Almost disturbance decoupling and tracking control for multi-input multi-output non-linear uncertain systems: Application to a half-car active suspension system

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Almost disturbance decoupling and tracking control for multi-input multi-output non-linear uncertain systems: application to a half-car active suspension system

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Abstract: This study presents a novel feedback linearization control of non-linear multi-input multi-output uncertain systems for the tracking and almost disturbance decoupling performances. The main contribution of this study is to construct a controller, under appropriate conditions, such that the resulting closed-loop system is valid for any initial condition and bounded tracking signal with the following characteristics: input-to-state stability with respect to disturbance inputs and almost disturbance decoupling. In addition, a new theorem on robust stability is proposed in this study to provide a new criterion for closed-loop stability. A typical case, which cannot be solved by any previous study on the almost disturbance decoupling problem, is proposed in this study to exploit the fact that the tracking and the almost disturbance decoupling performances can be easily achieved by the proposed approach. Finally, the proposed control law is simulated in a half-car active suspension system on which the effectiveness of the design is verified.

Keywords: almost disturbance decoupling, multi-input multi-output uncertain system, half-car active suspension system, feedback linearization approach, composite Lyapunov approach

1 INTRODUCTION

Stabilization and tracking are both important tasks in the solution of the control problem. The tracking task is generally more complicated than the stabilization task for non-linear control systems. Many approaches to these tasks have been proposed including feedback linearization, variable structure control (sliding mode control), backstepping, regulation control, non-linear H^∞ control, the internal model principle, and H^∞ adaptive fuzzy control. Richter *et al.* [**1**] have proposed the use of variable structure control to deal with non-linear systems. However, chattering behaviour caused by discontinuous switching and imperfect implementation that can drive the system into unstable regions is inevitable for variable structure control schemes [**2**]. Backstepping has proven to be a powerful tool for synthesizing controllers for non-linear systems [**3**]. However, a disadvantage of this approach is an explosion in the complexity which is a result of repeated differentiation of the non-linear functions [**4**, **5**]. An alternative approach is to utilize output regulation control in which the outputs are assumed to be excited by an exosystem [**6**]. However, this non-linear regulation approach requires the solution of difficult partial differential algebraic equations. Another difficulty is that the exosystem states need to be switched to describe changes in the output and this creates transient tracking errors [**7**]. In general, non-linear H^∞ control requires the solution of the Hamilton–Jacobi equation, which is a difficult non-linear partial differential equation [**8–11**]. Only for some particular non-linear systems it is possible to

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derive a closed-form solution [**12**]. The control approach that is based on the internal model principle converts the tracking problem into a non-linear output regulation problem [**13**]. This approach depends on solving a first-order partial differential equation of the centre manifold [**6**]. For some special non-linear systems and desired trajectories, the asymptotic solutions of this equation have been developed using ordinary differential equations [**14, 15**]. Recently, H^{∞} adaptive fuzzy control has been proposed to systematically deal with non-linear systems [**16**]. The drawback with H^{∞} adaptive fuzzy control is that the complex parameter update law makes this approach impractical in real-world situations. During the past decade significant progress has been made in researching control approaches for non-linear systems based on the feedback linearization theory [**17, 18**]. Moreover, the feedback linearization approach has been successfully applied to many real control systems. These include the control of an electromagnetic suspension system [**19**], pendulum system [**20**], spacecraft [**21**], electrohydraulic servosystem [**22**], car-pole system [**23**], bank-to-turn missile system [**24**], and a compact six-axis magnetic levitation stage [**25**].

It is difficult to obtain completely accurate mathematical models for many practical control systems. Thus, there are inevitable uncertainties in their models. Therefore, the design of a robust controller that deals with the uncertainties of a control system is of considerable interest. This study presents a systematic analysis and a simple design scheme that guarantees the globally asymptotic stability of a feedback-controlled uncertain system and achieves output tracking and almost disturbance decoupling performances for a class of non-linear control systems with uncertainties.

