Development of an adaptive learning case recommendation approach for problem-based e-learning on mathematics teaching for students with mild disabilities

Hui-Chuan Chu [a], Tsung-Yi Chen [b], Chia-Jou Lin [c], Min-Ju Liao [d], Yuh-Min Chen [c,*]

[a] Department of Special Education, National University of Tainan, Tainan, Taiwan, ROC
[b] Department of Electronic Commerce Management, Nan-Hua University, Chia-Yi, Taiwan, ROC
[c] Institute of Manufacturing Information and Systems, National Cheng Kung University, Taiwan, ROC
[d] College of Humanity and Social Science, National Chiao-Tung University, Taiwan, ROC

ARTICLE INFO

ABSTRACT

Most e-learning platforms offer theoretical knowledge content but not practical knowledge required for problem solving. This study proposed a problem-based e-learning (PBeL) model which incorporates the problem-based learning (PBL) theory, social constructivism, and situated learning theories to assist regular and special education teachers in effectively developing knowledge for mathematics teaching for students with mild disabilities. To support adaptive case-based learning in the proposed PBeL and to adequately address the real complexity and diversity of the learning problems of students' with mild disabilities, this research also developed an adaptive case recommendation approach which identifies the most suitable authentic learning cases based on the characteristics of learners (teachers), the strengths, weaknesses, and types of disabilities of their students, the teaching problems of various mathematical topics, and the teaching context in order to facilitate adaptive case-based learning in the context of problem-based e-learning for regular and special education teachers' knowledge development. Clustering and information retrieval techniques were used to construct the context and content maps for case-based reasoning with the capability of semantics identification. The adaptive recommendation approach not only enables the realization of adaptive PBeL, but also enhances teachers' practical knowledge and assists them to solve students' learning problems.

© 2008 Elsevier Ltd. All rights reserved.

1. Introduction

e-Learning provides learners with another learning channel that enables the learner to break free from constraints on time and space, and to engage in distance-based, non-synchronized learning activities. However, most e-learning platforms emphasize the convenience offered by digital knowledge content, without integrating suitable learning theory into the e-learning. Therefore, these e-learning platforms degenerate into knowledge dissemination tools, and neglect learning theory and practice. Additionally, as far as the development of knowledge and competence of pre-service and in-service teachers of special education are concerned, current e-learning platforms provide only knowledge and teaching materials related to formal teaching knowledge, but not sufficient practical knowledge required for solving students' learning problems.

Problem-solving is knowledge intensive. It involves acquiring relevant knowledge to identify the core causes of a problem, develop solutions, and taking appropriate actions to solve the problem (Liu & Ke, 2007). Although e-learning easily provides learning resources, without taking into account the characteristics of problems being encountered, the large amount of learning resources will result in cognitive overload or disorientation.

Case-based reasoning (CBR) method has been widely used to provide knowledge for problem solving by adapting solutions from that of historical problems (Carrascosa, Bajo, Julian, Corchado, & Botti, 2008; Liu & Ke, 2007; Yang, Han, & Kim, 2004). However, conventional case-based reasoning approaches have their limitations in handling semantics of knowledge, thus decreasing possibilities for knowledge dissemination. Moreover, as oppose to conventional case-based reasoning that merely focuses on the characteristics of the problems being encountered, in the areas of teaching or students' learning problem solving, characteristics of both students and learners (i.e. teachers) are being considered in the learning case recommendation.

In the trend of integrating students with mild disabilities into regular education classes and non-categorical resource programs, advancing mathematics achievement of the students with mild

* Corresponding author.
E-mail address: ymchen@mail.ncku.edu.tw (Y.-M. Chen).

disabilities, including the students with learning disabilities, high-functioning autism, Asperger syndrome, mild mental retardation, emotional and behavior disorders, or Attention Deficit Hyperactivity Disorder, becomes even more challenging for regular education and special education teachers. Research suggest that the mathematics underachievement of students with mild disabilities is the result of a complex interplay of cognitive, emotional/behavioral, physical, sensory, communication, social factors and mathematics (Montague & Applegate, 2000).

To adequately address the real complexity and diversity of learning problems of students with mild disabilities, this study (1) designed an e-learning model that featured the situated learning as a theoretical basis to integrate learning theories of social constructivism and case-based learning along a problem-based learning approach, and (2) developed an adaptive case recommendation approach which identifies the most suitable authentic learning cases based on the characteristics of learners (teachers), the strengths, weaknesses, and types of disabilities of their students, the teaching problems of various mathematical topics, and the teaching context. Clustering and information retrieval techniques were employed for construction of context and content maps for case-based reasoning with the capability of semantics identification.

In particular, this study focused on reasoning of learning cases on the mathematics teaching for students with mild disabilities, and considered the relevant needs for knowledge development and for improving problem-solving capabilities of teachers in realistic teaching situations. The results of this research not only strengthened teacher's knowledge for practical teaching, but also assisted teachers to solve students' learning problems, which would in turn to improve teaching quality.

2. Problem-based e-learning

This section presents a proposed problem-based e-learning model with the capability for adaptive learning case recommendation. This model incorporates problem-based learning, contextual learning and social constructivism as the underlying theoretical bases.

2.1. Theoretical framework

This study referred to the "course of cognitive skill acquisition" (Renkl & Atkinson, 2003; Patel, Kinshuk, & Russell, 2000; VanLehn, 1996) as the basis of the proposed e-learning process which includes stages of concept knowledge development, problem-solving knowledge development, and overall professional judgment knowledge development.

Problem-based learning (PBL) is to solve real-world problems by assembling and focusing problems, and guide the learner in utilizing his or her understanding, problem solving skills, and judgments (Barrows, 1996; Levin, 2001). It provides the learner with pragmatical experiences, allowing the user to achieve both "knowing" and "knowing how" (Delisle, 1997) through the learning context that much resembles an authentic learning context for the learner, as emphasized by the situated learning theory (Souders & Prescott, 1999). It is believed that problem-based learning may augment the structure of the learner's understanding, as well as his or her ability to integrate new knowledge (Robertson et al. 2000).

Nevertheless, Fenwick and Parsons (1998) mentioned that while PBL is able to improve the learner's motivation, solely utilizing such pedagogy may sometimes cause knowledge gaps. Advocates of situated learning also stressed the need for professional cognitive apprenticeship, and contended that the learner must engage in an active, participatory learning process within a scaffold provided by a teacher, an expert or a more experienced learner, who plays the role of a coach or facilitator to the learner.

Pedagogies based on social constructivism perceives learning as a collective thinking process that involves teacher–student or student–student interactions to solve problems, learn new knowledge and concepts, and make appropriate decisions. Students have to discuss with each other, in order to negotiate meanings or forge a consensus (Rogoff, 1990). Learning through discussions is highly valued and supported by social constructivists. When the learner interacts with other learners or the teacher, concepts are formed in a natural way because through talking to each other, they have collectively created a world that can be described and discussed as well as a common framework under which communication takes place (Solomon, 1987).

The case-based learning approach used in the field of teacher professional development involves narration of teaching practices based on a real classroom case, and helps the learner to link theories with practice (Chin & Lin, 2000; Merseth, 1996; Richardson, 1993) to stimulate introspections (Richert, 1991) and effective constructions of teaching knowledge.

According to the above discussions, we believe that providing a learner with a learning context similar to the teaching problems encountered by the learner may better enhance teachers' professional development. Therefore, this study has chosen the situated learning as a theoretical basis to integrate learning theories of social constructivism and case-based reasoning along a problem-based learning approach. The theoretical framework of this study is illustrated as Fig. 1.

2.2. Problem-based e-learning model

In accordance with the previous theoretical framework, this study designed an e-learning model with problem-based learning as its core and social constructivism and situated learning as its auxiliary theories. In the spirit of PBL, this model includes the stages of analysis, design, development, and practice (refer to Fig. 2).

