國 立 交 通 大 學

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中介變項統計分析

Statistical Mediation Analysis

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中 華 民 國 一 百 零 三 年 六 月

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摘要

我們觀察到，傳統中介變項的分析中的直接影響力和中介質有關，而我們認為的直接影響力是不經由中介質所產生的。而後，我們提出了一個新的中介變相分析，此分析中的影響力可分解出直接影響與間接影響，其中新的直接影響是不透過中介值的。我們新的假設檢定力相較於傳統 Baron 和 Kenny 的假設檢定力是有明顯改善。最後我們將中介變項的分析和交互作用做聯結。

關鍵字： 中介變項；中介值；分解效應；交互作用。

Statistical Mediation Analysis

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Abstract

We observe that the classical mediation analysis gives the direct effect involving the inputs of mediated variable leading that the direct effect and indirect effect do not serve their roles appropriately. We then propose a new mediation analysis with interaction identification for determining a clean direct effect for defining the total effect and then the effect decomposition. Power comparison between the Baron and Kenny's test and a new test based on our approach for indirect effect detection has been done and their corresponding efficiencies for detection are displayed. A new indirect effect proportion then is proposed for further investigation.

Key words: Mediation; Mediator; Effect Decomposition; Interaction.

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Abstract

We observe that the classical mediation analysis gives the direct effect involving the inputs of mediated variable leading that the direct effect and indirect effect do not serve their roles appropriately. We then propose a new mediation analysis with an interaction identification for determining a clean direct effect for defining the total effect and then the effect decomposition. Power comparison between the Baron and Kenny's test and a new test based on our approach for indirect effect detection has been done and their corresponding efficiencies for detection are displayed. A new indirect effect proportion then is proposed for further investigation.

1. Introduction

Since Woodworth (1928), effect decomposition (mediation analysis), with decompose the total effect of an exposure (independent) variable on the response (effect) variable into the effect that go directly (direct effect) and the effect that is influenced by a mediator variable (indirect effect), has been extensively studied and used in psychological science for more than 80 years. It is now also very popular in social science (Geneletti (2007)) and fast growing in medical and epidemiology studies relevant to the design of clinical and public health interventions (Laan and Petersen (2008) and Richiardi, Bellocco and Zugna (2013)).

In many studies of estimating and testing the mediation effects, Baron and Kenny (1986) proposed the causal steps regression approach that requires conditions for establishing mediation. Because of its simplicity for understanding and implementing, this approach is very influential and widely used. Once these conditions are satisfied, it needs to quantify the indirect effect to be tested for significance (Sobel (1982)). There are criticisms for this traditional approach such as bias effect estimates and without natural extension to non-linear models are observed (Robins and Greenland (1992) and Richiardi, Bellocco and Zugna (2013)). In a Monte Caro simulation by MacKinnon et al. (2002), it is observed that the condition requiring that response and exposure variables has to be correlated is not correct indicating that this approach may miss some true mediation effects. This problem has also been tackled extensively in the causal inference literature by using counterfactual framework (Robins and Greenland (1992), Pearl (2001), Robins (2003) and van der Lann and Petersen (2008)). This approach is also debatable for that it involves many untestable assumptions (Geneletti (2007)).

Although considerable effects has been devoted to the possibility of undesired behavior in statistical inference for Baron and Kenny's conditions and induced inferences methods for indirect effect detection, there is no satisfactory alternative mediation analysis technique for use. From our view, the disadvantages are resulted from the fact that we have not known enough to the unknown mechanism of cuasual relationship for creation of these effects. We propose a parametric study of mediation analysis by introducing the underlying distribution of involved random variables into the regression framework, not been treated in this field, that allows us to structure analysis in two parts. First, for this traditional mediation analysis, the correct conditions for presence of indirect effect can be drawn theoretically. We observe that significance for correlation between exposure and mediator variables is the only condition to be satisfied when the underlying distribution is true. This indicates that conditions for this presence must be case by case and conservative approach of maximizing the number of conditions in the classical one is not surprised in sacrificing its power performance. We then propose a corrected version of Baron and Kenny's approach for effect decomposition and inference methods development. As a consequence of parametric study, the unknown total, direct and indirect effects can be estimated with best asymptotically normal estimators and tests for statistical hypotheses of these effects can be developed with the derived asymptotic distributions.

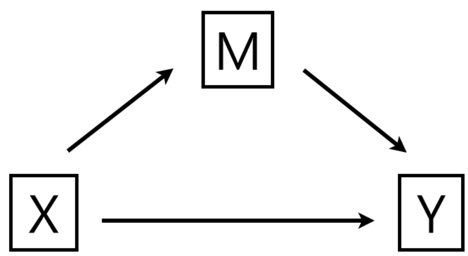
Second and most importantly, the traditional approach interprets the exposure coefficient for (multiple) regression model with conditioning on values of exposure and mediator variables as a direct effect. But this parametric approach shows that the size of this exposure coefficient is dependent on the true values of the distributional parameters of mediator variable indicating that statistical mechanism for mediation not only through variable's given values but also its parameters. This traditionally unaware unclean direct effect then induce the indirect effect also unclean. We then develop a refined version of Baron and Kenny's approach for developing conditions for clean indirect effect and use them to construct inference methods for this indirect effect.

2. Statistical Theory for Classical Effect Decomposition Methods

2.1. Verification of A New Baron and Kenny's Conditions

Suppose that we have response (effect) variable Y, exposure (independent) variable X and mediator (intermediate) variable M.

The paths that exposure variable X and mediation variable M affect the response variable Y can be described in the following figure.



