國 立 交 通 大 學

電信工程研究所

碩 士 論 文

多輸入多輸出雙使用者干擾通道在傳送端沒有通道狀態的資訊下使用功率劃分的自由度

Degrees of freedom for MIMO interference channel using power split with no channel state information at transmitters

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摘　要

在這篇論文中我們探討多輸入多輸出雙使用者干擾通道在傳送端沒有通道資訊下的自由度區域。我們延用 Huang, Jafar 和 Shamai 的研究，從不同的傳送端和接收端天線數分成三個類別。我們使用功率劃分方法以及接續干擾消除解碼器來得到自由度區域。同時我們也讓 Huang 等三人的研究中其中一個類別的外界變得更好。我們並延伸討論到多輸入多輸出雙使用者干擾通道在傳送端沒有通道資訊下有完美接收端合作的自由度區域，比較和多輸入多輸出雙使用者干擾通道在傳送端沒有通道資訊下沒有合作的自由度範圍的差異。

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Abstract

In this thesis we investigate the degree-of-freedom (DoF) region of a two-user MIMO interference channel without channel state information at the transmitters. Following the work of Huang, Jafar, and Shamai, we distinguish three types of channels based on the numbers of transmit and receive antennas. We derive the DoF region for a power-split scheme followed by a SIC decoder. We also improve an outer bound by Huang et. al. on the DoF region for a specific type of MIMO interference channels. We extend our discussion to two-user MIMO interference channels with perfect receiver cooperation and compare the result to two-user interference without cooperation.

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Chapter 1

Introduction

1.1 MIMO Systems and Degrees of Freedom (DoF)

In wireless communication, multiple-input-multiple-output (MIMO) systems have the ability to provide remarkable increase of capacity compared to single-input-single-output (SISO) systems. One of the key benefit from MIMO systems is multiplexing signal in space. For a MIMO system, the ability of multiplexing signals is measured by the *spatial multiplexing gain* [1], also known as degrees of freedom (DoF).

Let us consider a point-to-point MIMO system, the transmitter has n_t antennas, and the receiver has n_r antennas. The transmitter can transmit at most n_t independent streams simultaneously, and the receiver can receiver at most n_r independent streams simultaneously. Thus, it is easy to see that the maximal DoF of a point-to-point MIMO system is $\min(n_t, n_r)$. The standard definition of maximal DoF is

$$d_{max} := \lim_{\mathrm{SNR}\to\infty} \frac{C(\mathrm{SNR})}{\log \mathrm{SNR}}, \tag{1.1}$$

where $C(\mathrm{SNR})$ is the capacity of the point-to-point MIMO system given by

$$C(\mathrm{SNR}) = \mathbb{E}\left[\log\det\left(\mathbf{I}_{n_r} + \frac{\mathrm{SNR}}{n_t}\mathbf{H}\mathbf{H}^{\dagger}\right)\right], \tag{1.2}$$

and the SNR is the signal-to-noise power ratio. The elements of $\mathbf{H}$ and $\mathbf{N}$ are circularly symmetric complex Gaussian random variables with zero mean and unit variance. The equation (1.2) can be written as

$$C(\mathrm{SNR}) = \mathbb{E}\left[\sum_{i=1}^{n_{min}} \log\left(1 + \frac{\mathrm{SNR}}{n_t}\lambda_i^2\right)\right], \tag{1.3}$$

where $\lambda_1 \leq \lambda_2 \ldots < \lambda_{n_{min}}$ are the ordered singular values of $\mathbf{H}$, and $n_{min} = \min(n_t, n_r)$. By Jensen's inequality, we have that

$$\mathbb{E}\left[\sum_{i=1}^{n_{min}} \log\left(1 + \frac{\mathrm{SNR}}{n_t}\lambda_i^2\right)\right] \leq n_{min} \log\left(1 + \frac{\mathrm{SNR}}{n_t}\left[\frac{1}{n_{min}}\sum_{i=1}^{n_{min}} \lambda_i^2\right]\right). \tag{1.4}$$

At high SNR, the capacity approximates

$$C(\mathrm{SNR}) \approx n_{min} \log \frac{\mathrm{SNR}}{n_t} + \sum_{i=1}^{n_{min}} \mathbb{E}\left[\log \lambda_i^2\right]. \tag{1.5}$$

So we have the maximal DoF is

$$d_{max} = \lim_{\mathrm{SNR}\to\infty} \frac{C(\mathrm{SNR})}{\log \mathrm{SNR}} = n_{min}. \tag{1.6}$$

Thus we can know that the maximal DoF of a point-to-point MIMO system is $\min(n_t, n_r)$.

In [2], it has been shown that for the point-to-point MIMO communication, the absence of channel state information at transmitters, i.e., CSIT, does not reduce the DoF. But in a MIMO network with distributed processing units, [3] shows that in the absence of CSIT, the DoF may be lost.

1.2 Two-User MIMO Interference Channel

In this section, we consider the two-user MIMO interference channel. We have two transmitters and two receivers. Transmitter 1 and 2 are equipped with n_{t1}, n_{t2} antennas, respectively. Receiver 1 and 2 are equipped with n_{r1}, n_{r2} antennas, respectively. The channel is described as

$$\underline{\mathbf{Y}}_1[n] = \mathbf{H}_{11}[n]\underline{\mathbf{X}}_1[n] + \mathbf{H}_{12}[n]\underline{\mathbf{X}}_2[n] + \underline{\mathbf{Z}}_1[n] \tag{1.7}$$

$$\underline{\mathbf{Y}}_2[n] = \mathbf{H}_{21}[n]\underline{\mathbf{X}}_1[n] + \mathbf{H}_{22}[n]\underline{\mathbf{X}}_2[n] + \underline{\mathbf{Z}}_2[n], \tag{1.8}$$

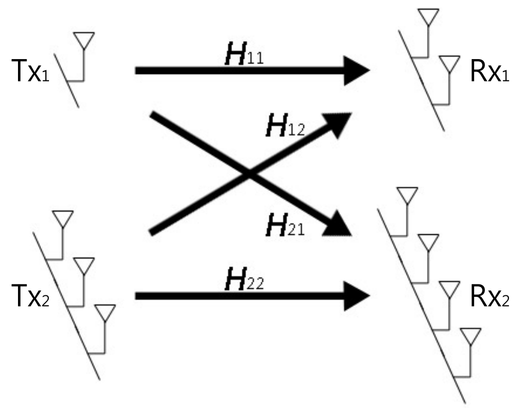

Figure 1.1: Two-user MIMO interference channel

where at the n-th channel use, $\underline{\mathbf{Y}}_j[n]$ and $\underline{\mathbf{Z}}_j[n]$ are the $n_{rj} \times 1$ vectors representing the channel output and additive white Gaussian noise at receiver j, $\mathbf{H}_{ji}[n]$ is the $n_{rj} \times n_{ti}$ channel matrix corresponding to receiver j and transmitter i, and $\underline{\mathbf{X}}_i[n]$ is the $n_{ti} \times 1$, $i, j \in 1, 2$. The elements of $\mathbf{H}_{ji}[n]$ and $\underline{\mathbf{Z}}_j[n]$ are i.i.d. circularly symmetric complex Gaussian random variables with zero mean and unit variance. The transmit power constraint is

$$\mathbf{E}[\underline{\mathbf{X}}_i^2] \leq \text{SNR}, i = 1, 2 \tag{1.9}$$

Fig. 1.1 is an example of two-user MIMO interference channel.

The capacity region $C(\text{SNR})$ for the two-user MIMO interference channel is the set of all rate pairs (R_1, R_2) for which the probability of error can be driven arbitrarily close to zero by using suitable long codewords. Thus, the DoF region for two-user MIMO interference channel is defined as

$$\begin{aligned}\mathbf{D} := \Big\{&(d_1, d_2) \in \mathbb{R}_2^+ : \exists (R_1(\text{SNR}), R_2(\text{SNR})) \in C(\text{SNR}) \text{ s.t.} \\ &d_i = \lim_{\text{SNR}\to\infty} \frac{R_i(\text{SNR})}{\log(\text{SNR})}, i = 1, 2. \Big\}\end{aligned} \tag{1.10}$$

Except in [4], in [5] and [6] also had considered the DoF region of two-user MIMO interference channel with no CSIT. In[7] and [8], the DoF region of two-user MIMO interference channel with perfect CSIT and CSIR had been analyzed.

1.3 Motivation

The DoF region of a two-user MIMO interference channel is determined by the values of n_{t1}, n_{t2}, n_{r1} and n_{r2}. In [4], Huang and Jafar distinguish the values of n_{t1}, n_{t2}, n_{r1} and n_{r2} as several types and analyze the DoF region of each type based on an information theoretic approach by introducing an auxiliary random variable. Yet, there is one type of the two-user MIMO interference channel whose DoF region remains unknown in [4], except for an outer bound.

In this thesis, we follow [4] to distinguish the types of the two-user MIMO interference channel by the associated values of n_{t1}, n_{t2}, n_{r1} and n_{r2} and determine the DoF region of each type based on a power-split transmission scheme followed by a successive-interference-cancellation (SIC) decoder. We will also try to reduce the outer bound of the unknown DoF region to tighten the bound.

1.4 Thesis Outline

In this thesis, we will consider the DoF region of two-user MIMO interference channel with no CSIT and perfect CSIR, we will use the power-split scheme and apply SIC decoder to characterize the DoF region. Our technique is closely related to the characterization of the DoF for a general two-user multiple-access channel (MAC), by which we mean the case when the two users can have different number of transmit antennas and different power constraint. The corresponding result will be given in Chapter 2. In Chapter 3, we will apply the results in Chapter 2 to the two-user MIMO interference channel, and investigate the DoF region for two-user MIMO interference channel. Also we will provide an outer bound for a specific two-user MIMO interference channel. In Chapter 4, we will consider two-user MIMO interference channel with perfect receiver cooperation. The results provide an outer bound for DoF region derived in Chapter 3. Chapter 5 is the conclusion.

Chapter 2

General Two-User MAC Sum Rate

In this chapter, we consider a two-user multiple access channel (MAC). The two transmitters have n_{t1}, n_{t2} antennas, respectively. The receiver has n_r antennas. We assume that the transmitters do not have the channel state information, i.e., no CSIT. We develope the theorem of general MAC sum rate, which will be applied to the two-user MIMO interference channel.

2.1 Sum Rate for General Two-User MAC

In this section, we consider a two-user MAC channel, the channel model is described as

$$\underline{\mathbf{Y}}[n] = \mathbf{H}_1[n]\underline{\mathbf{X}}_1[n] + \mathbf{H}_2[n]\underline{\mathbf{X}}_2[n] + \underline{\mathbf{Z}}[n] \tag{2.1}$$

Assuming the power constraints for the two users are given by

$$\mathbb{E}\ \|\underline{\mathbf{X}}_i[n]\|^2 \ \leq\ \mathrm{SNR}^{\alpha_i}, \quad i = 1, 2$$

the MAC sum rate at high SNR regime is

$$C(\mathrm{SNR}) = \mathbb{E}[\log\det(I_{n_r} + \mathrm{SNR}^{\alpha_1}\mathbf{H}_1\mathbf{H}_1^\dagger + \mathrm{SNR}^{\alpha_2}\mathbf{H}_2\mathbf{H}_2^\dagger)] \tag{2.2}$$

Here we give the theorem of general two-user MAC sum rate.

Theorem 2.1 *For* $\alpha_1 \geq \alpha_2 \geq 0$*,*

$$\begin{aligned} C(\mathrm{SNR}) &= \mathbb{E}[\log\det(I_{n_r} + \mathrm{SNR}^{\alpha_1}\boldsymbol{H}_1\boldsymbol{H}_1^\dagger + \mathrm{SNR}^{\alpha_2}\boldsymbol{H}_2\boldsymbol{H}_2^\dagger)] \\ &= \begin{cases} \alpha_1 n_r \log\mathrm{SNR} + O(1), & \text{if } n_r \leq n_{t1} \\ (\alpha_1 n_{t1} + \alpha_2(n_r - n_{t1}))\log\mathrm{SNR} + O(1), & \text{if } n_{t1} < n_r < n_{t1} + n_{t2} \\ (\alpha_1 n_{t1} + \alpha_2 n_{t2})\log\mathrm{SNR} + O(1), & \text{if } n_{t1} + n_{t2} \leq n_r \end{cases} \end{aligned}$$

The proof of Theorem 2.1 is given in the next section. From Theorem 2.1, we can simply extend it to the DoF.

Corollary 2.2 *For* $\alpha_1 \geq \alpha_2 \geq 0$*, the maximum DoF* d_{max} *for nonzero diversity gain is given by*

$$d_{max} = \begin{cases} \alpha_1 n_r, & \text{if } n_r \leq n_{t1} \\ \alpha_1 n_{t1} + \alpha_2(n_r - n_{t1}), & \text{if } n_{t1} < n_r < n_{t1} + n_{t2} \\ \alpha_1 n_{t1} + \alpha_2 n_{t2}, & \text{if } n_{t1} + n_{t2} \leq n_r \end{cases} \tag{2.3}$$

2.2 Proof of Theorem 2.1

We consider a two-user MAC channel, the transmitter 1 and 2 have n_{t1} and n_{t2} antennas, respectively. The receiver has n_r antennas. The channel model is described as

$$\underline{\mathbf{Y}}[n] = \mathbf{H}_1[n]\underline{\mathbf{X}}_1[n] + \mathbf{H}_2[n]\underline{\mathbf{X}}_2[n] + \underline{\mathbf{Z}}[n], \tag{2.4}$$

where the channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ are complex Gaussian random matrices with $\mathbb{C}\mathcal{N}(0,1)$. The sum rate at high SNR regime is

$$C = \mathbb{E}\left[\log\det(I_{n_r} + \mathrm{SNR}^{\alpha_1}\mathbf{H}_1\mathbf{H}_1^\dagger + \mathrm{SNR}^{\alpha_2}\mathbf{H}_2\mathbf{H}_2^\dagger)\right] \tag{2.5}$$

Let we consider two lemmas first.