The almost disturbance decoupling problem, that is the design of a controller that attenuates the effect of the disturbance on the output terminal to an arbitrary degree of accuracy, was originally developed for linear and non-linear control systems by Willems [**26**] and Marino *et al.* [**27**] respectively. The problem has attracted considerable research attention and many significant results have been developed for both linear and non-linear control systems [**28–30**]. The almost disturbance decoupling problem of non-linear single-input single-output (SISO) systems was investigated in Marino *et al.* [**27**] by using a state feedback approach and solved in terms of sufficient conditions for systems with non-linearities that are not globally Lipschitz and disturbances being linear but possibly actually being multiples of non-linearities. The resulting state feedback control is constructed following a singular perturbation approach. The sufficient conditions in Marino *et al.* [**27**] require that the non-linearities multiplying the disturbances satisfy structural triangular conditions. Marino *et al.* [**27**] show that for non-linear SISO systems the almost disturbance decoupling problem may not be solvable, as is the case for

$$\dot{x}_1(t)=\tan^{-1}(x_2)+\theta(t),\quad \dot{x}_2(t)=u,\quad y=x_1$$

where u and y denote the input and output respectively and θ is the disturbance. However, this example can be easily solved via the approach proposed in this paper and this approach has also been successfully used to derive a tracking controller with almost disturbance decoupling for a half-car active suspension system. Throughout the paper, the notation $\|\cdot\|$ denotes the usual Euclidean norm or the corresponding induced matrix norm.

2 TRACKING AND ALMOST DISTURBANCE DECOUPLING CONTROLLER DESIGN

The following non-linear uncertain control system with disturbances is considered

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \cdot\\ \cdot\\ \dot{x}_n\end{bmatrix}=\begin{bmatrix}f_1(x_1,x_2,\ldots,x_n)\\ f_2(x_1,x_2,\ldots,x_n)\\ \cdot\\ \cdot\\ f_n(x_1,x_2,\ldots,x_n)\end{bmatrix}+[g_1(x_1,x_2,\ldots,x_n)\; g_2(x_1,x_2,\ldots,x_n)\cdots g_m(x_1,x_2,\ldots,x_n)]\begin{bmatrix}u_1(x_1,x_2,\ldots,x_n)\\ u_2(x_1,x_2,\ldots,x_n)\\ \cdot\\ \cdot\\ u_m(x_1,x_2,\ldots,x_n)\end{bmatrix}+\sum_{j=1}^{p}q_j^{*}\theta_{jd}+\begin{bmatrix}\Delta f_1(x_1,x_2,\ldots,x_n)\\ \Delta f_2(x_1,x_2,\ldots,x_n)\\ \cdot\\ \cdot\\ \Delta f_n(x_1,x_2,\ldots,x_n)\end{bmatrix} \tag{1a}$$

$$\begin{bmatrix} y_1(x_1,x_2,\ldots,x_n) \\ y_2(x_1,x_2,\ldots,x_n) \\ \cdot \\ \cdot \\ y_m(x_1,x_2,\ldots,x_n) \end{bmatrix} = \begin{bmatrix} h_1(x_1,x_2,\ldots,x_n) \\ h_2(x_1,x_2,\ldots,x_n) \\ \cdot \\ \cdot \\ h_m(x_1,x_2,\ldots,x_n) \end{bmatrix} \tag{1b}$$

that is

$$\dot{\boldsymbol{X}}(t) = \boldsymbol{f}(\boldsymbol{X}(t)) + \boldsymbol{g}(\boldsymbol{X}(t))\boldsymbol{u} + \sum_{j=1}^{p} q_j^* \theta_{jd} + \Delta\boldsymbol{f}$$

$$\boldsymbol{y}(t) = \boldsymbol{h}(\boldsymbol{X}(t))$$

where $\boldsymbol{X}(t) \equiv [x_1(t)x_2(t)\ldots x_n(t)]^{\mathrm{T}} \in \Re^n$ is the state vector, $\boldsymbol{u} \equiv [u_1u_2\ldots u_m]^{\mathrm{T}} \in \Re^m$ is the input vector, $\boldsymbol{y} \equiv [y_1y_2\ldots y_m]^{\mathrm{T}} \in \Re^m$ is the output vector, $\theta_{\mathrm{d}} \equiv [\theta_{1\mathrm{d}}(t)\theta_{2\mathrm{d}}(t)\ldots\theta_{p\mathrm{d}}(t)]^{\mathrm{T}}$ is a bounded time-varying disturbances vector, and $\Delta\boldsymbol{f} \equiv [\Delta f_1 \Delta f_2 \ldots \Delta f_n] \in \Re^n$ is an unknown non-linear function representing uncertainty such as modelling error. Let $\Delta\boldsymbol{f}$ be defined as