Effect decomposition (mediation analysis) involves to identify the total effect of X on Y, the part of total effect because X influences M which in turn influences Y (indirect effect) and the effect of X unexplained by this variables M (direct effect). The direct and indirect effect together form the total effect of X on Y. The approach of Baron and Kenny (1986) for mediation analysis is most widely-used that considers a series of tests for regression coefficients of all paths (regression models) for inferencing the existence of indirect effect that is summarized by Howell (2009) as follows:

Step 1. Test hypothesis $H_{0a} : \beta_{1a} = 0$ vs $H_{1a} : \beta_{1a} \neq 0$ for significance of the simple linear regression model

$$y(x) = \beta_{0a} + \beta_{1a} x + \epsilon_a \tag{2.1}$$

describing the path $(X \to Y)$ requiring H_{0a} to be rejected to confirm that X is a significant predictor of the response variable Y.

Step 2. Test hypothesis $H_{0b} : \beta_{1b} = 0$ vs $H_{1b} : \beta_{1b} \neq 0$ for significance of the simple linear regression model

$$m(x) = \beta_{0b} + \beta_{1b}x + \epsilon_b. \tag{2.2}$$

describing the path $(X \to M \to Y)$ requiring H_{0b} to be rejected to confirm that X is a significant predictor of the mediator M.
Step 3. Consider the following multiple linear regression model

$$y(x, m) = \beta_{0c} + \beta_{1c}x + \beta_{2c}m + \epsilon_c. \tag{2.3}$$

Performing the test for hypothesis $H_{0c} : \beta_{2c} = 0$ vs $H_{1c} : \beta_{2c} \neq 0$ requiring H_{0c} to be rejected to confirm that the partial effect of M must be significant.

The effect relationships among variables following this series of tests may be explained in the followings (Howell (2009) and Hayes (2009)):
(a) If it shows significant evidence to reject H_{0a} in step 1, it defines β_{1a} as total effect for possible decomposition into direct and indirect effects.
(b) If one or more hypothesis in steps 1 - 3 are not rejected, researchers usually conclude that indirect effect does not exist.
(c) If three hypothesis in steps 1 - 3 are rejected, indirect effect exists. If hypothesis $H_{0c} : \beta_{1c} = 0$ vs $H_{1c} : \beta_{1c} \neq 0$ is not rejected, there is complete mediation and if it is rejected, there is partial mediation.

We consider here the problem that how many conditions is required for presence of indirect effect. With decision error generated when a hypothesis is tested, the more tests in order to claim an indirect effect, the more errors to be generated. This situation of lower power (Fritz and MacKinnon (2007) and MacKinnon et al. (2002)) could be even worse when number of exposures or mediators increases. Our concern is correct since as observed by MacKinnon et al. (2002) with Monte Carol simulation that the condition that Y and X has to be correlated is not correct.

Within the framework of series of regression models (2.1)-(2.3), it is seen the following coefficient decomposition

$$\beta_{1a} = \beta_{1c} + \beta_{1b}\beta_{2c} \tag{2.4}$$

holds when the underlying distribution of variables Y, X and M are jointly normal. Then, following the path analysis, usage of models leads to the following effects identification (decomposition):

$$\begin{aligned} &\text{Total effect: } T_{BK} = \beta_{1c} + \beta_{1b}\beta_{2c} \\ &\text{Direct effect: } D_{BK} = \beta_{1c} \\ &\text{Indirect effect: } ID_{BK} = \beta_{1b}\beta_{2c} \end{aligned} \tag{2.5}$$

satisfying that the sum of direct effect and indirect effect is equal to the total effect.

Now suppose that Y, X and M has a joint normal distribution as

$$\begin{pmatrix} Y \\ X \\ M \end{pmatrix} \sim N_3(\begin{pmatrix} \mu_y \\ \mu_x \\ \mu_m \end{pmatrix}, \begin{pmatrix} \sigma_y^2 & \sigma_{yx} & \sigma_{ym} \\ \sigma_{xy} & \sigma_x^2 & \sigma_{xm} \\ \sigma_{my} & \sigma_{mx} & \sigma_m^2 \end{pmatrix}). \tag{2.6}$$

for verification of Baron and Kenny's conditions. Denote

$$\beta_0(\theta) = \mu_y - \frac{(\sigma_{yx}\sigma_m^2 - \sigma_{ym}\sigma_{xm})\mu_x}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2} + \frac{(\sigma_{yx}\sigma_{xm} - \sigma_{ym}\sigma_x^2)\mu_m}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2}$$

$$\beta_1(\theta) = \frac{\sigma_{yx}\sigma_m^2 - \sigma_{ym}\sigma_{xm}}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2}, \ \beta_2(\theta) = \frac{\sigma_{ym}\sigma_x^2 - \sigma_{yx}\sigma_{xm}}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2}$$

where $\theta = (\mu_y, \mu_x, \mu_m, \sigma_y^2, \sigma_x^2, \sigma_m^2, \sigma_{yx}, \sigma_{ym}, \sigma_{xm})$.

The following theorem gives parametrized regression parameters of models (2.1)-(2.3).