Lemma 2.3 *Let* $A, B \in M_{m,n}(\mathbb{C})$*, then*

$$\log\det(I_m + AA^\dagger + BB^\dagger) \leq \log\det(I_m + AA^\dagger) + \log\det(I_m + BB^\dagger) \tag{2.6}$$

and

$$\log\det(I_m + AA^\dagger + BB^\dagger) \geq \max\{\log\det(I_m + AA^\dagger), \log\det(I_m + BB^\dagger)\} \tag{2.7}$$

Proof: The upper bound follows from

$$\begin{aligned}
\log\det(I_m + AA^\dagger + BB^\dagger) &= \log\det\left(\begin{bmatrix} I_n + A^\dagger A & A^\dagger B \\ B^\dagger A & I_n + B^\dagger B \end{bmatrix}\right) && (2.8)\\
&\le \log[\det(I_n + A^\dagger A)\det(I_n + B^\dagger B)] && (2.9)\\
&= \log\det(I_n + A^\dagger A) + \log\det(I_n + B^\dagger B), && (2.10)
\end{aligned}$$

where the inequality is due to Fischer's inequality on positive definite matrices. The lower bound simply follows from

$$\det(I_m + AA^\dagger + BB^\dagger) \ge \det(I_m + AA^\dagger), \qquad (2.11)$$

and

$$\det(I_m + AA^\dagger + BB^\dagger) \ge \det(I_m + BB^\dagger), \qquad (2.12)$$

since $AA^\dagger$ and $BB^\dagger$ are nonnegative definite matrices.

Lemma 2.4

$$\mathbb{E}[\log\det(I_{n_r} + \mathrm{SNR}^{\alpha_1}\boldsymbol{H}_1\boldsymbol{H}_1^\dagger + \mathrm{SNR}^{\alpha_2}\boldsymbol{H}_2\boldsymbol{H}_2^\dagger)] \le (\alpha_1 K_1 + (\alpha_2)^+ K_2)\log(\mathrm{SNR}) + O(1), \qquad (2.13)$$

where $K_1 = \min(n_{t1}, n_r)$ *and* $K_2 = \min(n_{t2}, n_r)$.

Proof: It follows from previous lemma that

$$\begin{aligned}
C &= \mathbb{E}[\log\det(I_{n_r} + \mathrm{SNR}^{\alpha_1}\mathbf{H}_1\mathbf{H}_1^\dagger + \mathrm{SNR}^{\alpha_2}\mathbf{H}_2\mathbf{H}_2^\dagger)] && (2.14)\\
&\le \mathbb{E}[\log\det(I_{n_{t1}} + \mathrm{SNR}^{\alpha_1}\mathbf{H}_1\mathbf{H}_1^\dagger)] + \mathbb{E}[\log\det(I_{n_{t2}} + \mathrm{SNR}^{\alpha_2}\mathbf{H}_2\mathbf{H}_2^\dagger)] && (2.15)\\
&= (\alpha_1 K_1 + (\alpha_2)^+ K_2)\log(\mathrm{SNR}) + O(1). && (2.16)
\end{aligned}$$

Now we start with the proof of the Theorem 2.1.

Proof:

1. Case $n_r \le n_{t1}$: By Lemma 1, we observe that for any combination of $n_{t1}, n_{t2}, n_r > 0$ that

$$\begin{aligned}
C &= \mathbb{E}[\log\det(I_{n_r} + \mathrm{SNR}^{\alpha_1}\mathbf{H}_1\mathbf{H}_1^\dagger + \mathrm{SNR}^{\alpha_2}\mathbf{H}_2\mathbf{H}_2^\dagger)] && (2.17)\\
&\ge \max(\alpha_1 K_1, \alpha_2 K_2)\log\mathrm{SNR}. && (2.18)
\end{aligned}$$

If $n_{t1} \geq n_{t2}$, then $K_1 \geq K_2$ and hence $\max(\alpha_1 K_1, \alpha_2 K_2) = \alpha_1 K_1 = \alpha_1 n_r$. On the othre hand, if $n_{t1} < n_{t2}$, we have $K_2 = K_1 = n_r$. Thus, it follows that $\max(\alpha_1 K_1, \alpha_2 K_2) = \alpha_1 n_r$. To show the converse, note that $\log\det(.)$ is strictly convex, so we have

$$
\begin{aligned}
C &= \mathbb{E}[\log\det(I_{n_r} + \mathrm{SNR}^{\alpha_1}\mathbf{H}_1\mathbf{H}_1^\dagger + \mathrm{SNR}^{\alpha_2}\mathbf{H}_2\mathbf{H}_2^\dagger)] && (2.19)\\
&\leq \log\det(\mathbb{E}[I_{n_r} + \mathrm{SNR}^{\alpha_1}\mathbf{H}_1\mathbf{H}_1^\dagger + \mathrm{SNR}^{\alpha_2}\mathbf{H}_2\mathbf{H}_2^\dagger]) && (2.20)\\
&= \log\det(I_{n_r} + n_{t1}\mathrm{SNR}^{\alpha_1} I_{n_r} + n_{t2}\mathrm{SNR}^{\alpha_2} I_{n_r}) && (2.21)\\
&\doteq \alpha_1 n_r \log \mathrm{SNR}. && (2.22)
\end{aligned}
$$

The case of $n_r \leq n_{t1}$ is proved.

2. Case $n_{t1} < n_r < n_{t1} + n_{t2}$ Define

$$
G = \begin{bmatrix} \sqrt{\mathrm{SNR}^{\alpha_1}}\mathbf{H}_1 & \sqrt{\mathrm{SNR}^{\alpha_2}}\mathbf{H}_2 \end{bmatrix} = \begin{bmatrix} G_L & G_R \end{bmatrix} \qquad (2.23)
$$

$$
G_L = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_{2L} \end{bmatrix} \begin{bmatrix} \sqrt{\mathrm{SNR}}^{\alpha_1} I_{n_{t1}} & \\ & \sqrt{\mathrm{SNR}}^{\alpha_2} I_{n_r - n_{t1}} \end{bmatrix} = H_L\sqrt{\Lambda}, \qquad (2.24)
$$

where G_L is a submatrix of G of size $n_r \times n_r$ and G_R is of size $n_r \times (n_{t1} + n_{t2} - n_r)$. Note that $\mathrm{rank}(G_L) = n_r$ with probability one. It then follows that

$$
\begin{aligned}
C &= \mathbb{E}[\log\det(I_{n_r} + \mathrm{SNR}^{\alpha_1}\mathbf{H}_1\mathbf{H}_1^\dagger + \mathrm{SNR}^{\alpha_2}\mathbf{H}_2\mathbf{H}_2^\dagger)] && (2.25)\\
&= \mathbb{E}[\log\det(I + GG^\dagger)] && (2.26)\\
&= \mathbb{E}[\log\det(I + G_L G_L^\dagger + G_R G_R^\dagger)] && (2.27)\\
&\geq \mathbb{E}[\log\det(I + G_L G_L^\dagger)] && (2.28)\\
&= \mathbb{E}[\log\det(I + \mathbf{H}_L \lambda \mathbf{H}_L^\dagger)] && (2.29)\\
&\geq \mathbb{E}[\log(1 + \det(\lambda)\det(H_L H_L^\dagger))], && (2.30)
\end{aligned}
$$

where the last inequality follows from $\det(I + AA^\dagger) \geq 1 + det(AA^\dagger)$. Now let us take expectation of both sides show

$$
\begin{aligned}
C &= \mathbb{E}[\log\det(I_{n_r} + \mathrm{SNR}^{\alpha_1}\mathbf{H}_1\mathbf{H}_1^\dagger + \mathrm{SNR}^{\alpha_2}\mathbf{H}_2\mathbf{H}_2^\dagger)] && (2.31)\\
&> \log\det(\lambda) + \mathbb{E}[\log\det(H_L H_L^\dagger)] && (2.32)\\
&= (\alpha_1 n_{t1} + \alpha_2(n_r - n_{t2}))\log \mathrm{SNR} + O(1). && (2.33)
\end{aligned}
$$

For the converse, we have the following inequalities

$$
\begin{aligned}
C &= \mathbb{E}[\log\det(I_{n_r} + \mathrm{SNR}^{\alpha_1}\mathbf{H}_1\mathbf{H}_1^\dagger + \mathrm{SNR}^{\alpha_2}\mathbf{H}_2\mathbf{H}_2^\dagger)] && (2.34)\\
&= \mathbb{E}[\log\det(I_{n_r} + G_L G_L^\dagger + G_R G_R^\dagger)] && (2.35)\\
&\overset{(a)}{=} \mathbb{E}_{G_L}[\mathbb{E}_{G_R}[\log\det(I_{n_r} + G_L G_L^\dagger + G_R G_R^\dagger)]] && (2.36)\\
&\overset{(b)}{<} \mathbb{E}_{G_L}[\log\det(\mathbb{E}_{G_R}[I_{n_r} + G_L G_L^\dagger + G_R G_R^\dagger])] && (2.37)\\
&\overset{(c)}{\doteq} \mathbb{E}_{G_L}[\log\det(I_{n_r} + G_L G_L^\dagger + \mathrm{SNR}^{\alpha_2} I_{n_r})] && (2.38)\\
&= n_r \log\mathrm{SNR} + \mathbb{E}_{G_L}[\log\det(I_{n_r} + (1+\mathrm{SNR}^{\alpha_2})^{-1} G_L G_L^\dagger)] && (2.39)\\
&= \alpha_2 n_r \log\mathrm{SNR} + \mathbb{E}_{G_L}[\log\det(I_{n_r} + \frac{\mathrm{SNR}_{\alpha_1}}{1+\mathrm{SNR}_{\alpha_2}}\mathbf{H}_1\mathbf{H}_1^\dagger && (2.40)\\
&+ \frac{\mathrm{SNR}_{\alpha_2}}{1+\mathrm{SNR}_{\alpha_2}}\mathbf{H}_{2L}\mathbf{H}_{2L}^\dagger)] && (2.41)\\
&\overset{(d)}{\doteq} \alpha_2 n_r \log\mathrm{SNR} + (\alpha_1 - \alpha_2) n_{t1} \log\mathrm{SNR} + O(1). && (2.42)
\end{aligned}
$$

(a) follows from conditional expectation.

(b) follows from that log det(.) is strictly concave on positive definite matrices.

(c) follows from $\mathbb{E}_{G_R}[G_R G_R^\dagger] = \mathrm{SNR}^{\alpha_2}(2n_{t2} - n_r) I_{n_r}$ and $(2n_{t2} - n_r) \doteq 1$.

(d) follows from $\frac{\mathrm{SNR}^{\alpha_2}}{1+\mathrm{SNR}^{\alpha_2}} \doteq 1$ for $\alpha_2 > 0$ and from Lemma 2.

3. Case $n_r \geq n_{t1} + n_{t2}$ Define

$$
G = \begin{bmatrix} \sqrt{\mathrm{SNR}^{\alpha_1}}\mathbf{H}_1 & \sqrt{\mathrm{SNR}^{\alpha_2}}\mathbf{H}_2 \end{bmatrix} = \begin{bmatrix} G_U \\ G_D \end{bmatrix}, \qquad (2.43)
$$

where G_U is of size $n_s \times n_s$ and G_D is of size $(n_r - n_s) \times n_s$. Then note that

$$
\begin{aligned}
C &= \mathbb{E}[\log\det(I_{n_r} + \mathrm{SNR}^{\alpha_1}\mathbf{H}_1\mathbf{H}_1^\dagger + \mathrm{SNR}^{\alpha_2}\mathbf{H}_2\mathbf{H}_2^\dagger)] && (2.44)\\
&= \mathbb{E}[\log\det(I_{n_s} + G^\dagger G)] && (2.45)\\
&= \mathbb{E}[\log\det(I_{n_s} + G_U^\dagger G_U + G_D^\dagger G_D)] && (2.46)\\
&\geq \mathbb{E}[\log\det(I_{n_s} + G_U^\dagger G_U)]. && (2.47)
\end{aligned}
$$

Now note that G_U is of size $n_s \times n_s$ and the result follows from the previous case with $n_r = n_s$. So we have

$$\begin{aligned} C &= \mathbb{E}[\log\det(I_{n_r} + \mathrm{SNR}^{\alpha_1}\mathbf{H}_1\mathbf{H}_1^{\dagger} + \mathrm{SNR}^{\alpha_2}\mathbf{H}_2\mathbf{H}_2^{\dagger})] \quad (2.48) \\ &\geq (\alpha_1 n_{t1} + \alpha_2 n_{t2}) \log \mathrm{SNR}. \quad (2.49) \end{aligned}$$

The converse follows from lemma 2.

Hence, the proof is complete.

Chapter 3

DoF region of Two-User MIMO Interference Channels

In this chapter, we will investigate the DoF region of two-user MIMO interference channel with no CSIT.

3.1 From two-user MAC Sum Rate to Two-User MIMO Interference Channel

In this section, we discuss how to extend Theorem 1 in Chapter 2 to the two-user MIMO interference channel with the following channel model

$$\underline{\mathbf{Y}}_1[n] = \mathbf{H}_{11}[n]\underline{\mathbf{X}}_1[n] + \mathbf{H}_{12}[n]\underline{\mathbf{X}}_2[n] + \underline{\mathbf{Z}}_1[n] \tag{3.1}$$

$$\underline{\mathbf{Y}}_2[n] = \mathbf{H}_{21}[n]\underline{\mathbf{X}}_1[n] + \mathbf{H}_{22}[n]\underline{\mathbf{X}}_2[n] + \underline{\mathbf{Z}}_2[n] \tag{3.2}$$

We can view the above interference channel as two two-user MACs. The receivers of the two MACs are n_{r1} and n_{r2}, respectively. For simplicity, let us denote the number of receive antennas at the receiver by n_r. Also, we will consider the use of successive-interference-cancellation (SIC) scheme for decoding. Intuitively, we decode the message with stronger power and treat the message with weaker power as noise first. After we decode the message with stronger power successfully, we can subtract it out and then decode the message with weaker power. Assume the power constraints for the two users are given by

$$\mathbb{E}\left\|\underline{\mathbf{X}}_i[n]\right\|^2 \;\leq\; \mathrm{SNR}^{\alpha_i}, \quad i = 1, 2$$

For the case of $\alpha_1 \geq \alpha_2$, we shall decode $\underline{\mathbf{X}}_1$ first. Thus the resulting rate R_1 for the first user is given by

$$R_1 \;\leq\; \mathbb{E}[\log\det(I_{n_r} + \mathrm{SNR}^{\alpha_1}\mathbf{H}_1\mathbf{H}_1^\dagger + \mathrm{SNR}^{\alpha_2}\mathbf{H}_2\mathbf{H}_2^\dagger)] - \mathbf{E}[\log\det(I_{n_r} + \mathrm{SNR}^{\alpha_1}\mathbf{H}_1\mathbf{H}_1^\dagger]$$

Hence by Theorem 1 in Chapter 2 the corresponding DoF is

$$d_1 \leq \begin{cases} \alpha_1 n_r - \alpha_2 \min(n_{t2}, n_r), & \text{if } n_r \leq n_{t1} \\ \alpha_1 n_{t1} + \alpha_2(n_r - n_{t1}) - \alpha_2 \min(n_{t2}, n_r), & \text{if } n_{t1} < n_r < n_{t1} + n_{t2} \\ \alpha_1 n_{t1} + \alpha_2 n_{t2} - \alpha_2 \min(n_{t2}, n_r), & \text{if } n_{t1} + n_{t2} \leq n_r \end{cases} \tag{3.3}$$

and then we subtract $\underline{\mathbf{X}}_1$ off and decode $\underline{\mathbf{X}}_2$. We have

$$d_2 \leq \alpha_2 \min(n_{t2}, n_r). \tag{3.4}$$

it should be noted that (3.3) and (3.4) are the DoF constraint of two-user MAC when we apply SIC at receiver.

