behaviors and some chaotic behaviors. To observe these complex phenomena, the open-loop system behavior with $u = 0$ was simulated with $p = 0.4$, $p_1 = -1.1$, $p_2 = 1.0$ and $w = 1.8$. The phase plane plots with an initial condition (0, 0) are shown in Fig. 1a, b for $q = 2.1$ (chaotic) and $q = 7.0$ (period 1), respectively. It is shown that the uncontrolled chaotic dynamic system has different chaotic trajectories for different values of q (Chen and Dong 1993). The control objective is to find a control law so that the chaotic trajectory can track the desired periodic orbit. Assume that all the parameters of system (Eq. 1) are exactly known, the design of ideal backstepping control for the chaotic dynamic system is described step-by-step as follows (Hsu et al. 2006; Slotine and Li 1991).

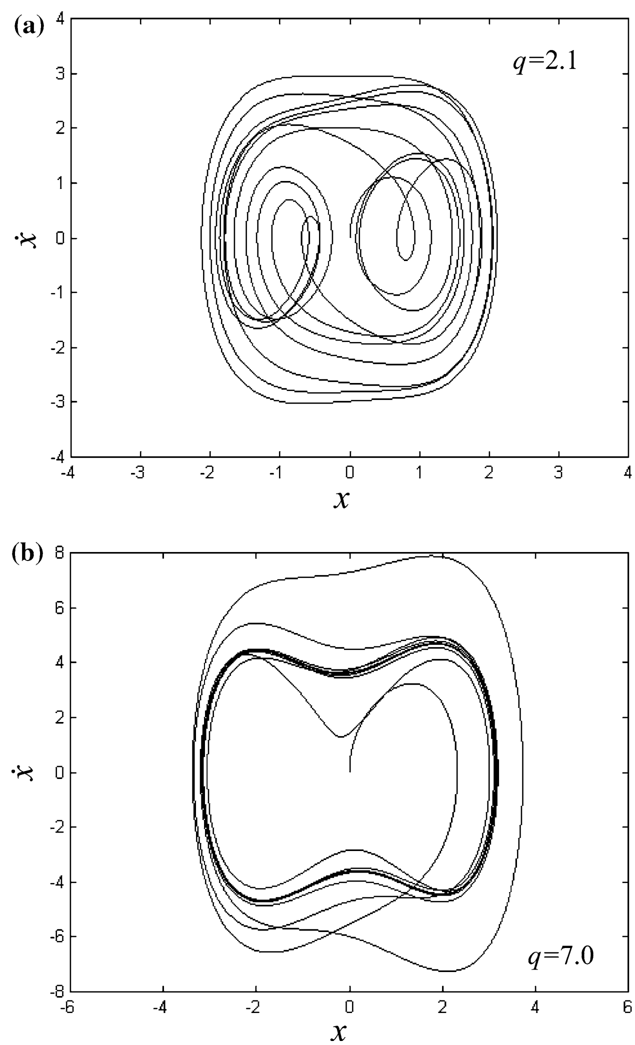

Fig. 1 Typical chaotic orbits of the chaotic dynamic system

Step 1 Define the tracking error

$$e_1 = x - x_c \tag{2}$$

where x_e is the command trajectory. The time derivative of tracking error is defined as

$$\dot{e}_1 = \dot{x} - \dot{x}_c. \tag{3}$$

The $\dot{x}$ can be viewed as a virtual control in the equation. Define the following stabilizing function

$$\alpha = -\tau_1 e_1 + \dot{x}_c \tag{4}$$

where τ_1 is a positive constant.

Step 2 Define

$$e_2 = \dot{x} - \alpha. \tag{5}$$

Then the time derivative of e_2 is expressed as

$$\dot{e}_2 = \ddot{x} - \dot{\alpha} = \ddot{x} - (-\tau_1 \dot{e}_1 + \ddot{x}_c) = \ddot{e}_1 + \tau_1 \dot{e}_1. \tag{6}$$

Step 3 If the dynamic system is known, an ideal backstepping controller can be obtained as (Hsu et al. 2006)

$$u_{ib} = \ddot{x}_c - f - \tau_1 \dot{e}_1 - \tau_2 e_2 - e_1 \tag{7}$$

where τ_2 is a positive constant. Substituting Eq. 7 into 1 yields

$$\dot{e}_2 = -\tau_2 e_2 - e_1. \tag{8}$$

Step 4 Define the Lyapunov function as

$$V_1(e_1, e_2) = \frac{e_1^2}{2} + \frac{e_2^2}{2}. \tag{9}$$

Differentiating Eq. 9 with respect to time and using Eqs. 3 and 8, yields

$$\begin{aligned} \dot{V}_1(e_1, e_2) &= e_1 \dot{e}_1 + e_2 \dot{e}_2 \\ &= e_1(e_2 - \tau_1 e_1) + e_2(-\tau_2 e_2 - e_1) \\ &= -\tau_1 e_1^2 - \tau_2 e_2^2 \leq 0 \end{aligned} \tag{10}$$

This implies that e_1 and e_2 converge to zero as $t \to \infty$ (Slotine and Li 1991). Therefore, the ideal backstepping controller in Eq. 7 will asymptotically stabilize the system.

3 Self-organizing fuzzy system (SFS)

Since the system dynamic function f may be unknown or perturbed in practical application, the ideal backstepping controller (Eq. 7) cannot be precisely obtained. To solve this problem, a SFS with the structure and parameter adaptation learning algorithm is developed to on-line estimate the system dynamic function f in this paper.

3.1 Description of SFS

Consider a two-input one-output fuzzy system wherein the IF-THEN rules are of the following form (Wang 1997)

$$\text{Rule}_{i_1,i_2}: \text{IF } X_1 \text{ is } F_1^{i_1} \text{ and } X_2 \text{ is } F_2^{i_2}, \text{ THEN } f \text{ is } \alpha_{i_1,i_2} \tag{11}$$

where X_1 and X_2 are the input variables; f is the output variable; α_{i_1,i_2} is the crisp singleton consequent; $F_j^{i_j}, j = 1, 2$ are the fuzzy sets characterized by the fuzzy membership function $F_j^{i_j}(X_j)$ with $i_j \in \{1, 2, \ldots, N_j\}$ being the number of membership functions of X_j. Define a set Ω which collects all possible fuzzy rules:

$$\mathbf{\Omega} = \{\text{Rule}_{i_1,i_2} | i_1 = 1, 2, \ldots, N_1; i_2 = 1, 2, \ldots, N_2\}. \tag{12}$$

The output of SFS can be expressed as (Wang 1997)

$$f = \frac{\sum_{\text{Rule}_{i_1,i_2} \in \mathbf{\Omega}_{sub}} \alpha_{i_1,i_2} \left[\prod_{j=1}^{2} \mu_{F_j^{i_j}}(X_j)\right]}{\sum_{\text{Rule}_{i_1,i_2} \in \mathbf{\Omega}_{sub}} \left[\prod_{j=1}^{2} \mu_{F_j^{i_j}}(X_j)\right]} \tag{13}$$

where $\mathbf{\Omega}_{sub} \subseteq \mathbf{\Omega}$ is the rule base. Then, SFS can be represented as a linear combination of fuzzy basis functions defined as

$$\xi_{i_1,i_2} = \frac{\Pi_{j=1}^{2} \mu_{F_j^{i_j}}(X_j)}{\sum_{\text{Rule}_{i_1,i_2} \in \mathbf{\Omega}_{sub}} \left[\Pi_{j=1}^{2} \mu_{F_j^{i_j}}(X_j)\right]}, i_j \in \{1, 2, \ldots, N_j\}, \quad j = 1, 2. \tag{14}$$

In this study, a Gaussian membership function is defined as

$$\mu_{F_j^{i_j}}(X_j, c_j^{i_j}, \sigma_j^{i_j}) = \exp\left[\frac{(X_j - c_j^{i_j})^2}{\sigma_j^{i_j 2}}\right] \tag{15}$$

where $c_j^{i_j}$ and $\sigma_j^{i_j}$ are the mean and standard deviation of Gaussian function, respectively. However, $\sigma_j^{i_j}$ may become to zero in the training procedure and thus the firing weight $\mu_{F_j^{i_j}}(\cdot)$ will not be defined. To avoid this problem, this paper consider a novel form of membership function as

$$\mu_{F_j^{i_j}}(X_j, c_j^{i_j}, \sigma_j^{i_j}) = \exp\left[\frac{(X_j - c_j^{i_j})^2}{\sigma_j^{i_j 2} + \bar{w}}\right] \tag{16}$$

where $\bar{w}$ is a small positive constant.