The analysis stage involves assessing a learner's (i.e. teacher's) knowledge of the students with mild disabilities, pedagogical content knowledge of mathematics, knowledge of modifications of curriculum, teaching methods, materials, techniques, and learning environments for teaching students with mild disabilities, mathematical content knowledge, and then diagnosing the students' learning problems. The learning goal for the learner is then translated into "solving students' learning problems". The design stage identifies the learner's background information and teaching objectives in order to outline a personalized learning plan. The
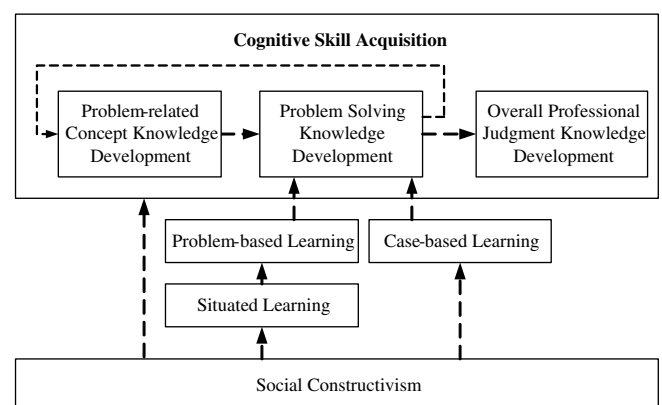

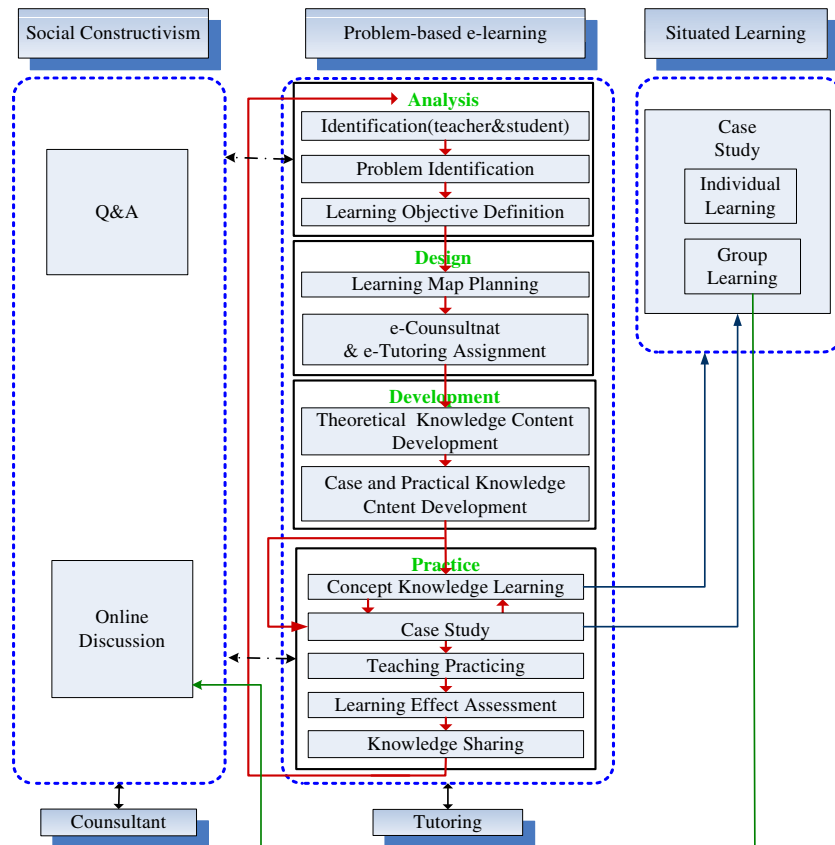**Fig. 1.** The theoretical framework.

Fig. 2. The problem-based e-learning model.

development stage develops contents, such as concepts and cases, for the personalized learning plan. Finally, the practice stage guides the learner to initiate learning activities, such as concept learning, case studies, practical teaching, feedback on teaching experience and knowledge sharing. After the learner has completed the concept learning and case studies, he/she is required to begin realistic teaching, by applying learned knowledge to realistic teaching context. Lastly, the system knowledge content can continue to expand and update as the learners would share their knowledge and thoughts.

A learner undertaking case studies may select either "individual learning" or "group learning". The "group learning" takes the learner to a learning mode based on social constructivism, where the learner may initiate a group discussion and direct questions to experts or learners with related experience in any phase of the case study. During Q&A sessions or online discussions in this forum, an experienced teacher or expert plays the role of an e-consultant to guide the learners to complete their learning processes.

2.3. Learning case structure

Development of an e-learning platform requires not only the design of functional learning activities, but also the provision of suitable learning materials. Moreover, a successful design of learning objects necessitates incorporation of instructional design and learning theories (Roderick & Baden, 2005).

Learning cases which were developed by expert teachers according to their own teaching narrations and were analyzed through actual teaching, observation, discussion, and assessed by experts were used to store practical teaching knowledge. Each case contained the parts of "teaching context" and "teaching narration". The teaching context contained the learner's demographic data, the

student's personal data, and the statement of student's learning problems. The teaching narration included three sections of teaching objectives, teaching units and learning assessments. Fig. 3 shows the learning case model defined in terms of UML (Booch, Rumbaugh, & Jacobson, 1999) notations, where a box represents a class of learning objects and a diamond indicates a composite class, which is composed of its component classes.

To store, organize, manage and use the case contents effectively, this study defined the instances of each class in the case model as learning objects. The bold rounded squares stand for the retrievable learning objects, while plain squares denote non-retrievable learning objects.

3. Adaptive case recommendation

This section describes the framework of proposed adaptive case recommendation approach for supporting case-based learning in problem-based e-learning.

3.1. Requirements

The requirements for the proposed adaptive case recommendation approach are identified as follows to ensure that the recommended learning cases conform to the learner's learning requirements.

(1) Adaptability: Research on adaptive learning indicated that learner's interests, ability, and cognitive characteristics greatly influence learning effectiveness (Kalyuga, 2007). Furthermore, the nature and scope of the problem, the type of disability, and the characteristics of students should be taken into consideration for developing solutions to
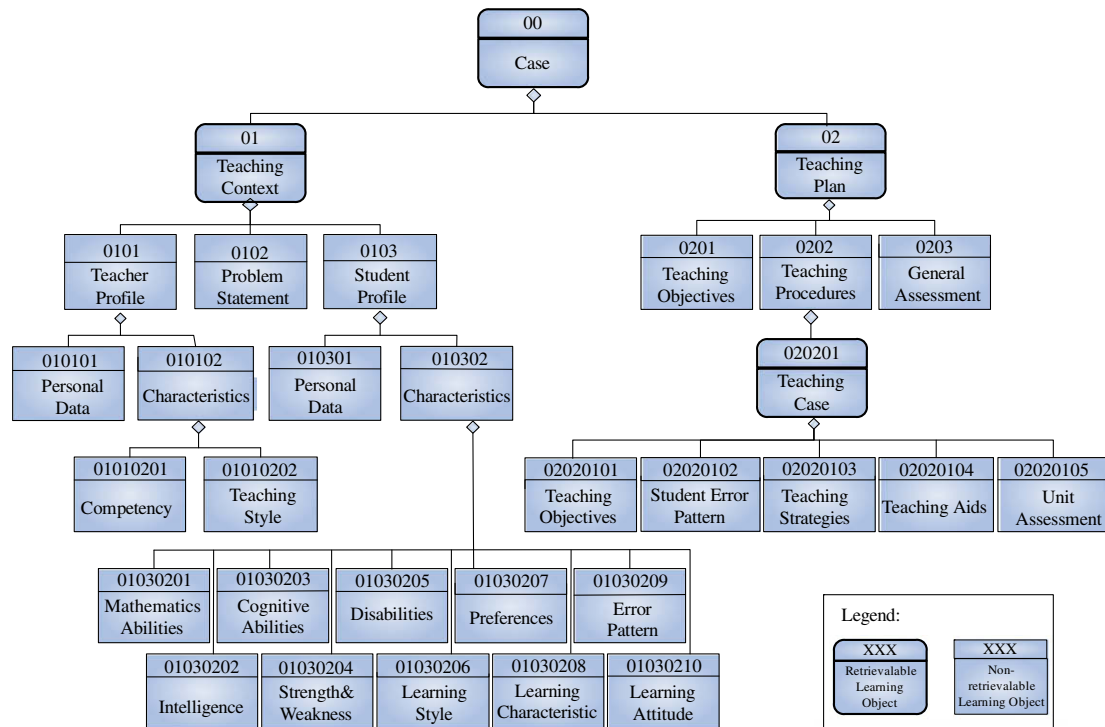
Fig. 3. Learning cases structure.

students' learning problems (Steele, 2006). Therefore, identifications of the characteristics of both the learner (i.e. teacher) and the student are required for case recommendation to facilitate learning adaptation and thus maximize the benefit to the learner and the student.

