CHAPTER 3 TOOTH SEGMENTATION

In this chapter, we will describe how to segment teeth from tooth regions. Our experimental images can be classified into two types. One is a line-aligned image and the other is a parabola-aligned image. We adopt different projection-based segmentation methods which project into different project axes. We will state segmentation methods of two types of images in section 3.1 and 3.2. In section 3.3, we will describe how to label teeth.

3.1 Segmentation of Line-aligned Images

In tooth regions, a valley exists between two neighboring teeth in Fig. 3.1.1. The valley separates two neighboring teeth. Therefore, segmenting two neighboring teeth is to find the position of the valley, i.e. find the concave of cure.

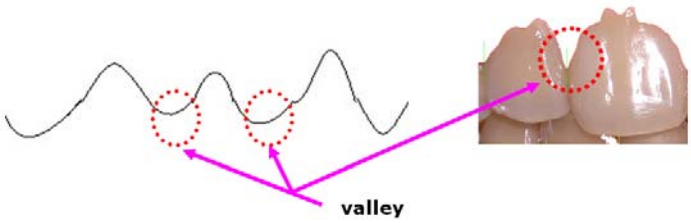


Fig. 3.1.1 Valleys

How to find a valley? We obtain a strip of height values from height of edges in the upper teeth. The strip of height values is such as follows:

5	5	4	4	3	2	1	0	2	5	6	7

Fig. 3.1.2 Strip of height values

The first derivative values are gained by subtracting the preceding value from the present value. Change from negative to positive is the valley illustrated in Fig. 3.1.3.

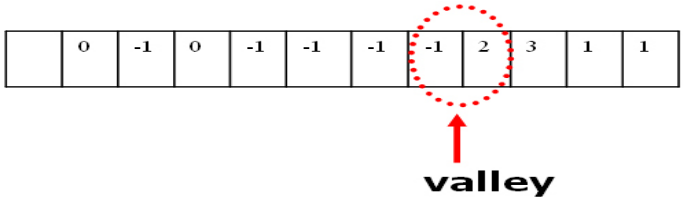


Fig. 3.1.3 Strip of first derivative

We realize how to find the valley; nevertheless, many valley candidates which are not the precise one separating two neighboring teeth are found in Fig. 3.1.4. Some are crevice between two teeth, and the others are not.

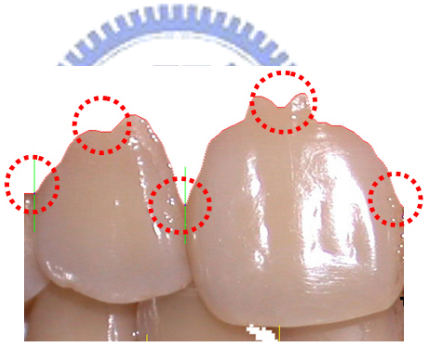


Fig. 3.1.4 Valley candidates

How to select correct valleys we wanted from many candidates is our concern. Hence, we solve the problem by finding the minimum valley in a specific region (window).

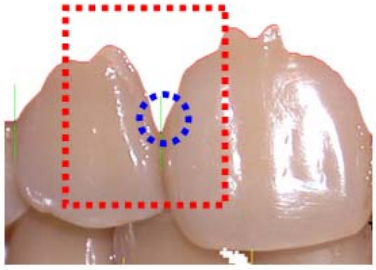


Fig. 3.1.5　A specific window

However, it is also not perfect enough. We will miss some valleys if the window size is not proper in Fig 3.1.6.

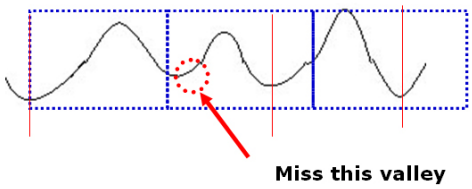


Fig. 3.1.6　Miss some valley

Consequently, we use a dynamic window. Then how to decide window size is emphasized. Our method of deciding window size bases on three ways. First, the present window size is obtained by the average of the previous window sizes. That is the *i-th* window size, R_i, is the average of from $Size_0$ to $Size_i$, where we define $Size_0$ as the smallest tooth size from the training images, i.e. define the window size from left to right according to the average of previous tooth sizes. In the front view, sizes of a tooth increase from molar to incisor. After passing the first incisor, sizes of a tooth decrease from incisor to molar. Therefore, we add a variable ε to control increasing or decreasing window size. ε is a predefined constant. We can describe them in a formula as follows:

$$R_i = s * \varepsilon + \frac{1}{i+1}\sum_{j=0}^{j=i} Size_j, i = 1,2,...,n.$$

$$\begin{cases} s = 1 & \text{when s at left part of the front view} \\ s = -1 & \text{when s at right part of the front view} \end{cases}$$

Second, if the right-hand side of a window does not locate at the curve top, move it left or right to the top according to its slope. If the right-hand side of a window locates at positive slope (increasing slope), we move it right toward the top of mountain. We illustrate the concept in Fig. 3.1.7.