$$\Delta\boldsymbol{f} = \sum_{i=1}^{p} q_i^* \theta_{iu}$$

where $\boldsymbol{\theta}_u \equiv [\theta_{1u}(t)\theta_{2u}(t)\ldots\theta_{pu}(t)]^{\mathrm{T}}$ is a bounded time-varying vector. $\boldsymbol{f} \equiv [f_1f_2\ldots f_n]^{\mathrm{T}} \in \Re^n$, $\boldsymbol{g} \equiv [g_1g_2\ldots g_m] \in \Re^{n\times m}$, and $\boldsymbol{h} \equiv [h_1h_2\ldots h_m]^{\mathrm{T}} \in \Re^m$ are smooth vector fields. The nominal system is then defined as follows:

$$\dot{\boldsymbol{X}}(t) = \boldsymbol{f}(\boldsymbol{X}(t)) + \boldsymbol{g}(\boldsymbol{X}(t))\boldsymbol{u} \tag{2a}$$

$$\boldsymbol{y}(t) = \boldsymbol{h}(\boldsymbol{X}(t)) \tag{2b}$$

The nominal system of the form (2) is assumed to have the vector relative degree $\{r_1, r_2, \ldots, r_m\}$ [**31**], i.e. the following conditions are satisfied for all $\boldsymbol{X} \in \Re^n$.

1. $$L_{g_j} L_f^k h_i(\boldsymbol{X}) = 0 \tag{3}$$

   for all $1 \leqslant i \leqslant m$, $1 \leqslant j \leqslant m$, $k < r_i - 1$, where the operator L is the Lie derivative [**31**] and $r_1 + r_2 + \cdots + r_m = r$.
2. The $m \times m$ matrix

$$\mathbf{A} \equiv \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(\boldsymbol{X}) & \cdots & L_{g_m} L_f^{r_1-1} h_1(\boldsymbol{X}) \\ L_{g_1} L_f^{r_2-1} h_2(\boldsymbol{X}) & \cdots & L_{g_m} L_f^{r_2-1} h_2(\boldsymbol{X}) \\ \vdots & & \vdots \\ L_{g_1} L_f^{r_m-1} h_m(\boldsymbol{X}) & & L_{g_m} L_f^{r_m-1} h_m(\boldsymbol{X}) \end{bmatrix} \tag{4}$$

   is non-singular.

The desired output trajectory y_{d}^i, $1 \leqslant i \leqslant m$ and its first r_i derivatives are all uniformly bounded and

$$\left\| \left[y_{\mathrm{d}}^i, y_{\mathrm{d}}^{i(1)}, \cdots, y_{\mathrm{d}}^{i(r_i)} \right] \right\| \leqslant B_{\mathrm{d}}^i, \quad 1 \leqslant i \leqslant m \tag{5}$$

where B_{d}^i is some positive constant. Under the assumption of well-defined vector relative degree, it has been shown [**31**] that the mapping

$$\phi : \Re^n \to \Re^n \tag{6}$$

defined as

$$\boldsymbol{\xi}_i \equiv \begin{bmatrix} \xi_1^i \\ \xi_2^i \\ \vdots \\ \xi_{r_i}^i \end{bmatrix} \equiv \begin{bmatrix} \phi_1^i \\ \phi_2^i \\ \vdots \\ \phi_{r_i}^i \end{bmatrix} \equiv \begin{bmatrix} L_f^0 h_i(\boldsymbol{X}) \\ L_f^1 h_i(\boldsymbol{X}) \\ \vdots \\ L_f^{r_i-1} h_i(\boldsymbol{X}) \end{bmatrix} \tag{7}$$

$$i = 1, 2, \ldots, m$$

$$\phi_k(\boldsymbol{X}(t)) \equiv \eta_k(t), \quad k = r+1, r+2, \ldots, n \tag{8}$$

and satisfying

$$L_{g_j}\phi_k(\boldsymbol{X}(t)) = 0, \quad k = r+1, r+2, \ldots, n, \quad 1 \leqslant j \leqslant m \tag{9}$$

is a diffeomorphism onto image, if the following hold.