(2) Natural language processing: The accuracy of case retrieval is highly dependent on the keys for searching. Conventionally, keywords are commonly used in information as well as case retrieval (Hsu & Wang, 2004). However, since students' learning problems can not be fully described in terms of keywords, a better way for students' learning problem definition that can be in turn used for case retrieval is required. This research allowed learners to describe students' learning problems using natural language so as to improve the accuracy of case retrieval. Therefore, the adaptive case recommendation approach needs to analyze the content of learning problems described in natural language and develop the keys for case retrieval.

(3) Semantic searching: Since both teaching problems and learning cases were stated in natural language, synonyms and homographs appeared frequently to cause semantic variations in learning case searching. Hence a method to handle semantic variations was required to promote the search accuracy.

3.2. The framework of adaptive case recommendation

The adaptive recommendation framework included layers of functional tasks, knowledge map and database as shown in Fig. 4.

By referencing the cycle of case-based reasoning (Agnar & Enric, 1994), tasks in the functional task layers included characteristic identification, student's learning problem analysis, learning case retrieval and learning case recommendation. Characteristic identification included analysis and identification of characteristics of teachers and students. The learning problem analyses, including

functions for teaching context identification and learning case content matching, were responsible for identifying the learner's requirements. The layer of learning case retrieval retrieved relevant learning cases according to the results of characteristic identification and learning problem analysis. Learning case recommendation layer selected the most suitable learning case from the retrieved learning cases.

The knowledge map, which was a structured knowledge representation, depicted the knowledge structure of learning cases and was used as a knowledge search guide to facilitate learning cases retrieval.

The database layer contained a user model base and a learning case repository. According to the learning case structure, the learning case repository was divided into areas for teaching contexts, teaching plans, and teaching cases. The user model was designed based on the personal characteristics, abilities, and preferences to define the learner's demographic data and background information. The user models can be further categorized into learner models and student models. The former contained learner's demographic data, characteristics, and learning profile, while the latter contained student's demographic data and characteristics information.

4. Definition and development of knowledge map

This section defines and develops the knowledge map of learning case repository. According to the content of learning cases, the knowledge map is split into a context map and a content map as discussed below.

4.1. Definition and development of context map

4.1.1. Definition of context map

The context map model contains clusters and occurrences as shown in Fig. 5. A cluster aggregated learning cases with the sim-
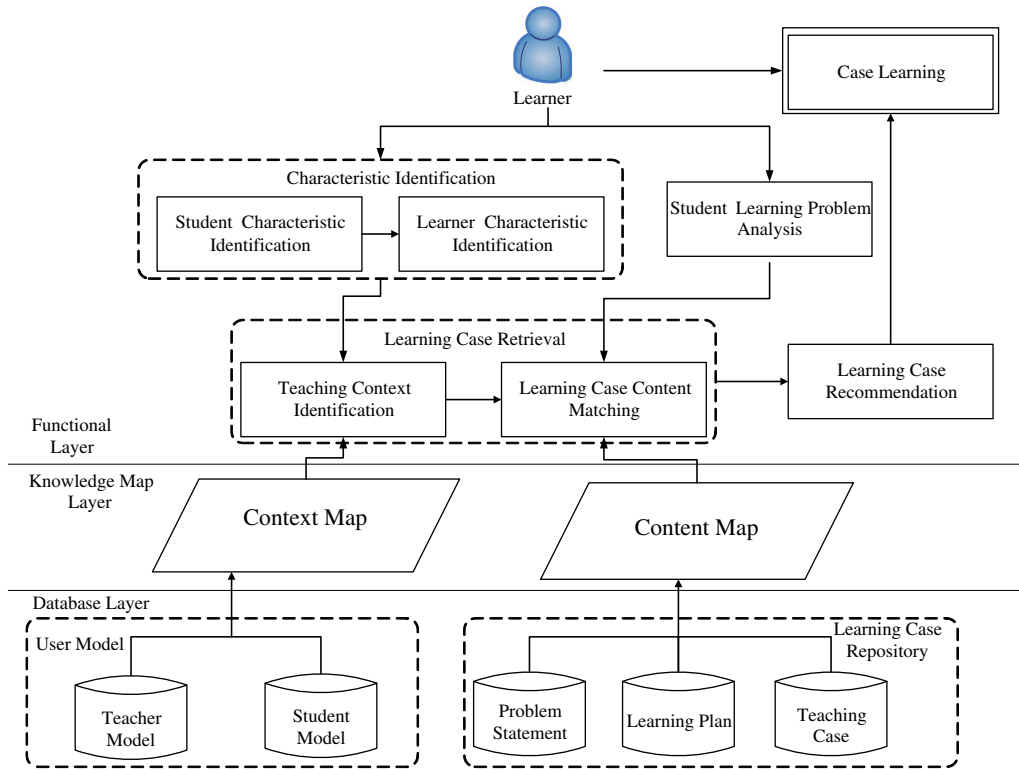
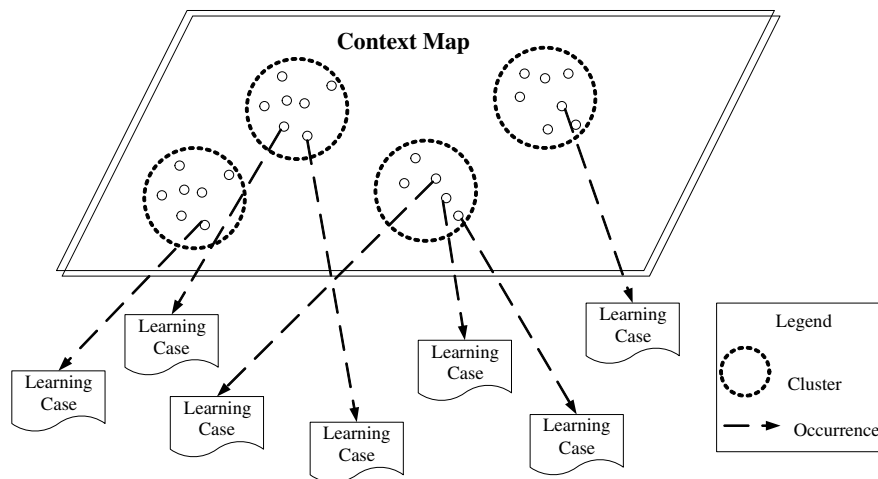**Fig. 4.** The adaptive case recommendation framework.

Fig. 5. The teaching context map model.

ilar teaching contexts. The occurrence was the index of a corresponding learning case.

The schema of context maps was designed based on the learning case structure and the context map model, as presented in Fig. 6. The cluster ID distinguished each cluster, while the occurrences recorded the IDs of learning cases in this cluster. In other words, the occurrence linked a cluster with its related learning cases. The centroid recorded the characteristics of clusters and contains the characteristics of teachers and students. The characteristics of a teacher consist of his/her competence and teaching style. The characteristics of a student included age, grade, mathematic abilities, types of disabilities, strengths, weaknesses, learning style, and learning preferences.

4.1.2. Development of context map

The learning cases were clustered according to the context feature tuples, which contained the attributes of the teacher and student profiles in leaning cases. Table 1 presented the context feature tuple of the learning cases. The context feature tuple included numeric and categorical attributes.