3.1.1 Types of Two-User MIMO Interference Channel

We divide the analysis of DoF region into three different types, depending on the values of n_{t1}, n_{t2}, n_{r1} and n_{r2}. The condition $n_{r1} \leq n_{r2}$ is assumed, and the result can be easily extended to $n_{r1} > n_{r2}$. In the following, we distinguish three types:

1. $n_{t2} \leq n_{r1}$:
   In this type, it is known [4] that the absence of CSIT does not reduce the DoF region. To simplify the analysis, we divide this type into three sub-types. From the result of Theorem 1, we can divide type 1 into three sub-types naturally:

   (a) $n_{r1} \leq n_{t1}$.

   (b) $n_{t1} < n_{r1} < n_{t1} + n_{t2}$.

(c) $n_{t1} + n_{t2} \le n_{r1}$.

2. $n_{r1} < n_{t2}$ and $n_{r1} \le n_{t1}$:

   In this type, the DoF region shrinks due to the absence of CSIT. We will use power-split scheme and apply SIC to characterize the DoF region.

3. $n_{t1} < n_{r1} \le n_{t2}$:

   In this type, the absence of CSIT will also reduce the DoF region. But unfortunately, we still do not know the exact DoF region of this type. There is an outer bound given in [4]. We will use a power-split transmission scheme and apply SIC to get a DoF region, and compare it to the outer bound to see the gap between them. Finally, we use a genie-aided two-user interference channel to obtain an outer bound that is tighter than the one in [4].

We discuss the DoF region of each type in the following three sections.

3.2 DoF region when $n_{t2} \le n_{r1}$

In this section, we discuss the DoF region of the type 1, where the antenna distribution satisfies $n_{t2} \le n_{r1}$. The transmit power can be expressed as SNR^{α_1} and SNR^{α_2}, where $0 \le \alpha_1 \le 1$ and $0 \le \alpha_2 \le 1$, for transmitter 1 and transmitter 2, respectively. Intuitively, the receiver decodes the message with stronger power first and treat the other as noise, i.e., if $\alpha_2 \le \alpha_1$, we decode the $\underline{\mathbf{X}}_1$ first, both at the receiver 1 and receiver 2, else we reverse the decoding order. After we decode the message with stronger power at both receivers successfully, we can subtract it off from the received signal, hence, the remaining is the message with weaker power and we can easily decode it at its intended receiver. We discuss the three sub-types of type 1 in the following three subsections.

3.2.1 DoF region when $n_{r1} \le n_{t1}$

We have two cases in the following:

1. $\alpha_1 \geq \alpha_2$.

2. $\alpha_2 \geq \alpha_1$.

We start with the case $\alpha_1 \geq \alpha_2$. At the receiver 1, we just treat the $\underline{\mathbf{X}}_2$ as noise and decode thew $\underline{\mathbf{X}}_1$. We have the following DoF constraint of d_1:

$$
\begin{aligned}
d_1 &\leq \alpha_1 n_{r1} - \alpha_2 \min(n_{t2}, n_{r1}) && (3.5)\\
&= \alpha_1 n_{r1} - \alpha_2 n_{t2}. && (3.6)
\end{aligned}
$$

At receiver 2, we also treat the $\underline{\mathbf{X}}_2$ as noise and decode $\underline{\mathbf{X}}_1$, then we subtract $\underline{\mathbf{X}}_1$ off and decode $\underline{\mathbf{X}}_2$ at receiver 2. Since we do two decoding operations at receiver 2, we have two constraints on DoF, one for d_1 and one for d_2. We have three sub-cases for this type depending the the values of n_{r1} and n_{t2}.

1. $n_{r2} \leq n_{t1}$:

$$
\begin{aligned}
d_1 &\leq \alpha_1 n_{r2} - \alpha_2 \min(n_{t2}, n_{r2}) && (3.7)\\
&= \alpha_1 n_{r2} - \alpha_2 n_{t2}. && (3.8)
\end{aligned}
$$

2. $n_{t1} \leq n_{r2} \leq n_{t1} + n_{t2}$:

$$
\begin{aligned}
d_1 &\leq \alpha_1 n_{t1} + \alpha_2 (n_{r2} - n_{t1}) - \alpha_2 \min(n_{t2}, n_{r2}) && (3.9)\\
&= \alpha_1 n_{t1} + \alpha_2 (n_{r2} - n_{t1} - n_{t2}). && (3.10)
\end{aligned}
$$

3. $n_{t1} + n_{t2} \leq n_{r2}$:

$$
\begin{aligned}
d_1 &\leq \alpha_1 n_{t1} + \alpha_2 n_{t2} - \alpha_2 \min(n_{t2}, nr2) && (3.11)\\
&= \alpha_1 n_{t1}. && (3.12)
\end{aligned}
$$

After we decode $\underline{\mathbf{X}}_1$ at receiver 2 successfully, we can remove it and decode $\underline{\mathbf{X}}_2$, the constraint of d_2 is

$$
\begin{aligned}
d_2 &\leq \alpha_2 \min(n_{t2}, n_{r2}) && (3.13)\\
&= \alpha_2 n_{t2}. && (3.14)
\end{aligned}
$$

The above gives the DoF region of type 1.(a) when $\alpha_1 \geq \alpha_2$.

Now we consider the case when $\alpha_2 \geq \alpha_1$. We treat $\underline{\mathbf{X}}_1$ as noise, decode $\underline{\mathbf{X}}_2$ first, then subtract $\underline{\mathbf{X}}_2$ off and decode $\underline{\mathbf{X}}_1$ at receiver 1. At receiver 1, since $n_{t2} \leq n_{r1} \leq n_{t1} + n_{t2}$, we have that

$$
\begin{aligned}
d_2 &\leq \alpha_2 n_{t2} + \alpha_1(n_{r1} - nt2) - \alpha_1 \min(n_{t1}, n_{r1}) && (3.15)\\
&= (\alpha_2 - \alpha_1) n_{t2} && (3.16)\\
d_1 &\leq \alpha_1 \min(n_{t1}, n_{r1}) && (3.17)\\
&= \alpha_1 n_{r1}. && (3.18)
\end{aligned}
$$

At receiver 2, we just treat $\underline{\mathbf{X}}_1$ as noise and decode $\underline{\mathbf{X}}_2$ directly, since $n_{t2} \leq n_{r2} \leq n_{t1} + n_{t2}$ and $n_{t1} + n_{t2} \leq n_{r2}$ are both possible, we need to consider two conditions of DoF constraint.

1. $n_{t2} \leq n_{r2} \leq n_{t1} + n_{t2}$:

$$
d_2 \leq \alpha_2 n_{t2} + \alpha_1(n_{r2} + n_{t2}) - \alpha_1 \min(n_{t1}, n_{r2}). \quad (3.19)
$$

2. $n_{t1} + n_{t2} \leq n_{r2}$:

$$
\begin{aligned}
d_2 &\leq \alpha_2 n_{t2} + \alpha_1 n_{t1} - \alpha_1 \min(n_{t1}, n_{r2}) && (3.20)\\
&= \alpha_2 n_{t2}. && (3.21)
\end{aligned}
$$

These give the DoF region of type 1.(a) when $\alpha_2 \geq \alpha_1$. Now we have the complete constraints of the DoF region of type 1.(a), but how to use these constraints to characterize the exact DoF region? Here we provide a simple way to get an insight into the DoF region. First, we consider the case of $\alpha_1 \geq \alpha_2$. We try to find the corner points on the DoF region, so we consider the extreme values of (α_1, α_2), that is, $(\alpha_1, \alpha_2) = (1, 0)$ and $(\alpha_1, \alpha_2) = (1, 1)$, respectively. $(\alpha_1, \alpha_2) = (1, 0)$ means that we give almost all power to transmitter 1 only when SNR is very high. We start with the case when $\alpha_1 \geq \alpha_2$, and replace the $(\alpha_1, \alpha_2) = (1, 0)$ to equation (3.5)-(3.14), then, at receiver 1, the DoF constraint becomes

$$
d_1 \leq n_{r1}. \quad (3.22)
$$

At receiver 2, we need to decode the $\underline{\mathbf{X}}_1$ reliably and then decode $\underline{\mathbf{X}}_2$, the DoF constraints of d_1 is following:

1. $n_{r2} \leq n_{t1}$:

$$d_1 \leq n_{r2}. \tag{3.23}$$

2. $n_{t1} \leq n_{r2} \leq n_{t1} + n_{t2}$:

$$d_1 \leq n_{t1}. \tag{3.24}$$

3. $n_{t1} + n_{t2} \leq n_{r2}$

$$d_1 \leq n_{t1}. \tag{3.25}$$

And since $\alpha_2 = 0$, $d_2 \leq 0$. From equation (3.15)-(3.21), we have 4 constraints on d_1, and we know that $n_{r1} \leq n_{t1}$ and $n_{r1} \leq n_{t2}$. We can only choose the lower rate in order to have reliable communication for both two streams, so here we need

$$d_1 \leq n_{r1} \tag{3.26}$$

The above shows that $(d_1, d_2) = (n_{r1}, 0)$ is a corner point in the DoF region. Then we substitute $(\alpha_1, \alpha_2) = (1, 1)$ into equation (3.5)-(3.14). At receiver 1, we have

$$d_1 \leq n_{r1} - n_{t2}. \tag{3.27}$$

At receiver 2, we have

1. $n_{r2} \leq n_{t1}$:

$$d_1 \leq n_{r2} - n_{t2}. \tag{3.28}$$

2. $n_{t1} \leq n_{r2} \leq n_{t1} + n_{t2}$:

$$d_1 \leq n_{r2} - n_{t2}. \tag{3.29}$$

3. $n_{t1} + n_{t2} \leq n_{r2}$

$$d_1 \leq n_{t1}. \tag{3.30}$$

From (3.23)-(3.26), we can see that $d_1 \leq n_{r1} - n_{t2}$ can yield reliable communication for both streams. After we decode $\underline{\mathbf{X}}_1$ successfully, we can subtract it off at receiver 2 then decode $\underline{\mathbf{X}}_2$. The DoF constraint is:

$$d_2 \leq n_{t2}. \tag{3.31}$$

Thus we can achieve $(d_1, d_2) = (n_{r1} - n_{t2}, n_{t2})$. This is also one of the corner point.

Now we consider the case when $\alpha_2 \geq \alpha_1$. If we replace $(\alpha_1, \alpha_2) = (0, 1)$ in (3.15)-(3.21), at receiver 1 we have that

$$d_2 \leq n_{t2}. \tag{3.32}$$

Then we can decode $\underline{\mathbf{X}}_1$ by subtracting the component $\underline{\mathbf{X}}_2$ off at receiver 1, but here the power of $\underline{\mathbf{X}}_1$ is vanishingly small compared to SNR, so the DoF constraint of d_1 is

$$d_1 \leq 0. \tag{3.33}$$

At receiver 2, we treat the $\underline{\mathbf{X}}_1$ as noise and decode the $\underline{\mathbf{X}}_2$ directly, then we have

1. $n_{t2} \leq n_{r2} \leq n_{t1} + n_{t2}$:
$$d_2 \leq n_{t2} \tag{3.34}$$

2. $n_{t1} + n_{t2} \leq n_{r2}$:
$$d_2 \leq n_{t2} \tag{3.35}$$

Hence, now we have third corner point, which is $(d_1, d_2) = (0, n_{t2})$. Finally, we consider $(\alpha_1, \alpha_2) = (1, 1)$. We follow the same procedure before. We replace $(\alpha_1, \alpha_2) = (1, 1)$ in (3.15)-(3.21). For receiver 1, we have

$$d_2 \leq 0. \tag{3.36}$$

This is a surprising fact. It means that if we want to decode the $\underline{\mathbf{X}}_2$ at receiver 1 when the powers of both $\underline{\mathbf{X}}_1$ and $\underline{\mathbf{X}}_2$ are almost the same, the reliable communication rate will approach 0. With this condition, we can get $d_1 \leq n_{r1}$. But since the constraint of $d_2 \leq 0$, we cannot have a better rate for $\underline{\mathbf{X}}_2$ at receiver 2. The DoF we can achieve in this situation is only $(d_1, d_2) = (n_{r1}, 0)$, which coincides with one of the three corner

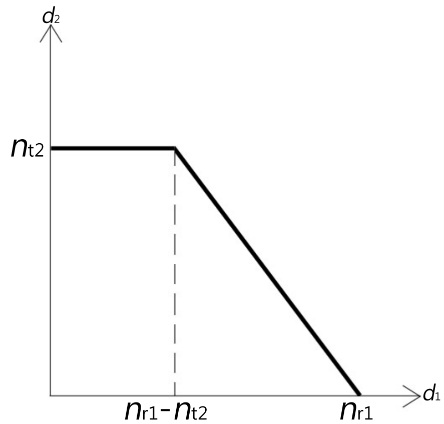

Figure 3.1: The DoF region of type 1.(a)

points before. We then use a time-sharing scheme to connect the 3 corner points, and see that this is the exact DoF region, which corresponds to the result of [4] as shown In Fig. 3.1.