1. The distribution

   $$G \equiv \mathrm{span}\{g_1, g_2, \ldots, g_m\} \tag{10}$$

   is involutive.
2. The vector fields

   $$\boldsymbol{Y}_j^k, \quad 1 \leqslant j \leqslant m, \quad 1 \leqslant k \leqslant r_j \tag{11}$$

   are complete, where

   $$\boldsymbol{Y}_j^k \equiv (-1)^{k-1} ad_{\tilde{f}}^{k-1} \tilde{g}_j, \quad 1 \leqslant j \leqslant m, \quad 1 \leqslant k \leqslant r_j \tag{12}$$

$$\tilde{\boldsymbol{f}}(\boldsymbol{X}) \equiv \boldsymbol{f}(\boldsymbol{X}) - \boldsymbol{g}(\boldsymbol{X})\mathbf{A}^{-1}(\boldsymbol{X})\boldsymbol{b}(\boldsymbol{X}) \tag{13}$$

$$\boldsymbol{b}(\boldsymbol{X}) \equiv \begin{bmatrix} L_f^{r_1} h_1(\boldsymbol{X}) \\ L_f^{r_2} h_2(\boldsymbol{X}) \\ \vdots \\ L_f^{r_m} h_m(\boldsymbol{X}) \end{bmatrix} \tag{14}$$

$$\tilde{\boldsymbol{g}} \equiv [\tilde{g}_1 \quad \tilde{g}_2 \quad \cdots \quad \tilde{g}_m] \equiv \boldsymbol{g}(\boldsymbol{X})\mathbf{A}^{-1}(\boldsymbol{X}) \tag{15}$$

$$ad_f^k \boldsymbol{g} \equiv \left[\boldsymbol{f} \quad ad_f^{k-1}\boldsymbol{g}\right] \tag{16}$$

$$[\boldsymbol{f} \quad \boldsymbol{g}] \equiv \frac{\partial \mathbf{g}}{\partial \boldsymbol{X}}\boldsymbol{f}(\boldsymbol{X}) - \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{X}}\boldsymbol{g}(\boldsymbol{X}) \tag{17}$$

For the sake of convenience, defining the trajectory error to be

$$e_j^i \equiv \xi_j^i - y_{\mathrm{d}}^{i(j-1)}, \quad i = 1,2,\ldots,m, \quad j = 1,2,\ldots,r_i \tag{18}$$

$$\boldsymbol{e}^i \equiv \left[e_1^i e_2^i \cdots e_{r_i}^i\right]^{\mathrm{T}} \in \Re^{r_i} \tag{19}$$

and the trajectory error to be multiplied with some adjustable positive constant ε

$$\overline{e_j^i} \equiv \varepsilon^{j-1} e_j^i, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, r_i \tag{20}$$

$$\overline{\boldsymbol{e}^i} \equiv \left[\overline{e_1^i e_2^i} \cdots \overline{e_{r_i}^i}(t)\right]^{\mathrm{T}} \in \Re^{r_i} \tag{21}$$

$$\bar{\boldsymbol{e}} \equiv \begin{bmatrix} \overline{\boldsymbol{e}^1} \\ \overline{\boldsymbol{e}^2} \\ \vdots \\ \overline{\boldsymbol{e}^m} \end{bmatrix} \in \Re^r \tag{22}$$

and

$$\xi \equiv \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_r \end{bmatrix} \in \Re^r \tag{23}$$

$$\boldsymbol{\eta}(t) \equiv \left[\eta_{r+1}(t)\eta_{r+2}(t) \cdots \eta_n(t)\right]^{\mathrm{T}} \in \Re^{n-r} \tag{24}$$

$$\begin{aligned} \boldsymbol{q}(\xi(t),\boldsymbol{\eta}(t)) &\equiv \left[L_f\phi_{r+1}(t)L_f\phi_{r+2}(t) \cdots L_f\phi_n(t)\right]^{\mathrm{T}} \\ &\equiv [q_{r+1} \quad q_{r+2} \quad \cdots \quad q_n]^{\mathrm{T}} \end{aligned} \tag{25}$$

