From performance tests on 27 registered people, it is clear that most genuine signatures can be successfully verified and most forgeries can be pointed out. With suitable setting of the order of LPC cepstrum, verification threshold, the number of hidden nodes in MLP and the number of frames of a word in the signature, this verification scheme performs very well. In addition, because we logically equip each registered person with a number of single-output MLP's, the verification system can be expanded by simply equipping MLP's for each new customer and training these MLP's independently. This verification scheme thus possesses the merits of flexibility, scalability and system expansion.

Although the term "signature" is generally known in western countries to refer to a handwritten name written in an alphabetic script, there is no doubt that this term could also refer to handwritten names in character form, such as Chinese characters. Section IV uses Chinese signatures, which are often written in character-by-character form, for simulation. But, in fact, this work can easily be adapted to other types of signatures.

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Similarity Measures Between Vague Sets and Between Elements

Shyi-Ming Chen

***Abstract*—This paper presents some similarity measures between vague sets and between elements. An example is also presented to illustrate the application of the proposed similarity measures in handling behavior analysis problems. The proposed method can provide a useful way in handling the behavior analysis problems.**

I. INTRODUCTION

Since the theory of fuzzy sets [15] was proposed in 1965, many measures of similarity between fuzzy sets have been developed in the literature [2], [4], [6], [12], [13], [17]. In [2], we presented a similarity measure for weighted reasoning for medical diagnosis. In [4], we presented a method for calculating the degree of similarity between fuzzy sets for handling fuzzy decision-making problems. In [12], Leekwang *et al*. presented two similarity measures for behavior analysis. In [13], Pappis *et al*. made a comparative assessment of measures of similarity of fuzzy values. In [6], we extended the work of [13] to present and compare the properties of some similarity measures of fuzzy values. In [17], Zwick reviewed 19 similarity measures of fuzzy sets and compared their performance in an experiment.

Roughly speaking, a fuzzy set is a class with fuzzy boundaries. A fuzzy set A of the universe of discourse U, $U = \{u_1, u_2, \ldots, u_n\}$, can be represented by

$$A = \mu_A(u_1)/u_1 + \mu_A(u_2)/u_2 + \cdots + \mu_A(u_n)/u_n \tag{1}$$

where μ_A is the membership function of the fuzzy set A, $\mu_A\colon U \to [0,1]$, and $\mu_A(u_i)$ indicates the grade of membership of u_i in the fuzzy set A. When the universe of discourse U is an infinite set, then a fuzzy set A is often written in the form

$$A = \int_U \mu_A(u_i)/u_i, \quad \forall u_i \in U. \tag{2}$$

It is obvious that $\forall u_i \in U$, the membership value $\mu_A(u_i)$ is a single value between zero and one. In [7], Gau *et al*. pointed out that this single value combined the evidence for $u_i \in U$ and the evidence against $u_i \in U$, without indicating how much there is of each. Therefore, in [7], Gau *et al*. presented the concepts of vague sets, where the notion of vague set is similar to that of intuitionistic fuzzy sets [1]. They used a truth-membership function t_A and a false-membership function f_A to characterize the lower bound on μ_A. The lower bounds are used to create a subinterval on $[0,1]$, namely $[t_A(u_i), 1 - f_A(u_i)]$, to generalize the $\mu_A(u_i)$ of fuzzy sets, where $t_A(u_i) \le \mu_A(u_i) \le 1 - f_A(u_i)$. For example, if $[t_A(u_i), 1 - f_A(u_i)] = [0.5, 0.7]$, then we can see that $t_A(u_i) = 0.5$, $1 - f_A(u_i) = 0.7$, and $f_A(u_i) = 0.3$. It can be interpreted as "the degree that object u_i belongs to the vague set A is 0.5, the degree that object u_i does not belong to the vague set A is 0.3." As another example, in a voting model, the vague value $[0.5, 0.7]$ can be interpreted as "the vote for resolution is 5 in favor,

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The author is with the Department of Computer and Information Science, National Chiao Tung University, Hsinchu 300, Taiwan, R.O.C.