Huang (1998) proposed a k-prototypes algorithm, which integrates the k-means and k-modes processes to cluster data with mixed numeric and categorical values. However, the attributes can only be a single value. In this study, certain context featured with categorical values was multi-values. For example, the values of disability were dyslexia, reading disability and mathematic disability. The context featured with categorical values in this study
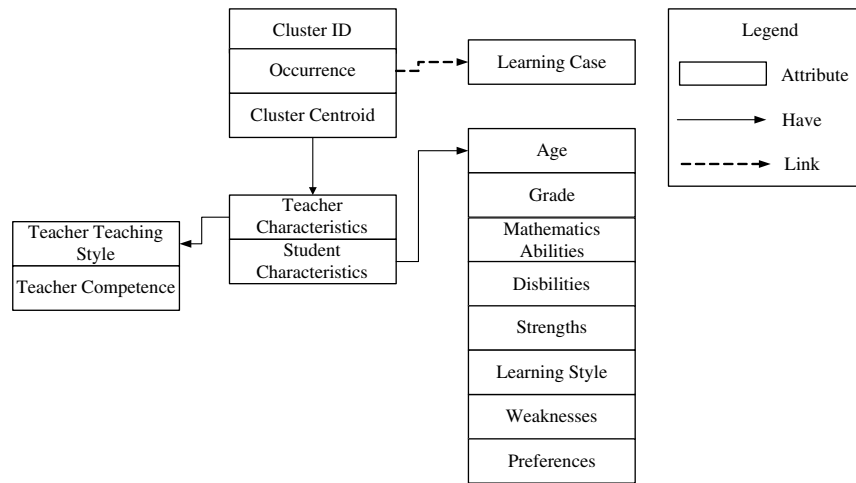
Fig. 6. The teaching context map schema.

Table 1
The context features

User	Attribute name	Value	Type
Teacher characteristics	Teaching style	(T1) Symbolic language teaching and individual	CS
	Learning style	V	CS
	Teaching seniority	6	N
	Teaching course	Special education	CM
Student characteristics	Grade	3	N
	Age	9	N
	Math abilities – number	35.5	N
	Math abilities – fractions	39.9	N
	Math abilities – figures and space	39.6	N
	Math abilities – addition and subtraction	37.3	N
	Math abilities – multiplication and division	37.7	N
	Math abilities – the four fundamental operations of arithmetic	0	N
	Math abilities – measure	36	N
	Math abilities – time and Calculation	34.5	N
	Math abilities – statistics and statistical chart	37.7	N
	Math abilities – a calculating machine and strategies on math word problems	31.8	N
	Disabilities	(D4) Dyslexia	CM
		(D7) Reading disability	
		(D11) Math disability	
	Learning style	(L2) Diverger	CS
	Preferences	(P4) Computer game,	CM
		(P5) Liking work of images and drafting,	
		(P6) Singing	
	Strengths	(A1) Superior observer	CM
		(A14) Good ability to express in speech	
	Weaknesses	(w8) Limited vocabulary	CM
		(w14) Difficulty in grasping important information	

*N – Numerical data type.
CS – categorical single-value data type.
CM – categorical multi-value data type.

can be classified into single-value and multi-value categorical attributes. Therefore, this study proposed an enhanced algorithm that used the k-prototypes paradigm to cluster data with numerical, categorical single-values and categorical multi-values.

Let $X = \{x_1, x_2, \ldots, x_n\}$ denote a set of n learning cases, and let k be a positive integer. The partition of X into k clusters to minimize the within-groups sum of dissimilarity can be discovered by the following optimization model:

Minimize $\quad E(W, M) = \sum_{j=1}^{k} \sum_{i=1}^{n} w_{ij} D(X_i, M_j)$

Subject to $\quad \sum_{j=1}^{k} w_{ij} = 1, \quad 1 \leqslant i \leqslant n$

$\quad\quad\quad\quad w_{ij} \in \{0, 1\}, \quad 1 \leqslant i \leqslant n, \ 1 \leqslant j \leqslant k$

where W is an $n \times k$ partition matrix; M is the centroid of Cluster, and $M = \{M_1, M_2, \ldots, M_k\}$, $D(X_i, M_j)$ is the dissimilarity between two objects.

To solve the above model, the enhanced k-prototypes algorithm (EKP) applies the new dissimilarity measure approach, which not only clusters multi-type data, but also gives equal importance to each context feature. This is because the dissimilarity value with every context feature is between 0 and 1. Furthermore, the new dissimilarity measure approach also considers the relative frequencies of categorical attributes in each cluster in order to enhance the accuracy of the clustering results (He, Xu, Deng, & Dong, 2004). The dissimilarity measure approach of IKP is described below:

Let $X = \{x_1, x_2, \ldots, x_p, \ldots, x_m\}$ and $Y = \{y_1, y_2, \ldots, y_p, \ldots, y_m\}$ be two mixed-type objects described by m attributes, where the first p

attributes are numeric values, and the rest are categorical values. Suppose Y is the centroid of the cluster C_q. The dissimilarity measure between X and Y can be described as

$$D(X, Y) = Dn + Dc$$

The first term, namely the summation of the numeric attributes, can be calculated as

$$Dn = \sum_{i=1}^{p} \left(\frac{x_i - y_i}{i_{max} - i_{min}} \right)^2$$

The second term, which is the summation of the categorical attributes, is computed as

$$Dc = \sum_{i=p+1}^{m} d(x_i, y_i)$$

If the ith attribute is single-value category, then it can be derived as

$$d(x_i, y_i) = \begin{cases} 1 - f(j_i = x_i | C_q) & (x_i = y_i) \\ 1 & (x_i \neq y_i) \end{cases}$$

where $f(j_i = x_i | C_q)$ represents the frequency of the data objects, of which the ith attribute is equal to x_i in C_q. This function is calculated as

$$f(i = A_k | C_q) = \frac{n_{A_{k,i}}}{N_q}$$

where N_q is the number of objects in cluster C_q, and $n_{A_{k,i}}$ is the number of objects which the category value of the ith attribute is A_k.

If the ith attribute is a multi-value category, $x_i = \{x_{i1}, x_{i2}, \ldots, x_{in}\}$ and $y_i = \{y_{i1}, y_{i2}, \ldots, y_{im}\}$, then it is obtained as

$$d(x_i, y_i) = 1 - \frac{\sum f(y_{im} = x_{in} | C_q)}{|x_i \cup y_i|}$$

where $f(y_{im} = x_{in} | C_q)$ represents the frequency of the element of objects, of which the mth element in the ith attribute equals x_{in} in C_q, and is obtained as

$$f(i = A_m | C_q) = \frac{n_{A_{m,i}}}{N_q}$$

where $n_{A_{m,i}}$ is the number of objects in which the category value of the ith attribute contains A_m.

Additionally, the IKP algorithm also defines an approach to find centroid M from a given set. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of mixed-type objects containing attributes $A_1, A_2, \ldots, A_p, \ldots, A_m$, where the first p attributes are numeric, and the rest are categorical, and X belongs to cluster C_j. The values of the first p attributes in centroid M_j are calculated by the mean approach, and can be determined by

$$m_z = \frac{\sum_{i=1}^{n} x_{zi}}{Nj} \quad 1 \leqslant z \leqslant p$$

where N_j is the total number of objects in cluster C_j, and x_{zi} is the zth attribute in the ith object.

The categories single-value of attributes in centroid M_j are calculated by the mode approach. Objects with the same multi-value category attributes do not necessarily have the same number of attribute values. Therefore, the mean is first calculated to determine the number of attribute values, and the mode is then computed to select the elements of attribute values. For instance, $x_{1i} = \{a, b, c\}$, $x_{2i} = \{a, c, d, f\}$, $x_{3i} = \{a, f\}$ are the ith attribute in object x_1, x_2 and x_3, respectively of cluster C_j. The ith attribute in M_j has three elements according to the mean approach, $3 + 4 + 2/3 = 3$. Then the mode approach is applied to calculate the frequency of each element, $f(a) = 1$, $f(b) = 1/3$, $f(c) = 2/3$, $f(d) = 1/3$, $f(f) = 2/3$. Hence, the ith attribute in M_j is $\{a, c, f\}$.

The IKP algorithm was utilized to construct the context map. Let $X = \{x_1, x_2, \ldots, x_n\}$ denote a set of n learning cases, and $x_i = [x_{i1}, x_{i2}, \ldots, x_{ip}, \ldots, x_{im}]^T$ be a learning case represented by m values of attributes, where the first p elements are numeric values, and the rest are categorical values. To confirm the optimal clusters number, the experience rule was used to determine the range of numbers of cluster as $k_{max} \leqslant \sqrt{n}$ (Ramze, Lelieveldt, & Reiber, 1998), where n is the amount of learning cases. Finally, the number of clusters k which minimizes the within-groups sum of dissimilarity is the optimal result of the cluster. Table 2 describes the context map construction algorithm.