Define a phase-variable canonical matrix $\mathbf{A}_{\mathrm{c}}^i$ to be

$$\mathbf{A}_{\mathrm{c}}^i \equiv \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\alpha_1^i & -\alpha_2^i & -\alpha_3^i & \cdots & -\alpha_{r_i}^i \end{bmatrix}_{r_i \times r_i}$$
$$1 \leqslant i \leqslant m \tag{26}$$

where $\alpha_1^i, \alpha_2^i, \ldots, \alpha_{r_i}^i$ are any chosen parameters such that $\mathbf{A}_{\mathrm{c}}^i$ is Hurwitz and the vector $\boldsymbol{B}^i$ will be

$$\boldsymbol{B}^i \equiv [0 \quad 0 \quad \cdots \quad 0 \quad 1]_{r_i \times 1}^{\mathrm{T}}, \quad r \leqslant i \leqslant m \tag{27}$$

Let $\mathbf{P}^i$ be the positive definite solution of the following Lyapunov equation

$$\left(\mathbf{A}_{\mathrm{c}}^i\right)^{\mathrm{T}}\mathbf{P}^i + \mathbf{P}^i\mathbf{A}_{\mathrm{c}}^i = -\mathbf{I}, \quad 1 \leqslant i \leqslant m \tag{28}$$

$$\lambda_{\max}\left(\mathbf{P}^i\right) \equiv \text{the maximum eigenvalue of } \mathbf{P}^i \quad 1 \leqslant i \leqslant m \tag{29}$$

$$\lambda_{\min}\left(\mathbf{P}^i\right) \equiv \text{the minimum eigenvalue of } \mathbf{P}^i \quad 1 \leqslant i \leqslant m \tag{30}$$

$$\lambda_{\max}^* \equiv \min\{\lambda_{\max}\left(\mathbf{P}^1\right), \lambda_{\max}\left(\mathbf{P}^2\right), \ldots, \lambda_{\max}\left(\mathbf{P}^m\right)\} \tag{31}$$

$$\lambda_{\min}^* \equiv \min\{\lambda_{\min}\left(\mathbf{P}^1\right), \lambda_{\min}\left(\mathbf{P}^2\right), \ldots, \lambda_{\min}\left(\mathbf{P}^m\right)\} \tag{32}$$

Assumption 1

For all $t \geqslant 0$, $\boldsymbol{\eta} \in \Re^{n-r}$ and $\xi \in \Re^r$, there exists a positive constant M such that the following inequality holds

$$\left\|\boldsymbol{q}_{22}(t, \boldsymbol{\eta}, \bar{\boldsymbol{e}}) - \boldsymbol{q}_{22}(t, \boldsymbol{\eta}, 0)\right\| \leqslant M(\|\bar{\boldsymbol{e}}\|) \tag{33}$$

where $\boldsymbol{q}_{22}(t, \boldsymbol{\eta}, \bar{\boldsymbol{e}}) \equiv \boldsymbol{q}(\xi, \boldsymbol{\eta})$.

For the sake of stating precisely the investigated problem, defining

$$d_{ij} \equiv L_{g_j}L_f^{r_i-1}h_i(\boldsymbol{X}), \quad 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant m \tag{34}$$

$$c_i \equiv L_f^{r_i}h_i(\boldsymbol{X}), \quad 1 \leqslant i \leqslant m \tag{35}$$

and

$$\overline{\overline{\boldsymbol{e}^{i}}} \equiv \alpha_1^i \, \overline{e_1^i} + \alpha_2^i \, \overline{e_2^i} + \cdots + \alpha_{r_i}^i \overline{e_{r_i}^i}, \quad 1 \leqslant i \leqslant m \tag{36}$$