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3 against, and 2 abstentions." Let U be the universe of discourse, $U = \{u_1, u_2, \ldots, u_n\}$, and let t_A and f_A be the truth-membership function and the false-membership function of the vague set A. A vague set A of the universe of discourse U can be represented by

$$A = [t_A(u_1), 1 - f_A(u_1)]/u_1 + [t_A(u_2), 1 - f_A(u_2)]/u_2 + \cdots + [t_A(u_n), 1 - f_A(u_n)]/u_n \quad (3)$$

where $0 \leq t_A(u_i) \leq 1 - f_A(u_i) \leq 1$ and $1 \leq i \leq n$. When the universe of discourse U is an infinite set, then the vague set A can be represented by

$$A = \int_U [t_A(u_i), 1 - f_A(u_i)]/u_i, \quad \forall u_i \in U. \quad (4)$$

In [3], we have presented two similarity measures for measuring the degree of similarity between vague sets. In [5], we have presented new techniques for handling multicriteria fuzzy decision-making problems based on vague set theory. In [12], Leekwang *et al*. has considered an example of behavior analysis in an organization. In this paper, we will extend the work of [12] to apply the vague set theory in behavior analysis of an organization.

The rest of this paper is organized as follows. In Section II, we introduce some basic definitions of vague values and vague sets. In Section III, we present some similarity measures between vague sets and between elements. In Section IV, we present an example to apply the proposed similarity measures in behavior analysis of an organization. The conclusions are discussed in Section V.

II. Basic Definitions of Vague Values and Vague Sets

In this section, we review some basic definitions of vague values and vagues sets from [3], [5], and [7].

Definition 2.1: Let x be a vague value, $x = [t_x, 1 - f_x]$, where $0 \leq t_x \leq 1 - f_x \leq 1$. The vague value x can be divided into three parts: the truth-membership part (i.e., t_x), the false-membership part (i.e., f_x), and the unknown part (i.e., $1 - t_x - f_x$).

Definition 2.2: Let x be a vague value, where $x = [t_x, 1 - f_x]$. If $t_x = 1$ and $f_x = 0$ (i.e., $x = [1, 1]$), then x is called a unit vague value.

Definition 2.3: Let x be a vague value, where $x = [t_x, 1 - f_x]$. If $t_x = 0$ and $f_x = 1$ (i.e., $x = [0, 0]$), then x is called a zero vague value.

Definition 2.4: Let x and y be two vague values, where $x = [t_x, 1 - f_x]$ and $y = [t_y, 1 - f_y]$. If $t_x = t_y$ and $f_x = f_y$, then the vague values x and y are called equal (i.e., $[t_x, 1 - f_x] = [t_y, 1 - f_y]$).

Definition 2.5: Let A and B be vague sets of the universe of discourse U, $U = \{u_1, u_2, \ldots, u_n\}$, where

$$A = [t_A(u_1), 1 - f_A(u_1)]/u_1 + [t_A(u_2), 1 - f_A(u_2)]/u_2 + \cdots + [t_A(u_n), 1 - f_A(u_n)]/u_n$$
$$B = [t_B(u_1), 1 - f_B(u_1)]/u_1 + [t_B(u_2), 1 - f_B(u_2)]/u_2 + \cdots + [t_B(u_n), 1 - f_B(u_n)]/u_n.$$

If $\forall i$, $[t_A(u_i),\ 1 - f_A(u_i)] = [t_B(u_i),\ 1 - f_B(u_i)]$, then the vague sets A and B are called equal, where $1 \leq i \leq n$.

Definition 2.6: Let A be a vague set of the universe of discourse U, $U = \{u_1, u_2, \ldots, u_n\}$, where

$$A = [t_A(u_i), 1 - f_A(u_1)]/u_1 + [t_A(u_2), 1 - f_A(u_2)]/u_2 + \cdots + [t_A(u_n), 1 - f_A(u_n)]/u_n.$$

If $\forall i$, $t_A(u_i) = 1$ and $f_A(u_i) = 0$, then A is called a unit vague set, where $1 \leq i \leq n$.