4.2. Definition and development of content map

4.2.1. Definition of content map

The content map presents the knowledge structure of the problem statements, the teaching plans and the teaching cases in learning cases. It facilitates semantic searching of the adaptive recommendation approach. Components of the content map model include concepts and the occurrences, as shown in Fig. 7. The concepts are the knowledge concepts appearing in the learning problem statements, the teaching plans and the teaching cases in learning cases. The occurrences link concepts and learning cases, meaning that an occurrence indexes a concept appearing in a certain learning case. Since the importance of a concept varies with learning cases, learning problems, and teaching plans, each occurrence has three weights, W_{Pi}, W_{Ni} and W_{Ci}. W_{Pi} is the importance of the ith concept in the learning problem, W_N is the importance of the ith concept in a teaching plan, and W_C is the importance of the ith concept in a teaching case.

Fig. 8 illustrates the content map schema, which is designed based on the learning case structure and the proposed content map model. The Concept name indicates the name of a knowledge concept in the learning case. The occurrence links a concept with a learning case. The occurrence contains three weight attributes, the weight in problem statement, the weight in teaching plan and the weight in teaching case.

Table 2
Context map construction algorithm

Context map construction algorithm
Input: a set of n learning cases Output: leaning cases were assigned into their corresponding clusters

```
begin
    let case features Xi = {x1,…,xp,…,xh}, i = 1 ~ n, the first p elements are
  numeric values and the rest are categorical values;
    let k be a positive integer;
    for 1≤k≤√n do
  begin
    select k as the number of initial cluster centroids randomly, each centroids
  corresponds to one and only one cluster;
      while new cluster centroids are not equal to existing cluster centroids do
      begin
        for 1≤i≤n do
        begin
            Dij(Xi, Mj) = ∑ᵖₜ₌₁ ( (xit - mjt)/(tmax - tmin) )² + ∑ʰₜ₌ₚ₊₁ d(xit, mit) for 1≤j≤k do
        select min Dij(Xi,Mj);
        let case i belong to cluster j;
      end;
          calculate new cluster centroids for each cluster;
      end;
        Ek(W, M) = ∑ᵏⱼ₌₁ ∑ⁿᵢ₌₁ Dij(Xi, Mj) for each Xi ∈ Cj
    end;
    take minEk as the optimal clustering result;
  end;
```
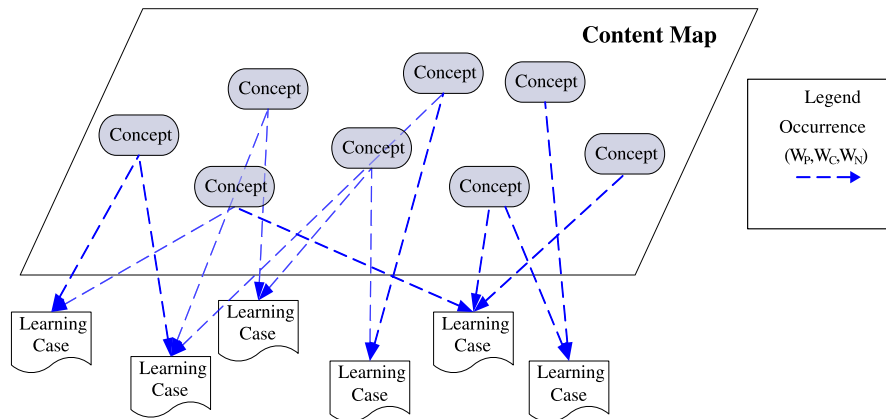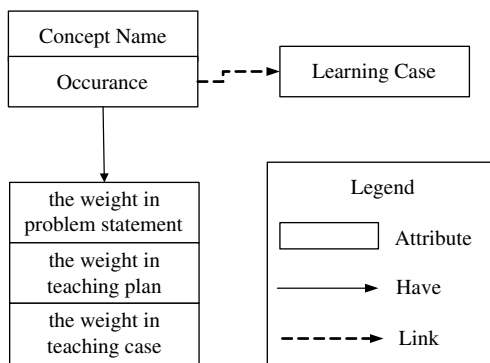
Fig. 7. The content map model.

Fig. 8. The content map schema.

4.2.2. Development of content map

The first step in developing the content map is pre-process, which includes tasks of sentence breaking, word breaking, word tagging and concept parsing.

Sentence breaking: As learning case is described in natural language and the sentence is the basic unit of natural language, the sentence breaking techniques is first performed to break the content of learning cases into sentences. This study defined sentence breaking as periods, commas and semicolons.

Word breaking and tagging: The purpose of this step was to retrieve the core concepts in the learner's teaching problem for adaptive learning case recommendation. This study used the AutoTag techniques developed by CKIP projects in Academia Sinica to segmentalize phrases, to transform each sentence into a series of words, and to tag each word to determine its grammatical attribute.

Concept parsing: Concept parsing was performed primarily to retrieve the core concepts from the tagged word in each sentence. Since the meaning of a sentence was usually determined by nouns, concept parsing selected nouns in the problem statement, the teaching plan and the teaching cases of a learning case as core knowledge concepts to represent the learning case.

Unstructured learning cases were transformed into structured data after pre-process. The collection of learning cases were capsulated in three $m \times n$ co-occurrence matrices M_p, M_n and M_c, where $M_p(c,l_c)$ represents the number of occurrences of a concept c in the problem statement of learning case l_c; $M_n(c,l_c)$ represents the number of occurrences of a concept c in the teaching g plan of learning case l_c, and $M_c(c,l_c)$ represents the number of occurrences of a concept c in the teaching case of learning case l_c.

To deal with polysemous words, and to distinguish explicitly between different meanings and word usages, the Probability La-

tent Semantic Analysis (PLSA) was applied to model the relationships between concepts and learning cases.

PLSA is a statistical latent class model or aspect model (Hofmann, 1999b; Hofmann, 1999a). The model was fitted to a training corpus by the Expectation Maximization (EM) algorithm (Dempster, Laird, & Rubin, 1977). PLSA obtained the joint probability of a document d and a word w based on a latent class variable z:

The model assumes that word w and document d are independent if the latent class z is given, i.e., $P(w|z,d) = P(w|z)$.

Table 3 depicts the content map construction algorithm with PLSA. This study initially set $z = 100$ latent classes. The content map can gauge very accurately which learning cases are relevant to student's learning problem features, even for features do not appear in a learning case.

5. Adaptive learning case recommendation approach

This section describes the process of adaptive learning case recommendation as the basis for technological development. The process retrieves adaptive learning cases for the learner from the learning case repository based on student's learning problem of the learner. As shown in Fig. 9, the adaptive recommendation procedure includes three phases, namely (i) analysis and identification of teaching context, (ii) definition and establishment of teaching problem features for the student's learning problem, and (iii) searching, matching and ranking of the adaptive learning cases.

5.1. Analysis and identification of teaching context

This phase includes steps of characteristic assessment and adaptive teaching context identification as discussed below.

5.1.1. The assessments

The assessments which include teacher assessment and student assessment aimed to identify the characteristics of both teacher and his/her student for adaptive teaching context identification. The results of the assessments are presented in a context feature tuple stored in user model.

5.1.1.1. The teacher assessment. The teacher assessment conducts identification of learner's teaching style, leaning style and competence. This study adopted the VARK Questionnaire (VARK) to assess the learner's learning style, and the CORD Questionnaire (CORD, 2005) to assess the learner's teaching style. Scores on the pedagogical content knowledge of mathematics, education of students with mild disabilities, and mathematics knowledge tests were adopted as criteria to assess the teacher's competence.