*Definition 1 [**32**]*

Consider the system $\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \ \boldsymbol{x}, \ \boldsymbol{\theta})$, where $\boldsymbol{f}$:[0, $\infty] \times \Re^n \times \Re^n \rightarrow \Re^n$ is piecewise continuous in t and locally Lipschitz in $\boldsymbol{x}$ and $\boldsymbol{\theta}$. This system can be regarded as input-to-state stable if there exists a class *KL* function β, a class K function γ, and positive constants k_1 and k_2 such that for any initial state $\boldsymbol{x}(t_0)$ with $\|\boldsymbol{x}(t_0)\| < k_1$and any bounded input $\boldsymbol{\theta}(t)$ with $\sup_{t \geqslant t_0} \|\boldsymbol{\theta}(t)\| < k_2$, the state exists and satisfies

$$\|\boldsymbol{x}(t)\| \leqslant \beta(\|\boldsymbol{x}(t_0)\|, \, t - t_0) + \gamma\left(\sup_{t_0 \leqslant \tau \leqslant t} \|\boldsymbol{\theta}(\tau)\| \right) \tag{37}$$

for all $t \geqslant t_0 \geqslant 0$. The tracking problem with almost disturbance decoupling is now formulated as follows.

*Definition 2 [**29**]*

The tracking problem with almost disturbance decoupling is said to be globally solvable by the state feedback controller $\boldsymbol{u}$ for the transformed-error system by a global diffeomorphism (6), if the controller $\boldsymbol{u}$ enjoys the following properties.

1. It is input-to-state stable with respect to disturbance inputs.
2. For any initial value $\bar{\boldsymbol{x}}_{e0} \equiv [\bar{\boldsymbol{e}}(t_0) \;\; \boldsymbol{\eta}(t_0)]^{\mathrm{T}}$, for any $t \geqslant t_0$ and for any $t_0 \geqslant 0$

$$|y(t) - y_{\mathrm{d}}(t)| \leqslant \beta_{11}(\|\boldsymbol{x}(t_0)\|, \, t - t_0) + \frac{1}{\sqrt{\beta_{22}}} \beta_{33}\left(\sup_{t_0 \leqslant \tau \leqslant t} \|\boldsymbol{\theta}(\tau)\| \right) \tag{38}$$

and

$$\int_{t_0}^{t} [y(\tau) - y_{\mathrm{d}}(\tau)]^2 \mathrm{d}\tau \leqslant \frac{1}{\beta_{44}} \left[\beta_{55}(\|\bar{\boldsymbol{x}}_{e0}\|) + \int_{t_0}^{t} \beta_{33}\left(\|\boldsymbol{\theta}(\tau)\|^2\right) \mathrm{d}\tau \right] \tag{39}$$

where β_{22} and β_{44} are positive constants, β_{33} and β_{55} are class K functions, and β_{11} is a class *KL* function.

Theorem 1

Suppose that there exists a continuously differentiable function $\boldsymbol{V}$:$\Re^{n-r} \rightarrow \Re^+$ such that the following three inequalities hold for all $\boldsymbol{\eta} \in \Re^{n-r}$.

1. $$\omega_1 \|\boldsymbol{\eta}\|^2 \leqslant \boldsymbol{V}(\boldsymbol{\eta}) \leqslant \omega_2 \|\boldsymbol{\eta}\|^2, \quad \omega_1, \omega_2 > 0 \tag{40a}$$

2. $$\nabla_t \boldsymbol{V} + \left(\nabla_{\boldsymbol{\eta}} \boldsymbol{V}\right)^{\mathrm{T}} \boldsymbol{q}_{22}(t, \boldsymbol{\eta}, 0) \leqslant -2\alpha_x \|\boldsymbol{\eta}\|,^2 \quad \alpha_x > 0 \tag{40b}$$

3. $$\|\nabla_{\boldsymbol{\eta}} \boldsymbol{V}\| \leqslant \varpi_3 \|\boldsymbol{\eta}\|, \quad \varpi_3 > 0 \tag{40c}$$

then the tracking problem with almost disturbance decoupling is globally solvable by the controller defined by

$$\boldsymbol{u} = \mathbf{A}^{-1}\{-\boldsymbol{b} + \boldsymbol{v}\} \tag{41}$$

$$\boldsymbol{b} \equiv \left[L_f^{r_1} h_1 \quad L_f^{r_2} h_2 \quad \cdots \quad L_f^{r_m} h_m \right]^{\mathrm{T}} \tag{42}$$

$$\boldsymbol{v} \equiv [\, v_1 \quad v_2 \quad \cdots \quad v_m \,]^{\mathrm{T}} \tag{43}$$