Definition 2.7: Let A be a vague set of the universe of discourse U, $U = \{u_1, u_2, \ldots, u_n\}$, where

$$A = [t_A(u_1), 1 - f_A(u_1)]/u_1 + [t_A(u_2), 1 - f_A(u_2)]/u_2 + \cdots + [t_A(u_n), 1 - f_A(u_n)]/u_n.$$

If $\forall i, t_A(u_i) = 0$ and $f_A(u_i) = 1$, then A is called a zero vague set, where $1 \leq i \leq n$.

Definition 2.8: Let A be a vague set of the universe of discourse U, $U = \{u_1, u_2, \ldots, u_n\}$, where

$$A = [t_A(u_1), 1 - f_A(u_1)]/u_1 + [t_A(u_2), 1 - f_A(u_2)]/u_2 + \cdots + [t_A(u_n), 1 - f_A(u_n)]/u_n.$$

If $\forall i$, $t_A(u_i) = 0$ and $f_A(u_i) = 0$, then A is called an empty vague set, where $1 \leq i \leq n$.

III. Similarity Measures Between Vague Sets and Between Elements

In this section, we present some similarity measures between vague sets and between elements.

A. Similarity Measure Between Vague Sets

Let x and y be two vague values

$$x = [t_x, 1 - f_x]$$
$$y = [t_y, 1 - f_y]$$

$0 \leq t_x \leq 1 - f_x \leq 1$, and $0 \leq t_y \leq 1 - f_y \leq 1$. Based on the score function we presented in [3] and [5], the score of the vague values x and y can be evaluated by the score function S, respectively,

$$S(x) = t_x - f_x \quad (5)$$
$$S(y) = t_y - f_y \quad (6)$$

where $S(x) \in [-1, 1]$ and $S(y) \in [-1, 1]$. Then, based on the function M we presented in [3], the degree of similarity between the vague values x and y can be evaluated, where

$$M(x, y) = 1 - \frac{|S(x) - S(y)|}{2} = 1 - \frac{|t_x - f_x - (t_y - f_y)|}{2} = 1 - \frac{|t_x - t_y - (f_x - f_y)|}{2} \quad (7)$$

where $M(x, y) \in [0, 1]$. The larger the value of $M(x, y)$, the more the similarity between the vague values x and y.

Example 3.1: Let x and y be two vague values, where $x = [1, 1]$ and $y = [0, 0]$. That is, $t_x = 1,\ f_x = 0,\ t_y = 0$, and $f_y = 1$. Then, based on (7), the degree of similarity between the vague values x and y can be evaluated as follows:

$$M(x, y) = 1 - \frac{|1 - 0 - (0 - 1)|}{2} = 0. \quad (8)$$

That is, the degree of similarity between the vague values x and y is equal to 0.

Example 3.2: Let x and y be two vague values, where $x = [1, 1]$ and $y = [1, 1]$. That is, $t_x = 1,\ f_x = 0,\ t_y = 1$, and $f_y = 0$. Based on (7), the degree of similarity between the vague values x and y can be evaluated as follows

$$M(x, y) = 1 - \frac{|1 - 1 - (0 - 0)|}{2} = 1. \quad (9)$$

That is, the degree of similarity between the vague values x and y is equal to 1.

Example 3.3: Let x and y be two vague values, where $x = y = [a, a]$ and $0 \le a \le 1$. That is, $t_x = t_y = a$ and $f_x = f_y = 1 - a$. Then, based on (7), the degree of similarity between the vague values x and y can be evaluated as follows:

$$M(x, y) = 1 - \frac{|a - a - (1 - a - (1 - a))|}{2} = 1. \tag{10}$$

That is, if the vague values x and y are equal (i.e., $x = y$), then $M(x, y) = 1$.