Furthermore, in Fig. 3.2 we consider the case $n_{t1} = 3, n_{t2} = 1, n_{r1} = 2$ and $n_{r2} = 4$. For a given d_1, we vary the value of α_1 and α_2 to compute the greatest d_2, where $0 \le d_1 \le \min(n_{t1}, n_{r1})$, in this example $\min(n_{t1}, n_{r1}) = 2$. The DoF region is given in Fig. 3.2.

3.2.2 DoF region when $n_{t1} < n_{r1} < n_{t1} + n_{t2}$

In this subsection we discuss the DoF region of type 1.(b). We follow the similar way in the previous subsection. We can use Theorem 1 and power-split scheme to calculate the DoF constraints. Again we start with the case of $\alpha_1 \ge \alpha_2$. So we treat $\underline{\mathbf{X}}_2$ as noise and decode $\underline{\mathbf{X}}_1$. After we extract $\underline{\mathbf{X}}_1$ at receiver 2, we can apply SIC to decode $\underline{\mathbf{X}}_2$. At receiver 1, we have

$$\begin{aligned} d_1 &\le \alpha_1 n_{t1} + \alpha_2(n_{r1} - n_{t1}) - \alpha_2 \min(n_{t2}, n_{r1}) && (3.37)\\ &= \alpha_1 n_{t1} + \alpha_2(n_{r1} - n_{t1} - n_{t2}). && (3.38)\end{aligned}$$

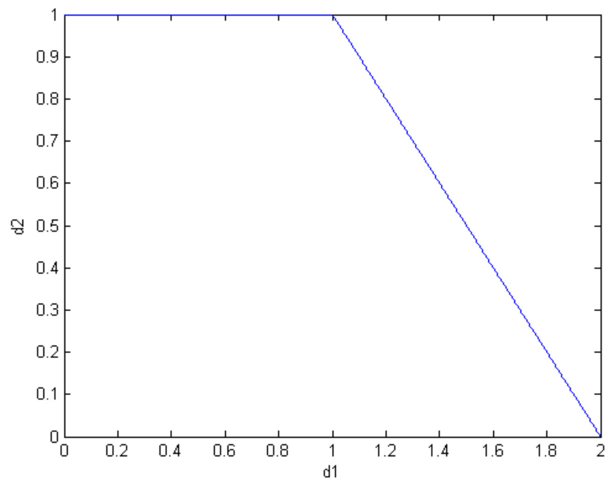

Figure 3.2: The DoF region of type 1.(a) with $n_{t1} = 3, n_{t2} = 1, n_{r1} = 2$ and $n_{r2} = 4$

At receiver 2, we have to consider the distribution of antenna being whether $n_{t1} < n_{r2} < n_{t1} + n_{t2}$ and $n_{t1} + n_{t2} \leq n_{r2}$. We decode $\underline{\mathbf{X}}_1$ and treat $\underline{\mathbf{X}}_2$ as noise. Then we have

1. $n_{t1} < n_{r2} < n_{t1} + n_{t2}$:

$$\begin{aligned} d_1 &\leq \alpha_1 n_{t1} + \alpha_2(n_{r2} - n_{t1}) - \alpha_2 \min(n_{t2}, n_{r2}) && (3.39) \\ &= \alpha_1 n_{t1} + \alpha_2(n_{r2} - n_{t1} - n_{t2}) && (3.40) \end{aligned}$$

2. $n_{t1} + n_{t2} \leq n_{r2}$:

$$\begin{aligned} d_1 &\leq (\alpha_1 n_{t1} + \alpha_2 n_{t2}) - \alpha_2 \min(n_{t2}, n_{r1}) && (3.41) \\ &= \alpha_1 n_{t1}. && (3.42) \end{aligned}$$

After we decode $\underline{\mathbf{X}}_1$ at receiver 2 successfully, we can apply SIC to decode the $\underline{\mathbf{X}}_2$. The DoF constraint of d_2 is simply

$$\begin{aligned} d_2 &\leq \alpha_2 \min(n_{t2}, n_{r2}) && (3.43) \\ &= \alpha_2 n_{t2}. && (3.44) \end{aligned}$$

Now let us consider the case when $\alpha_2 \geq \alpha_1$. In this case we decode the $\underline{\mathbf{X}}_2$ first, both at receiver 1 and receiver 2. At receiver 1, we have

$$\begin{aligned} d_2 &\leq \alpha_2 n_{t2} + \alpha_1(n_{r1} - n_{t2}) - \alpha_1 \min(n_{t1}, n_{r1}) && (3.45) \\ &= \alpha_2 n_{t2} + \alpha_1(n_{r1} - n_{t1} - n_{t2}). && (3.46) \end{aligned}$$

Then we use SIC to decode $\underline{\mathbf{X}}_1$ and have

$$d_1 \leq \alpha_1 \min(n_{t1}, n_{r1}) = \alpha_1 n_{t1}. \quad (3.47)$$

At receiver 2, we deocode $\underline{\mathbf{X}}_2$ directly, but there are two possible different antenna distributions.

1. $n_{t2} < n_{r2} < n_{t1} + n_{t2}$:

$$\begin{aligned} d_2 &\leq \alpha_2 n_{t2} + \alpha_1(n_{r2} - n_{t2}) - \alpha_1 \min(n_{t1}, n_{r2}) && (3.48) \\ &= \alpha_2 n_{t2} + \alpha_1(n_{r2} - n_{t1} - n_{t2}). && (3.49) \end{aligned}$$

2. $n_{t1} + n_{t2} \leq n_{r2}$:

$$d_2 \leq (\alpha_2 n_{t2} + \alpha_1 n_{t1}) - \alpha_1 \min(n_{t1}, n_{r2}) \quad = \quad \alpha_2 n_{t2}. \quad (3.50)$$

Now, we have the full DoF constraints of the type1.(b), then we can follow the same step in last subsection. We use the extreme values of α_1 and α_2 to find the corner point in the DoF region. The results are given below. When $\alpha_1 \geq \alpha_2$, we have

1. $(\alpha_1, \alpha_2) = (1, 0)$:

$$\begin{aligned} d_1 &\leq n_{t1} && (3.51) \\ d_2 &\leq 0. && (3.52) \end{aligned}$$

2. $(\alpha_1, \alpha_2) = (1, 1)$:

$$\begin{aligned} d_1 &\leq n_{r1} - n_{t2} && (3.53) \\ d_2 &\leq n_{t2}. && (3.54) \end{aligned}$$

When $\alpha_2 \ge \alpha_1$, we have

1. $(\alpha_1, \alpha_2) = (0, 1)$:

$$\begin{aligned} d_1 &\le 0 && (3.55) \\ d_2 &\le n_{t2}. && (3.56) \end{aligned}$$

2. $(\alpha_1, \alpha_2) = (1, 1)$:

$$\begin{aligned} d_1 &\le n_{t1} && (3.57) \\ d_2 &\le n_{r1} - n_{t1}. && (3.58) \end{aligned}$$

From (3.51)-(3.58), we have the four corner points of the DoF region for type 1.(b). Then we can apply the time-sharing scheme to connect the four corner points to get the entire DoF region for type 1.(b). This DoF region is the same as in [4]. Fig. 3.3 shows the DoF region of type 1.(b)

We also consider an example of type 1.(b)by setting $n_{t1} = 3, n_{t2} = 2, n_{r1} = 4$ and $n_{r2} = 4$. Fig. 3.4 shows the DoF region of type 1.(b) by the computer calculation.

3.2.3 DoF region when $n_{t1} + n_{t2} \le n_{r1}$

In this subsection, we discuss the DoF region of type 1.(c). We use the similar way as in the previous two subsections. We start with the case $\alpha_1 \ge \alpha_2$, and then the case $\alpha_2 \ge \alpha_1$. Type 1.(c) is relatively simple since we have both $n_{t1} + n_{t2} \le n_{r1}$ and $n_{t1} + n_{t2} \le n_{r2}$.

When $\alpha_1 \ge \alpha_2$, we decode $\underline{\mathbf{X}}_1$ first and then apply SIC to cancel out the component of $\underline{\mathbf{X}}_1$ and decode $\underline{\mathbf{X}}_2$. At receiver 1, we have

$$\begin{aligned} d_1 &\le \alpha_1 n_{t1} + \alpha_2 n_{t2} - \alpha_2 \min(n_{t2}, n_{r1}) && (3.59) \\ &= \alpha_1 n_{t1}. && (3.60) \end{aligned}$$

At receiver 2, we also decode $\underline{\mathbf{X}}_1$ first, and then use SIC to remove the component of

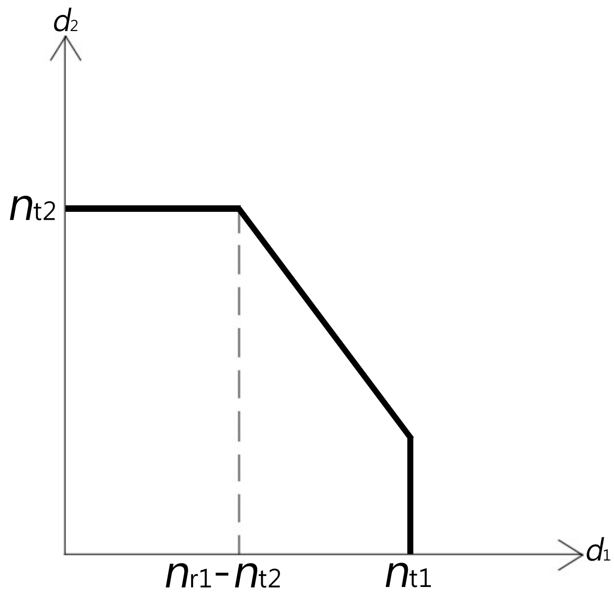

Figure 3.3: The DoF region of type 1.(b)

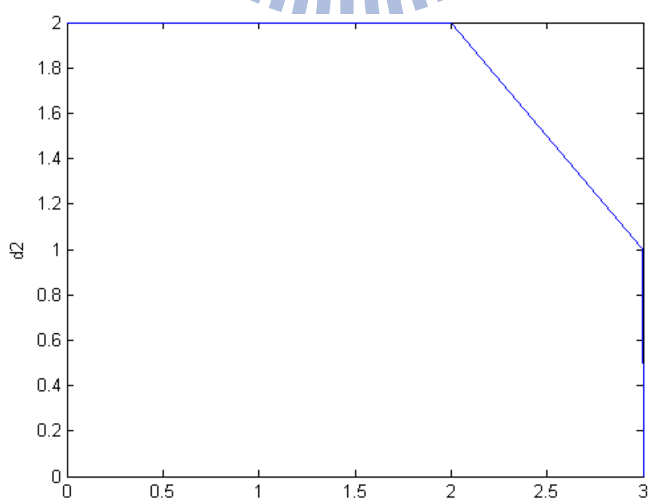

Figure 3.4: The DoF region result of type 1.(b) with $n_{t1} = 3, n_{t2} = 2, n_{r1} = 4$ and $n_{r2} = 4$

$\underline{\mathbf{X}}_1$, then we can decode $\underline{\mathbf{X}}_2$, the DoF constraint at receiver 2 is below.

$$\begin{aligned} d_1 &\le \alpha_1 n_{t1} + \alpha_2 n_{t2} - \alpha_2 \min(n_{t2}, n_{r2}) && (3.61)\\ &= \alpha_1 n_{t1}. && (3.62)\end{aligned}$$

After we decode $\underline{\mathbf{X}}_1$ successfully and then apply SIC, we have

$$d_2 \le \alpha_2 \min(n_{t2}, n_{r2}) = \alpha_2 n_{t2}. \tag{3.63}$$

Now we consider the case when $\alpha_2 \ge \alpha_1$. We decode $\underline{\mathbf{X}}_2$ first and apply SIC at receiver 1. At receiver 1, we have

$$\begin{aligned} d_2 &\le \alpha_2 n_{t2} + \alpha_1 n_{t1} - \alpha_2 \min(n_{t1}, n_{r1}) && (3.64)\\ &= \alpha_1 n_{t2}. && (3.65)\end{aligned}$$

Then we apply SIC to decode $\underline{\mathbf{X}}_2$. The result is simply

$$\begin{aligned} d_1 &\le \alpha_1 \min(n_{t1}, n_{r1}) && (3.66)\\ &= \alpha_1 n_{t1}. && (3.67)\end{aligned}$$

At receiver 2, we have

$$\begin{aligned} d_2 &\le \alpha_2 n_{t2} + \alpha_1 n_{t1} - \alpha_2 \min(n_{t1}, n_{r2}) && (3.68)\\ &= \alpha_1 n_{t2}. && (3.69)\end{aligned}$$

Now we have the entire DoF constraint of type 1.(c). We then use the extreme values of α_1 and α_2 to find the corner points of DoF region. Both the case $\alpha_1 \ge \alpha_2$ and $\alpha_2 \ge \alpha_1$ have the same corner points. The result is below. When $\alpha_1 \ge \alpha_2$, we have

1. $(\alpha_1, \alpha_2) = (1, 0)$:

$$\begin{aligned} d_1 &\le n_{t1} && (3.70)\\ d_2 &\le 0. && (3.71)\end{aligned}$$

2. $(\alpha_1, \alpha_2) = (1, 1)$:

$$d_1 \leq n_{t1} \tag{3.72}$$

$$d_2 \leq n_{t2}. \tag{3.73}$$

When $\alpha_2 \geq \alpha_1$, we have

1. $(\alpha_1, \alpha_2) = (0, 1)$:

$$d_1 \leq 0 \tag{3.74}$$

$$d_2 \leq n_{t2}. \tag{3.75}$$

2. $(\alpha_1, \alpha_2) = (1, 1)$:

$$d_1 \leq n_{t1} \tag{3.76}$$

$$d_2 \leq n_{t2}. \tag{3.77}$$

From the above result, we can clearly see that $(d_1, d_2) = (n_{t1}, 0), (0, n_{t2})$ and (n_{t1}, n_{t2}) are the three corner points on the DoF region. Thus, the DoF region is a rectangle. This is the same as in [4]. Fig. 3.5 shows the DoF of type 1.(c).

An example of type 1.(c) is a two-user MIMO interference channel with $n_{t1} = 2, n_{t2} = 2, n_{r1} = 4$ and $n_{r2} = 5$. Fig. 3.6 shows the DoF region of type 1.(c) by the computer calculation.