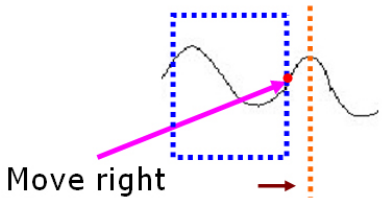


Fig. 3.1.7 Move the right-hand side of a window right

If the right-hand side of a window locates at negative slope (decreasing slope), we move it left toward the top of mountain. We illustrate the concept in Fig. 3.1.8.

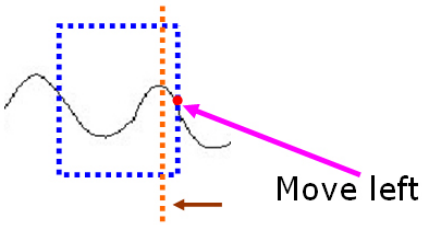


Fig. 3.1.8 Move the right-hand side of a window left

Third, if the size of a dynamic window is larger than 2*(average tooth size), we divide it into two windows.

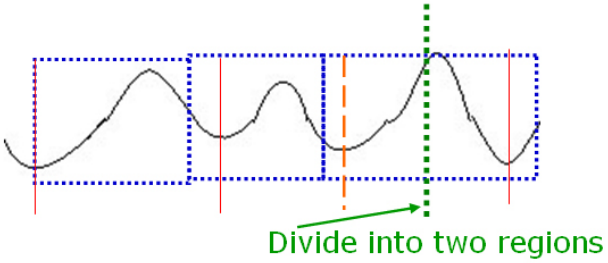
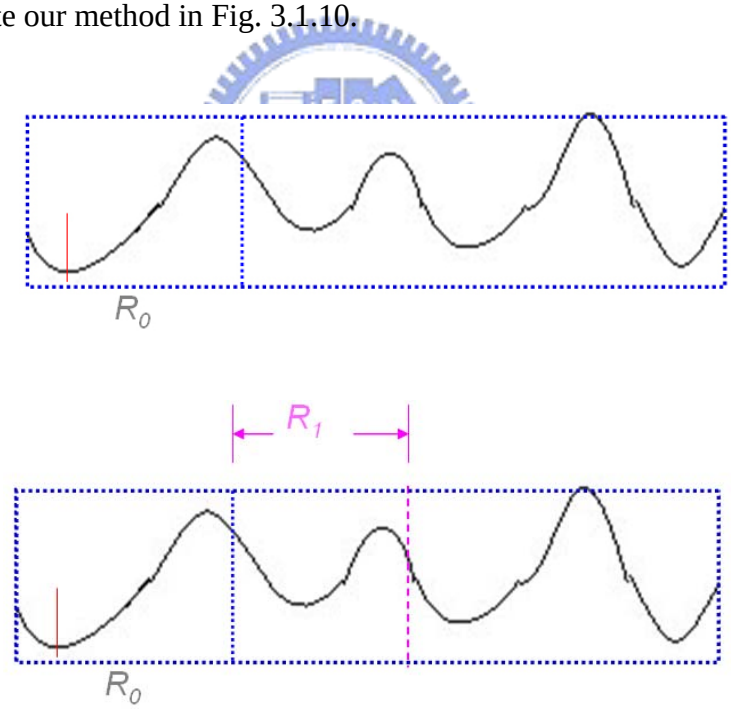
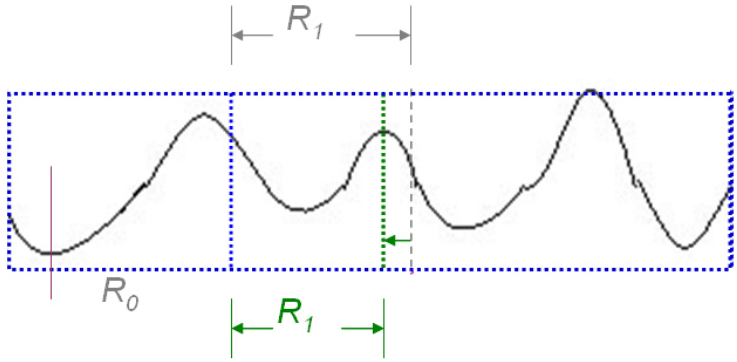
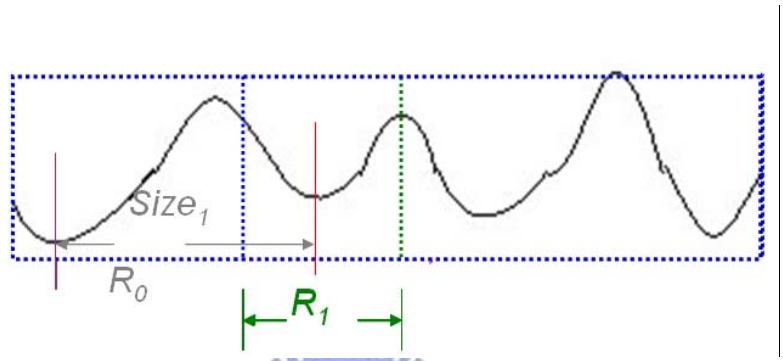