Let A and B be vague sets of the universe of discourse U, $U = \{u_1, u_2, \ldots, u_n\}$, where

$$A = [t_A(u_1), 1 - f_A(u_1)]/u_1 + [t_A(u_2), 1 - f_A(u_2)]/u_2 + \cdots + [t_A(u_n), 1 - f_A(u_n)]/u_n$$
$$B = [t_B(u_1), 1 - f_B(u_1)]/u_1 + [t_B(u_2), 1 - f_B(u_2)]/u_2 + \cdots + [t_B(u_n), 1 - f_B(u_n)]/u_n.$$

Then, based on the function T we presented in [3] and (7), the degree of similarity between the vague sets A and B can be evaluated as follows

$$T(A, B) = \frac{1}{n}\sum_{k=1}^{n} M([t_A(u_k), 1 - f_A(u_k)], [t_B(u_k), 1 - f_B(u_k)]) = \frac{1}{n}\sum_{k=1}^{n}\left(1 - \frac{|t_A(u_k) - t_B(u_k) - (f_A(u_k) - f_B(u_k))|}{2}\right) \tag{11}$$

where $T(A, B) \in [0, 1]$. The larger the value of $T(A, B)$, the more the similarity between the vague sets A and B.

We can see some properties of the function T:

(1) The similarity degree is bounded, i.e., $0 \le T(A, B) \le 1$.
(2) If the vague sets A and B are equal (i.e., $A = B$), then $T(A, B) = 1$.
(3) If A is a unit vague set and B is a zero vague set, then $T(A, B) = 0$.
(4) The similarity measure is commutative, i.e., $T(A, B) = T(B, A)$.

Let x and y be two vague values, where

$$x = [t_x, 1 - f_x]$$
$$y = [t_y, 1 - f_y].$$

The weighted scores of the vague values x and y can be evaluated by the weighted score function S_w, respectively,

$$S_w(x) = a * t_x + b * f_x + c * (1 - t_x - f_x) \tag{12}$$
$$S_w(y) = a * t_y + b * f_y + c * (1 - t_y - f_y) \tag{13}$$

where a, b, and c represent the weight of the truth-membership part, the weight of the false-membership part, and the weight of the unknown part of the vague values, respectively, $a \ge c \ge 0 \ge b$, $S_w(x) \in [b, a]$, and $S_w(y) \in [b, a]$. It is obvious that if $a = 1$, $b = -1$, and $c = 0$, then (12) and (13) will be reduced into (5) and (6), respectively. The weighted similarity measure between the vague values x and y can be evaluated by the function M_w

$$M_w(x, y) = 1 - \frac{|S_w(x) - S_w(y)|}{a - b} = 1 - \frac{|a * (t_x - t_y) + b * (f_x - f_y) + c * (t_y + f_y - (t_x + f_x))|}{a - b} \tag{14}$$

where $M_w(x, y) \in [0, 1]$. The larger the value of $M_w(x, y)$, the more the similarity between the vague values x and y.

Example 3.4: Let x and y be two vague values, where $x = [0.2, 0.6]$ and $y = [0.3, 0.6]$. That is, $t_x = 0.2$, $f_x = 0.4$, $t_y = 0.3$, and $f_y = 0.4$. Then:

Case 1: If the weight of the truth-membership part, the weight of the false-membership part, and the weight of the unknown part of the vague values are 1, -1, and 0, respectively, then based on (14), the degree of similarity between the vague values x and y can be evaluated as shown in (15) at the bottom of page. That is, the degree of similarity between the vague values x and y is equal to 0.95.

Case 2: If the weight of the truth-membership part, the weight of the false-membership part, and the weight of the unknown part of the vague values are 2, -1, and 0, respectively, then based on (14), the degree of similarity between the vague values x and y can be evaluated as shown in (16) at the bottom of page. That is, the degree of similarity between the vague values x and y is about 0.933.

Let A and B be two vague sets in the universe of discourse $U, U = \{u_1, u_2, \ldots, u_n\}$, where

$$A = [t_A(u_1), 1 - f_A(u_1)]/u_1 + [t_A(u_2), 1 - f_A(u_2)]/u_2 + \cdots + [t_A(u_n), 1 - f_A(u_n)]/u_n$$
$$B = [t_B(u_1), 1 - f_B(u_1)]/u_1 + [t_B(u_2), 1 - f_B(u_2)]/u_2 + \cdots + [t_B(u_n), 1 - f_B(u_n)]/u_n.$$

Assume that the weight of the truth-membership part, the weight of the false-membership part, and the weight of the unknown part of the vague values are a, b, and c, respectively, where $a \ge c \ge 0 \ge b$, then the degree of similarity between the vague sets A and B can be evaluated by the function T_w (see (17) at the bottom of the next page) where $T_w(A, B) \in [0, 1]$. The larger the value of $T_w(A, B)$, the more the similarity between the vague set A and B. It is obvious that if $a = 1$, $b = -1$, and $c = 0$, then (17) will be reduced into (11).