3.3 DoF region when $n_{r1} < n_{t2}$ and $n_{r1} \leq n_{t1}$

In this section, we discuss the type 2, in this type, we still have the exact DoF region when the CSIT is not available. Also, we consider two cases, $\alpha_1 \geq \alpha_2$ and $\alpha_2 \geq \alpha_1$, respectively. If $\alpha_1 \geq \alpha_2$, we decode $\underline{\mathbf{X}}_1$ first at both and then apply SIC at receiver 2 to decode $\underline{\mathbf{X}}_2$. At receiver 1, we have $n_{r1} \leq n_{t1}$, so the DoF constraint of d_1 is

$$\begin{aligned} d_1 &\leq \alpha_1 n_{r1} - \alpha_2 \min(n_{t2}, n_{r1}) && (3.78) \\ &= \alpha_1 n_{r1} - \alpha_2 n_{r1}. && (3.79) \end{aligned}$$

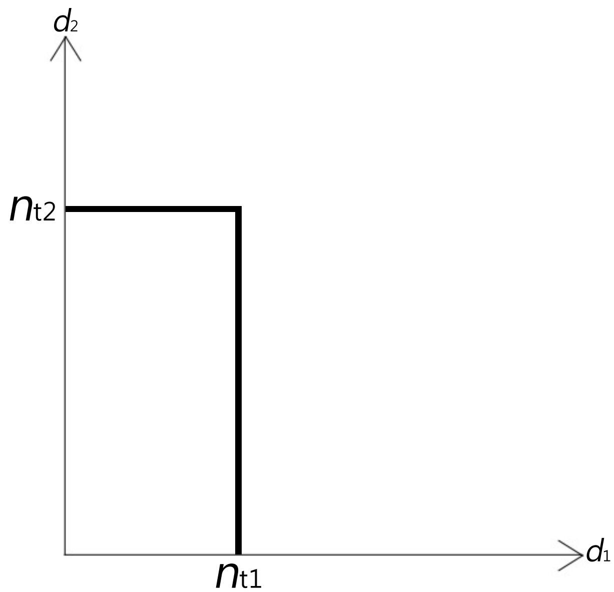

Figure 3.5: The DoF region of type 1.(c)

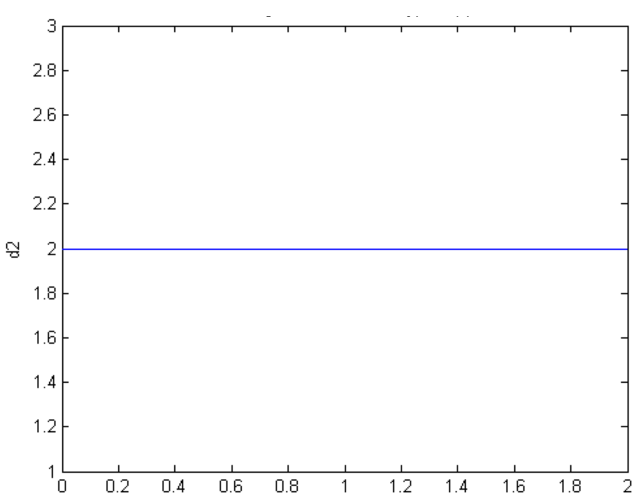

Figure 3.6: The DoF region of type 1.(c) with $n_{t1} = 2, n_{t2} = 2, n_{r1} = 4$ and $n_{r2} = 5$

At receiver 2, unfortunately, $n_{r2} \leq n_{t1}$, $n_{t1} < n_{r2} < n_{t1} + n_{t2}$ and $n_{t1} + n_{t2} \leq n_{r2}$ are all possible in type 2, so we need to give DoF constraint for three different antenna distributions.

1. $n_{r2} \leq n_{t1}$:

$$d_1 \leq \alpha_1 n_{r2} - \alpha_2 \min(n_{t2}, n_{r2}). \tag{3.80}$$

2. $n_{t1} < n_{r2} < n_{t1} + n_{t2}$:

$$d_1 \leq \alpha_1 n_{t1} + \alpha_2(n_{r2} - n_{t1}) - \alpha_2 \min(n_{t2}, n_{r2}). \tag{3.81}$$

3. $n_{t1} + n_{t2} \leq n_{r2}$:

$$\begin{aligned} d_1 &\leq \alpha_1 n_{t1} + \alpha_2 n_{t2} - \alpha_2 \min(n_{t2}, n_{r2}) && (3.82) \\ &= \alpha_1 n_{t1}. && (3.83) \end{aligned}$$

After we decode $\underline{\mathbf{X}}_1$ successfully at receiver 2, we apply SIC and decode $\underline{\mathbf{X}}_2$. The DoF constraint is

$$d_2 \leq \alpha_2 \min(n_{t2}, n_{r2}). \tag{3.84}$$

Note that we have $n_{t2} \leq n_{r2}$ only when $n_{t1} + n_{t2} \leq n_{r2}$. Then we consider the case when $\alpha_2 \geq \alpha_1$. We decode $\underline{\mathbf{X}}_2$ first at both receivers. At receiver 1, we have

$$\begin{aligned} d_2 &\leq \alpha_2 n_{r1} - \alpha_1 \min(n_{t1}, n_{r1}) && (3.85) \\ &= \alpha_2 n_{r1} - \alpha_1 n_{r1}. && (3.86) \end{aligned}$$

After we decode $\underline{\mathbf{X}}_2$ successfully at receiver 1, we can apply SIC and decode $\underline{\mathbf{X}}_1$. The DoF constraint is

$$\begin{aligned} d_1 &\leq \alpha_1 \min(n_{t1}, n_{r1}) && (3.87) \\ &= \alpha_1 n_{r1}. && (3.88) \end{aligned}$$

At receiver 2, we have

1. $n_{r2} \leq n_{t2}$:

$$d_2 \leq \alpha_2 n_{r2} - \alpha_1 \min(n_{t1}, n_{r2}). \tag{3.89}$$

2. $n_{t2} < n_{r2} < n_{t1} + n_{t2}$:

$$d_2 \le \alpha_2 n_{t2} + \alpha_1(n_{r2} - n_{t2}) - \alpha_1 \min(n_{t1}, n_{r2}). \tag{3.90}$$

3. $n_{t1} + n_{t2} \le n_{r2}$:

$$\begin{aligned} d_2 &\le \alpha_1 n_{t2} + \alpha_1 n_{t1} - \alpha_1 \min(n_{t1}, n_{r2}) && (3.91) \\ &= \alpha_2 n_{t2}. && (3.92) \end{aligned}$$

Note that again we have $n_{t1} \le n_{r2}$ only when $n_{t1} + n_{t2} \le n_{r2}$, otherwise we do not know whether $n_{t1} \le n_{r2}$ or $n_{r2} \le n_{t1}$. So far, we have all DoF constraints of the type 2, then we can replace the extreme values to above DoF constraints to get the corner points in the DoF region. We skip the calculations and show the results directly. When $\alpha_1 \ge \alpha_2$, we have

1. $(\alpha_1, \alpha_2) = (1, 0)$:

$$\begin{aligned} d_1 &\le n_{r1} && (3.93) \\ d_2 &\le 0. && (3.94) \end{aligned}$$

2. $(\alpha_1, \alpha_2) = (1, 1)$:

$$\begin{aligned} d_1 &\le 0 && (3.95) \\ d_2 &\le \min(n_{t2}, n_{r2}). && (3.96) \end{aligned}$$

When $\alpha_2 \ge \alpha_1$, we have

1. $(\alpha_1, \alpha_2) = (0, 1)$:

$$\begin{aligned} d_1 &\le 0 && (3.97) \\ d_2 &\le n_{r1}. && (3.98) \end{aligned}$$

2. $(\alpha_1, \alpha_2) = (1, 1)$:

$$\begin{aligned} d_1 &\le n_{r1} && (3.99) \\ d_2 &\le 0. && (3.100) \end{aligned}$$

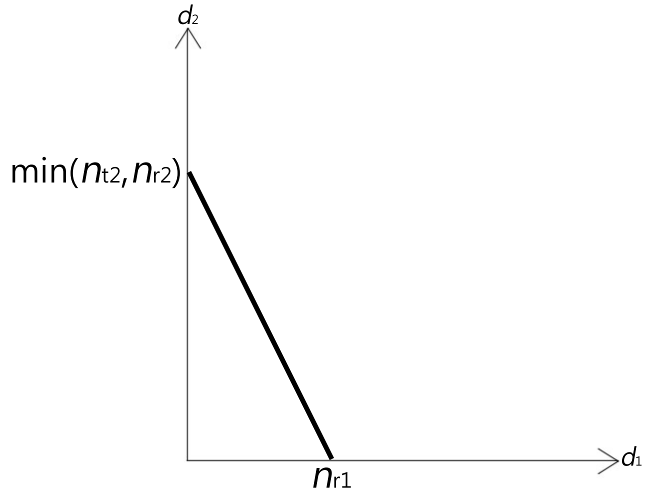

Figure 3.7: The DoF region of type 2

We see that (3.96) and (3.98) give different DoF constraints for d_2 when $d_1 \leq 0$. Since we always try to maximize the rate, we should choose (3.96) as our rate constraint for d_2, because $\min(n_{t2}, n_{r2}) \geq n_{r1}$. Thus we have two corner points in DoF region here, $(d_1, d_2) = (n_{r1}, 0)$ and $(0, \min(n_{t2}, n_{r2}))$. We can use time-sharing scheme to connect these two corner points to get the exact DoF region, which corresponds to the result in [4]

In type 2, we can see that if we want to achieve the maximum value of d_1, we need d_2 to be zero, and vice versa. The entirely DoF region is given by time sharing scheme between the maximum of d_1 and d_2. The shape of DoF region is a triangle. Fig. 3.7 shows the DoF region of type 2.

An example of type 2 is a two-user MIMO interference channel with $n_{t1} = 3, n_{t2} = 4, n_{r1} = 2$ and $n_{r2} = 4$. Fig. 3.8 shows the DoF region of type 2 by the computer calculation.

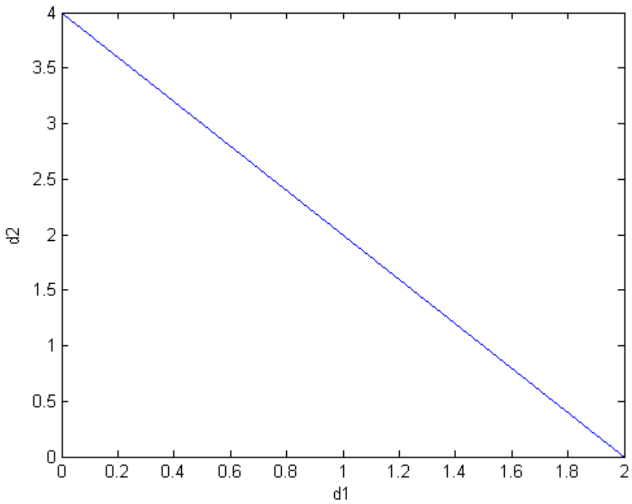

Figure 3.8: The DoF region of type 2 with $n_{t1} = 3, n_{t2} = 4, n_{r1} = 2$ and $n_{r2} = 4$

3.4 DoF region when $n_{r1} < n_{t2}$ and $n_{t1} < n_{r1}$

In this section, we consider the DoF region of type 3, note that type 2 and type 3 are only differ by $n_{r1} \leq n_{t1}$ or $n_{r1} > n_{t1}$. From [4], there is only an outer bound for this type.

In type 3, we no longer decode the stream with higher power first. Instead, we will consider two decoding strategies at both receivers, regardless of the power of the two streams. For user 1, we want to transmit $\underline{\mathbf{X}}_1$ to receiver 1 reliably, so we have that at receiver 1:

1. Treating $\underline{\mathbf{X}}_2$ as noise and decoding $\underline{\mathbf{X}}_1$ directly.

2. First treating $\underline{\mathbf{X}}_1$ as noise and decoding $\underline{\mathbf{X}}_2$, then applying SIC to decode $\underline{\mathbf{X}}_1$.

For user 2, the concept is same and at receiver 2 we have

1. Treating $\underline{\mathbf{X}}_1$ as noise and decoding $\underline{\mathbf{X}}_2$ directly.

2. First treating $\underline{\mathbf{X}}_2$ as noise and decoding $\underline{\mathbf{X}}_1$, then applying SIC to decode $\underline{\mathbf{X}}_2$.