Theorem 2.1. Suppose that the underlying distribution is normal.
(a) The regression parameters for regression function of Y given $X = x$ of (2.1) includes

$$\beta_{0a} = \mu_y - \frac{\sigma_{yx}}{\sigma_x^2}\mu_x, \ \beta_{1a} = \frac{\sigma_{yx}}{\sigma_x^2}$$

and $\epsilon_a \sim N(0, \sigma_a^2)$ where $\sigma_a^2 = \sigma_y^2 - \frac{\sigma_{yx}^2}{\sigma_x^2}$.
(b) The regression parameters for regression function of M given $X = x$ of (2.2) includes

$$\beta_{0b} = \mu_m - \frac{\sigma_{mx}}{\sigma_x^2}\mu_x, \ \beta_{1b} = \frac{\sigma_{mx}}{\sigma_x^2}$$

and $\epsilon_b \sim N(0, \sigma_b^2)$ where $\sigma_b^2 = \sigma_m^2 - \frac{\sigma_{mx}^2}{\sigma_x^2}$.
(c) The regression function for Y given $X = x$ and $M = m$ is

$$\mu(x, m; \theta) = \beta_0(\theta) + \beta_1(\theta)x + \beta_2(\theta)m \tag{2.7}$$

Hence $\beta_{0c} = \beta_0(\theta), \beta_{1c} = \beta_1(\theta), \beta_{2c} = \beta_2(\theta)$. and $\epsilon_c \sim N(0, \sigma_c^2)$ where $\sigma_c^2 = \sigma_y^2 - (\sigma_{yx}, \sigma_{ym}) \begin{pmatrix} \sigma_x^2 & \sigma_{xm} \\ \sigma_{mx} & \sigma_m^2 \end{pmatrix}^{-1} \begin{pmatrix} \sigma_{yx} \\ \sigma_{ym} \end{pmatrix}$.
Proof. The result in (c) are induced from Chen et al. (2013) and the others are trivial. □

We now are ready to give a theoretical verification of Baron and Kenny's conditions for existence of indirect effect and mediation.

Theorem 2.2. Suppose that the underlying distribution is normal. The effects can be decomposed following the Baron and Kenny (1986)'s approach as

$$\begin{aligned}
&\text{Total effect } (\beta_{1c} + \beta_{1b}\beta_{2c}) : \\
&\quad T_{BK} = \frac{(\sigma_{ym}\sigma_x^2 - \sigma_{yx}\sigma_{xm})}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2}\frac{\sigma_{xm}}{\sigma_x^2} + \frac{(\sigma_{yx}\sigma_m^2 - \sigma_{ym}\sigma_{xm})}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2} \\
&\qquad = \frac{\sigma_{yx}}{\sigma_x^2} \\
&\text{Direct effect } (\beta_{1c}) : D_{BK} = \frac{\sigma_{yx}\sigma_m^2 - \sigma_{ym}\sigma_{xm}}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2} \\
&\text{Indirect effect } (\beta_{1b}\beta_{2c}) : ID_{BK} = \frac{(\sigma_{ym}\sigma_x^2 - \sigma_{yx}\sigma_{xm})}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2}\frac{\sigma_{xm}}{\sigma_x^2}
\end{aligned} \tag{2.8}$$

We first examine the Baron and Kenny's condition for presence of indirect effect.

Theorem 2.3. The indirect effect under the Baron and Kenny's conditions are:

$$ID_{BK} = \begin{cases} \frac{\sigma_{ym}\sigma_{xm}}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2} & \text{if } \sigma_{yx} = 0 \\ 0 & \text{if } \sigma_{xm} = 0 \\ \frac{\sigma_{yx}\sigma_m^2}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2} & \text{if } \sigma_{ym} = 0 \end{cases}$$

Proof. It is straight forward but careful re-arrangements. □

This shows that the existence of partial mediation or complete mediation (Howell (2009)) does not require all hypothesis in three steps to be rejected.

This also confirms the observation of MacKinnon et al. (2002) that Y and X are not necessary to be associated but in addition that Y and M are not necessary for presence of indirect effect. Combining the results in (a) and (c), a new Baron and Kenny's rule for indirect effect identification is:

$$\textbf{Indirect effect } ID_{BK}\textbf{exists if } H_{0b} \textbf{ for model (2.2) is rejected}. \quad (2.9)$$

We consider $(\mu_y, \mu_x, \mu_m) = (2, 3, 3)$ and $\sigma_y^2 = \sigma_x^2 = \sigma_m^2 = 2$ to conduct a Monte Carlo simulation to evaluate the powers of the classical Baron and Kenny's three conditions test and the above new test for claiming an indirect effect. The simulated results are displayed in Table 1.

Table 1. Power performance for refined and classical indirect effect detection

σ_{xm}	$\sigma_{yx} = \sigma_{ym}$	B-K	Revised B-K
$\sigma_{xm} = 0$	0	0.0480	0.050
$\sigma_{xm} = 0.2$	0.3	0.049	0.081
	0.7	0.060	0.081
	0.9	0.076	0.081
$\sigma_{xm} = 0.8$	0.3	0.055	0.611
	0.7	0.127	0.611
	0.9	0.250	0.611
$\sigma_{xm} = 1.0$	0.3	0.055	0.825
	0.7	0.131	0.827
	0.9	0.256	0.826
$\sigma_{xm} = 1.2$	0.3	0.055	0.955
	0.7	0.122	0.955
	0.9	0.228	0.955

From our investigation, expressing effect decomposition in terms of distributional parameters is desired for each specific underlying distribution for developing correct conditions for improving power performance for claiming an indirect effect.

When both direct and indirect effects are identified, a measure of the proportion mediated is sometimes calculated as the ratio of the indirect effect to the total effect (Ditlevsen, et al. (2005) and Hafeman (2009)). This measure in some sense captures how important the pathway through the intermediate is in explaining the actual operation of the effect of the exposure on the outcome. This implicitly assumed that all effects are positive values. In the following table, we present the total and indirect effect when the underlying distribution is normal.