$$\begin{aligned} v_i \equiv\; & y_{\mathrm{d}}^{i(r_i)} - \varepsilon^{-r_i} \alpha_1^i \left[L_f^0 h_i(\boldsymbol{X}) - y_{\mathrm{d}}^i \right] \\ & - \varepsilon^{1-r_i} \alpha_2^i \left[L_f^1 h_i(\boldsymbol{X}) - y_{\mathrm{d}}^{i(1)} \right] - \cdots \\ & - \varepsilon^{-1} \alpha_{r_i}^i \left[L_f^{r_i - 1} h_i(\boldsymbol{X}) - y_{\mathrm{d}}^{i(\gamma_i - 1)} \right], \quad 1 \leqslant i \leqslant m \end{aligned} \tag{44}$$

Moreover, the influence of disturbances on the L_2-norm of the tracking error can be arbitrarily attenuated by increasing the following adjustable parameter $N_2 > 1$.

$$\begin{aligned} k_{11} \equiv\; & \frac{k}{2\varepsilon} - \frac{k^2 \left\|\boldsymbol{\varphi}_{\xi}^1\right\|^2 \|\mathbf{P}^1\|^2}{\varepsilon^2} - \cdots \\ & - \frac{k^2 \left\|\boldsymbol{\varphi}_{\xi}^m\right\|^2 \|\mathbf{P}^m\|^2}{\varepsilon^2} - 4 \end{aligned} \tag{45a}$$

$$k_{22} \equiv 2\alpha_x - \frac{\omega_3^2 M^2}{16} - \omega_3^2 \|\boldsymbol{\phi}_{\eta}\|^2 \tag{45b}$$

$$N_2 \equiv \min\{k_{11}, \, k_{22}\} \tag{45c}$$

$$N_1 \equiv \frac{m+1}{4} \left(\sup_{t_0 \leqslant \tau \leqslant t} \|\boldsymbol{\theta}_{\mathrm{d}}(\tau) + \boldsymbol{\theta}_{\mathrm{u}}(\tau)\| \right)^2 \tag{45d}$$

$$\boldsymbol{\phi}_{\xi}^{i}(\varepsilon)\equiv\begin{bmatrix}\varepsilon\frac{\partial}{\partial X}h_i q_1^* & \cdots & \varepsilon\frac{\partial}{\partial X}h_i q_p^* \\ \vdots & & \vdots \\ \varepsilon^{r_i}\frac{\partial}{\partial X}L_f^{r_i-1}h_i q_1^* & \cdots & \varepsilon^{r_i}\frac{\partial}{\partial X}L_f^{r_i-1}h_i q_q^*\end{bmatrix}$$
$$1\leqslant i\leqslant m \tag{45e}$$

$$\boldsymbol{\phi}_{\eta}(\varepsilon)\equiv\begin{bmatrix}\frac{\partial}{\partial X}\phi_{r+1}q_1^* & \cdots & \frac{\partial}{\partial X}\phi_{r+1}q_p^* \\ \vdots & & \vdots \\ \frac{\partial}{\partial X}\phi_n q_1^* & \cdots & \frac{\partial}{\partial X}\phi_n q_q^*\end{bmatrix} \tag{45f}$$

where $k(\varepsilon):\Re^+\rightarrow\Re^+$ is any continuous function satisfies

$$\lim_{\varepsilon\to 0}k(\varepsilon)=0 \text{ and } \lim_{\varepsilon\to 0}\frac{\varepsilon}{k(\varepsilon)}=0 \tag{45g}$$

Proof. Applying the coordinate transformation equation (6) yields

$$\begin{aligned}\dot{\xi}_1^1&=\frac{\partial\phi_1^1}{\partial X}\frac{\mathrm{d}X}{\mathrm{d}t}=\frac{\partial h_1}{\partial \boldsymbol{X}}\left[\boldsymbol{f}+\boldsymbol{g}\cdot\boldsymbol{u}+\sum_{j=1}^{p}q_j^*\theta_{\mathrm{jd}}+\Delta\boldsymbol{f}\right]\\&=\frac{\partial h_1}{\partial \boldsymbol{X}}\boldsymbol{f}+\sum_{j=1}^{p}\frac{\partial h_1}{\partial \boldsymbol{X}}q_j^*(\theta_{\mathrm{jd}}+\theta_{\mathrm{ju}})\\&=\xi_2^1+\sum_{j=1}^{p}\frac{\partial h_1}{\partial X}q_j^*(\theta_{\mathrm{jd}}+\theta_{\mathrm{ju}})\end{aligned} \tag{46}$$