Each receiver should choose the best decoding strategy for a given power allocation and antenna distribution. At receiver 1, if we decode the $\underline{\mathbf{X}}_1$ directly, we have

1. $n_{t1} < n_{r1} < n_{t1} + n_{t2}$:

$$d_1 \leq \alpha_1 n_{t1} + \alpha_2(n_{r1} - n_{t1}) - \alpha_2 \min(n_{t2}, n_{r1}). \tag{3.101}$$

2. $n_{t1} + n_{t2} \leq n_{r1}$:

$$d_1 \leq \alpha_1 n_{t1} + \alpha_2 n_{t2} - \alpha_2 \min(n_{t2}, n_{r1}). \tag{3.102}$$

If we decode the $\underline{\mathbf{X}}_2$ first and then apply SIC to decode $\underline{\mathbf{X}}_1$, the DoF constraint of d_2 is

$$d_2 \leq \alpha_2 n_{r1} - \alpha_1 \min(n_{t1}, n_{r1}) \tag{3.103}$$

and d_1 is simply

$$d_1 \leq \alpha_1 \min(n_{t1}, n_{r1}). \tag{3.104}$$

At receiver 2, we also have two decoding strategies, but the situation becomes more complicated since we do not know $n_{r2} \leq n_{t2}$ or $n_{t2} < n_{r2}$, thus we need to list all the possible conditions. We consider the strategy of decoding $\underline{\mathbf{X}}_2$ directly first:

1. $n_{r2} \leq n_{t2}$:

$$d_2 \leq \alpha_2 n_{r2} - \alpha_1 \min(n_{t1}, n_{r2}). \tag{3.105}$$

2. $n_{t2} < n_{r2} < n_{t1} + n_{t2}$:

$$d_2 \leq \alpha_2 n_{t2} + \alpha_2(n_{r2} - n_{t2}) - \alpha_1 \min(n_{t1}, n_{r2}). \tag{3.106}$$

3. $n_{t1} + n_{t2} < n_{r2}$:

$$d_2 \leq \alpha_2 n_{t2} + \alpha_1 n_{t1} - \alpha_1 \min(n_{t1}, n_{r2}). \tag{3.107}$$

The above are the three conditions when we decode $\underline{\mathbf{X}}_2$ directly at receiver 2. Now we consider the strategy of decoding the $\underline{\mathbf{X}}_1$ first and then applying SIC to decode $\underline{\mathbf{X}}_2$. The DoF constraint of d_1 is:

1. $n_{t1} < n_{r2} < n_{t1} + n_{t2}$:

$$d_1 \leq \alpha_1 n_{t1} + \alpha_2(n_{r2} - n_{t1}) - \alpha_2 \min(n_{t2}, n_{r2}). \tag{3.108}$$

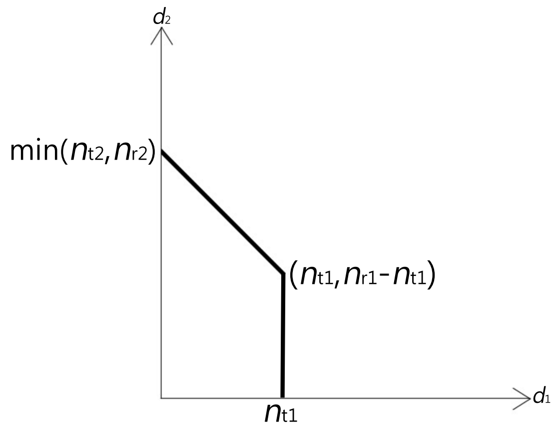

Figure 3.9: The DoF region of type 3

2. $n_{t1} + n_{t2} \le n_{r2}$:

$$d_1 \le \alpha_1 n_{t1} + \alpha_2 n_{t2} - \alpha_2 \min(n_{t2}, n_{r2}) \tag{3.109}$$

and d_2 is simply

$$d_2 \le \alpha_2 \min(n_{t2}, n_{r2}). \tag{3.110}$$

The above (3.101)-(3.110) is the whole DoF constraints of the type 3. From the work in [4], we have an outer bound of DoF region, denoted as $\mathbf{D}_{out}$,

$$\mathbf{D}_{out} = \left\{ (d_1, d_2) \in \mathbb{R}_2^+ : \frac{d_1}{n_{r1}} + \frac{d_2}{\min(n_{t2}, n_{r2})} \right\}. \tag{3.111}$$

But this outer bound seems loose. Consider a genie-aided two-user MIMO interference channel, by which we mean there is a genie at both receivers, informing the receiver the interference from the unintended user completely. The point $(d_1, d_2) = (n_{r1}, 0)$ is still not achievable. We can achieve only $(d_1, d_2) = (n_{t1}, 0)$ since $n_{t1} < n_{r1}$. The DoF region of the two-user MIMO interference channel must be bounded by the one of the genie-aided two-user MIMO interference channel, so we can make the outer bound tighter by the genie-aided 2-user MIMO interference channel. This is a new outer bound for type 3. Fig. 3.9 shows the DoF region of type 3.

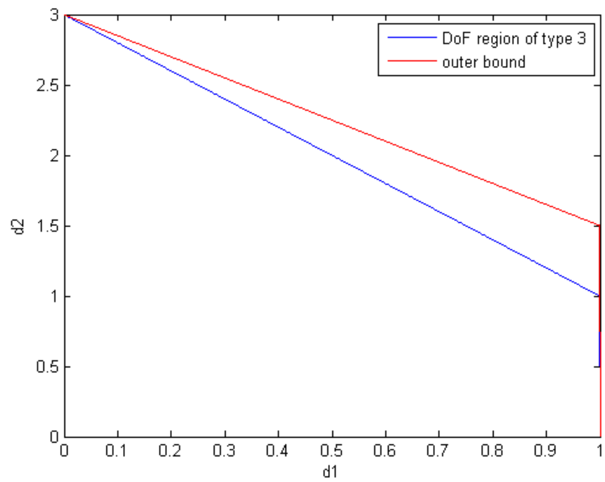

Figure 3.10: The DoF region of type 3 with $n_{t1} = 1, n_{t2} = 3, n_{r1} = 2$ and $n_{r2} = 4$ compared to the new outer bound

An example of type 3 is $n_{t1} = 1, n_{t2} = 3, n_{r1} = 2$ and $n_{r2} = 4$. Fig. 3.10 shows the gap between our DoF region and the new outer bound.

Although we do not know that for a two-user Rayleigh fading channel, the gap between the DoF region and new outer bound is achievable or not, the work in [9] had already shown that for a two-user isotropic and independent (or block-wise independent) fading channel, the exact DoF region of this type is just the same as the DoF region in our work, which implies that the gap between the DoF region and the new outer bound may not be achievable, no matter whether beamforming or interference alignment is used.

Chapter 4

DoF region of Two-User MIMO Interference Channels with Perfect Receiver Cooperation

In this chapter, we investigate the DoF region of two-user MIMO interference channel with perfect receiver cooperation and no CSIT. Here, perfect receiver cooperation means that two receivers have a noiseless and interference-free link to communicate with each other. For example, if we decode $\underline{\mathbf{X}}_1$ successfully at receiver 1, then we can send it to receiver 2, so receiver 2 can know $\underline{\mathbf{X}}_1$ completely and remove the component of $\underline{\mathbf{X}}_1$ perfectly. The remaining signal is only $\underline{\mathbf{X}}_2$ plus noise, as the interference from $\underline{\mathbf{X}}_1$ has been removed. We can also use Theorem 1 in Chapter 2 for the general MAC sum rate to analyze the DoF region in this chapter. We then compare the results in the previous chapter, where we do not have any cooperation. We use the same way to divide the antenna distributions into three types. The three types are listed below.

1. $n_{t2} \leq n_{r1}$
   (a) $n_{r1} \leq n_{t1}$
   (b) $n_{t1} < n_{r1} < n_{t1} + n_{t2}$
   (c) $n_{t1} + n_{t2} \leq n_{r1}$

2. $n_{r1} < n_{t2}$ and $n_{r1} \leq n_{t1}$

3. $n_{r1} < n_{t2}$ and $n_{t1} \leq n_{r2}$

The condition $n_{r1} \leq n_{r2}$ is assumed, and the result can be easily extended to $n_{r1} > n_{r2}$. We discuss the DoF region of each type in the following three sections.

4.1 DoF region with perfect receiver cooperation when $n_{t2} \leq n_{r1}$

In this section, we consider the DoF region of type 1. The transmit power of transmitter 1 and transmitter 2 are SNR^{α_1} and SNR^{α_2}, respectively. Note that $0 \leq \alpha_1 \leq 1$ and $0 \leq \alpha_2 \leq 1$. We decode the message with stronger power at its intended receiver first. Since the two receivers have perfect cooperation, we can transmit the decoded message(the stronger power message) to the other receivers. Thus the interference can be remove easily. We discuss the three sub-types of type 1 in the following three subsections

4.1.1 DoF region with perfect receiver cooperation when $n_{r1} \leq n_{t1}$

We have two cases in the following

1. $\alpha_1 \geq \alpha_2$.

2. $\alpha_2 \geq \alpha_1$.

We start with the case $\alpha_1 \geq \alpha_2$. At receiver 1, we treat $\underline{\mathbf{X}}_2$ as noise and decode $\underline{\mathbf{X}}_1$ directly. The constraint of d_1 is

$$\begin{aligned} d_1 &\leq \alpha_1 n_{r1} - \alpha_2 \min(n_{t2}, n_{r1}) && (4.1) \\ &= \alpha_1 n_{r1} - \alpha_2 n_{t2}. && (4.2) \end{aligned}$$

If d_1 satisfies the above constraint, we can decode $\underline{\mathbf{X}}_1$ reliably at receiver 1. Since we assume that the two receivers have perfect cooperation, the receiver 2 can know $\underline{\mathbf{X}}_1$. Thus receive 2 can subtract it off, then we can decode $\underline{\mathbf{X}}_2$ at receiver 2. The DoF constraint of $\underline{\mathbf{X}}_2$ is

$$\begin{aligned} d_2 &\leq \alpha_2 \min(n_{t2}, n_{r2}) && (4.3) \\ &= \alpha_2 n_{t2}. && (4.4) \end{aligned}$$

Let us consider the case $\alpha_2 \geq \alpha_1$ now. We decode $\underline{\mathbf{X}}_2$ at receiver 2 first. Here, since $n_{t2} < n_{r2} < n_{t1} + n_{t2}$ and $n_{t1} + n_{t2} \leq n_{r2}$ are both possible, we have to consider both of them. If $n_{t2} < n_{r2} < n_{t1} + n_{t2}$, at receiver 2, we have the constraint of d_2 is

$$d_2 \leq \alpha_2 n_{t2} + \alpha_1 (n_{r2} - n_{t2}) - \alpha_1 \min(n_{t2}, n_{r2}). \quad (4.5)$$

If $n_{t1} + n_{t2} \leq n_{r2}$, the constraint of d_2 is:

$$\begin{aligned} d_2 &\leq \alpha_2 n_{t2} + \alpha_1 n_{t1} - \alpha_1 \min(n_{t1}, n_{r2}) && (4.6) \\ &= \alpha_2 n_{t2}. && (4.7) \end{aligned}$$

By the receiver cooperation, receiver 1 can easily subtract the component of $\underline{\mathbf{X}}_2$. The DoF constraint of d_1 is

$$\begin{aligned} d_1 &\leq \alpha_1 \min(n_{t1}, n_{r1}) && (4.8) \\ &= \alpha_1 n_{r1}. && (4.9) \end{aligned}$$

We see that if $n_{t1} + n_{t2} \leq n_{r2}$, we can achieve $d_1 = n_{r1}$ and $d_2 = n_{t2}$ simultaneously. This DoF pair is the maximum of the two-user MIMO interference channel. If $n_{t2} < n_{r2} < n_{t1} + n_{t2}$, we can achieve $(d_1, d_2) = (n_{r1}, n_{r2} - \min(n_{t1}, n_{r2}))$. This result is better than no cooperation in section 3.1.1. Fig. 4.1 shows the DoF region of type 1.(a), both no cooperation and perfect receiver cooperation are considered.

An example of type 1.(a) is a two-user MIMO interference channel with $n_{t1} = 3, n_{t2} = 1, n_{r1} = 2$ and $n_{r2} = 4$. Fig. 4.2 shows the DoF region of type 1.(a) with perfect receiver cooperation by computer calculation.

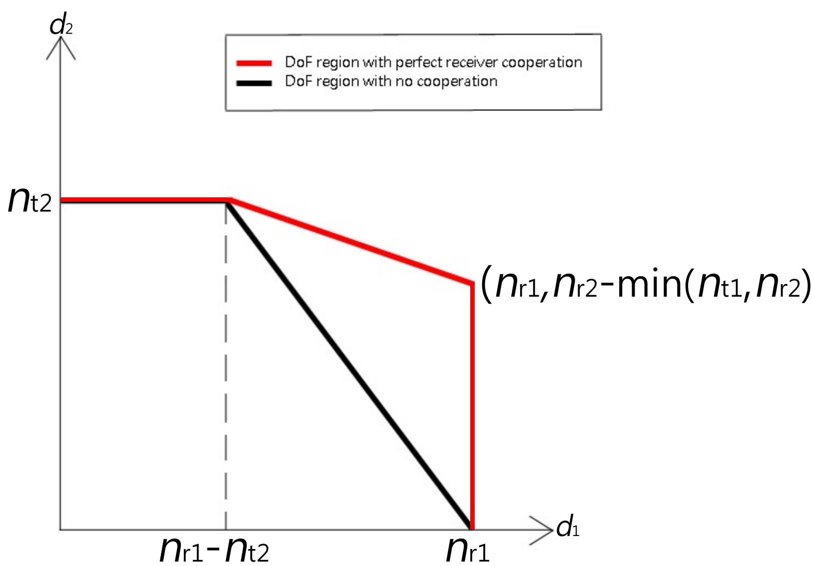

Figure 4.1: The DoF region of the DoF region of type 1.(a) with perfect receiver cooperation compared to the DoF region with no cooperation.

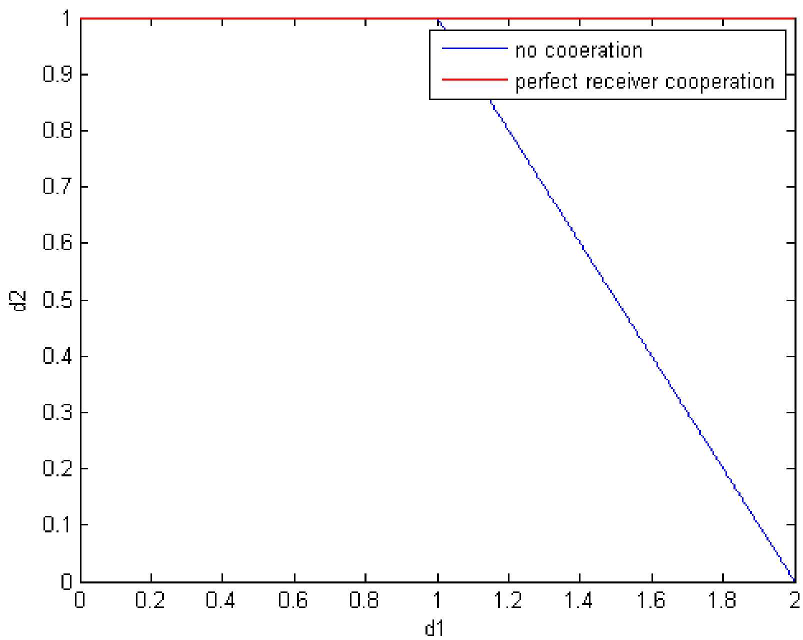

Figure 4.2: The DoF region of type 1.(a) with $n_{t1} = 3, n_{t2} = 1, n_{r1} = 2$ and $n_{r2} = 4$

4.1.2 DoF region with perfect receiver cooperation when $n_{t1} < n_{r1} < n_{t1} + n_{t2}$

In this subsection, we discuss the DoF region of type 1.(b). The method is just the same as previous subsection. Again we start with the case $\alpha_1 \geq \alpha_2$. At receiver 1, we have the constraint of d_1 is

$$\begin{aligned} d_1 &\leq \alpha_1 n_{t1} + \alpha_2(n_{r1} - n_{t1}) - \alpha_2 \min(n_{t2}, n_{r1}) && (4.10) \\ &= \alpha_1 n_{t1} + \alpha_2(n_{r1} - n_{t1} - n_{t2}). && (4.11) \end{aligned}$$