Table 2. Total and indirect effect

$(\sigma_{ym}, \sigma_{yx})$	T_{BK}	ID_{BK}
$\sigma_{xm} = 0.2$		
$(0.6, 0.8)$	0.4	0.026
$(0.8, 0.2)$	0.1	0.039
$(0.8, 0.4)$	0.2	0.038
$(0.8, 0.8)$	0.4	0.036
$\sigma_{xm} = 0.6$		
$(0.8, 0.2)$	0.1	0.122
$(0.8, 0.4)$	0.2	0.112
$(0.8, 0.6)$	0.3	0.102
$\sigma_{xm} = 0.8$		
$(0.2, 0.6)$	0.3	-0.010
$(0.2, 0.8)$	0.4	-0.029
$(0.6, 0.2)$	0.1	0.124
$(0.8, 0.2)$	0.1	0.171
$\sigma_{xm} = -0.2$		
$(0.6, 0.6)$	0.3	-0.033
$(0.6, 0.8)$	0.4	-0.034
$(0.8, 0.6)$	0.3	-0.043
$(0.8, 0.8)$	0.4	-0.044
$\sigma_{xm} = -0.8$		
$(0.2, 0.2)$	0.1	-0.067
$(0.4, 0.4)$	0.2	-0.133
$(0.6, 0.6)$	0.3	-0.200
$(0.8, 0.8)$	0.4	-0.267

We have two comments:

(a) Indirect effect could be negative value when direct effect is larger than the total effect. Then the ratio of indirect effect is also negative when β_{1b} is small or direct effect is large enough.

(b) Indirect effect could have value larger than the total effect such that the ratio of indirect effect is larger than one when $\beta_{1c} < 0$.

We often call the presence of mediation effect (Fairchild and MacKinnon

(2009)) if there is nonzero indirect effect. It is seen that it is not always dangerous for presence of mediation effect, depending on its sign.

Definition 2.4. We say that there is synergistic mediation effect if $ID_{BK} > 0$ and antagonistic mediation effect if $ID_{BK} < 0$.

2.2. Statistical Properties for Estimators of Baron and Kenny's Effects

Effects of (2.5) are generally estimated by least squares estimators of the corresponding parameters and hypothesis testing for effect parameters are done by assuming that the parameter estimators are normally distributed to develop the scale estimator of the effect estimator. For example, to deal with hypothesis of presence of indirect effect as $H_0 : \beta_{1b}\beta_{2c} = 0$, researchers have done (Sobel (1982), Aroian (1944) and Goodman (1960)) to develop the asymptotic distribution of the product of two normal distributions of least squares estimators of β_{1b} and β_{2c}. With our approach scale estimator of any effect estimator or their functions are much easier to develop.

We denote the sample means $(\bar{y}, \bar{x}, \bar{m})' = \frac{1}{n}\sum_{i=1}^{n}(y_i, x_i, m_i)'$ and the sample covariance matrix $\begin{pmatrix} s_y^2 & s_{yx} & s_{ym} \\ s_{xy} & s_x^2 & s_{xm} \\ s_{my} & s_{mx} & s_m^2 \end{pmatrix} = \frac{1}{n-1}\sum_{i=1}^{n}\begin{pmatrix} y_i - \bar{y} \\ x_i - \bar{x} \\ m_i - \bar{m} \end{pmatrix}\begin{pmatrix} y_i - \bar{y} \\ x_i - \bar{x} \\ m_i - \bar{m} \end{pmatrix}'$. Let $\hat{\theta}$ be the maximum likelihood estimator of parameters θ. The maximum likelihood estimators of direct and indirect effect are, respectively,

$$\hat{D}_{BK,mle} = \frac{s_{yx}s_m^2 - s_{ym}s_{xm}}{s_x^2 s_m^2 - s_{xm}^2}$$

$$\hat{ID}_{BK,mle} = \frac{(s_{ym}s_x^2 - s_{yx}s_{xm})}{s_x^2 s_m^2 - s_{xm}^2}\frac{s_{xm}}{s_x^2}$$

The following theorem states the asymptotic distributional theory for the maximum likelihood estimators.

Theorem 2.5. (a) We have $n^{1/2}(\hat{D}_{BK,mle} - D_{BK})$ convergent in distribution to a normal distribution $N(0, \Sigma_d)$ with asymptotic covariance matrix $\Sigma_d = \frac{\partial D_{BK}}{\partial \theta'} V_\theta \frac{\partial D'_{BK}}{\partial \theta}$, where $V_\theta = -[E(\partial^2 log\phi_N(X,Y)/\partial\theta\partial\theta')]^{-1}$ is the Cramer-Rao's lower bound for θ and $\phi_N(Y, X, M)$ is the probability density function of the normal distribution for Y, X and M. Hence $\hat{D}_{BK,mle}(x)$ forms a best asymptotically normal estimator of unknown $D_{BK}(x)$.

(b) We have $n^{1/2}(I\hat{D}_{BK,mle} - ID_{BK})$ convergent in distribution to a normal distribution $N(0, \Sigma_{id})$ with asymptotic covariance matrix $\Sigma_{id} = \frac{\partial ID_{BK}}{\partial \theta'} V_\theta \frac{\partial ID'_{BK}}{\partial \theta}$. Hence $I\hat{D}_{BK,mle}$ forms a best asymptotically normal estimator of unknown ID_{BK}.

Optimal properties of the least squares estimators for direct and indirect effects are implied from the following theorem when normality assumption holds.

Theorem 2.6. Let $\hat{\beta}_{0b}$ and $\hat{\beta}_{1b}$ be the least squares estimators of β_{0b} and β_{1b}. Then, under the normality assumption, $\hat{D}_{BK} = \hat{\beta}_{1c}$ and $I\hat{D}_{BK} = \hat{\beta}_{1b}\hat{\beta}_{2c}$ and then they are also best asymptotically normal.