In the following, we present the weighted similarity measure between vague sets based on (16). Let A and B be vague sets of the universe of discourse U, $U = \{u_1, u_2, \ldots, u_n\}$, where

$$A = [t_A(u_1), 1 - f_A(u_1)]/u_1 + [t_A(u_2), 1 - f_A(u_2)]/u_2 + \cdots + [t_A(u_n), 1 - f_A(u_n)]/u_n$$
$$B = [t_B(u_1), 1 - f_B(u_1)]/u_1 + [t_B(u_2), 1 - f_B(u_2)]/u_2 + \cdots + [t_B(u_n), 1 - f_B(u_n)]/u_n.$$

$$M_w(x, y) = 1 - \frac{|1 * (0.2 - 0.3) + (-1) * (0.4 - 0.4) + 0 * (0.3 + 0.4 - (0.2 + 0.4)|}{1 - (-1)} = 0.95. \tag{15}$$

$$M_w(x, y) = 1 - \frac{|2 * (0.2 - 0.3) + (-1) * (0.4 - 0.4) + 0 * (0.3 + 0.4 - (0.2 + 0.4)|}{2 - (-1)} = 0.933. \tag{16}$$

TABLE I
TABULATION OF VAGUE SETS

	A_1	$\cdots$	A_k	$\cdots$	A_m
u_1	$[t_{A_1}(u_1), 1-f_{A_1}(u_1)]$	$\cdots$	$[t_{A_k}(u_1), 1-f_{A_k}(u_1)]$	$\cdots$	$[t_{A_m}(u_1), 1-f_{A_m}(u_1)]$
u_2	$[t_{A_1}(u_2), 1-f_{A_1}(u_2)]$	$\cdots$	$[t_{A_k}(u_2), 1-f_{A_k}(u_2)]$	$\cdots$	$[t_{A_m}(u_2), 1-f_{A_m}(u_2)]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
u_i	$[t_{A_1}(u_i), 1-f_{A_1}(u_i)]$	$\cdots$	$[t_{A_k}(u_i), 1-f_{A_k}(u_i)]$	$\cdots$	$[t_{A_m}(u_i), 1-f_{A_m}(u_i)]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
u_j	$[t_{A_1}(u_j), 1-f_{A_1}(u_j)]$	$\cdots$	$[t_{A_k}(u_j), 1-f_{A_k}(u_j)]$	$\cdots$	$[t_{A_m}(u_j), 1-f_{A_m}(u_j)]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
u_n	$[t_{A_1}(u_n), 1-f_{A_1}(u_n)]$	$\cdots$	$[t_{A_k}(u_n), 1-f_{A_k}(u_n)]$	$\cdots$	$[t_{A_m}(u_n), 1-f_{A_m}(u_n)]$

Assume that the weight of the element u_i in the universe of discourse U is w_i, respectively, where $w_i \in [0, 1]$ and $1 \leq i \leq n$, and assume that the weight of the truth-membership part, the weight of the false-membership part, and the weight of the unknown part of the vague values are a, b, and c, respectively, where $a \geq c \geq 0 \geq b$, then the degree of similarity between the vague sets A and B can be evaluated by the weighting function W, (see (18) at the bottom of the page) where $W(A, B) \in [0, 1]$. The larger the value of $W(A, B)$, the more the similarity between the vague sets A and B.