If the number of the fuzzy rules is chosen too large, the computation loading is heavy so that they are not suitable for online practical applications. If the number of the fuzzy rules is chosen too small, the learning performance may be not good enough to achieve desired performance. In order to avoid the time consuming process of constructing proper fuzzy rules, a structure adaptation algorithm is developed. Before building fuzzy rules, every input space $S(X_j)$ is partitioned into several overlapping clusters to construct the fuzzy sets of X_j. Traditionally, this work is done by humans. However, even with experienced experts, it could happen that for some incoming X_j, the membership degrees of all its fuzzy sets are quite small, i.e., $F_j^{i_j}(X_j), i_j = 1, 2, \ldots, N_j$ are quite small, as depicted in Fig. 2a. This implies that the input space $S(X_j)$ is not properly clustered. Hence, the fundamental concept of the growing of fuzzy rules is developed to adjust the inappropriate clustering. Initially, create one initial fuzzy rule for the given initial state as

$$\text{Rule}_{1,1} \text{ IF } X_1 \text{ is } F_1^1 \text{ and } X_2 \text{ is} F_2^1 \text{ THEN } f \text{ is } \alpha_{1,1} \tag{17}$$

where the membership functions for F_j^1, $j = 1, 2$, are defined with the initial input, $X_j(0)$, as the following form

$$\mu_{F_j^1}(X_j) = \exp\left\{-\frac{[X_j - X_j(0)]^2}{\sigma_j^{12}}\right\}. \tag{18}$$

The fuzzy approximator will be operated with this single rule. Define the growing criterion as

$$\mu_j^{\max} < \Theta_g, \quad j = 1, 2 \tag{19}$$

where $\mu_j^{\max} = \max_{i_j=1,2,\ldots} \mu_{F_j^{i_j}}(X_j)$ is the max membership function degree of X_j and $\Theta_g \in (0, 1)$ is a given threshold. During the control process, if the growing criterion (Eq. 18) is satisfied for new incoming X_j, $1 \le j \le 2$, an new membership functions with the mean and standard deviation are created as (Lin et al. 2005)

$$c_j^{N_j+1} = X_j \tag{20}$$

$$\sigma_j^{N_j+1} = \bar{\sigma} \tag{21}$$

where $\bar{\sigma}$ is a predefined positive constant. The created membership function is shown as Fig. 2b. For example, if one new membership function is created for X_1, N_1 new fuzzy rules will be generated according to the new membership function.

$$\begin{aligned}
&\text{Rule}_{N_1+1}: \text{IF } X_1\ F_j^{N_1+1} && \text{and } X_2 \text{ is } F_2^1 \text{ THEN} f \text{ is } \alpha_{N_1+1,1} \\
&\text{Rule}_{N_j+1,2}: \text{IF } X_1 \text{ is } F_1^{N_1+1} && \text{and } X_2 \text{ is } F_2^2 \text{ THEN} f \text{ is } \alpha_{N_1+1,2} \\
&&& \vdots \\
&\text{Rule}_{N_j+1,N_2}: \text{IF } X_1 \text{is } F_1^{N_1+1} && \text{and } X_2 \text{ is } F_2^{N_2}, \text{THEN} f \text{ is } \alpha_{N_1+1,N_2}
\end{aligned} \tag{22}$$

where $\alpha_{N_1+1,1}, \alpha_{N_1+1,2}, \ldots, \alpha_{N_1+1,N_2}$ are initialized from zeros. If one new membership function is created for X_2, N_1 new fuzzy rules will be generated in the way similar to Eq. 22. Next the structure learning phase is considered to determine whether or not to eliminate the inappropriate existing fuzzy rules. The contribution made by kth rule on the output f can be defined in 2-norm sense as

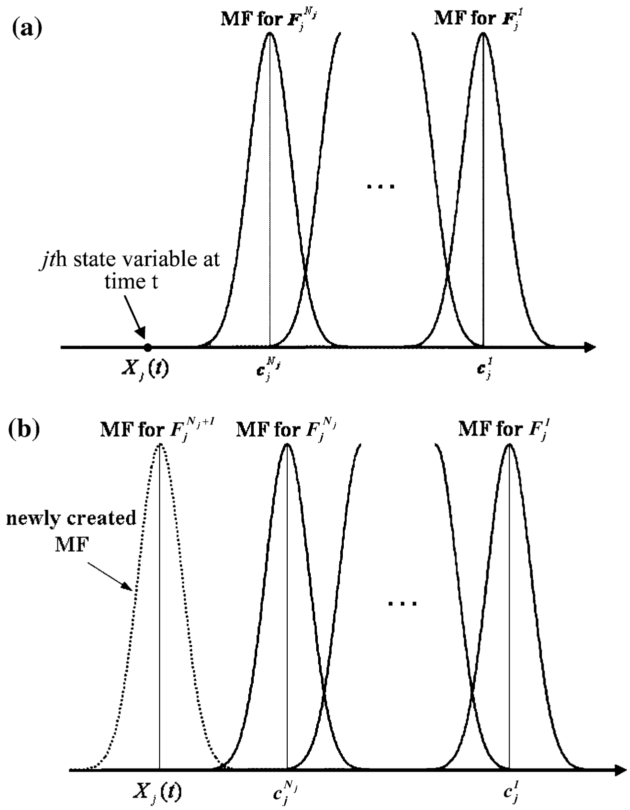

Fig. 2 **a** Improper fuzzy clustering of input variable X_j; **b** Newly created membership functions

$$C_k = \frac{|f_k|}{\sum_{k=1}^{n} |f_k|}, \quad k = 1, 2, \ldots, n \tag{23}$$

where $f_k = \alpha_k \xi_k$ and n is the number of the existing fuzzy rule. A significance index which determines the importance of the k-th rules is given as follows

$$S_k = \begin{cases} S_k^{rc}\tau, & \text{if } C_k < \eta \\ S_k^{rc}, & \text{if } C_k \geq \eta \end{cases}, \quad k = 1, 2, \ldots, n \tag{24}$$

where S_k^{rc} is the most recent S_k, $\tau \in (0, 1)$ is a decay constant, and $\eta \in (0, 1)$ is a given constant. The initial value of S_k is 1. It can be observed from Eq. 18 that S_k declines when C_k is smaller than η. A decaying significance index implies that the associated rule is becoming less and less important and should be pruned. The pruning criterion of kth fuzzy rule is thus defined as follows.

$$S_k < \Theta_p, \quad k = 1, 2, \ldots, n \tag{25}$$

where $\Theta_p \in (0, 1)$ is a selected threshold. If the pruning criterion (Eq. 25) is satisfied for S_k, the associated kth rule is pruned. An exponential function was used to calculate the value of significant index for each existing fuzzy rule in [0, 1]; however, the computation load would be large (Hsu 2007). In this study, decay constant is used to calculate the value of significant index. It is better than exponential function used in Hsu (2007), since it is simple to implement and easy to design.

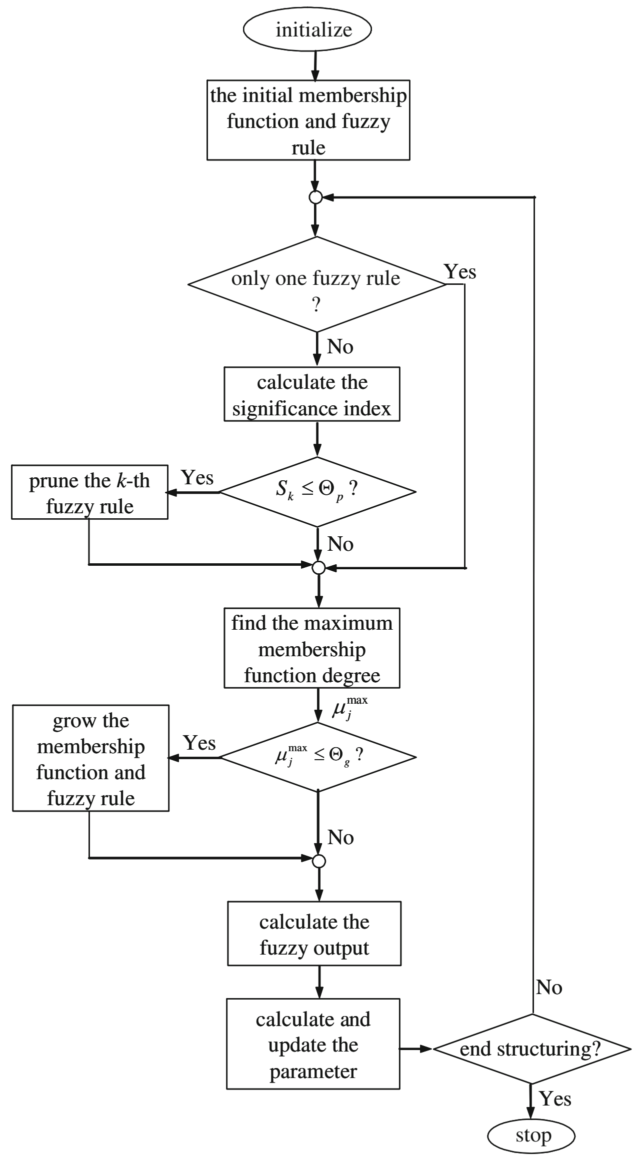

Fig. 3 Flowchart of the structure learning algorithm

In summary, the flowchart of the structure learning algorithm for the SFS is summarized in Fig. 3. The major contributions of the SFS are: (1) fuzzy rules can be automatically generated and pruned by the structure learning approach; (2) the concepts of growing and pruning of rules are quite simple and intuitive and thus only little computational load is caused by the structure learning; (3) no prior knowledge is needed to determine the fuzzy partitions of input spaces and the number of fuzzy rules; the rule base is automatically constructed from a single rule which is determined according to the initial input signal to the SFS; (4) the computational load caused by the ineffective fuzzy rules can be relieved.