Proof. The least squares estimators $\{\hat{\beta}_{0c}, \hat{\beta}_{1c}, \hat{\beta}_{2c}\}$ for regression model of (2.3) are also maximum likelihood estimators for this regression model summing that ϵ_c is normal of zero mean and constant variance. On the other hand, the maximum likelihood estimator $\hat{\theta}_{mle}$ for distribution of Y, X and M in (2.6) makes $\{\hat{\beta}_0(\theta), \hat{\beta}_1(\theta), \hat{\beta}_2(\theta)\} = \{\beta_0(\hat{\theta}_{mle}), \beta_1(\hat{\theta}_{mle}), \beta_2(\hat{\theta}_{mle})\}$ the maximum likelihood estimator of $\{\beta_0(\theta), \beta_1(\theta), \beta_2(\theta)\}$ for regression model (2.7) that has a normal error variable. This indicates that $\{\hat{\beta}_0(\theta), \hat{\beta}_1(\theta), \hat{\beta}_2(\theta)\}$ and $\{\hat{\beta}_{0c}, \hat{\beta}_{1c}, \hat{\beta}_{2c}\}$ are identical which further implies that $\hat{\beta}_{1c} = \hat{\beta}_1(\theta)$ (direct effect) and $\hat{\beta}_{2c} = \hat{\beta}_2(\theta)$. Analogous discussion for model of M given $X = x$ can be done to finish the theorem. □

We have several comments from the above theorem:

(a) The least squares estimator of the product of coefficients to be best asymptotically holds only occasionally. If the distribution of underlying distribution of variables involved is no-longer normal, the theory may be different. However, the optimality properties always hold for $\hat{D}_{BK}$ and $I\hat{D}_{BK}$ if they are derived from the process stated in this paper for any underlying distribution.

(b) The test statistics based on least squares estimators of coefficients such as the commonly used one of Sobel (1982) and some others as Aroian (1944) and Goodman (1960) all assume that this product of least squares estimators is asymptotically normal. However our theory verified that this is certain when the underlying distribution is normal but not certain for other situations.

Let $\hat{\Sigma}_{id}$ be the maximum likelihood estimator of Σ_{id}. A test for hypothesis

$H_0 : ID_{BK}(x) = 0$ is:

$$\text{rejecting } H_0 \text{ if } \frac{|n^{1/2}\hat{ID}_{BK}|}{\sqrt{\hat{\Sigma}_{id}}} \geq t_\alpha$$

$$\gamma = \frac{ID_{BK}}{T_{BK}}$$

3. A New Mediation Analysis

Is the classical specification of total, direct and indirect effects statistical appropriate? When the variables Y, X and M follows the normal distribution (2.6), the direct effect and total effect are respectively as

$$D_{BK} = \frac{\sigma_{yx}\sigma_m^2 - \sigma_{ym}\sigma_{xm}}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2} \text{ and } T_{BK} = \frac{\sigma_{yx}}{\sigma_x^2} \tag{3.1}$$

If there is a unit change in X, it is expected that the direct effect measures only the direct (no-mediation) impact on response Y, unfortunately, the magnitude of this impact D_{BK} involves distribution parameters $\{\sigma_{ym}, \sigma_{xm}, \sigma_m^2\}$ related to mediator M indicating that this effect is mixed with effect of association between X and M. On the other hand, the total effect is supposed to contain effect of X mediated and not-mediated by variable M. Unfortunately this magnitude influenced by X T_{BK} dose not involve distributional parameters related to mediator M indicating absence of mediation. This classical effect specification does not give precise direct and indirect effects. This also indicates that the commonly used test statistics of Sobel (1982), Aroian (1944) and Goodman (1960) and many others may lead to in-correct conclusion for indirect effect. Beneficial from parametrization of regression function, uncontroversial specification of indirect effect can be specified with correct derivation of statistical interaction.

We assume that Y, X and M have a joint distribution with probability density function $f(y, x, m, \theta)$ where θ is vector of unknown parameters. We further assume that the parameter vector θ my be partitioned based on association between variables into no-association vectors $\{\theta_y, \theta_x, \theta_m\}$, two-variables association vectors $\{\theta_{yx}, \theta_{ym}, \theta_{xm}\}$ and three-variables association vector $\{\theta_{yxm}\}$ where $\theta_{z_j:j\in A}$ is set of association parameters involved all variables $Z_j, j \in A$. This criterion for parameters partition work for most interesting multivariate distributions. For examples, if the joint distribution of Y, X and M is multivariate normal or multivariate t distribution, the parameter vector θ is vector of population mean vector $\begin{pmatrix} \mu_y \\ \mu_x \\ \mu_m \end{pmatrix}$ and covariance matrix $\begin{pmatrix} \sigma_y^2 & \sigma_{yx} & \sigma_{ym} \\ \sigma_{xy} & \sigma_x^2 & \sigma_{xm} \\ \sigma_{my} & \sigma_{mx} & \sigma_m^2 \end{pmatrix}$. Then we have the no-association parameter vectors $\{\theta_y, \theta_x, \theta_m\} = \{\begin{pmatrix} \mu_y \\ \sigma_y^2 \end{pmatrix}, \begin{pmatrix} \mu_x \\ \sigma_x^2 \end{pmatrix}, \begin{pmatrix} \mu_m \\ \sigma_m^2 \end{pmatrix}\}$, two-variables association parameter vectors $\{\theta_{yx}, \theta_{ym}, \theta_{xm}\} = \{\{\sigma_{yx}\}, \{\sigma_{ym}\}, \{\sigma_{xm}\}\}$ and empty set for three-variables association parameters.