B. Similarity Measure Between Elements

Let $A_1, A_2, \ldots$, and A_m be vague sets in the universe of discourse U, $U = \{u_1, u_2, \ldots, u_n\}$, where

$$\begin{aligned} A_1 &= [t_{A_1}(u_1), 1 - f_{A_1}(u_1)]/u_1 + [t_{A_1}(u_2), 1 - f_{A_1}(u_2)]/u_2 \\ &\quad + \cdots + [t_{A_1}(u_n), 1 - f_{A_1}(u_n)]/u_n \\ A_2 &= [t_{A_2}(u_1), 1 - f_{A_2}(u_1)]/u_1 + [t_{A_2}(u_2), 1 - f_{A_2}(u_2)]/u_2 \\ &\quad + \cdots + [t_{A_2}(u_n), 1 - f_{A_2}(u_n)]/u_n \\ &\vdots \\ A_m &= [t_{A_m}(u_1), 1 - f_{A_m}(u_1)]/u_1 + [t_{A_m}(u_2), 1 - f_{A_m}(u_2)]/u_2 \\ &\quad + \cdots + [t_{A_m}(u_n), 1 - f_{A_m}(u_n)]/u_n. \end{aligned}$$

These vague sets can be tabulated as shown in Table I. Assume that the weight of the truth-membership part, the weight of the false-membership part, and the weight of the unknown part of the vague values are a, b, and c, respectively, where $a \geq c \geq 0 \geq b$, the similarity measure between two elements u_i, u_j in fuzzy set $A_k \in U$, $k = 1, 2, \ldots, m$, is defined as shown in (19) (see top of the next page) where $S_e(u_i, u_j) \in [0, 1]$. The larger the value of $S_e(u_i, u_j)$, the more the similarity between the elements u_i and u_j. It is obvious that if $a = 1$, $b = -1$, and $c = 0$, then (19) will be reduced into

$$S_e(u_i, u_j) = \frac{1}{m} \sum_{k=1}^{m} \left(1 - \frac{|t_{A_k}(u_i) - t_{A_k}(u_j) - (f_{A_k}(u_i) - f_{A_k}(u_j))|}{2}\right). \tag{20}$$

The similarity measure of (19) satisfies the following properties:

1) $0 \leq S_e(u_i, u_j) \leq 1$.

$$\begin{aligned} T_w(A, B) &= \frac{1}{n} \sum_{k=1}^{n} M_w([t_A(u_k), 1 - f_A(u_k)], [t_B(u_k), 1 - f_B(u_k)]) \\ &\frac{1}{n} \sum_{k=1}^{n} \left(1 - \frac{|a * (t_A(u_k) - t_B(u_k)) + b * (f_A(u_k) - f_B(u_k)) + c * (t_B(u_k) + f_B(u_k) - (t_A(u_k) + f_A(u_k))|}{a - b}\right) \end{aligned} \tag{17}$$

$$\begin{aligned} W(A, B) &= \frac{\sum_{k=1}^{n} w_k * M_w([t_A(u_k), 1 - f_A(u_k)], [t_B(u_k), 1 - f_B(u_k)])}{\sum_{k=1}^{n} w_k} \\ &= \frac{\sum_{k=1}^{n} w_k * \left(1 - \frac{|a*(t_A(u_k) - t_B(u_k)) + b*(f_A(u_k) - f_B(u_k)) + c*(t_B(u_k) + f_B(u_k) - (t_A(u_k) + f_A(u_k))|}{a-b}\right)}{\sum_{k=1}^{n} w_k} \end{aligned} \tag{18}$$

$$S_e(u_i, u_j) = \frac{1}{m}\sum_{k=1}^{n}(M_w([t_{A_k}(u_i), 1 - f_{A_k}(u_i)], [t_{A_k}(u_j), 1 - f_{A_k}(u_j)])$$
$$= \frac{1}{m}\sum_{k=1}^{m}\left(1 - \frac{|a * (t_{A_k}(u_i) - t_{A_k}(u_j)) + b * (f_{A_k}(u_i) - f_{A_k}(u_j)) + c * (t_{A_k}(u_j) + f_{A_k}(u_j) - (t_{A_k}(u_i) + f_{A_k}(u_i))|}{a - b}\right) \tag{19}$$