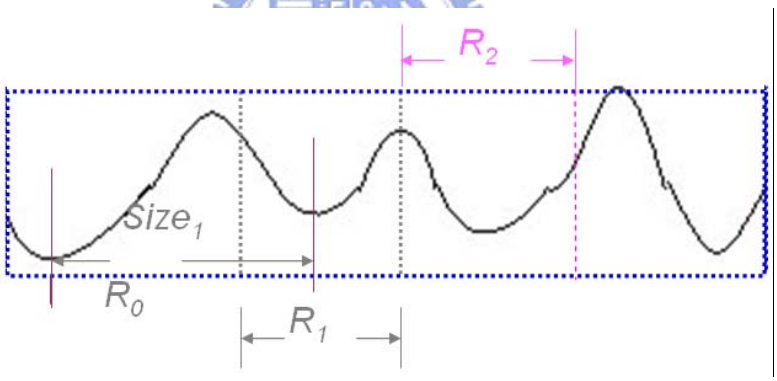
Fig. 3.1.9 Divide too larger window into to windows

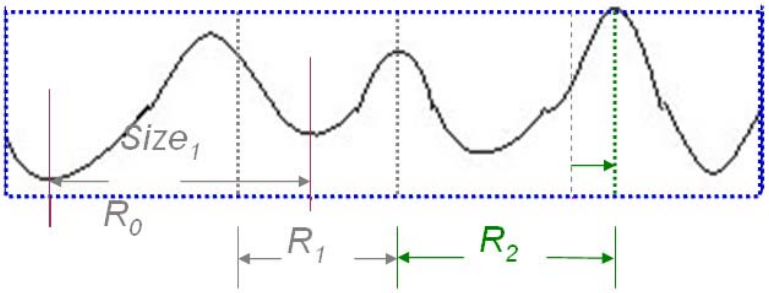
We illustrate our method in Fig. 3.1.10.




R_1
R_0
R_1


$Size_1$
R_0
R_1


R_2
$Size_1$
R_0
R_1


$Size_1$
R_0
R_1
R_2

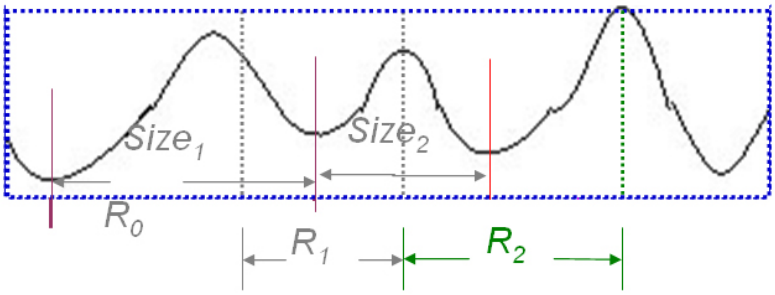


Fig. 3.1.10 Illustrate our method

In the same way, segmentation in lower tooth regions is similar to segmentation in upper tooth regions. Here, we find the hill-tops instead of finding the valley in Fig. 3.1.11. How to find the hill-top is similar to find the valleys.

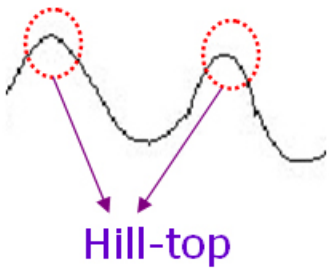


Fig. 3.1.11 Hill-top

We obtain a strip of height values from height of edges in the lower teeth. The strip of height values is such as follows:

2	3	4	5	3	2	1	0	2	5	6	7

Fig. 3.1.12 Strip of height values

The first derivative values are gained by subtracting the preceding value from the present value. Change from positive to negative is the valley illustrated in Fig. 3.1.13.

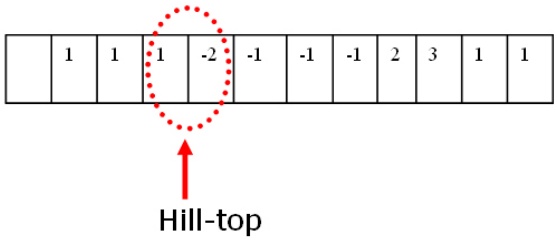


Fig. 3.1.13 Strip of first derivative

3.2 Segmentation of Parabola-aligned Images

A method of segmentation of parabola-aligned images is that we project teeth into a parabola axis to obtain a waveform of teeth. After we have the waveform of teeth, we can apply the method similar to the one of section 3.1 to segment teeth.