Let $\mu(x, m; \theta)$ be the conditional mean of Y given $X = x$ and $M = m$. We consider additivity of control values $X = x$ and $M = m$ for specification of total effects of explanatory and mediation variables.

Definition 3.1 We say that regression function is total effect decomposable if the regression function is

$$\mu(x, m; \theta) = g(\theta) + T_x(\theta)x + T_m(\theta)m. \tag{3.2}$$

In this case, we say that $T_x(\theta)$ and $T_m(\theta)$ are, respectively, the total effects of exposure and mediator variables.

A common feature of decomposition in linear model of effects contributed by explanatory variables is that their combined effect (sum of separated effects) has to be the conditional mean of response variable Y. For example, the group mean in analysis of variance is the sum of main effects and interactions and, in multiple linear regression model, the regression function is the sum of univariate terms x_1 and x_2 (for main effects) and product term x_1x_2 (for interaction). The specification of total effects in (3.2) does confirm this expectation. But the Baron and Kenny's framework does not indicates this expectation.

If the total effect $T_x(\theta)$ involves distributional parameters of mediator M, it contains effect mediated by M requiring for disentangling the interrelationships between variables from total effect decomposable regression function of (3.2). Denoting parameter sets $\theta_{yx} = \{\theta_x, \theta_{yx}\}$ and $\theta_{ym} = \{\theta_m, \theta_{ym}\}$, obviously θ_{yx} and θ_{ym} respectively are sets of distributional parameters that are considered with contribution of X on Y and of M on Y, not mediated by other variables.

An application for interaction identification in Chen et al. (2013) is to used for direct and indirect effects identification.

Definition 3.2. (a)Suppose that there exists a minimal function of parameters $G(\theta)$, denoting its dimension as c, such that the regression function given $G(\theta) = 0_c$ can be decomposed as

$$\mu(x, m, \theta|G(\theta) = 0_c) = g(\theta) + D_x(\theta_{yx})x + D_m(\theta_{ym})m. \tag{3.3}$$

We say that $\mu(x, m, \theta|G(\theta) = 0_c)$ is the no-interaction regression function and $G(\theta)$ is the intercorrelation parameter set. We call $D_x(\theta_{yx})$ and $D_m(\theta_{ym})$, respectively, the direct effects of exposure and mediator variables.
(b) We call $ID_x = T_x - D_x(\theta_{yx})$ and $ID_m = T_m - D_m(\theta_{ym})$ the indirect effects of X and M.
(c) We say that there is no mediation effect if $T_x = D_x(\theta_{yx})$ leading to $ID_x = 0$.

The intercorrelation parameter set contributes the indirect effects of exposure X and mediator M. But ID_x is pure indirect effect of exposure X.

Now, consider the normal distribution for variables Y, X and M of (2.6).

Theorem 3.3. (a) Under the normality assumption, we have the total effects as

$$T_x = \frac{\sigma_{yx}\sigma_m^2 - \sigma_{ym}\sigma_{xm}}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2} \text{ and } T_m = \frac{\sigma_{ym}\sigma_x^2 - \sigma_{yx}\sigma_{xm}}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2}$$

Renoting $T_x = T(\theta)$, hence, T_x is the total effect.
(b) The ineraction parameter is $G(\theta) = \{\sigma_{xm}\}$ and the induced regression function is

$$\mu(x, m, ; \theta) = \mu_y + \frac{\sigma_{yx}}{\sigma_x^2}(x - \mu_x) + \frac{\sigma_{ym}}{\sigma_m^2}(m - \mu_m)$$

indicating that the direct effect is

$$D_x = \frac{\sigma_{yx}}{\sigma_x^2} \text{ and } D_m = \frac{\sigma_{ym}}{\sigma_m^2} \tag{3.4}$$

which is a function of θ_{yx} only and the indirect effects are $ID_x = T_x - D_x$ with

$$ID_x = \frac{\sigma_{yx}\sigma_m^2 - \sigma_{ym}\sigma_{xm}}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2} - \frac{\sigma_{yx}}{\sigma_x^2} \text{ and } ID_m = \frac{\sigma_{ym}\sigma_x^2 - \sigma_{yx}\sigma_{xm}}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2} - \frac{\sigma_{ym}}{\sigma_m^2}$$

Proof. The conditional mean of M given $X = x$ is $\mu(x, \theta) = \mu_m + \frac{\sigma_{xm}}{\sigma_x^2}(x - \mu_x)$ that gives (a). □

This direct effect $D_x = \sigma_{yx}/\sigma_x^2$ is identical to β_{1a} of (2.1) that makes sense for it measures the effect of X on Y in the environment that mediator is not involved. Similarly, $D_m = \sigma_{ym}/\sigma_m^2$ is identical to the regression coefficient β_{1d} in model $y = \beta_{0d} + \beta_{1d}m + \epsilon_c$ that measures the effect of mediator M on Y in the environment that X is not involved.

We denote the total effect as a ratio between the new total effect and classical total effect in the following

$$r_t = \frac{T_{BK}}{T_x}.$$

Table 3. Total effect ratio and indirect effects of new decompositions

$(\sigma_{ym}, \sigma_{yx})$	r_t	ID_x
$\sigma_{xm} = 0.6$		
$(0.4, 0.4)$	0.765	-0.046
$(0.4, 0.6)$	0.87	-0.036
$(0.4, 0.8)$	0.932	-0.026
$(0.6, 0.2)$	0.1	-0.089
$(0.6, 0.4)$	0.6	-0.079
$(0.6, 0.6)$	0.76	-0.069
$\sigma_{xm} = 0.8$		
$(0.6, 0.4)$	0.45	-0.104
$(0.6, 0.6)$	0.7	-0.085
$(0.6, 0.8)$	0.832	-0.066
$(0.8, 0.2)$	0.235	-0.171
$(0.8, 0.6)$	0.553	-0.133

The total, direct and indirect effects adjusted with interaction are generally varying in the underlying distribution. The derived indirect effect allows us to establish new Baron and Kenny's conditions of mediation analysis for identification of clean indirect effect.