TABLE II
FOUR MEMBERS AND THREE GROUPS IN AN ORGANIZATION

	A	B	C
u_1	[0.4, 0.6]	[0.6, 0.7]	[0, 0]
u_2	[0.8, 0.9]	[0.3, 0.5]	[0.4, 0.6]
u_3	[0.9, 0.9]	[0, 0]	[0.8, 0.9]
u_4	[0, 0]	[0.5, 0.8]	[1, 1]

2) If $u_i = u_j$, then $S_e(u_i, u_j) = 1$.
3) If $\forall k$, the grade of membership of u_i in A_k is a unit vague value (i.e., $[1, 1]$) and the grade of membership of u_j in A_k is a zero vague value (i.e., $[0, 0]$), then $S_e(u_i, u_j) = 0$, where $1 \leq k \leq m$.
4) The measure is commutative

$$S_e(u_i, u_j) = S_e(u_j, u_i).$$

IV. An Application in Behavior Analysis in an Organization

Let us consider an example of behavior analysis problems. The example is essentially a modification of the one shown in [12]. Assume that there are four members (u_1, u_2, u_3, u_4) and three groups (A, B, C). A member is involved in groups with the membership degrees represented by vague values shown in Table II.

Based on (17) and (19), we can answer the following two types of questions [12]:

Type 1: At what degree the groups A and B can be cooperated?

Type 2: At what degree the members u_2 and u_3 can be in the same group?

In the following, we assume that the weight of the truth-membership part, the weight of the false-membership part, and the weight of the unknown part of the vague values are 1, -1, and 0, respectively. By applying (17), we can answer the Type 1 question. In this case, the degree that the groups A and B can be cooperated is evaluated as follows:

$$\begin{aligned}T_w(A, B) = \frac{1}{4}\Bigg[&\left(1 - \frac{|(0.4 - 0.6) - (0.4 - 0.3)|}{2}\right)\\ &+\left(1 - \frac{|(0.8 - 0.3) - (0.1 - 0.5)|}{2}\right)\\ &+\left(1 - \frac{|(0.9 - 0) - (0.1 - 1)|}{2}\right)\\ &+\left(1 - \frac{|(0 - 0.5) - (1 - 0.2)|}{2}\right)\Bigg]\\ = 0.4625.\end{aligned} \tag{21}$$

That is, the degree that the groups A and B can be cooperated is equal to 0.4625.

By applying (19), we can answer the Type 2 question. For example, consider the elements u_2 and u_3 shown in Table II. By applying (19), the degree of similarity between u_2 and u_3 can be evaluated as follows:

$$\begin{aligned}S_e(u_2, u_3) = \frac{1}{3}\Bigg[&\left(1 - \frac{|(0.8 - 0.9) - (0.1 - 0.1)|}{2}\right)\\ &+\left(1 - \frac{|(0.3 - 0) - (0.5 - 1)|}{2}\right)\\ &+\left(1 - \frac{|(0.4 - 0.8) - (0.4 - 0.1)|}{2}\right)\Bigg]\\ = 0.7333.\end{aligned} \tag{22}$$

That is, the degree that the members u_2 and u_3 can be in the same group is about 0.7333.

V. Conclusion

In this paper, we have presented some similarity measures between vague sets and between elements. We also used an example to illustrate the application of the proposed similarity measures in handling the behavior analysis problems. The proposed similarity measures can provide a useful way for behavior analysis in a vague environment, where the degree that each member belongs to each group is represented by a vague value rather than a fuzzy value presented in [12]. Consequently, our method for behavior analysis is more flexible than the one presented in [12].

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Handwritten Word Recognition with Character and Inter-Character Neural Networks

Paul D. Gader, Magdi Mohamed, and Jung-Hsien Chiang

***Abstract*— An off-line handwritten word recognition system is described. Images of handwritten words are matched to lexicons of candidate strings. A word image is segmented into primitives. The best match between sequences of unions of primitives and a lexicon string is found using dynamic programming. Neural networks assign match scores between characters and segments. Two particularly unique features are that neural networks assign confidence that pairs of segments are compatible with character confidence assignments and that this confidence is integrated into the dynamic programming. Experimental results are provided on data from the U.S. Postal Service.**

I. Introduction

An off-line, handwritten word recognition algorithm has two inputs: a digital image (assumed to be an image of a word), and a list of strings called a lexicon, representing possible identities for the word image. The goal is to assign a match score to each candidate in the lexicon.