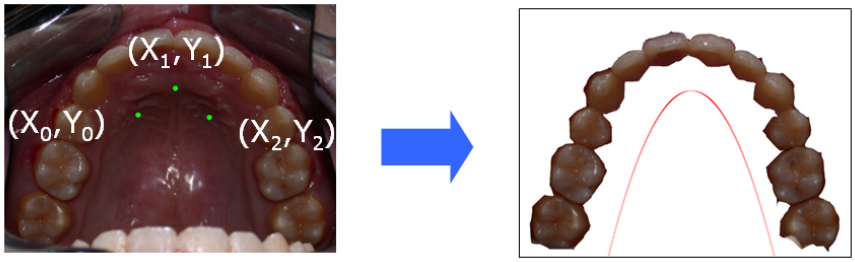
In our experiment, we found that only using one parabola, we can't cut properly in middle regions of teeth due to sharp change of the slope. Hence, we project the middle tooth regions into another parabola, called the second parabola, in order to

segment teeth correctly. Therefore, we use the combination of two parabolas instead of only one parabola.

3.2.1 Construct the first parabola

(a) An initial parabola

We concern how to obtain the first parabola to form the parabola axis. We gain the parabola initially from training period. In samples, we mark three points to represent three points in a parabola in Fig. 3.2.1 (a). Therefore, we can obtain the parabola in Fig. 3.2.1 (b) from the three points in Fig. 3.2.1 (a).



(a) (b)

Fig. 3.2.1 Three points and the parabola

(b) Adjust the initial parabola

After obtaining the parabola, sometimes the one was not at the proper position such as in Fig. 3.2.2. So we need move the parabola to the center position of teeth regions.

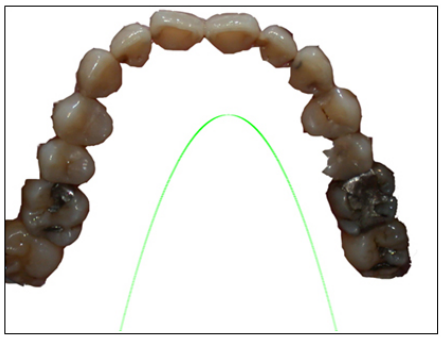


Fig. 3.2.2 An improper parabola

First, we can obtain the center point of tooth region by calculating mean of right side and left side in Fig. 3.2.3. The method of calculating mean in Fig. 3.2.3 can be described in Formula 3.2.1. Then we can move the center point of the initial parabola to the center point of tooth region.

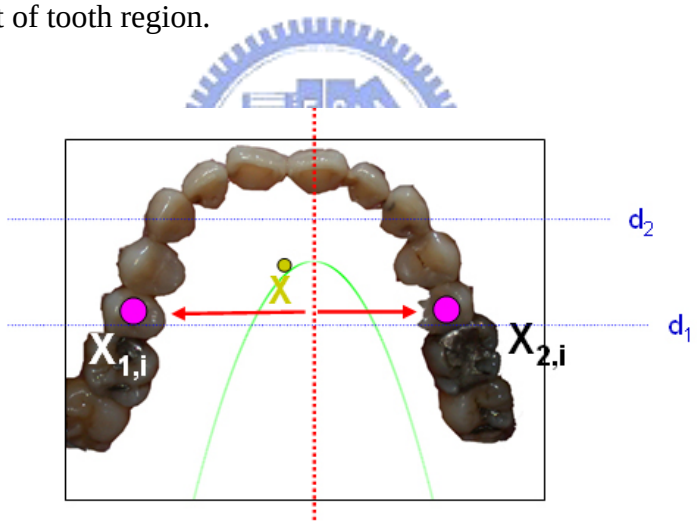


Fig. 3.2.3 Calculate the mean

$$X = \frac{1/2\sum_{i=d_1}^{d_2}(x_{1,i} + x_{2,i})}{d_2 - d_1 + 1}$$

Formula 3.2.1

Second, we check the new parabola if it locates on the regions of teeth. If the top

of the parabola touch tooth regions, we move down the position of top point. If the right and left side touch tooth regions, we shrink the parabola by moving toward the center point of tooth regions. Before moving the parabola is shown in Fig. 3.2.4 (a) and after moving the parabola is displayed in Fig. 3.2.4 (b).