After we decode $\underline{\mathbf{X}}_1$ at receiver 1 successfully, the receiver 2 can know $\underline{\mathbf{X}}_1$ by the perfect receiver cooperation. Thus we can subtract $\underline{\mathbf{X}}_1$ off at receiver 2 and then decode $\underline{\mathbf{X}}_2$, the constraint of d_2 is

$$\begin{aligned} d_2 &\leq \alpha_2 \min(n_{t2}, n_{r2}) && (4.12) \\ &= \alpha_2 n_{t2}. && (4.13) \end{aligned}$$

Now we consider the case $\alpha_2 \geq \alpha_1$. In this case we simply decode $\underline{\mathbf{X}}_2$ at receiver 2 first. But here $n_{t2} < n_{r2} < n_{t1} + n_{t2}$ and $n_{t1} + n_{t2} \leq n_{r2}$ are both possible, we need to consider the two possibilities, respectively. If $n_{t2} < n_{r2} < n_{t1} + n_{t2}$, the constraint of d_2 is

$$\begin{aligned} d_2 &\leq \alpha_2 n_{t2} + \alpha_1(n_{r2} - n_{t2}) - \alpha_1 \min(n_{t1}, n_{r2}) && (4.14) \\ &= \alpha_2 n_{t2} + \alpha_1(n_{r2} - n_{t1} - n_{t2}). && (4.15) \end{aligned}$$

If $n_{t1} + n_{t2} \leq n_{r2}$, the constraint of d_2 is

$$\begin{aligned} d_2 &\leq \alpha_2 n_{t2} + \alpha_1 n_{t1} - \alpha_1 \min(n_{t1}, n_{r2}) && (4.16) \\ &= \alpha_2 n_{t2}. && (4.17) \end{aligned}$$

After we decode $\underline{\mathbf{X}}_2$ at receiver 2 successfully, receiver 1 can know $\underline{\mathbf{X}}_2$ by the perfect receiver cooperation. Thus the constraint of d_1 is

$$\begin{aligned} d_1 &\leq \alpha_1 \min(n_{t1}, n_{r1}) && (4.18) \\ &= \alpha_1 n_{t1}. && (4.19) \end{aligned}$$

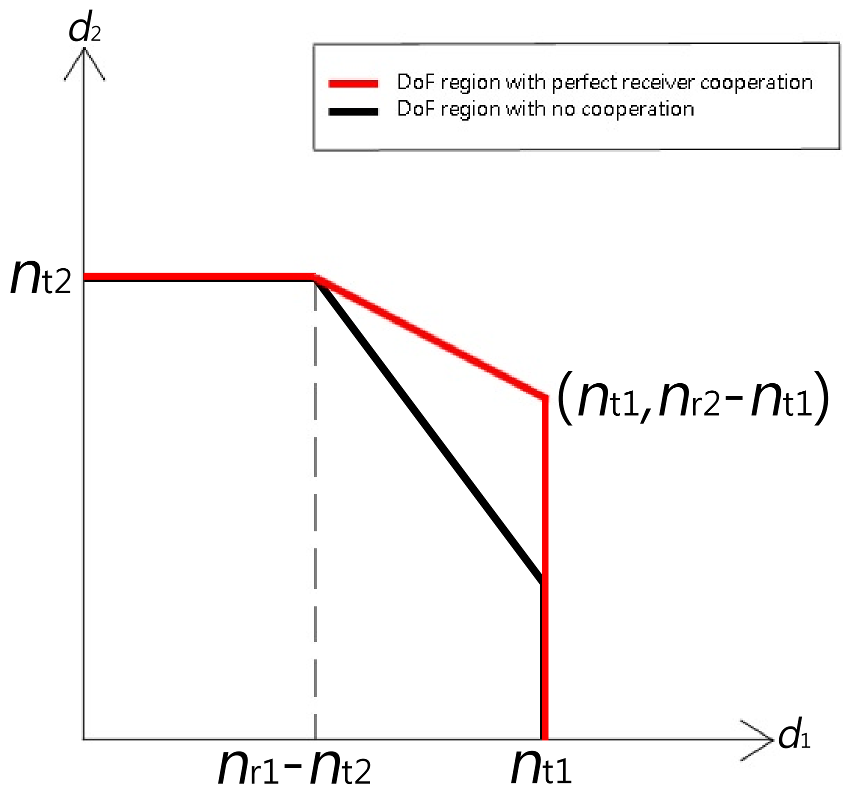

Figure 4.3: The DoF region of the DoF region of type 1.(b) with perfect receiver cooperation compared to the DoF region with no cooperation.

We can see that the result is almost the same as type 1.(a). The only difference is that when $n_{t2} < n_{r2} < n_{t1}+n_{t2}$. We know $\min(n_{t1}, n_{r2}) = n_{t1}$ in type 1.(b) but do not know in type 1.(a). If $n_{t2} < n_{r2} < n_{t1} + n_{t2}$, we can achieve $(d_1, d_2) = (n_{t1}, n_{r2} - n_{t1})$. This is better than $(d_1, d_2) = (n_{t1}, n_{r1} - n_{t1})$, the result of no cooperation. If $n_{t1}+n_{t2} \leq n_{r2}$, we can achieve $(d_1, d_2) = (n_{t1}, n_{t2})$, and both the two stream can achieve their maximum DoF. Fig. 4.3 shows the DoF region of type 1.(b), with and without receiver cooperation.

An example of type 1.(b) is a two-user MIMO interference channel with n_{t1} =

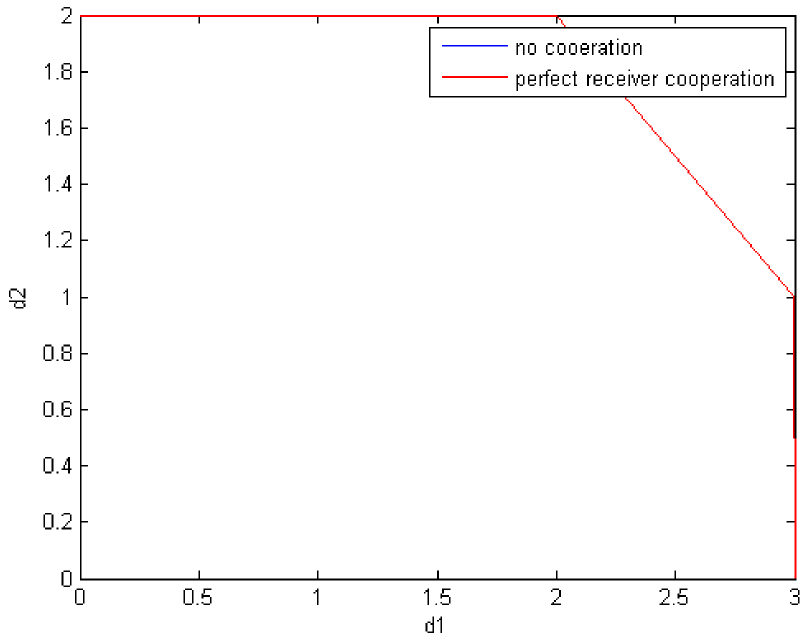

Figure 4.4: The DoF region of type 1.(b)

$3, n_{t2} = 2, n_{r1} = 4$ and $n_{r2} = 4$. Fig. 4.4 shows the DoF region of type 1.(b) with perfect receiver cooperation by computer calculation. In this example, we can see that the DoF regions of no cooperation and with perfect receiver cooperation are the same, since $n_{r1} = n_{r2} = 4$.

4.1.3 DoF region with perfect receiver cooperation when $n_{t1} + n_{t2} \le n_{r1}$

In this subsection, we discuss the DoF region of type 1.(c). Again we start with the case $\alpha_1 \le \alpha_2$. At receiver 1, we treat $\underline{\mathbf{X}}_2$ as noise and decode $\underline{\mathbf{X}}_1$ directly. Thus we have the constraint of d_1 is

$$\begin{aligned} d_1 &\le \alpha_1 n_{t1} + \alpha_2 n_{t2} - \alpha_2 \min(n_{t2}, n_{r1}) && (4.20) \\ &= \alpha_1 n_{t1}. && (4.21) \end{aligned}$$

After we decode $\underline{\mathbf{X}}_1$ at receiver 1 successfully, the receiver 2 knows $\underline{\mathbf{X}}_1$ by the perfect receiver cooperation. Then we have the constraint of d_2 is

$$\begin{aligned} d_2 &\leq \alpha_2 \min(n_{t2}, n_{r2}) &(4.22)\\ &= \alpha_2 n_{t2}. &(4.23) \end{aligned}$$

Now, let us consider the case $\alpha_2 \geq \alpha_1$. In this case, we decode $\underline{\mathbf{X}}_2$ at receiver 2 first, so we have the constraint of d_2 is

$$\begin{aligned} d_2 &\leq \alpha_2 n_{t2} + \alpha_1 n_{t1} - \alpha_1 \min(n_{t1}, n_{r2}) &(4.24)\\ &= \alpha_2 n_{t2}. &(4.25) \end{aligned}$$

After we decode $\underline{\mathbf{X}}_2$ at receiver 2 successfully, the receiver 1 knows $\underline{\mathbf{X}}_2$ by the perfect receiver cooperation. So the constraint of d_1 is simply

$$\begin{aligned} d_1 &\leq \alpha_1 \min(n_{t1}, n_{r1}) &(4.26)\\ &= \alpha_1 n_{t1}. &(4.27) \end{aligned}$$

From the above constraints of DoF, the corner points of the DoF region are $(d_1, d_2) = (n_{t1}, 0), (0, n_{t2})$ and (n_{t1}, n_{t2}). The DoF region with perfect receiver cooperation is the same as the DoF region with no cooperation in section 3.1.3. Fig. 4.5 shows the DoF region of type 1.(c), with and without receiver cooperation.

An example of type 1.(c) is two-user MIMO interference channel with $n_{t1} = 2, n_{t2} = 2, n_{r1} = 4$ and $n_{r2} = 5$. Fig. 4.6 shows the DoF region of type 1.(c) with perfect receiver cooperation by computer calculation.

4.2 DoF region with perfect receiver cooperation when $n_{r1} < n_{t2}$ and $n_{r1} \leq n_{t1}$

In this section, we consider the DoF region of type 2. The decoding strategy is the same as previous section. We start with the case $\alpha_1 \geq \alpha_2$.

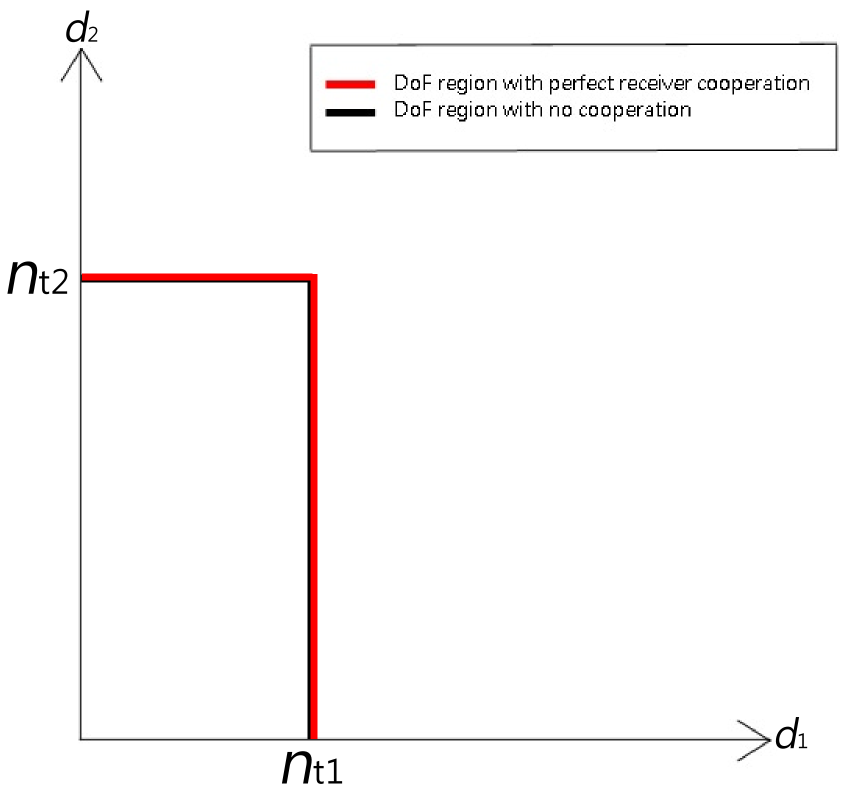

Figure 4.5: The DoF region of the DoF region of type 1.(c) with perfect receiver cooperation compared to the DoF region with no cooperation.

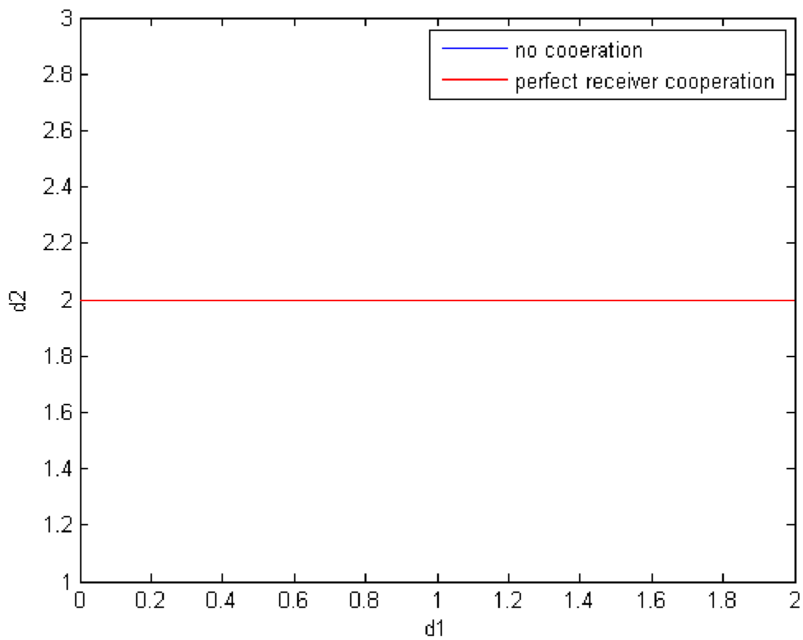

Figure 4.6: The DoF region of type 1.(c)

When $\alpha_1 \geq \alpha_2$, we treat $\underline{\mathbf{X}}_2$ as noise and decode $\underline{\mathbf{X}}_1$ at receiver 1. Thus we have the constraint of d_1 is

$$\begin{aligned} d_1 &\leq \alpha_1 n_{r1} - \alpha_2 \min(n_{t2}, n_{r1}) && (4.28) \\ &= (\alpha_1 - \alpha_2) n_{r1}. && (4.29) \end{aligned}$$

After we decode $\underline{\mathbf{X}}_1$ at receiver 1 successfully, the receiver 2 knows $\underline{\mathbf{X}}_1$ by the perfect receiver cooperation. The constraint of d_2 is simply

$$d_2 \leq \alpha_2 \min(n_{t2}, n_{r2}) \tag{4.30}$$

Note that we do not know $\min(n_{t2}, n_{r2}) = n_{t2}$ or n_{r2} here.