Remark 1 In this paper, a two-input one-output fuzzy system is considered. It is reasonable that the proposed SFS still works with larger input dimension. Though much more rules may be generated, the ineffective or redundant rules will be pruned by the proposed self-organizing approach.

Remark 2 If the computation load is the important issue, the threshold Θ_p should be chosen large enough so that more fuzzy rules are pruned, or the threshold Θ_g should be chosen small enough so that fewer fuzzy rules are generated.

3.2 Fuzzy approximation

For ease of notation, define vectors $\mathbf{c}$ and $\boldsymbol{\sigma}$ as

$$\mathbf{c} = [\mathbf{c}_1\ \mathbf{c}_2]^{\mathrm{T}} \tag{26}$$

$$\boldsymbol{\sigma} = [\boldsymbol{\sigma}_1\ \boldsymbol{\sigma}_2]^{\mathrm{T}} \tag{27}$$

where $\mathbf{c}_j = [c_j^1 \ldots c_j^{N_j}]^{\mathrm{T}}$ and $\boldsymbol{\sigma}_j = [\sigma_j^1 \ldots \sigma_j^{N_j}]^{\mathrm{T}}$ collect the means and standard deviations of the Gaussian membership functions of X_j, $j = 1, 2$, respectively. Equation 13 can be expressed in the vector form as

$$f = [\alpha_1\ \alpha_2\ \cdots \alpha_n]\begin{bmatrix}\xi_1\\ \xi_2\\ \vdots\\ \xi_n\end{bmatrix} = \boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{\xi}(\mathbf{X}, \mathbf{c}, \boldsymbol{\sigma}) \tag{28}$$

where $\mathbf{X} = [X_1X_2]$ is the input vector; α_k and ξ_k represent the singleton consequents and the fuzzy basis functions of kth fuzzy rule, respectively. It has been proven that there exists a fuzzy system so that it can uniformly approximate a nonlinear even time-varying function. Thus, an SFS is designed to estimate the system dynamics. To develop the control law, the fuzzy system described in Eq. 28 can approximate the unknown system dynamics as follows (Wang 1994; Wang and Lin 2000)

$$f = f^* + \Delta = \boldsymbol{\alpha}^{*\mathrm{T}}\boldsymbol{\xi}(\mathbf{X}, \mathbf{c}^*, \boldsymbol{\sigma}^*) + \Delta = \boldsymbol{\alpha}^{*\mathrm{T}}\boldsymbol{\xi}^* + \Delta \tag{29}$$

where $\boldsymbol{\alpha}^*$, $\mathbf{c}^*$, and $\boldsymbol{\sigma}^*$ are the bounded optimal vectors of $\boldsymbol{\alpha}$, $\mathbf{c}$, and $\boldsymbol{\sigma}$, and Δ denotes the approximation error. In fact, it is difficult to determine the optimal vectors to best approximate a nonlinear function. The optimal vectors even may not be unique. Therefore, a fuzzy approximator is defined as

$$\hat{f} = \hat{\boldsymbol{\alpha}}^{\mathrm{T}}\hat{\xi}(\mathbf{X}, \hat{\mathbf{c}}, \hat{\boldsymbol{\sigma}}) = \hat{\boldsymbol{\alpha}}^{\mathrm{T}}\hat{\xi} \tag{30}$$

where $\hat{\boldsymbol{\alpha}}$, $\hat{\mathbf{c}}$, and $\hat{\boldsymbol{\sigma}}$ are the estimation vectors of $\boldsymbol{\alpha}$, $\mathbf{c}$, and $\boldsymbol{\sigma}$, respectively. Hence, the modeling error, $\tilde{f}$, can be expressed as

$$\begin{aligned}\tilde{f} &= f - \hat{f} = \boldsymbol{\alpha}^{*\mathrm{T}}\boldsymbol{\xi}^* + \Delta - \hat{\boldsymbol{\alpha}}^{\mathrm{T}}\hat{\xi}\\ &= \tilde{\boldsymbol{\alpha}}^{\mathrm{T}}\hat{\xi} + \hat{\boldsymbol{\alpha}}^{\mathrm{T}}\tilde{\xi} + \tilde{\boldsymbol{\alpha}}^{\mathrm{T}}\tilde{\xi} + \Delta\end{aligned} \tag{31}$$

where $\tilde{\boldsymbol{\alpha}} = \boldsymbol{\alpha}^* - \hat{\boldsymbol{\alpha}}$ and $\tilde{\xi} = \boldsymbol{\xi}^* - \hat{\xi}$. In the following, some preliminaries will be made for adaptive online-tuning of the parameters of SFS, and thus favorable approximation can be achieved. To achieve this goal, the Taylor linearization technique is employed to transform the nonlinear fuzzy basis function into partially linear form as follows (Wang and Lin 2000)

$$\tilde{\xi} = \begin{bmatrix}\tilde{\xi}_1\\ \tilde{\xi}_2\\ \vdots\\ \tilde{\xi}_n\end{bmatrix} = \begin{bmatrix}\frac{\partial\xi_1}{\partial\mathbf{c}}\\ \frac{\partial\xi_2}{\partial\mathbf{c}}\\ \vdots\\ \frac{\partial\xi_n}{\partial\mathbf{c}}\end{bmatrix}\Bigg|_{\mathbf{c}=\hat{\mathbf{c}}}(\mathbf{c}^* - \hat{\mathbf{c}}) + \begin{bmatrix}\frac{\partial\xi_1}{\partial\boldsymbol{\sigma}}\\ \frac{\partial\xi_2}{\partial\boldsymbol{\sigma}}\\ \vdots\\ \frac{\partial\xi_n}{\partial\boldsymbol{\sigma}}\end{bmatrix}\Bigg|_{\boldsymbol{\sigma}=\hat{\boldsymbol{\sigma}}}(\boldsymbol{\sigma}^* - \hat{\boldsymbol{\sigma}}) + \mathbf{h} \tag{32}$$

or

$$\tilde{\xi} = \boldsymbol{\xi}_{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{c}} + \boldsymbol{\xi}_{\boldsymbol{\sigma}}^{\mathrm{T}}\tilde{\boldsymbol{\sigma}} + \mathbf{h} \tag{33}$$

where $\mathbf{h}$ represents the higher order term, $\tilde{\mathbf{c}} = \mathbf{c}^* - \hat{\mathbf{c}}$ and $\tilde{\sigma} = \boldsymbol{\sigma}^* - \hat{\boldsymbol{\sigma}}$. Substituting Eq. 33 into 31 yields

$$\begin{aligned}\tilde{f} &= \tilde{\boldsymbol{\alpha}}^{\mathrm{T}}\hat{\xi} + \hat{\boldsymbol{\alpha}}^{\mathrm{T}}\xi_{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{c}} + \hat{\boldsymbol{\alpha}}^{\mathrm{T}}\xi_{\boldsymbol{\sigma}}^{\mathrm{T}}\tilde{\boldsymbol{\sigma}} + \varepsilon\\ &= \tilde{\boldsymbol{\alpha}}^{\mathrm{T}}\hat{\xi} + \tilde{\mathbf{c}}^{\mathrm{T}}\xi_{\mathbf{c}}\hat{\boldsymbol{\alpha}} + \tilde{\boldsymbol{\sigma}}^{\mathrm{T}}\xi_{\boldsymbol{\sigma}}\hat{\boldsymbol{\alpha}} + \varepsilon\end{aligned} \tag{34}$$

where $\hat{\boldsymbol{\alpha}}^{\mathrm{T}}\boldsymbol{\xi}_{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{c}} = \tilde{\mathbf{c}}^{\mathrm{T}}\boldsymbol{\xi}_{\mathbf{c}}\hat{\boldsymbol{\alpha}}$ and $\hat{\boldsymbol{\alpha}}^{\mathrm{T}}\boldsymbol{\xi}_{\boldsymbol{\sigma}}^{\mathrm{T}}\tilde{\boldsymbol{\sigma}} = \tilde{\boldsymbol{\sigma}}^{\mathrm{T}}\boldsymbol{\xi}_{\boldsymbol{\sigma}}\hat{\boldsymbol{\alpha}}$ are used since they are scalars, and $\varepsilon = \hat{\boldsymbol{\alpha}}^{\mathrm{T}}\mathbf{h} + \tilde{\boldsymbol{\alpha}}^{\mathrm{T}}\tilde{\xi} + \Delta$ is the lumped uncertainty. Assume the lumped uncertainty ε is globally bounded by

$$|\varepsilon| \leq E \tag{35}$$

where E is a positive bound, which is difficult to determine in practical applications. This assumption is not strong because only the existence of E is required. Prior knowledge of E is not necessary in the development of FABC, and E will be estimated by a proposed estimation scheme.