Theorem 3.4. Baron and Kenny's conditions are true for $\sigma_{yx} = 0$ and $\sigma_{xm} = 0$. In specific, we have the followings:

(a) When $\sigma_{yx} = 0$ we have $ID_x = -\frac{\sigma_{ym}\sigma_{xm}}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2}$.

(b) When $\sigma_{xm} = 0$ we have $ID_x = 0$.

(c) When $\sigma_{ym} = 0$ we have

$$ID_x = \sigma_{yx}(\frac{\sigma_m^2}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2} - \frac{1}{\sigma_x^2})$$

This study gives a refined two steps tests of Baron and Kenny's approach to identify the clean indirect effect as follows:

$$\textbf{If hypotheses for model (2.2) is rejected, indirect effect exists.} \quad (3.5)$$

This refined direct and indirect effects are no longer equal to β_{1c} and $\beta_{1b}\beta_{2c}$, respectively. Hence, their asymptotic distributions are varied. The maximum likelihood estimators of direct and indirect effect may be defined as:

$$\hat{D}_x = \frac{s_{yx}}{s_x^2}$$

$$I\hat{D}_x = s_{yx}(\frac{s_m^2}{s_x^2 s_m^2 - s_{xm}^2} - \frac{1}{s_x^2})$$

Theorem 3.5. (a) We have $n^{1/2}(\hat{D}_x - D_x)$ convergent in distribution to a normal distribution $N(0, \Sigma_d)$ with asymptotic covariance matrix $\Sigma_d = \frac{\partial D_x}{\partial \theta'} V_\theta \frac{\partial D_x}{\partial \theta}$, where $V_\theta = -[E(\partial^2 log\phi_N(X, Y)/\partial\theta\partial\theta')]^{-1}$ is the Cramer-Rao's lower bound for θ and $\phi_N(Y, X, M)$ is the probability density function of the normal distribution for Y, X and M. Hence $\hat{D}_x$ forms a best asymptotically normal estimator of unknown D_x.

(b) We have $n^{1/2}(I\hat{D}_x - ID_x)$ convergent in distribution to a normal distribution $N(0, \Sigma_{id})$ with asymptotic covariance matrix $\Sigma_{id} = \frac{\partial ID_x}{\partial \theta'} V_\theta \frac{\partial ID_x}{\partial \theta}$. Hence $I\hat{D}_x$ forms a best asymptotically normal estimator of unknown ID_x.

Let $\hat{\Sigma}_{id}$ be the maximum likelihood estimator of Σ_{id}. A new test for presence of indirect effect based on estimator $I\hat{D}_x$ is as follows;

$$\text{rejecting } H_0 \text{ if } \sqrt{n}|I\hat{D}_x|/\sqrt{\hat{\Sigma}_{id}} \geq t \quad (3.6)$$

where t is the threshold assuring the size of the test is the significance level α.

We define the efficiencies of the test based on Baron and Kerry's condition and new test as

$$Eff_{BK} = \frac{\text{Power for B-K test}}{\max\{\text{Powers for B-K test and new test}\}},$$

$$Eff_{New} = \frac{\text{Power for new test}}{\max\{\text{Powers for B-K test and new test}\}}.$$

In the following, we display the efficiencies with $\alpha = 0.05$.

Table 4. Power performance for B-K condition test and test for presence of clean indirect effect

σ_{xm}	$\sigma_{yx} = \sigma_{ym}$	Eff_{BK}	Eff_{New}
$\sigma_{xm} = -0.5$	0.3	0.283	1
	0.5	0.052	1
	0.7	0.049	1
	0.9	0.269	1
$\sigma_{xm} = -1$	0.3	0.953	1
	0.5	0.874	1
	0.7	0.846	1
	0.9	0.835	1
$\sigma_{xm} = 0.5$	0.3	0.493	1
	0.5	0.377	1
	0.7	0.335	1
	0.9	0.311	1
$\sigma_{xm} = 1$	0.3	1	0.174
	0.5	1	0.320
	0.7	1	0.440
	0.9	1	0.549

Verify the situations that the indirect effect is negative showing that the direct effect needs not be smaller than the total effect.

4. Statistical Interaction Identification

It is a consensus that the statistical interaction represents the effect of inter-correlation between explanatory variables on the conditional mean of Y given the values of explanatory variables. Suppose that Y, X and M be respectively the response, explanatory and mediation variable. We denote $\mu(x, m; \theta)$ the conditional mean of Y given values $X = x$ and $M = m$. The direct effect, indirect effect and interaction effect must characterize are generally defined in the following way.

Definition 4.1. We consider X and M all explanatory variables.
(a) The direct effect of an explanatory variable is the change of mean function that is not mediated by the other one variable when this explanatory variable increases one unit and the other variable is held fixed.
(b) The indirect effect of an explanatory variable is the change of mean function that is purely mediated by other one variable when this variable increases one unit and the other variable is held fixed.
(c) The interaction effect is the the change of mean function when the explanatory variables X and M both increase one unit simultaneously.