Table 3
Content map construction algorithm

Content map construction algorithm
Input: concepts in the problem statement of learning case matrix $TP(c,lc)$ $c = 1{:}T$, $lc = 1{:}N$
concepts in the teaching plan of learning case matrix $TN(c,lc)$ $c = 1{:}T$, $lc = 1{:}N$
concepts in the teaching case of learning case matrix $TC(c,lc)$ $c = 1{:}T$, $lc = 1{:}N$
The number K of latent class z
Output: W_{Pi} the weight of the ith concept in the problem statement of learning case
W_{Ni} the weight of the ith concept in the teaching plan of learning case
W_{Ci} the weight of the ith concept in the teaching case of learning case
begin
set $M1$=matrix TP
set $M2$=matrix TN
Set $M3$=matrix TC
for m=1 to 3 do
begin
set $M(c,lc) = Mm$
initialize $P(c_i|z_k)$ randomly with numbers between [0 and 1] where columns should be normalized to 1;
initialize $P(z_k) = 1/k$
initialize $P(lc_i|z_k) = 1/k$
for i: = 1 to N
begin
for j: = 1 to T
begin
$P(lc_i,c_j) = \sum_{k=1}^{K}P(z_k)P(c_j|z_k)P(lc_i|z_k)$
end;
end;
while a convergence condition is not met do
begin
for i: = 1 to N
begin
for j: = 1 to T
begin
for k: = 1 to K
begin
E-Step $P(z_k|lc_i,c_j) = \frac{P(z_k)P(c_j|z_k)P(lc_i|z_k)}{\sum_{k=1}^{K}P(z_k)P(c_j|z_k)P(lc_i|z_k)}$
M-Step $P(c_j|z_k) = \frac{\sum_{i=1}^{N}n(lc_i,c_j)P(z_k|lc_i,c_j)}{\sum_{j'=1}^{T}\sum_{i=1}^{N}n(lc_i,c_j)P(z_k|lc_i,c_j)}$
$P(lc_i|z_k) = \frac{\sum_{j=1}^{T}n(lc_i,c_j)P(z_k|lc_i,c_j)}{\sum_{i=1}^{N}\sum_{j=1}^{T}n(lc_i,c_j)P(z_k|lc_i,c_j)}$
$P(z_k) = \frac{\sum_{i=1}^{N}\sum_{j=1}^{T}n(lc_i,c_{j'})P(z_k|lc_i,c_{j'})}{\sum_{i=1}^{N}\sum_{j=1}^{T}n(lc_i,c_j)}$
end;
end;
end;
end;
for i: = 1 to N
begin
for j: = 1 to T
begin
$w_{ii} = P(lc_i,c_j) = \sum_{k=1}^{K}P(z_k)P(c_j|z_k)P(lc_i|z_k)$
end;
end;
end;
end;

5.1.1.2. The student assessment. The student assessment identifies a student's disabilities, learning style, strengths and weaknesses, and mathematics abilities. These characteristics information can be provided by the learner, or determined though referral forms of student with special needs and the templates of Universal Design for Learning (UDL) (Rose & Meyer, 2002), respectively. A learning style measurement test (Kolb, 1999) and academic ability assessment were then performed to identify the student's learning style and to determine the current math ability level.

5.1.2. Teaching context identification

Based on the assessment results which are presented as a context feature tuple, a learner's suitable teaching context can be identified through the context map. In other words, the new context feature tuple is classified into the most similar cluster by comparing each cluster's centroid with the dissimilarity measure approach of EKP. Smaller degree dissimilarity between two objects indicates greater similarity. Therefore, the cluster with the smallest degree of dissimilarity is identified as the most suitable teaching context for the learner.

5.2. Definition and establishment of teaching problem features

The second phase transformed an unstructured description of a learner's teaching problem into structured problem features for learning case searching and matching. The problem analysis included five steps, i.e. sentence breaking, word breaking, word tagging, concept parsing and problem features weight calculation. The first four steps were as in Section 4.2.2. The weight of problem features was calculated as the probability of appearing in the teaching problem as

$$w_{pf_i} = P(pf_i) = \frac{n_{pf_i}}{N_{pf}}$$

where $P(pf_i)$ is the frequency of teaching problem feature i in the learner's teaching problem; n_{pf_i} is the number of teaching problem feature i in the learner's teaching problem and N_{pf} is the number of teaching problem features in the learner's teaching problem.

5.3. The searching, matching and recommendation of the adaptive learning case

The third phase retrieved the adaptive learning case according to the results of the previous two phases. This phase included steps of searching learning case candidates, matching learning content, ranking the learning cases, recommending and re-recommending.

5.3.1. Searching learning case candidates

This step searched relevant learning cases as learning cases candidates through the context map based on the teaching context identified by the first phase, and then proceeded to content matching.

5.3.2. Matching learning content

The matching model provided global matching and local matching according to the learner's preference. The global matching computed content similarity on all learning cases, while local matching computed content similarity on problem statement, teaching plan and teaching cases, respectively.

The matching types determined by setting control factors α_p, α_n and α_c were set for the student's learning problem statement, the teaching plan and the teaching case respectively. A control factor value was set to 1 if the learner selected local matching, and 0 otherwise. If global matching was chosen, all control factor values were set to 1.

The similarity measurement was then performed to calculate the degree of similarity between a learning case and the learner's teaching problem using the cosine-measure (Gerard, 1989). The learning case similarity, denoted as $Sim(j)$, is defined by

$$Sim(j) = \alpha_P \times CosSim(L_{Pj},P) + \alpha_N \times CosSim(L_{Nj},P) + \alpha_C \times CosSim(L_{Cj},P)$$

$$CosSim(L_{Pj},P) = \frac{\sum_{i=1}^{t}(w_{Pji} \cdot w_{pfi})}{\sqrt{\sum_{i=1}^{t}w_{Pji}^2 \cdot \sum_{i=1}^{t}w_{pfi}^2}}$$

$$= \frac{\sum_{i=1}^{t}(P(c_{Pji}|l_{pj})P(pf_i))}{\sqrt{\sum_{i=1}^{t}(P(c_{Pji}|l_{pj}))^2 \sum_{i=1}^{t}(P(pf_i))^2}}$$

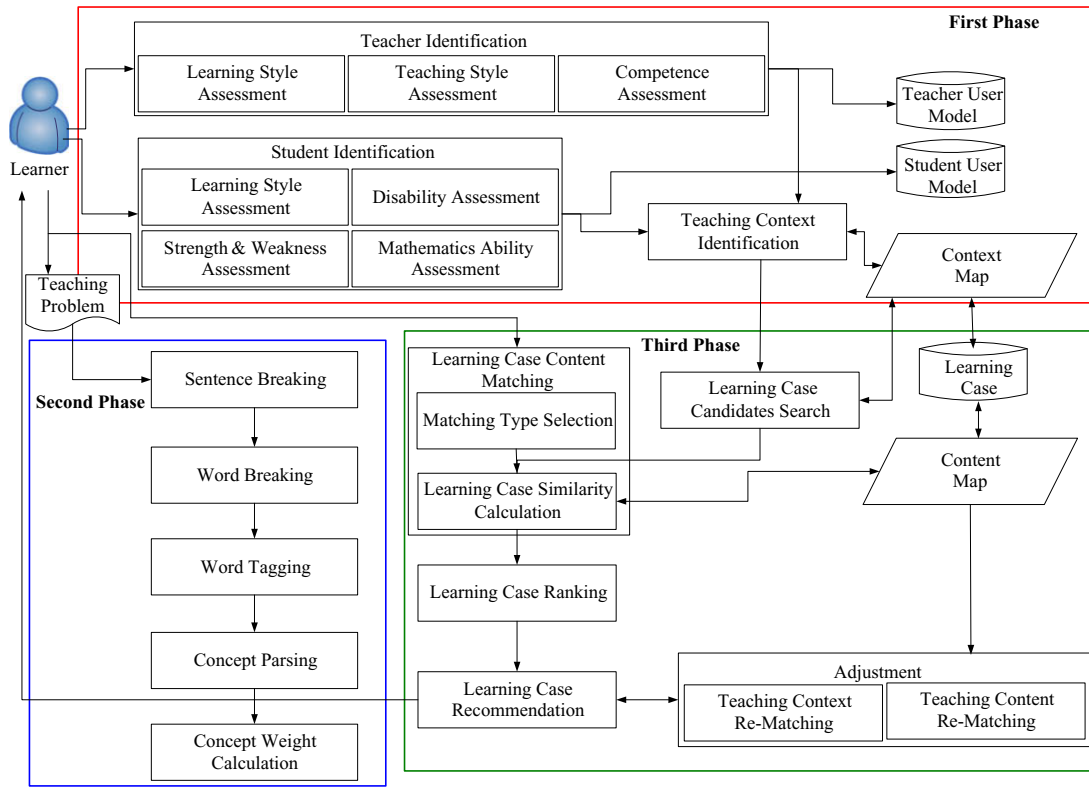**Fig. 9.** The process of adaptive case recommendation.