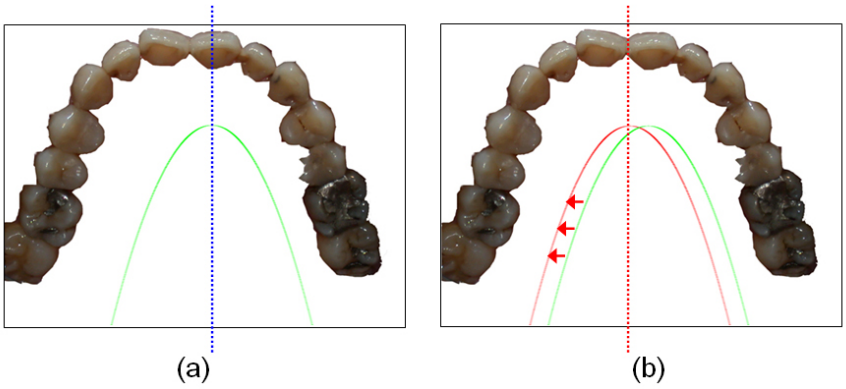


Fig. 3.2.4 (a) before adjusting the parabola (b) after adjusting the parabola

We need some tool of math formulas to construct a parabola. We want to project the teeth to a parabola. We need know the tangent on a parabola. Projecting teeth to a parabola means projecting teeth to one's tangent line of project point. Assume the equation of a parabola is $Y = aX^2 + bX + c$. Hence, the tangent line equation of the point (g, h) in a parabola is $y = (2a \times g + h)x + (h - 2g^2h + gh)$. And the slope, m, of the tangent line equation is $m = 2a \times g + b$. We project the point (c, d) on tooth regions into the tangent line, $y = ax + b$, of the parabola which project point is (e, f) . We illustrate as follow:

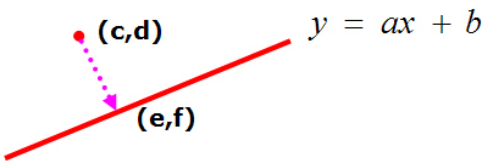


Fig. 3.2.5 Project a (*c*, *d*) to $y=ax+b$

The project point (e, f) is as follow:

$$\begin{cases} e = \dfrac{ad + c - ab}{1 + a^2} \\ f = \dfrac{a^2 d + ac - a^2 b}{1 + a^2} + b \end{cases}$$

After using the above formulas to project teeth to a parabola, we can obtain a diagram of projection. We gain the wave of the teeth with respect to the parabola. Segment the wave of the teeth with respect to the parabola and we obtain the cut-point (*i*, *j*) in the parabola which is the valley on the wave of teeth with respect to the parabola. We can obtain the cut-line which is a normal of tangent line on cut-point (*i*, *j*) of a parabola in Fig. 3.2.6. The equation of the cut-line is $y = -\frac{1}{a}x + c$. And the cut-line passes cut-point (*i*, *j*) which c is obtained from the formula of $c = j + \frac{1}{a} \times i$.

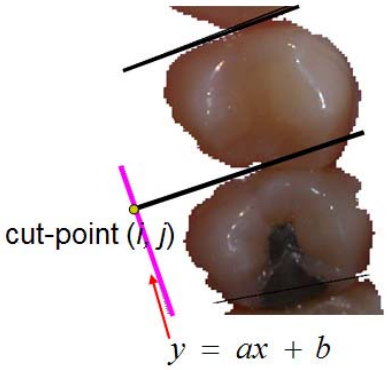


Fig. 3.2.6 The normal of the line: $y=ax+b$

3.2.2 Construct the second parabola

Only using one parabola, we cannot cut properly in middle regions of teeth due to sharp change of the slope. Therefore, we use two parabolas instead of only one parabola. We can project teeth in the middle regions into the second parabola in dental images. Next, we describe how to construct the second parabola. The second parabola can be reduce on the basis of three points (X_3 ,Y_3), (X_4 ,Y_4), (X_5 ,Y_5) in Fig. 3.2.7.

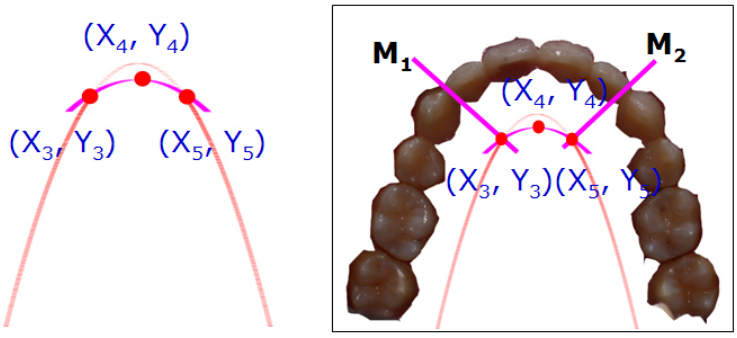


Fig. 3.2.7 Construct the second parabola

How do we find (X_3 ,Y_3) and (X_5 ,Y_5) in Fig. 3.2.7? In the first parabola, the absolute of the slope of cut-line referred in section 3.2.1 is increasing from left to the cut-point (X_3 ,Y_3) in Fig. 3.2.7. And the absolute of the slope of cut-line referred in section 3.2.1 is also increasing from right to the cut-point (X_5 ,Y_5) in Fig. 3.2.7. We find the two cut-points of which the slopes of the cut-lines in the first parabola are M1 and M2 in Fig. 3.2.7. The absolute values of slopes M1 and M2 are little less than 1.