Now let us consider the case $\alpha_2 \geq \alpha_1$. In this case, we treat $\underline{\mathbf{X}}_1$ as noise and decode $\underline{\mathbf{X}}_2$ at receiver 2 directly. But in type 2, $n_{r2} \leq n_{t2}$, $n_{t2} < n_{r2} < n_{t1} + n_{t2}$ are all possible. So we need to consider these three conditions separately.

1. $n_{r2} \leq n_{t2}$

$$d_2 \leq \alpha_2 n_{r2} - \alpha_1 \min(n_{t1}, n_{r2}). \tag{4.31}$$

2. $n_{t2} < n_{r2} < n_{t1} + n_{t2}$

$$d_2 \leq \alpha_2 n_{t2} + \alpha_1 (n_{r2} - n_{t2}) - \alpha_1 \min(n_{t1}, n_{r2}). \tag{4.32}$$

3. $n_{t1} + n_{t2} \leq n_{r2}$

$$\begin{aligned} d_2 &\leq \alpha_2 n_{t2} + \alpha_1 n_{t2} - \alpha_1 \min(n_{t1}, n_{r2}) && (4.33) \\ &= \alpha_2 n_{t2}. && (4.34) \end{aligned}$$

After we decode $\underline{\mathbf{X}}_2$ successfully at receiver 2, the receiver 1 can subtract the component of $\underline{\mathbf{X}}_2$ by the perfect receiver cooperation. Thus, we can decode $\underline{\mathbf{X}}_1$ at receiver 1 without the interference from $\underline{\mathbf{X}}_2$. The constraint of d_1 is

$$\begin{aligned} d_1 &\leq \alpha_1 \min(n_{t1}, n_{r1}) && (4.35) \\ &= \alpha_1 n_{r1} && (4.36) \end{aligned}$$

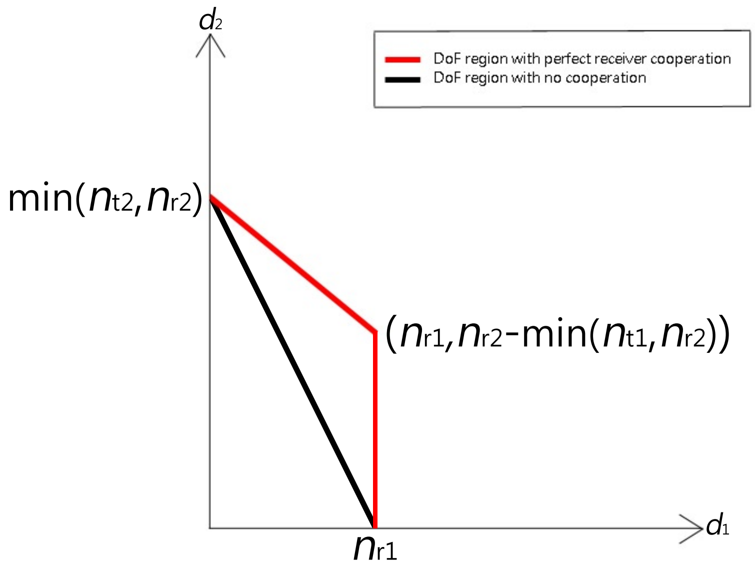

Figure 4.7: The DoF region of the DoF region of type 2 with perfect receiver cooperation compared to the DoF region with no cooperation.

These are all DoF constraints of type 2 with perfect receiver cooperation. We can see that if we want to achieve the maximum of d_2, i.e. $\min(n_{t2}, n_{r2})$, d_1 must be zero. But if we want to achieve the maximum of d_1, i.e. n_{r1}, d_2 can still be positive. This is the improvement given by the perfect receiver cooperation. Fig. 4.7 shows the DoF region of type 2, with and without receiver cooperation.

An example of type 2 is a two-user MIMO interference channel with $n_{t1} = 3, n_{t2} = 4, n_{r1} = 2$ and $n_{r2} = 4$. Fig. 4.8 shows the DoF region of type 2 with perfect receiver cooperation by computer calculation.

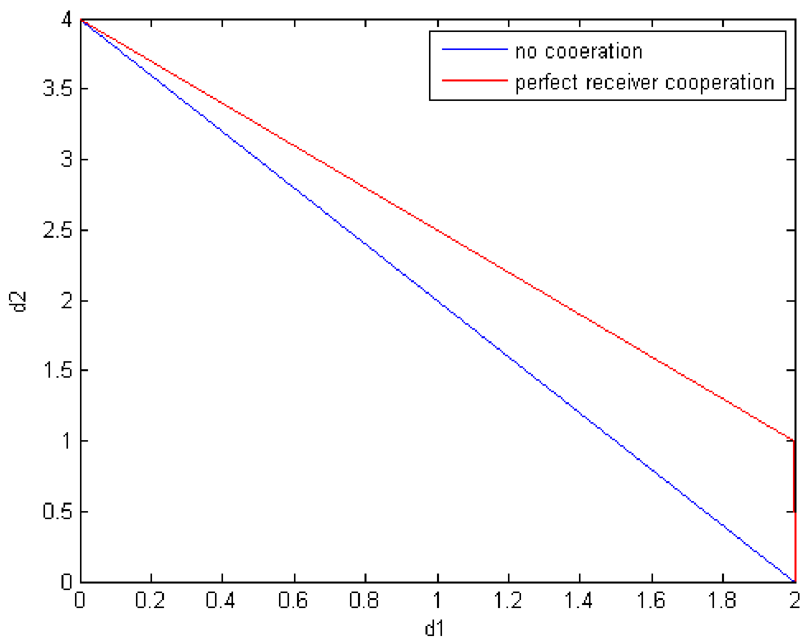

Figure 4.8: The DoF region of type 2

4.3 DoF region with perfect receiver cooperation when $n_{r1} < n_{t2}$ and $n_{t1} < n_{r1}$

In this section, we consider the DoF region of type 3. The decoding strategy is the same as previous two sections. Let us start with the case $\alpha_1 \geq \alpha_2$.

When $\alpha_1 \geq \alpha_2$, we treat $\underline{\mathbf{X}}_2$ as noise and decode $\underline{\mathbf{X}}_1$ at receiver 1. The constraint of d_1 is

$$\begin{aligned} d_1 &\leq \alpha_1 n_{t1} + \alpha_2(n_{r1} - n_{t1}) - \alpha_2 \min(n_{t2}, n_{r1}) && (4.37) \\ &= (\alpha_1 - \alpha_2) n_{t1}. && (4.38) \end{aligned}$$

The receiver 2 knows $\underline{\mathbf{X}}_1$ by the perfect receiver cooperation and can then subtract it off. Thus the constraint of d_2 is simply

$$d_2 \leq \alpha_2 \min(n_{t2}, n_{r2}). \quad (4.39)$$

Note that we do not know $\min(n_{t2}, n_{r2}) = n_{t2}$ or n_{r2} here.

Now let us consider the case $\alpha_2 \geq \alpha_1$. In this case, we treat $\underline{\mathbf{X}}_1$ as noise and decode $\underline{\mathbf{X}}_2$ at receiver 2. But in type 3, $n_{r2} \leq n_{t2}, n_{t2} < n_{r2} < n_{t1} + n_{t2}$ and $n_{t1} + n_{t2} \leq n_{r2}$ are all possible. So we need to consider these three conditions separately.

1. $n_{r2} \leq n_{t2}$

$$\begin{aligned} d_2 &\leq \alpha_2 n_{r2} - \alpha_1 \min(n_{t1}, n_{r2}) && (4.40) \\ &= \alpha_2 n_{r2} - \alpha_1 n_{t1}. && (4.41) \end{aligned}$$

2. $n_{t2} < n_{r2} < n_{t1} + n_{t2}$

$$\begin{aligned} d_2 &\leq \alpha_2 n_{t2} + \alpha_1(n_{r2} - n_{t2}) - \alpha_1 \min(n_{t1}, n_{r2}) && (4.42) \\ &= \alpha_2 n_{t2} + \alpha_1(n_{r2} - n_{t1} - n_{t2}). && (4.43) \end{aligned}$$

3. $n_{t1} + n_{t2} \leq n_{r2}$

$$\begin{aligned} d_2 &\leq \alpha_2 n_{t2} + \alpha_1 n_{t2} - \alpha_1 \min(n_{t1}, n_{r2}) && (4.44) \\ &= \alpha_2 n_{t2} && (4.45) \end{aligned}$$

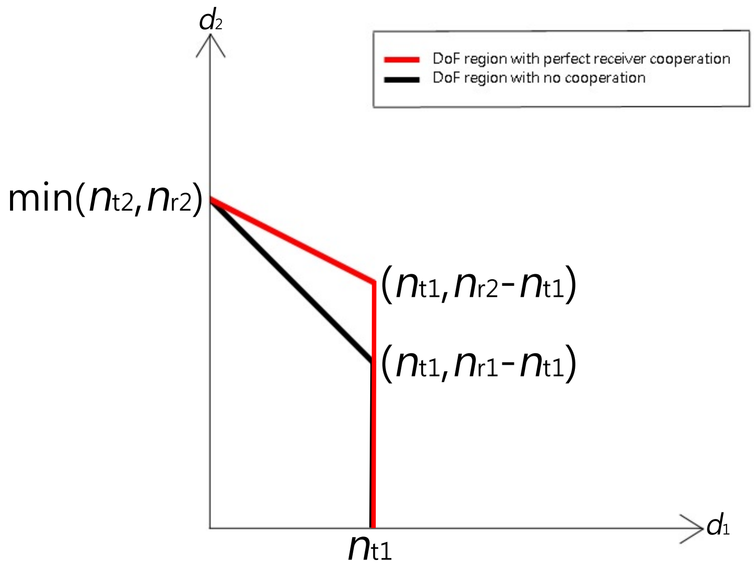

Figure 4.9: The DoF region of the DoF region of type 3 with perfect receiver cooperation compared to the DoF region with no cooperation.

The above characterize the DoF region of type 3 with perfect receiver cooperation. In type 3, we cannot have nonzero d_1 when we achieve the maximum of d_2. If we have perfect receiver cooperation, we can have better d_2 when we achieve the maximum of d_1, compared with no cooperation in Section 2.3. Fig. 4.9 shows the DoF region of type 3, with and without receiver cooperation.

An example of type 3 is a two-user MIMO interference channel with $n_{t1} = 1, n_{t2} = 3, n_{r1} = 2$ and $n_{r2} = 4$. Fig. 4.10 shows the DoF region of type 3 with perfect receiver cooperation by computer calculaton. We can see that if we have perfect receiver cooperation, the DoF region might be larger than the new outer bound of type 3 that we stated in Section 3.3.

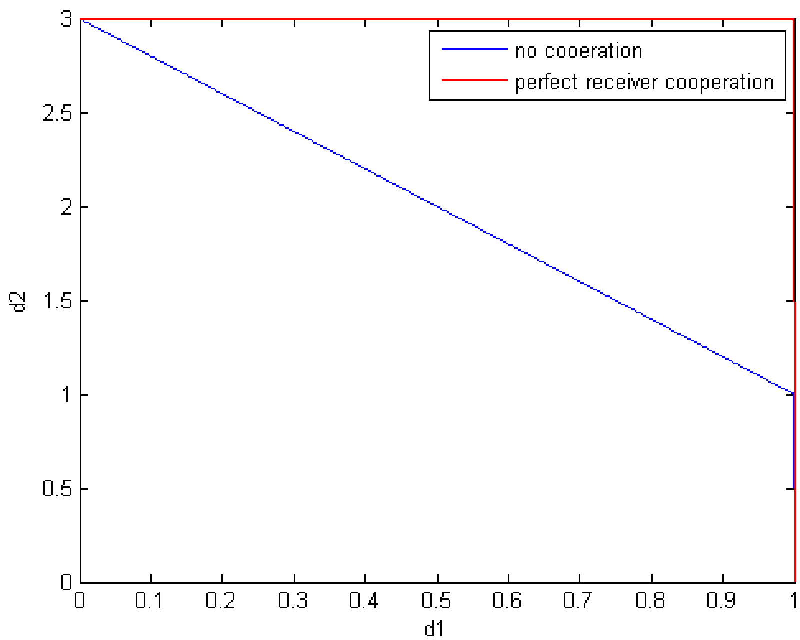

Figure 4.10: The DoF region of type 3

Chapter 5

Conclusion

In this thesis, we study the DoF region of the two-user MIMO interference channel without CSIT. By considering the power-split transmission scheme at the transmitters, we first view the interference channel as two two-user MAC and develop the DoF for the general two-user MAC. Following [4], we distinguish the two-user MIMO interference channel by the values of transmit and receive antennas and consider three types. For type 1 and type 2, we show that the power-split scheme with SIC can achieve the same DoF region as in [4]. For type 3, we reduce the outer bound in [4] by a two-user genie-aided MIMO interference channel. It is seen that for this type the gap between the power-split scheme with SIC and the outer bound is small. Also, from [9] it is seen that the exact DoF region of type 3 in the case of isotropic and independent fading is the same as our power-split scheme with SIC. Finally, we consider the DoF region of two-user MIMO interference channel with perfect receiver cooperation, and compare to the two-user MIMO interference channel without cooperation.

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