$$CosSim(L_{Nj}, P) = \frac{\sum_{i=1}^{t}(w_{Nji} \cdot w_{pfi})}{\sqrt{\sum_{i=1}^{t} w_{Nji}^2 \cdot \sum_{i=1}^{t} w_{pfi}^2}}$$

$$= \frac{\sum_{i=1}^{t}(P(c_{Nji}|l_{pj})P(pf_i))}{\sqrt{\sum_{i=1}^{t}(P(c_{Nji}|l_{pj}))^2 \sum_{i=1}^{t}(P(pf_i))^2}}$$

$$CosSim(L_{Cj}, P) = \frac{\sum_{i=1}^{t}(w_{Cji} \cdot w_{pfi})}{\sqrt{\sum_{i=1}^{t} w_{Cji}^2 \cdot \sum_{i=1}^{t} w_{pfi}^2}}$$

$$= \frac{\sum_{i=1}^{t}(P(c_{Cji}|l_{pj})P(pf_i))}{\sqrt{\sum_{i=1}^{t}(P(c_{Cji}|l_{pj}))^2 \sum_{i=1}^{t}(P(pf_i))^2}}$$

In the above equations, w_{Pji} is the weight of concept i in the problem statement of learning case j; w_{Cji} is the weight of concept i in the teaching case of learning case j; w_{Nji} is the weight of concept i in the teaching plan of learning case j, and w_{pfi} is the weight of problem feature i in the learner's teaching problem.

5.3.3. Re-recommendation the adaptive learning case

If the recommended learning case did not fit the learner's requirements, an adjustment was conducted and which is describe below.

The learning case with the most similarity to the learner's teaching context was measured by the dissimilarity approach of IKP from the specific cluster which was identified in the first phase. The learning case with the most similarity with the learner's teaching problem was measured by the equation of learning case similarity through the content map. These two learning cases were re-recommended to assist learning for the learner.

6. Implementation and demonstration

An adaptive learning case recommendation mechanism for problem-based e-learning, based on the proposed adaptive learning case recommendation approach, was implemented at the Enterprise Engineering and Integration Laboratory (EEIL) of National Cheng Kung University, Taiwan, ROC.

6.1. Case representation

To store, organize, manage, and use the case contents effectively, the instances of each class in the case model were defined as learning objects by Extensible Markup Language (XML). XML is a simple, cross-platform, extensible and flexible text-based standard for representing data (Sun Microsystems, 2002, Zhang, Sheng, Li, & Yao, 2002). Practical teaching knowledge can be represented and stored in XML documents by defining tags and the structural relationships among them. The PBeL platform can display knowledge content adaptively by enabling the same data to be published in different media.

In this study, a learning case was composed of three XML document classes, i.e. the teaching context, teaching plan, and teaching case. Part of the XML schema is shown as follows:

```
<Teaching Plan>
  <Teaching Procedure>
    <Teaching Case TCID="1" name=" able to say, read, write,
and count numbers up to 2000 and compare their values">
      <Teaching Procedure> 1–1 able to create numbers up to
2000 by adding 1, 10, 100 and 1000, and arrange them in a cor-
rect sequential order. 1–2 able to read correctly numbers up to
2000. 1–3 able to write correctly numbers up to 2000. 1–4 able
to comprehend the correspondence between digits and place
```

values of numbers up to 2000. 1–5 able to operate additions or subtractions for numbers up to 2000. 1–6 able to compare two numbers up to 2000 and to express such a relationship with <, >, or =. 1–7 able to operate additions and subtractions with coins for the sum of monetary values up to 2000. 1–8 able to make correct payment according to prices of the objects concerned.

</Teaching Procedure>

<Error Pattern EPID="**1**">(1) Having difficulty counting 10 continuous numbers in the correct sequential order; unable to identify the sequence for 1080-D1090-D( ) due to problem with numbers from 1090 to 1100. (2) Having incorrect comprehension of place values: missing "thousand, hundred, or '-thy'" when reading numbers, e.g. 1685 was spoken as "one thousand six hundred eight five" and 1035 as "one zero thirty five," while one thousand and five was written as 105. </Error Pattern>

<Error Pattern EPID="**2**"> able to compare numbers but cannot distinguish sign > from sign <. </Error Pattern>

<Teaching Approach and Strategy TASID="**1**">(1) Hands-on operations with visual aids – using decimal digit table, coins, and place value board for hands-on operations. When student places one more 10 on the TEN digit while counting from 1080 to 1090, teacher must emphasize the digit being added has a "TEN" place value to strengthen student's understanding of place values. Have student add one more "10" to 1090, making "ten 10s," which can be replaced by "one '100'". Through such hands-on operations with the aid of decimal digit table, coins, place value board, student can see clearly the changes in decimal digit values, thus facilitating the students' understanding of place values as an abstract concept through physical, visual stimulation. (2) Verbal hints: e.g. teacher says "one thousand 'and' seventy" for 1070, "one thousand 'and' eighty" for 1080, "one thousand 'and' ninety" for 1090, and "one thousand 'and' one hundred" for 1100. The emphasized "and" helps students avoid possible confusions about concepts related to a decimal digit system. </Teaching Approach and Strategy >

<Teaching Approach and Strategy TASID="**2**"> Work analysis (1) □ > ○: Have students produce visual cards 「>」and 「<」,

with the hint that A「-」B means A is " bigger" than B. likewise, in □>○, □ is a bigger and ○ is smaller; then, have students to fill in two numbers of their choosing, one before and the other after the sign, to personally experience the use and concept of 「>」. (2) Have students choose either > or < for a pair of arbitrary numbers, e.g.「8 □ 5」. Teacher provides visual cards of both > and< for students to choose from. Based on students' choice, teacher will be able to see if student understands > and <. (3) Teacher gives direct explanations for > and <, and ends with telling students directly that 8 > 5 means "8 is bigger than 5," and 「>」is read as "is bigger," whereas 5 < 8 means "5 is smaller than 8," and 「<」is read as "is smaller". </Teaching Approach and Strategy >

<Teaching Aids TAID="**3**"> decimal digit board</Teaching Tool >

<Objective>Hands-on operations of decimal digit board enhances students' comprehension of decimal values as a concept, and helps train students for the conversion between numbers and Chinese numerals as well as for the reading and speaking of numbers. </Objective >

<Instructional Practice>decimal digit board.jpg</Instructional Practice>

<Explanation> Chinese words for decimal digits are made into visual cards. Hands-on operations and arranging these word cards in the correct order improve students' conceptual understanding of decimal values and strengthen students' interchanging capability in between reading and writing. </Explanation>

</Teaching Case>

6.2. Example

Learners were required to enter their ID and password via the interface to access the PBeL platform. If a learner logged into the system at the first time, a user model would be built to contain the learner's demographic data, background information, and the teacher assessment results.
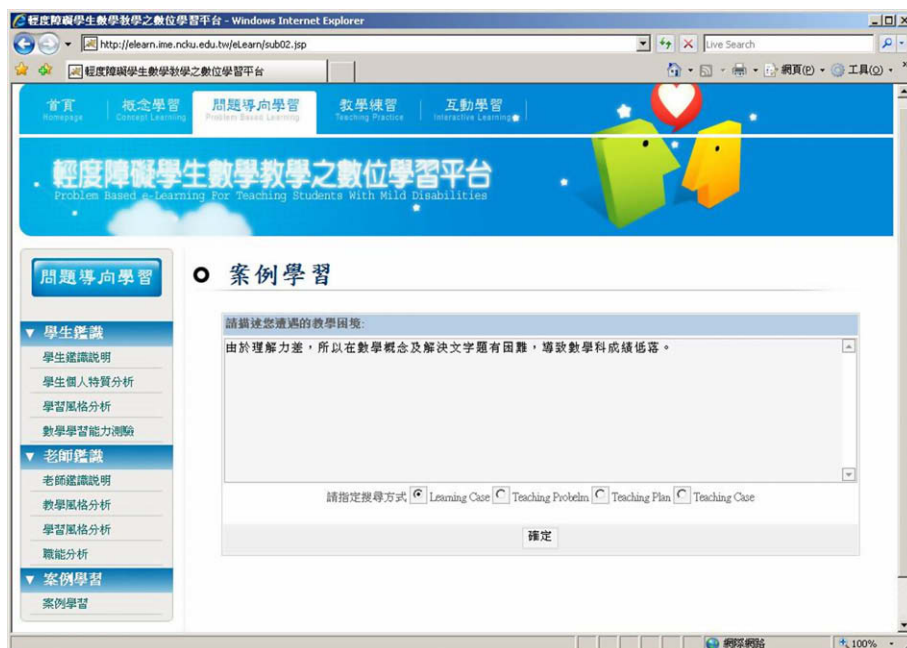

Fig. 10. Teaching problem description.