$$M_1 = \min(A), A = \{m \mid -1 < m < 0\}$$
$$M_2 = \max(A), A = \{m \mid 0 < m < 1\}$$

After obtaining the two points (X_3 ,Y_3) and (X_5 ,Y_5), we find the third point (X_4, Y_4). We can find the middle point C of (X_3 ,Y_3) and (X_5 ,Y_5). We add δ value to the Y value of middle point C, i.e.

$(X_4, Y_4) = ((X_3+X_5)/2, (Y_3+Y_5)/2 + \delta)$. In our experiment, δ = 30 pixels.

After gaining the three points (X_3, Y_3), (X_4, Y_4),and (X_5, Y_5) in Fig. 3.2.7, we can construction the second parabola.

After projecting teeth to the two parabolas axis, we obtain the wave of the teeth with respect to the parabolas in Fig. 3.2.8 (a). We compare two diagrams as listed the following:

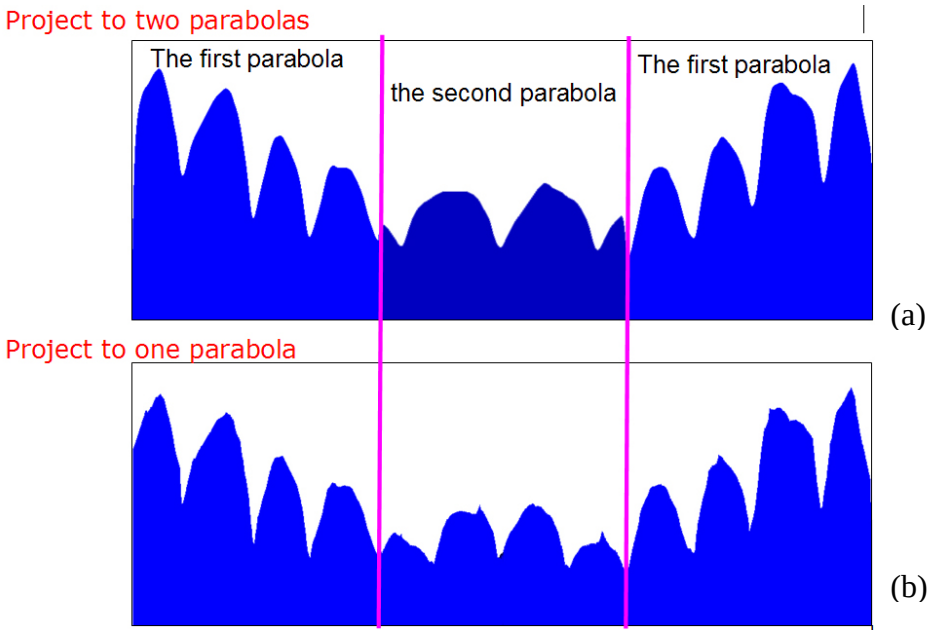


Fig. 3.2.8 Compare two kinds of projection

After gaining the wave of the teeth with respect to the parabolas, the method of section 3.1 can segment teeth in parabola-aligned images. Then how to extract a tooth region is our concern. Extract a tooth region in shape of bounding quadrangle. Using the concept linear programming can extract a tooth region in shape of bounding

quadrangle. Then we can extract a tooth region in shape of bounding quadrangle in Fig. 3.2.9. We select values of d_1 and d_2 which are large enough to include the whole tooth. In our experimental, d_1 and d_2 are predefined constant, which are 150 and 180 respectively. And we can obtain the four apexes of bounding quadrangle. We can obtain the tooth region by a formula of linear programming in Fig. 3.2.10.