How to measure these effects? The literature has a consistent treatment in a wider range of effect decomposition. Summarizing from the direct effect of Baron and Kenny (1986) and the second order interaction of Mullahy (1999) and Ai and Norton (2003), the direct effect and interaction are defined from derivatives of the mean function as follows:
(a) The direct effect of variable X: $\frac{\partial \mu}{\partial x}$.
(b) The direct effect of variable M: $\frac{\partial \mu}{\partial m}$.
(c) The interaction effect of variables X and M: $\frac{\partial^2 \mu}{\partial x \partial m}$.

Example 1. (a) Suppose that we have a regression model

$$y = \beta_{a0} + \beta_{a1} x + \beta_{a2} m + \epsilon_a.$$

We have the direct effects $D_x = \beta_{a1}$ and $D_m = \beta_{a2}$ and interaction effect $IA_{xm} = 0$.
(b) Suppose that we have a regression model

$$y = \beta_{b0} + \beta_{b1} x + \beta_{b2} m + \beta_{b3} xm + \epsilon_b.$$

We have the direct effects $D_x = \beta_{b1}$ and $D_m = \beta_{b2}$ and interaction effect $IA_{xm} = \beta_{b3}$.
(c) Suppose that we have a regression model

$$y = \beta_{c0} + \beta_{c1} x + \beta_{c2} x^2 + \beta_{c3} m + \beta_{c4} m^3 + \beta_{c5} xm + \beta_{c6} x^2 m^2 + \epsilon_c.$$

We have the direct effects $D_x = \beta_{c1} + 2\beta_{c2}x$ and $D_m = \beta_{c3} + 3\beta_{c4}m$ and interaction effect $IA_{xm} = \beta_{c5} + 4\beta_{c6}xm$.

The controversy of this classical way to characterize the interaction is that it is model dependent - various interaction to be specified when different models are applied.

Suppose now that Y, X and M have a joint normal distribution of (2.6). The regression model then is

$$y = \beta_0(\theta) + \beta_1(\theta)x + \beta_2(\theta)m + \epsilon \tag{4.1}$$

where

$$\beta_0(\theta) = \mu_y - \frac{(\sigma_{yx}\sigma_m^2 - \sigma_{ym}\sigma_{xm})\mu_x}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2} + \frac{(\sigma_{yx}\sigma_{xm} - \sigma_{ym}\sigma_x^2)\mu_m}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2}$$

$$\beta_1(\theta) = \frac{\sigma_{yx}\sigma_m^2 - \sigma_{ym}\sigma_{xm}}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2},\ \beta_2(\theta) = \frac{\sigma_{ym}\sigma_x^2 - \sigma_{yx}\sigma_{xm}}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2}$$

This is not model dependence since the regression model is solving determined by the analyzing distribution.

The classical effect decomposition method considers, the direct effects as $D_x = \beta_1(\theta)$ and $D_m = \beta_2(\theta)$ indicating that direct effect is not appropriate since variable X's direct effect involves the effect mediated by variable M due to the effect that involves M's parameters. Similarly variable M's direct effect involves the effect mediated by variable X.

Definition 4.2. The interaction effect of X and M is defined as the sum of indirect effects of X and M.

Theorem 4.3. When the normality assumption is made, the effects decomposition is:

$$g(\theta) = \mu_y$$

$$D_x(\theta_{yx}) = \frac{\sigma_{yx}}{\sigma_x^2},\ D_m(\theta_{ym}) = \frac{\sigma_{ym}}{\sigma_m^2}$$

$$ID_x(\theta) = \frac{(\sigma_{yx}\sigma_m^2 - \sigma_{ym}\sigma_{xm})}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2} - \frac{\sigma_{yx}}{\sigma_x^2}$$

$$ID_m(\theta) = \frac{(\sigma_{ym}\sigma_x^2 - \sigma_{y1}\sigma_{xm})}{\sigma_x^2\sigma_m^2 - \sigma_{xm}^2} - \frac{\sigma_{ym}}{\sigma_x^2}$$

Hence the size of interaction effect is $ID_x(\theta) + ID_m(\theta)$

Table 5. Full effect decomposition

Effect classified	Effect quantity
Combined effect	$\mu(x, m; \theta) = g(\theta) + T_x(\theta)x + T_m(\theta)m$
X's direct effect	$D_x(\theta_{yx})$
X's Total effect	$T_x(\theta)$
M's direct effect	$D_m(\theta_{ym})$
M's Total effect	$T_m(\theta)$
No Interaction	$\mu(x, m, \theta \| G(\theta) = 0_c) = g(\theta)$ $+D_x(\theta_{yx})x + D_m(\theta_{ym})m$
X's indirect effect	$ID_x = T_x(\theta) - D_x(\theta_{yx})$
M's indirect effect	$ID_m = T_m(\theta) - D_m(\theta_{ym})$
Interaction	$IA_{xm} = ID_x + ID_m$

The full decomposition model with two direct effects, two driving interactions and compounding interaction is formulated as

$$\begin{aligned} Y =& g(\theta) + D_x(\theta_{yx})x + D_m(\theta_{ym})m \\ &+ ID_x(\theta)x + ID_m(\theta)m + \epsilon \end{aligned} \tag{4.2}$$

where ϵ has a distribution with mean zero and variance $var(Y|X = x, M = m)$.

5. Concluding Remarks

In this paper, we have shown that the classical effect decomposition method is not statistically correct due to the fact that the derived direct effect involves the effect mediated by other variables. Our approach of effect decomposition not only provide clean effects but there effects can also be estimated this BAN estimation.

There are several related topics to be studied. First, a real data analysis to compare the direct and indirect effects of classical and new versions is desired. Second, the new concept of interaction is interesting but that requires further investigation.

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