A variety of approaches have been reported since 1990. Several researchers [1]–[7] have used hidden Markov models. Others have tried to use "wholistic approaches" in which a word is recognized as an entity. These algorithms do well at providing auxiliary information, but not as stand-alone recognizers [8]–[13]. Some of the most successful results have come from segmentation-based techniques that rely on dynamic programming [5], [14]–[20]. In these approaches, an optimal segmentation is generated for each lexicon string.

Our baseline system is based on dynamic programming and is illustrated in Fig. 1. A word image is segmented into subimages called *primitives* without using a lexicon. Each primitive ideally consists of a single character or a subimage of a single character. A segment is defined as either a primitive or a union of primitives and a segmentation as a sequence of segments using all the primitives. Dynamic programming is used to find the segmentation that matches a given string best. The match score is assigned by matching each segment in the segmentation to the corresponding character in the string using a character recognizer that returns confidence values.

This approach does not consider important inter-character relationships. For example, in Fig. 2, a segmentation of the word

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The authors are with the Department of Electrical and Computer Engineering, University of Missouri, Columbia, MO 65211 USA (email: gader@sunpg.ece.missouri.edu).

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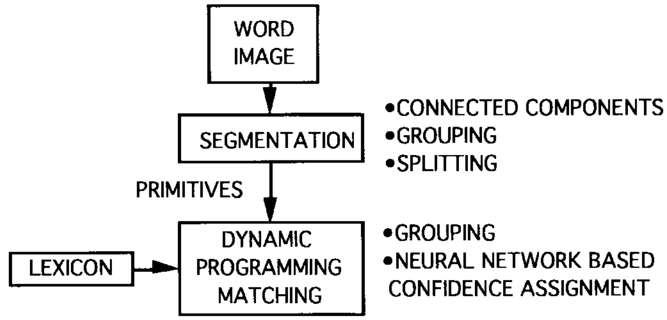

Fig. 1. Overview of the word recognition system.

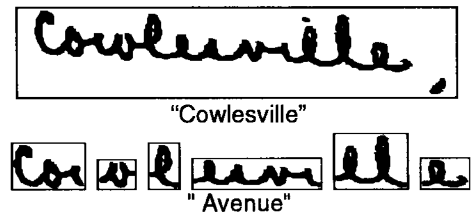

Fig. 2. The character recognition scores of the individual fifth and sixth segments match well against the characters "u" and "e," but the sizes of the segments are not spatially compatible.

"Cowlesville" matches well to the string "avenue". The fifth and sixth segments together do not look much like "ue" since the fifth segment is much larger than the sixth segment and "u" and "e" are about the same size. However, as individual characters, the fifth segment looks very much like "u" and the sixth segment very much like "e". Of course, the "ue" hypothesis is possible and should be assigned some nonzero confidence.

The spatial relationships and relative sizes between segments are cues that should be considered in assigning a match score between a word image and a lexicon string. One method for doing so is to use a post processor that modifies the match score after dynamic programming. This approach cannot correct for segmentation errors caused by bad matches.

The novel approach described here builds the confidence modification due to spatial relationships into the dynamic programming. A compatibility score is assigned to pairs of adjacent segments using a neural network. This compatibility score is combined with the character recognition score to assign match scores between segments and characters. A related concept was used by Obaidat and Macchairolo who used time intervals between typed characters to identify computer users [21]. We now describe the system and then provide experimental results for the character recognition and compatibility modules and the entire system.

II. Segmentation

The segmentation module is very similar to that described in [22] and we therefore do not discuss it much here. The segmentation process initially detects connected components. Some simple grouping and noise removal is performed. The results are referred to as the initial segments. An element of an initial segmentation is generally a significant connected component in the word, or a grouping of connected components. Those components which are not "bars" (such as the top of a "T" or the vertical bar in a "D") are sent to a splitting