Fig. 11. Part of the content of learning case.

The platform then guided the learner to assess his or her student's academic abilities, and analyzes the types of disabilities, learning style, and strengths and weaknesses of the students. A student model was then built based on the student assessment results.

The learner was then required to describe his/her student's learning problem in Chinese natural language, and chose the matching type as shown Fig. 10. The adaptive recommendation mechanism then recommended the most similar learning case according to the characteristics of the learner and the student, and the student's learning problem. The content of the recommended learning case displays as shown in Fig. 11.

7. Conclusion and future work

This study designed an e-learning model, with problem-based learning as its core and social constructivism and situated learning as its auxiliary theories, to assist teachers to effectively develop knowledge of teaching mathematics for students with mild disabilities.

Additionally, an adaptive recommendation approach was designed for the problem-based e-learning model to achieve the goal of adaptive learning. This approach recommended adaptive learning cases based on the characteristics of both the learner and the student, and has semantic searching capability to avoid information mistake and loss caused by semantic variations. To fulfill the requirements of clustering data with numerical, categorical single-values, and categorical multi-values, an enhanced k-prototypes algorithm was proposed.

Although IKP used in the recommendation approach can cluster data with different data types, the clustering results allow only one learning case in a cluster. Future work will combine fuzzy theory into the clustering method. Since fuzzy clustering applies membership degrees between 0 and 1, instead of crisp assignments, to represent the degree of membership of data to each cluster, it can identify more suitable teaching contexts for the learner.

References

Agnar, A., & Enric, P. (1994). Case-based reasoning: Foundational issues, methodological variations, and system approaches. *AI Communications, 7*(1), 39–59.

Barrows, H. S. (1996). Problem-based learning in medicine and beyond: A brief overview. In L. Wilkerson & H. GiJselaers (Eds.). *Bringing PBL to higher education: Theory and practice* (Vol. 68, pp. 3–11). San Francisco: Josssey Bass.

Booch, G., Rumbaugh, J., & Jacobson, I. (1999). *The unified modeling language user guide*. Reading: Addison-Wesley.

Carrascosa, C., Bajo, J., Julian, V., Corchado, J. M., & Botti, V. (2008). Hybrid multi-agent architecture as a real-time problem-solving model. *Expert Systems with Applications, 34*, 2–17.

Chin, C., & Lin, F. L. (2000). A case study of a mathematics teacher's pedagogical values: Using a methodological framework of interpretation and reflection. *Proceedings of the National Science Council, Part D: Mathematics, Science, and Technology Education, 10*(2), 99–101.

CORD (Teaching style inventory) (2005). <http://www.cord.org/teaching-styles-inventory/>.

Delisle, R. (1997). *How to use problem-based learning in the classroom*. Alexandria, Virginia: Association for Supervision and Curriculum Development.

Dempster, A. P., Laird, N. M., & Rubin, D. B. (1977). Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society, 39*(1), 1–21.

Fenwick, T. J., & Parsons, J. (1998). Boldly solving the world: A critical analysis of problem-based learning as a method of professional education. *Studies in the Education of Adults, 30*(1), 53–66.

Gerard, S. (1989). In M. A. Harrison (Ed.), *Automatic text processing: The transformation, analysis, and retrieval of information by computer*. Addison Wesley.

He, Z., Xu, X., Deng, S., & Dong, B. (2004). FK-modes: Improving k-modes algorithm considering frequencies of attribute values in mode. <http://www.paper.edu.cn>.

Hofmann, T. (1999a). Probabilistic latent semantic analysis. In *Proceedings of uncertainty in artificial intelligence, Stockholm, Sweden*.

Hofmann, T. (1999b). Probabilistic latent semantic indexing. In *Proceedings of SIGIR-99, Berkeley, CA* (pp. 35–44).

Hsu, S. H., & Wang, S. L. (2004). Evaluating the effectiveness of knowledge-retrieval aids for fault-diagnosis tasks. *Journal of the Chinese Institute of Industrial Engineers, 21*(5), 465–474.

Huang, Z. (1998). Extensions to the *k*-means algorithm for clustering large data sets with categorical values. *Data Mining and Knowledge Discovery, 2*, 283–304.

Kalyuga, S. (2007). Expertise reversal effect and its implications for learner-tailored instruction. *Educational Psychology Review, 19*(4), 509–539.

Kolb, D.A. (1999). *Learning Style Inventory, Version 3*: Technical specifications. Boston, MA: Hay Group, Hay Resources Direct. 116 Huntington Avenue, Boston.

Levin, B. B. (2001). *Energizing teacher education and professional development with problem-based learning.* Alexandria, VA: Association Supervision and Curriculum Development.

Liu, D. R., & Ke, C. K. (2007). Knowledge support for problem-solving in a production process: A hybrid of knowledge discovery and case-based reasoning. *Expert Systems with Applications, 33*, 147–161.

Merseth, K. K. (1996). Cases and case methods in teacher education. In J. Sikula (Ed.), *Handbook of research on teacher education (2nd)* (pp. 722–774). New York: Macmillan.

Montague, M., & Applegate, B. (2000). Middle school students' perceptions, persistence, and performance in mathematical problem solving. *Learning Disability Quarterly, 23*, 215–227.

Patel, A., Kinshuk, & Russell, D. (2000). Intelligent tutoring tools for cognitive skill acquisition in life long learning. *Educational Technology & Society, 3*(1), 32–40.

Ramze, R. M., Lelieveldt, B. P. F., & Reiber, J. H. C. (1998). A new cluster validity index for the fuzzy c-mean. *Pattern Recognition Letters, 19*, 237–246.

Renkl, A., & Atkinson, R. K. (2003). Structuring the transition from example study to problem solving in cognitive skills acquisition: A cognitive load perspective. *Educational Psychologist, 38*, 15–22.

Richardson, V. (1993). Use of cases in considering methods for motivating students. In H. Harrington & M. Thompson (Eds.), *Student motivation and case study manual* (pp. 5760). Boone, NC: Appalachian State University.

Richert, A. E. (1991). Using teacher cases for reflection and enhanced understanding. In A. Lieberman & L. Miller (Eds.), *Staff development for education in the 90's* (pp. 113–132). New York: Teachers College Press.

Roderick, A. F., & Baden, H. (2005). A situated learning perspective on learning object design, *ICAL2005 proceeding* (pp. 72–75).

Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context.* New York: Oxford.

Rose, D. H., & Meyer, A. (2002). *Teaching every student in the digital age: Universal design for learning.* Alexandria, Virginia: Association for Supervision and Curriculum Development.

Solomon, J. (1987). Social influences on the construction of pupils' understanding of science. *Studies in Science Education, 14*, 63–82.

Souders, J., & Prescott, C. (1999). A case for contextual learning. *High School Magazine, 7*(3), 38–43.

Steele, M. M. (2006). Teaching suggestions for students with learning problems. *TechTrends: Linking Research and Practice to Improve Learning, 50*(6), 32–35.

Sun Microsystems, Inc. (2002). Web services made easier: the java APIs and architectures for XML.

VanLehn, K. (1996). Cognitive skill acquisition. *Annual Review of Psychology, 42*, 513–539.

VARK questionnaire. <http://www.vark-learn.com/english/page.asp?p=questionnaire>.

Yang, B. S., Han, T., & Kim, Y. S. (2004). Integration of ART-Kohonen neural network and case-based reasoning for intelligent fault diagnosis. *Expert Systems with Applications, 26*, 387–395.

Zhang, D. M., Sheng, H. Y., Li, F., & Yao T. F. (2002). The model design of a case-based reasoning multilingual natural language interface for database. *Proceeding of the first international conference on machine learning and cybernetics* (pp. 1474–1478).