The tooth region locates on the area, which satisfies the conditions.

$$\begin{cases} y < a_1 x + b_1 \\ y > a_2 x + b_2 \\ y < a_3 x + b_3 \\ y > a_4 x + b_4 \end{cases}$$

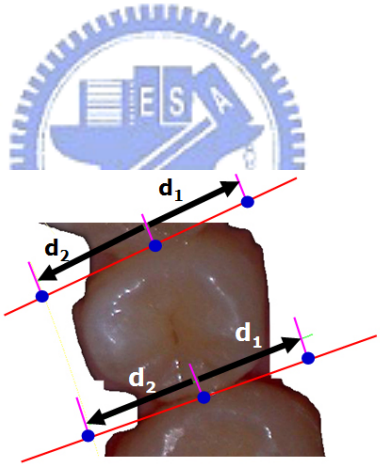


Fig. 3.2.9 Extract a tooth

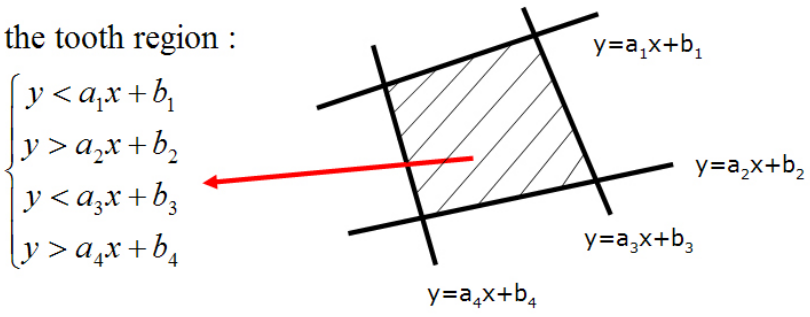


Fig. 3.2.10 Extract a tooth by formula

3.2.3 Accurate Segmentation

The above method segments teeth roughly. Maybe some cut-line in section 3.2 cannot segment teeth properly. We need segment teeth more accurately. We find valleys and tops from edges of a tooth in the neighbor of segmentation lines which are between the line L_1 and L_2. Then connect the position of the valley and its nearest top to form a new segmentation line.

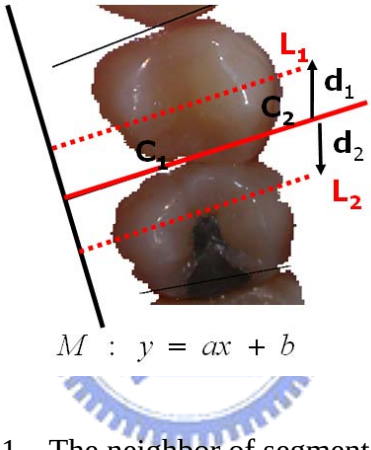


Fig. 3.2.11 The neighbor of segmentation lines

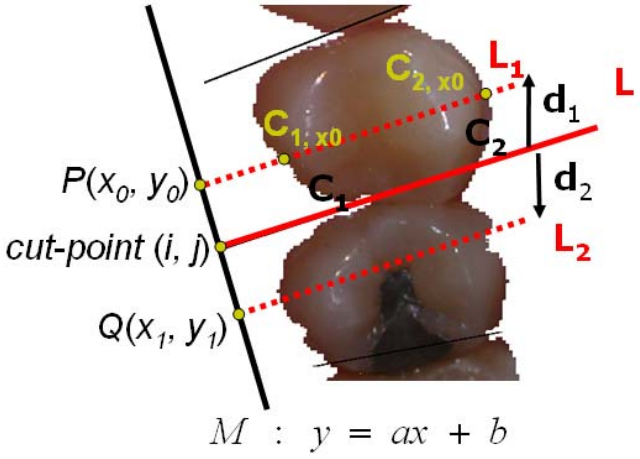


Fig. 3.2.12 Accurate segmentation

Our method of accurate segmentation:

First, we shift the preceding cut-line L up distance of d_1 and down that of d_2. Then we can find two curves C_1 and C_2 which are edges in tooth regions between L_1 and L_2. C_1 is the curve nearest the line M in Fig. 3.2.12. C_2 is the curve farthest the line M in Fig. 3.2.12. How do we obtain the cure C_1 and C_2? We can obtain C_2 by finding the points in tooth regions which distance to its project point (x_0, y_0) in the line *M:* $y=ax + b$ in Fig. 3.2.12 are minimum. In the same way, we can gain C_1 by finding the teeth of which distance from teeth to the line *M:* $y=ax + b$ in Fig. 3.2.12 for its project point (x_0, y_0) are maximum. We describe the method as a formula.

$$C_{1,x_0} = \min_{x_0}(distance(P,M))$$
$$C_{2,x_0} = \max_{x_0}(distance(P,M))$$
$$\therefore \begin{cases} C_1 = \{C_{1,x}\}, x \text{ is from } x_0 \text{ to } x_1 \\ C_2 = \{C_{2,x}\}, x \text{ is from } x_0 \text{ to } x_1 \end{cases}$$
where
$$P(x_0, y_0), M : ax + by + c = 0$$
$$distance(P,M) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

C_{1,x_0} : each pixel of which project - point are (x_0, y_0) in the line M
$\min_{x_0}$: each pixel of which distance are minimun for project - point (x_0, y_0)
$\max_{x_0}$: each pixel of which distance are maximun for project - point (x_0, y_0)