4 Design of FABC

Since the system dynamic function f may be unknown or perturbed in practical application, the ideal backstepping controller (Eq. 7) cannot be precisely obtained. Then, a fuzzy-identification-based adaptive backstepping control (FABC) scheme, as shown in Fig. 4, is proposed to achieve favorable tracking performance for a tracking problem of a chaotic dynamic system. The FABC system is composed of a backstepping controller and a robust controller. The backstepping controller, which uses an SFS with the structure and parameter learning phases to

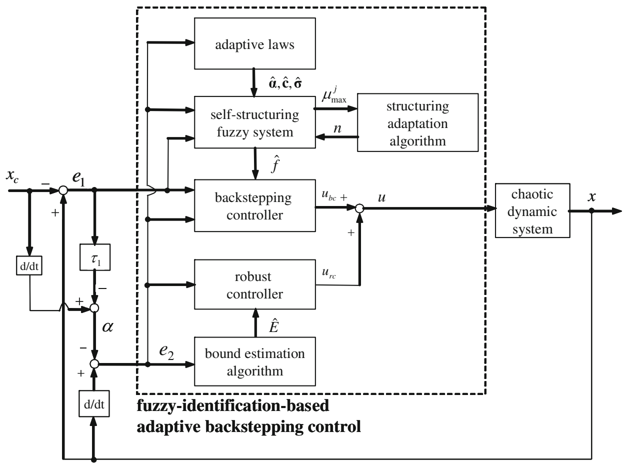

Fig. 4 The block diagram of the FABC system using a SFS

on-line estimate the controlled system dynamics, is the principal controller. The robust controller designed to dispel the effect of approximation introduced by the SFS. Define the tracking error e_1 as Eq. 2, a stabilizing function α as Eq. 4 and e_2 as Eq. 2. The control law of the FABC system defines as

$$u = u_{bc} + u_{rc} \tag{36}$$

where

$$u_{bc} = \ddot{x}_c - \hat{f} - \tau_1 \dot{e}_1 - \tau_2 e_2 - e_1 \tag{37}$$

and

$$u_{rc} = -\hat{E}\ \tanh\left(\frac{e_2}{\gamma}\right) \tag{38}$$

in which $\hat{f}$ is the output of SFS, $\hat{E}$ is the estimated bound value of the approximation error, $\tanh(\cdot)$ denotes the hyperbolic tangent function, and γ is a predefined positive constant. Substituting Eq. 36 into 1 yields

$$\dot{e}_2 = f - \hat{f} - \tau_2 e_2 - e_1 - \hat{E}\tanh\left(\frac{e_2}{\gamma}\right). \tag{39}$$

By using the modeling approximation error in Eq. 31, Eq. 39 can be rewritten as

$$\dot{e}_2 = \tilde{\boldsymbol{\alpha}}^{\mathrm{T}}\hat{\xi} + \tilde{\mathbf{c}}^{\mathrm{T}}\xi_c^{\mathrm{T}}\hat{\boldsymbol{\alpha}} + \tilde{\sigma}^{\mathrm{T}}\xi_\sigma^{\mathrm{T}}\hat{\boldsymbol{\alpha}} + \varepsilon - \tau_2 e_2 - e_1 - \hat{E}\tanh\left(\frac{e_2}{\gamma}\right). \tag{40}$$

Define the Lyapunov function as

$$V_2 = \frac{e_1^2}{2} + \frac{e_2^2}{2} + \frac{\tilde{\boldsymbol{\alpha}}^{\mathrm{T}}\tilde{\boldsymbol{\alpha}}}{2\beta_{\boldsymbol{\alpha}}} + \frac{\tilde{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{c}}}{2\beta_{\mathbf{c}}} + \frac{\tilde{\sigma}^{\mathrm{T}}\tilde{\sigma}}{2\beta_{\boldsymbol{\sigma}}} + \frac{\tilde{E}^2}{2\beta_E} \tag{41}$$

where $\tilde{E} \equiv E - \hat{E}$; $\beta_{\boldsymbol{\alpha}}, \beta_{\mathbf{c}}, \beta_{\boldsymbol{\sigma}}$ and β_E are positive constants. Differentiating Eq. 41 with respect to time and using Eq. 40 yields

$$\begin{aligned}
\dot{V}_2 &= e_1\dot{e}_1 + e_2\dot{e}_2 + \frac{\tilde{\boldsymbol{\alpha}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{\alpha}}}}{\beta_{\boldsymbol{\alpha}}} + \frac{\tilde{\mathbf{c}}^{\mathrm{T}}\dot{\tilde{\mathbf{c}}}}{\beta_{\mathbf{c}}} + \frac{\tilde{\sigma}^{\mathrm{T}}\dot{\tilde{\sigma}}}{\beta_{\boldsymbol{\sigma}}} + \frac{\tilde{E}\dot{\tilde{E}}}{\beta_E} = e_1(e_2 - \tau_1 e_1) \\
&\quad + e_2\Bigg[\tilde{\boldsymbol{\alpha}}^{\mathrm{T}}\hat{\xi} + \tilde{\mathbf{c}}^{\mathrm{T}}\xi_{\mathbf{c}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} + \tilde{\sigma}^{\mathrm{T}}\xi_{\boldsymbol{\sigma}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} + \varepsilon - \tau_2 e_2 \\
&\quad - e_1 - \hat{E}\tanh\left(\frac{e_2}{\gamma}\right)\Bigg] + \frac{\tilde{\boldsymbol{\alpha}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{\alpha}}}}{\beta_{\boldsymbol{\alpha}}} + \frac{\tilde{\mathbf{c}}^{\mathrm{T}}\dot{\tilde{\mathbf{c}}}}{\beta_{\mathbf{c}}} + \frac{\tilde{\sigma}^{\mathrm{T}}\dot{\tilde{\sigma}}}{\beta_{\boldsymbol{\sigma}}} + \frac{\tilde{E}\dot{\tilde{E}}}{\beta_E} \\
&= -\tau_1 e_1^2 - \tau_2 e_2^2 + \tilde{\boldsymbol{\alpha}}^{\mathrm{T}}\left(e_2\hat{\xi} + \frac{\dot{\tilde{\boldsymbol{\alpha}}}}{\beta_{\boldsymbol{\alpha}}}\right) + \tilde{\mathbf{c}}^{\mathrm{T}}\left(e_2\hat{\xi}_{\mathbf{c}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} + \frac{\dot{\tilde{\mathbf{c}}}}{\beta_{\mathbf{c}}}\right) \\
&\quad + \tilde{\sigma}^{\mathrm{T}}\left(e_2\hat{\xi}_{\boldsymbol{\sigma}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} + \frac{\dot{\tilde{\sigma}}}{\beta_{\boldsymbol{\sigma}}}\right) + e_2\varepsilon + e_2(\tilde{E} - E)\tanh\left(\frac{e_2}{\gamma}\right) \\
&\quad + \frac{\tilde{E}\dot{\tilde{E}}}{\beta_E} \le -\tau_1 e_1^2 - \tau_2 e_2^2 + \tilde{\boldsymbol{\alpha}}^{\mathrm{T}}\left(e_2\hat{\xi} + \frac{\dot{\tilde{\boldsymbol{\alpha}}}}{\beta_{\boldsymbol{\alpha}}}\right) \\
&\quad + \tilde{\mathbf{c}}^{\mathrm{T}}\left(e_2\hat{\xi}_{\mathbf{c}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} + \frac{\dot{\tilde{\mathbf{c}}}}{\beta_{\mathbf{c}}}\right) + \tilde{\sigma}^{\mathrm{T}}\left(e_2\hat{\xi}_{\boldsymbol{\sigma}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} + \frac{\dot{\tilde{\sigma}}}{\beta_{\boldsymbol{\sigma}}}\right) \\
&\quad + |e_2|E + e_2(\tilde{E} - E)\tanh\left(\frac{e_2}{\gamma}\right) + \frac{\tilde{E}\dot{\tilde{E}}}{\beta_E} \\
&= -\tau_1 e_1^2 - \tau_2 e_2^2 + \tilde{\boldsymbol{\alpha}}^{\mathrm{T}}\left(e_2\hat{\xi} + \frac{\dot{\tilde{\boldsymbol{\alpha}}}}{\beta_{\boldsymbol{\alpha}}}\right) + \tilde{\mathbf{c}}^{\mathrm{T}}\left(e_2\hat{\xi}_{\mathbf{c}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} + \frac{\dot{\tilde{\mathbf{c}}}}{\beta_{\mathbf{c}}}\right) \\
&\quad + \tilde{\sigma}^{\mathrm{T}}\left(e_2\hat{\xi}_{\boldsymbol{\sigma}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} + \frac{\dot{\tilde{\sigma}}}{\beta_{\boldsymbol{\sigma}}}\right) + \left[|e_2| - e_2\tanh\left(\frac{e_2}{\gamma}\right)\right]E \\
&\quad + \tilde{E}\left[e_2\tanh\left(\frac{e_2}{\gamma}\right) + \frac{\dot{\tilde{E}}}{\beta_E}\right]
\end{aligned} \tag{42}$$

The adaptive laws of SFS are designed as

$$\dot{\hat{\boldsymbol{\alpha}}} = -\dot{\tilde{\boldsymbol{\alpha}}} = \beta_{\boldsymbol{\alpha}}[e_2\boldsymbol{\xi} - \rho_{\boldsymbol{\alpha}}\hat{\boldsymbol{\alpha}}] \tag{43}$$

$$\dot{\hat{\mathbf{c}}} = -\dot{\tilde{\mathbf{c}}} = \beta_{\mathbf{c}}[e_2\boldsymbol{\xi}_{\mathbf{c}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} - \rho_{\mathbf{c}}\hat{\mathbf{c}}] \tag{44}$$

$$\dot{\hat{\boldsymbol{\sigma}}} = -\dot{\tilde{\sigma}} = \beta_{\boldsymbol{\sigma}}[e_2\boldsymbol{\xi}_{\boldsymbol{\sigma}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} - \rho_{\boldsymbol{\sigma}}\hat{\boldsymbol{\sigma}}] \tag{45}$$

and the adaptive laws of the approximation error bound is designed as

$$\dot{\hat{E}} = -\dot{\tilde{E}} = \beta_E\left[e_2\tanh\left(\frac{e_2}{\gamma}\right) - \rho_E\hat{E}\right] \tag{46}$$

where $\rho_{\boldsymbol{\alpha}}$, $\rho_{\mathbf{c}}$, $\rho_{\boldsymbol{\sigma}}$ and ρ_E are positive small constants. Every adaptive law in Eqs. 43–46 incorporates a leakage term based on σ-modification (Kim and Calise 2007; Wang and Hill 2006). These leakage terms are used to improve the robustness in the presence of approximation errors and prevent the parameter drifts of the system. The following inequality holds for any $v \in R$ and any $\gamma > 0$ (Park et al. 2003)

$$0 \le |v| - v\tanh\left(\frac{v}{\gamma}\right) \le \kappa\gamma \tag{47}$$

where κ is a constants satisfying $\kappa = \exp(-\kappa + 1)$, i.e., $\kappa = 0.2875$. Then, Eq. 42 can be expressed as

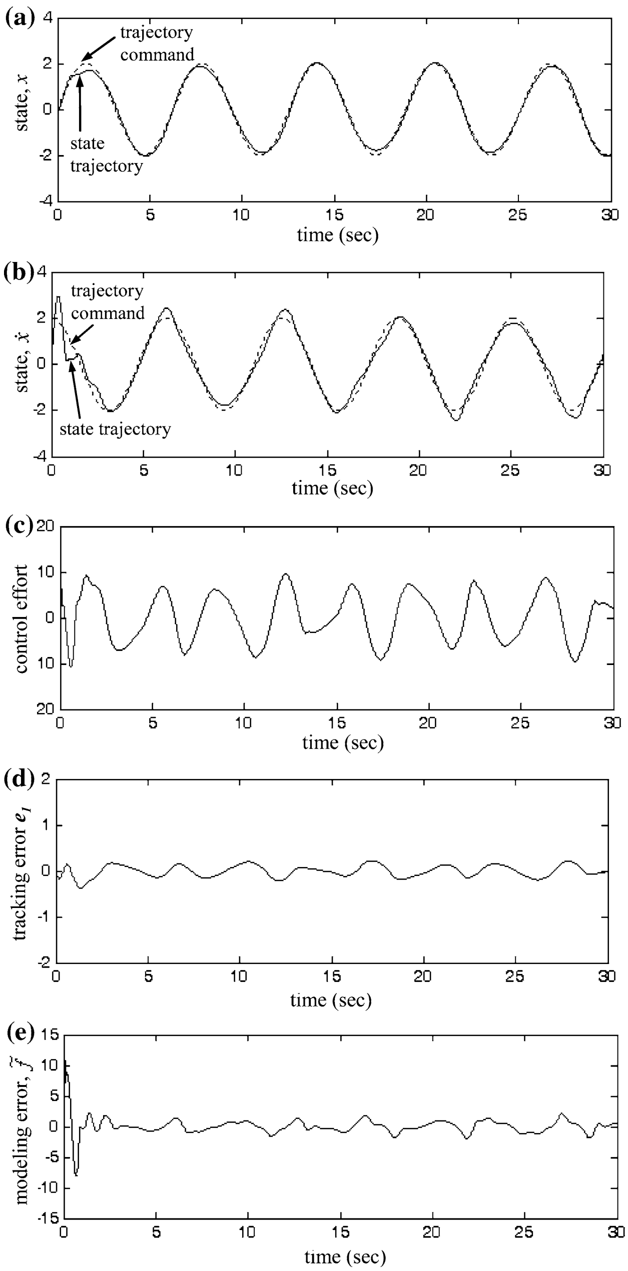

Fig. 5 Simulation results of FABC using 3 fuzzy rules for q = 7.0

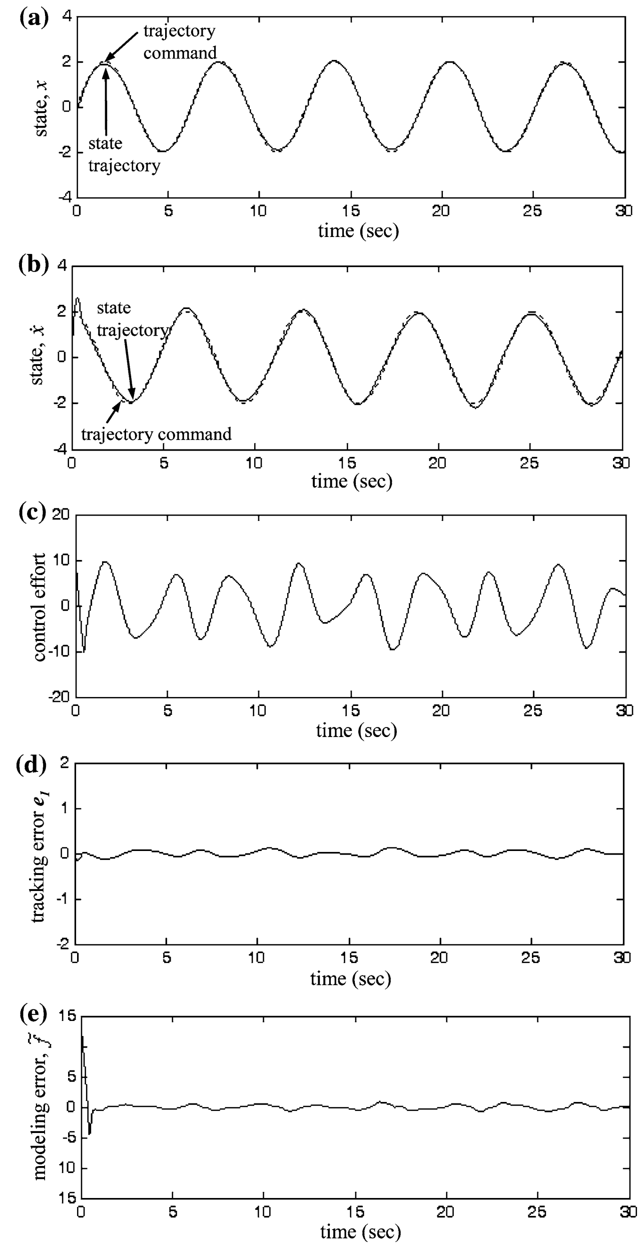

Fig. 6 Simulation results of FABC using 10 fuzzy rules for q = 7.0

$$\dot{V}_2 \leq -\tau_1 e_1^2 - \tau_2 e_2^2 + \rho_{\boldsymbol{\alpha}} \tilde{\boldsymbol{\alpha}}^{\mathrm{T}} \hat{\boldsymbol{\alpha}} + \rho_{\mathbf{c}} \tilde{\mathbf{c}}^{\mathrm{T}} \hat{\mathbf{c}} + \rho_{\boldsymbol{\sigma}} \tilde{\boldsymbol{\sigma}}^{\mathrm{T}} \hat{\boldsymbol{\sigma}} + \rho_E \tilde{E}^{\mathrm{T}} \hat{E} + \kappa\gamma E. \tag{48}$$

Completing the squares yields

$$\begin{aligned}
\dot{V}_2 \leq & -\tau_1 e_1^2 - \tau_2 e_2^2 \\
& + \frac{1}{2}\left\{\rho_{\boldsymbol{\alpha}} \|\tilde{\boldsymbol{\alpha}} + \hat{\boldsymbol{\alpha}}\|^2 + \rho_{\mathbf{c}} \|\tilde{\mathbf{c}} + \hat{\mathbf{c}}\|^2 + \rho_{\boldsymbol{\sigma}} \|\tilde{\sigma} + \hat{\boldsymbol{\sigma}}\|^2 + \rho_E (\tilde{E} + \hat{E})^2\right\} \\
& - \frac{1}{2}\left[\rho_{\boldsymbol{\alpha}} \|\tilde{\boldsymbol{\alpha}}\|^2 + \rho_{\mathbf{c}} \|\tilde{\mathbf{c}}\|^2 + \rho_{\boldsymbol{\sigma}} \|\tilde{\sigma}\|^2 + \rho_E \tilde{E}^2\right] \\
& - \frac{1}{2}\left[\rho_{\boldsymbol{\alpha}} \|\hat{\boldsymbol{\alpha}}\|^2 + \rho_{\mathbf{c}} \|\hat{\mathbf{c}}\|^2 + \rho_{\boldsymbol{\sigma}} \|\hat{\boldsymbol{\sigma}}\|^2 + \rho_E \hat{E}^2\right] + \kappa\gamma E \\
\leq & -\tau_1 e_1^2 - \tau_2 e_2^2 - \frac{1}{2}\left[\rho_{\boldsymbol{\alpha}} \|\tilde{\boldsymbol{\alpha}}\|^2 + \rho_{\mathbf{c}} \|\tilde{\mathbf{c}}\|^2 + \rho_{\boldsymbol{\sigma}} \|\tilde{\sigma}\|^2 + \rho_E \tilde{E}^2\right] \\
& + \frac{1}{2}\left[\rho_{\boldsymbol{\alpha}} \|\boldsymbol{\alpha}^*\|^2 + \rho_{\mathbf{c}} \|\mathbf{c}^*\|^2 + \rho_{\boldsymbol{\sigma}} \|\boldsymbol{\sigma}^*\|^2 + \rho_E E^2\right] + \kappa\gamma E \\
\leq & -sV_2 + z
\end{aligned} \tag{49}$$