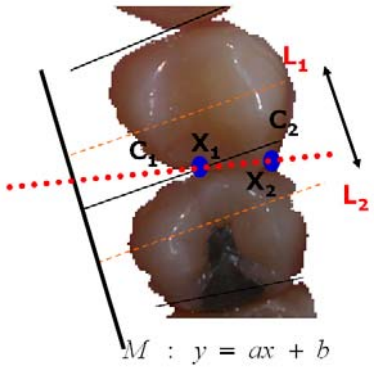


Fig. 3.2.13 The valley and hill-top

We find the valley in cure C_2 and hill-top in cure C_1 in the range from L_1 to L_2. X_1 locate on local maximum of C_1 and X_2 locate on local minimum of C_2 in Fig.3.2.13. We connect X_1 and X_2. Therefore, the line, from X_1 to X_2, is the line of segmentation.

3.3 Labeling

After segmentation of teeth, we label name of teeth in sequence by relation of teeth position.

(A) Label teeth in a front view

Due to pleasing to the eye, everyone always has center incisor whether is a real or fake tooth. In the front view, sizes of a tooth increase from molar to incisor. After passing the first incisor, sizes of a tooth decrease from incisor to molar. The size of the center incisor is the maximum in the front view. For example, changes of the teeth size are illustrated as follows:

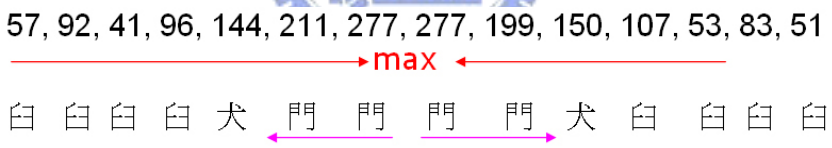


Fig. 3.3.1 Example of teeth size in the front

Therefore, we can start labeling from the center incisor (i.e11, 21, 31, 41).

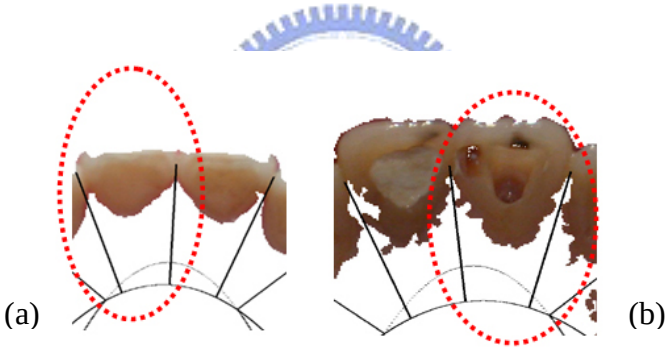
(B) Label teeth in a right view and left view

In the right view or left view samples, their first tooth starts from central incisor. Therefore, we can start labeling tooth from the central incisor (i.e11, 21, 31, 41).

(C) Label teeth in an upper view on the maxilla

In the upper view on the maxilla, we also start from the central incisor. Hence, we need to find which tooth is the central incisor. We perceive that the tooth which is inverse signs of two slopes of cut-line referred in section 3.2 is the central tooth. We find that the 11 is absolution value of the right slope smaller than the left. Inversely, the 21 is absolution value of the right slope larger than the left. We can describe as the following rule to label a tooth:

$$slope_{i-1} \times slope_i < 0$$
$$\begin{cases} if \left\| slope_{i-1} \right\| < \left\| slope_i \right\| \Rightarrow name = 11 \\ if \left\| slope_{i-1} \right\| > \left\| slope_i \right\| \Rightarrow name = 21 \end{cases}$$



(a) (b)

Fig. 3.3.2 Two slopes of a tooth in the upper view on the maxilla

(D) Label teeth in a lower view on the mandible

In the upper view on the mandible, we start from the central incisor. Hence, we need decide which tooth is the central incisor and how to find the central incisor? In a tooth region, there are inverse signs of two slopes of cut-line referred in section 3.2. We perceive that the 41 is absolution value of the right slope smaller than the left. Inversely, the 31 is absolution value of the right slope larger than the left. We can

decide as the following rule to label a tooth:

$$slope_{i-1} \times slope_i < 0$$
$$\begin{cases} if \left\| slope_{i-1} \right\| < \left\| slope_i \right\| \Rightarrow name = 41 \\ if \left\| slope_{i-1} \right\| > \left\| slope_i \right\| \Rightarrow name = 31 \end{cases}$$

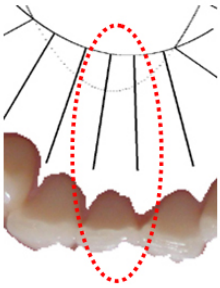


Fig. 3.3.3 Two slopes of a tooth in the upper view on the mandible