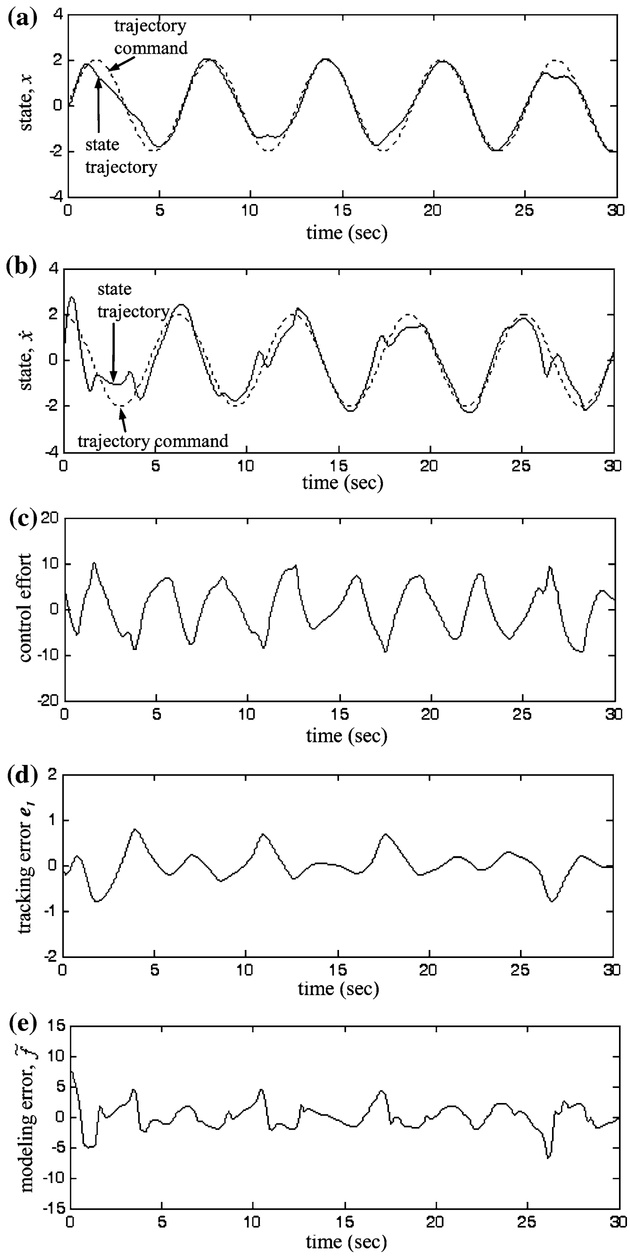

Fig. 7 Simulation results of FABC using 20 fuzzy rules for $q = 7.0$

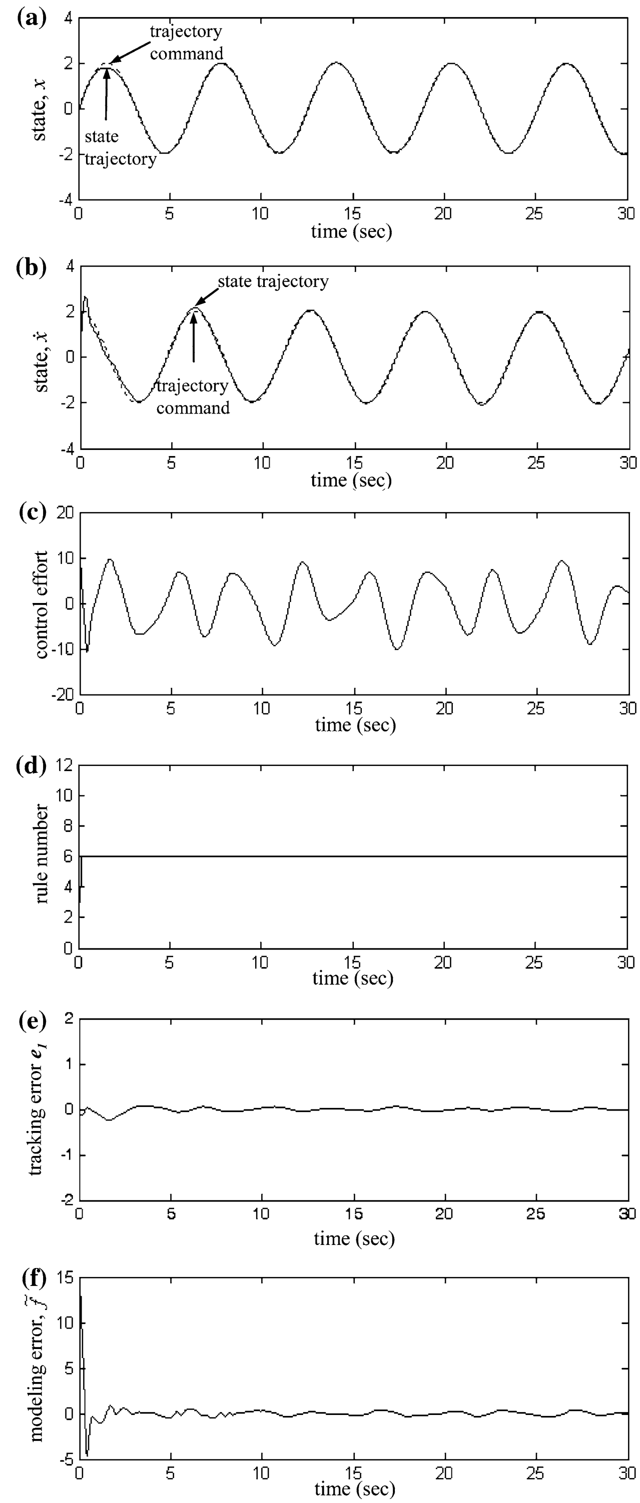

Fig. 8 Simulation results of FABC using SFS without rule pruning algorithm for $q = 7.0$

where s and z are positive constants given by

$$s = \min\{2\tau_1, 2\tau_2, \rho_{\boldsymbol{\alpha}}\beta_{\boldsymbol{\alpha}}, \rho_{\mathbf{c}}\beta_{\mathbf{c}}, \rho_{\boldsymbol{\sigma}}\beta_{\boldsymbol{\sigma}}, \rho_E\beta_E\} \tag{50}$$

$$z = \frac{1}{2}\left[\rho_{\boldsymbol{\alpha}}\|\boldsymbol{\alpha}^*\|^2 + \rho_{\mathbf{c}}\|\mathbf{c}^*\|^2 + \rho_{\boldsymbol{\sigma}}\|\boldsymbol{\sigma}^*\|^2 + \rho_E E^2\right] + \kappa\gamma E. \tag{51}$$

Define $\varsigma = z/s > 0$. Then, the solution of the differential inequality (Eq. 49) satisfies

$$0 \le V_2 \le \varsigma + [V_2(0) - \varsigma]\exp(-st) \tag{52}$$

where $V_2(0)$ is the initial value of V_2. Thus, $\mathbf{e}$, $\hat{\boldsymbol{\alpha}}$, $\hat{\mathbf{c}}$, and $\hat{\boldsymbol{\sigma}}$ are uniformly ultimately bounded according to the extensions of the Lyapunov theory (Wang and Hill 2006). From Eq. 40, it is obvious that $\frac{e_1^2}{2} \le V_2$ for any V_2. From Eq. 52, we obtain

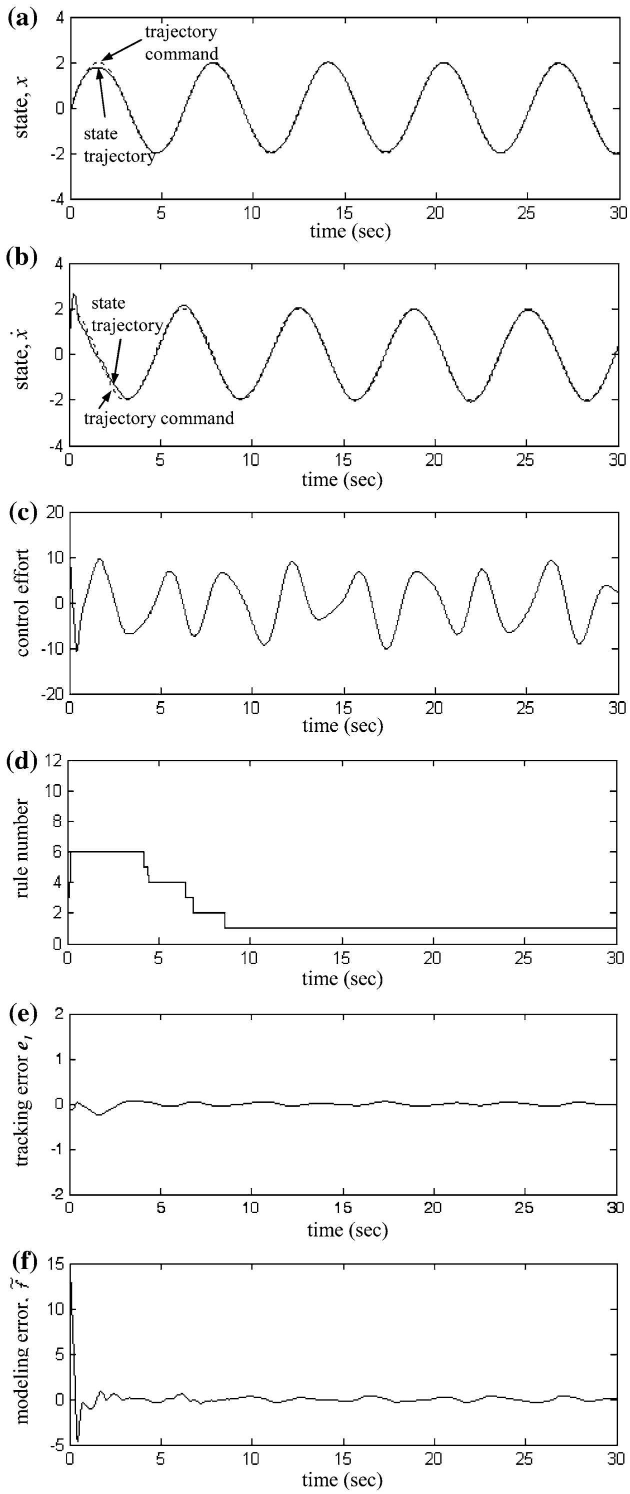

Fig. 9 Simulation results of FABC using proposed SFS for q = 7.0

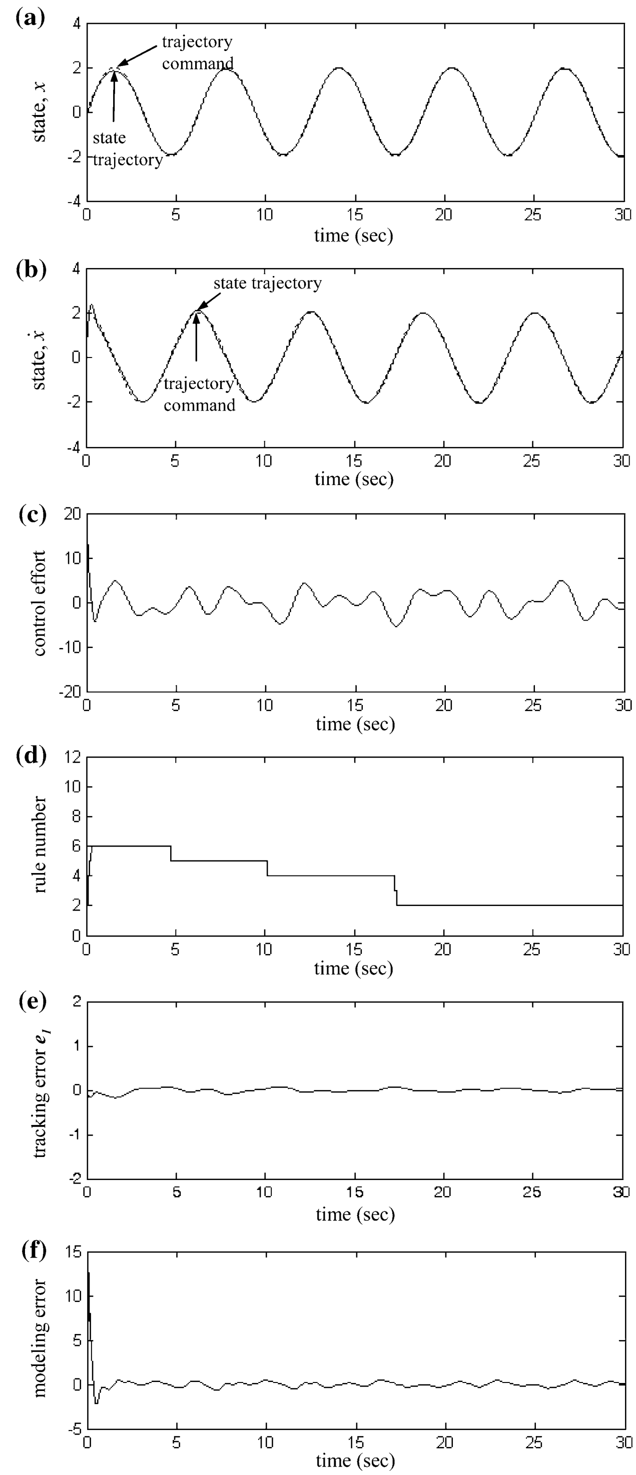

Fig. 10 Simulation results of FABC using proposed SFS for q = 2.1

$$\frac{e_1^2}{2} \leq V(t) \leq \varsigma + [V_2(0) - \varsigma] \exp(-st). \tag{53}$$

Then, Eq. 53 can be rearranged to yield the following inequality

$$|e_1| \leq \sqrt{2\varsigma + 2[V_2(0) - \varsigma] \exp(-st)}. \tag{54}$$

Note that the term $2[V_2(0) - \varsigma] \exp(-st)$ will decay gradually with time because $s > 0$. Therefore, Eq. 54 implies that for any given $\lambda > \sqrt{2\varsigma}$, there exists T so that the tracking error e_1 satisfies

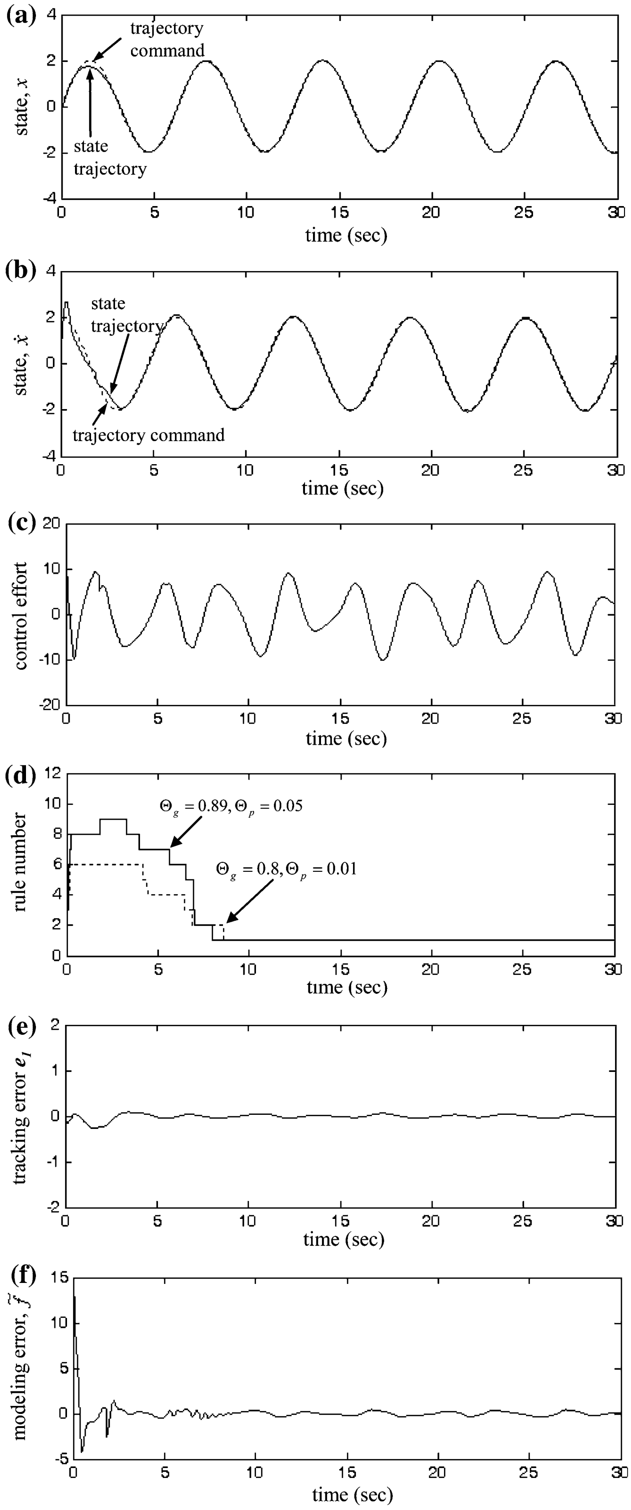

Fig. 11 Simulation results with SFS which tends to easily generate and prune rules for q = 7.0

$$|e_1| = |x_1 - x_c| < \lambda. \tag{55}$$

for all $t \geq T$. Then, the output of the FABC system can exponentially converge to a small neighborhood of the trajectory command.

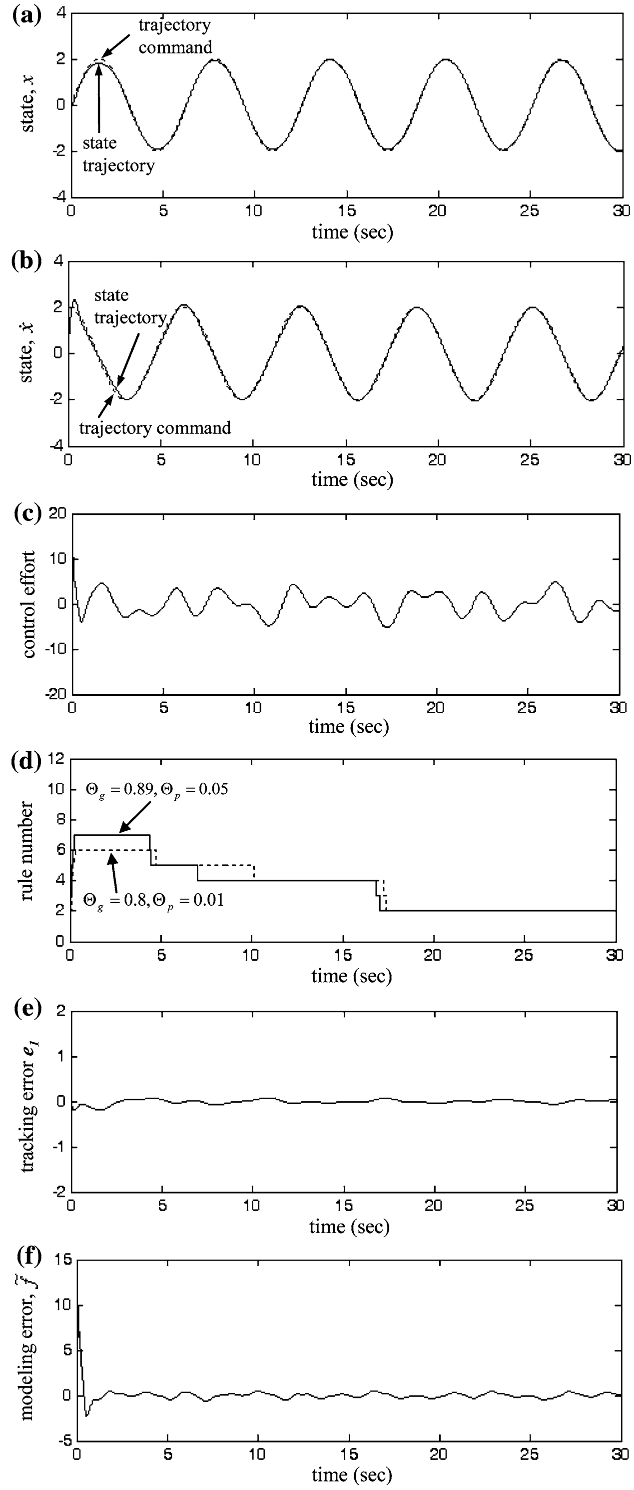

Fig. 12 Simulation results with SFS which tends to easily generate and prune rules for q = 2.1

5 Simulation results

The proposed FABC is applied to control a chaotic Duffing's equation to track a desired orbit. It should be emphasized that the prior knowledge of the dynamics of the controlled system is not necessary in the development of FABC. An SFS with the proposed structuring and parameter learning algorithms is utilized to on-line estimate the chaotic dynamics equation. Excessively small τ_1 and τ_2 will diminish the control effort and thus slow down the convergence of tracking error; on the contrary, over-large τ_1 and τ_2 may cause a variation of high gain control. If the learning rates $\beta_{\boldsymbol{\alpha}}$, $\beta_{\mathbf{c}}$, $\beta_{\boldsymbol{\sigma}}$ and β_E are small, the parameters convergence of the FABC scheme will be easily achieved but result in slow learning speed. However, if the learning rates $\beta_{\boldsymbol{\alpha}}$, $\beta_{\mathbf{c}}$ and $\beta_{\boldsymbol{\sigma}}$ are large, the learning speed will be fast, but the FABC scheme may become more unstable for parameter convergence. The parameters $\rho_{\boldsymbol{\alpha}}$, $\rho_{\mathbf{c}}$, $\rho_{\boldsymbol{\sigma}}$ and ρ_E should not be chosen too small to destroy their capabilities of preventing parameter estimations from drifting to vary large values. An excessively small γ is not suggested, because $\tanh(\cdot)$ may be driven to $\mathrm{sgn}(\cdot)$ and thus loses the power of $\tanh(\cdot)$ to smoothen the control efforts. To illustrate the effectiveness of the proposed design method, a comparison among fix-structure fuzzy systems, the proposed SFS without pruning algorithm, and the proposed SFS is made.

The simulation results of FABC using 3 fuzzy rules for q = 7.0 are shown in Fig. 5. The tracking response of x is shown in Fig. 5a; the tracking response of $\dot{x}$ is shown in Fig. 5b; the associated control effort is shown in Fig. 5c; the tracking error e_1 is shown in Fig. 5d; and the modeling error $\tilde{f}$ is shown in Fig. 5e, respectively. The simulation results show that the tracking performance is unsatisfactory when the number of fuzzy rules is chosen too small. Moreover, the simulation results of FABC using 10 and 20 fuzzy rules for $q = 7.0$ are shown in Figs. 6 and 7, respectively. The simulation results show that the satisfied tracking responses can be achieved when the number of fuzzy rules is chosen appropriately. In general, the better approximation capability of the fuzzy system can achieve as using the more fuzzy rules. However, some exceptions show that a fuzzy system with an over-large number of rules is likely to fail to achieve approximation performance. So it is not an easy task to determine an appropriate number of fuzzy rules to achieve favorable approximation performance.

Next, the proposed FABC is applied to chaotic dynamic system again. The parameters of FABC system are selected as $\tau_1 = \tau_2 = 1, \rho_{\boldsymbol{\alpha}} = \rho_{\mathbf{c}} = \rho_{\boldsymbol{\sigma}} = \rho_E = 0.005, \gamma = 0.05, \beta_{\boldsymbol{\alpha}} = 200, \beta_{\mathbf{c}} = \beta_{\boldsymbol{\sigma}} = 1, \beta_E = 0.1, \bar{\sigma} = 1.0, \Theta_g = 0.8, \Theta_p = 0.01$ and $\eta = 0.001$. Considering the requirement of stability and possible operating conditions, these values through some trials to achieve satisfactory control performance. To illustrate the effectiveness of the proposed design method, a comparison between a FABC with and without structure pruning algorithm is made. The simulation results without structure pruning algorithm for q = 7.0 are shown in Fig. 8. The tracking response of x is shown in Fig. 8a; the tracking response of $\dot{x}$ is shown in Fig. 8b; the associated control effort is shown in Fig. 8c; the number of the fuzzy rule is shown in Fig. 8d; the tracking error e_1 is shown in Fig. 8e; and the modeling error $\tilde{f}$ is shown in Fig. 8f, respectively. From the simulation results, the simulation results show that the favorable tracking performance is achieved. However, since the pruning algorithm of fuzzy rules is not considered in the structure learning, the number of the fuzzy rule may grow unboundedly.

Then, the proposed pruning algorithm is adopted. The simulation results with structure pruning algorithm for q = 7.0 and q = 2.1 are shown in Figs. 9 and 10, respectively. Simulation results prove that the proposed FABC system with structure adaptation algorithm can achieve favorable tracking performance under parameter variation. Moreover, the number of fuzzy rules increases rapidly at the beginning of control, and then gradually decreases to an invariant value at the end of control. The on-line structure adaptation algorithm can create new fuzzy rules to improve unsatisfactory approximation performance and prune insignificant fuzzy rules to reduce the computation load. Finally, the thresholds for growing an pruning rules (Θ_g and Θ_p) are the most significant pre-given constants that influence the number of rules. To illustrate the influence of them, the proposed FABC using SFS is applied to control the chaotic system again with $\Theta_g = 0.89$ and $\Theta_p = 0.05$. The simulation results are shown in Figs. 11 and 12 for q = 7.0 and q = 2.1, respectively. Simulation results prove that the proposed FABC system can achieve favorable tracking performance. Moreover, the fuzzy rules are generated and pruned more easily when the thresholds for growing and pruning rules are chosen larger.

6 Conclusion

To deal with the highly nonlinear property of chaotic dynamic system and the difficulty of obtaining precise system model, a fuzzy-identification-based adaptive backstepping control (FABC) system using self-organizing fuzzy system (SFS) is proposed in this paper. The FABC system is composed of a backstepping controller and a robust controller. The effectiveness of the FABC system using a SFS is verified by some simulations. The main contributions of this paper are: (1) a time-consuming trial-and-error tuning procedure for determining suitable number of fuzzy rules can be avoided, and rules of the SFS can be automatically generated and pruned by the structure learning algorithm; (2) the parameter and structure learning of the SFS are performed simultaneously instead of sequentially, and thus make the FABC system using a SFS suitable for on-line instead of off-line operation; (3) the overall closed-loop control system guarantees that the tracking error and parameter estimation error are uniformly ultimately bounded, and the tracking error can be asymptotically attenuated to a desired small level around zero by appropriate choices of parameters and learning rates.

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