$\vdots$

$$\begin{aligned}\dot{\xi}_{r_1-1}^1&=\frac{\partial\phi_{r_1-1}^1}{\partial X}\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t}=\frac{\partial L_f^{r_1-2}h_1}{\partial \boldsymbol{X}}\\&\quad\left[\boldsymbol{f}+\boldsymbol{g}\cdot\boldsymbol{u}+\sum_{j=1}^{p}q_j^*\theta_{\mathrm{jd}}+\Delta\boldsymbol{f}\right]\\&=\frac{\partial L_f^{r_1-2}h_1}{\partial X}\boldsymbol{f}+\sum_{j=1}^{p}\frac{\partial L_f^{r_1-2}h_1}{\partial \boldsymbol{X}}q_j^*(\theta_{\mathrm{jd}}+\theta_{\mathrm{ju}})\\&=L_f^{r_1-1}h_1+\sum_{j=1}^{p}\frac{\partial L_f^{r_1-2}h_1}{\partial X}q_j^*(\theta_{\mathrm{jd}}+\theta_{\mathrm{ju}})\end{aligned} \tag{47}$$

$$\begin{aligned}\dot{\xi}_{r_1}^1&=\frac{\partial\phi_{r_1}^1}{\partial \boldsymbol{X}}\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t}=\frac{\partial L_f^{r_1-1}h_1}{\partial \boldsymbol{X}}\left[\boldsymbol{f}+\boldsymbol{g}\cdot\boldsymbol{u}+\sum_{j=1}^{p}q_j^*\theta_{\mathrm{jd}}+\Delta\boldsymbol{f}\right]\\&=L_f^{r_1}h_1+L_{g_1}L_f^{r_1-1}h_1u_1+\cdots+L_{g_m}L_f^{r_1-1}h_1u_m\\&\quad+\sum_{j=1}^{p}\frac{\partial L_f^{r_1-1}h_1}{\partial X}q_j^*(\theta_{\mathrm{jd}}+\theta_{\mathrm{ju}})\\&=c_1+d_{11}u_1+\cdots+d_{1m}u_m+\\&\quad\sum_{j=1}^{p}\frac{\partial L_f^{r_1-1}h_1}{\partial X}q_j^*(\theta_{\mathrm{jd}}+\theta_{\mathrm{ju}})\end{aligned} \tag{48}$$

$\vdots$

$$\begin{aligned}\dot{\xi}_1^m&=\frac{\partial\phi_1^m}{\partial X}\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t}=\frac{\partial h_m}{\partial X}\left[\boldsymbol{f}+\boldsymbol{g}\cdot\boldsymbol{u}+\sum_{j=1}^{p}q_j^*\theta_{\mathrm{jd}}+\Delta\boldsymbol{f}\right]\\&=L_f^1h_m+\sum_{j=1}^{p}\frac{\partial h_1}{\partial X}q_j^*(\theta_{\mathrm{jd}}+\theta_{\mathrm{ju}})\\&=\xi_2^m+\sum_{j=1}^{p}\frac{\partial h_m}{\partial X}q_j^*(\theta_{\mathrm{jd}}+\theta_{\mathrm{ju}})\end{aligned} \tag{49}$$

$\vdots$

$$\begin{aligned}\dot{\xi}_{r_m-1}^m&=\frac{\partial\phi_{r_m-1}^m}{\partial \boldsymbol{X}}\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t}=\frac{\partial L_f^{r_m-2}h_m}{\partial \boldsymbol{X}}\\&\qquad\times\left[\boldsymbol{f}+\boldsymbol{g}\cdot\boldsymbol{u}+\sum_{j=1}^{p}q_j^*\theta_{\mathrm{jd}}+\Delta\boldsymbol{f}\right]\\&=L_f^{r_m-1}h_m+\sum_{j=1}^{p}\frac{\partial L_f^{r_m-2}h_m}{\partial X}q_j^*(\theta_{\mathrm{jd}}+\theta_{\mathrm{ju}})\\&=\xi_{r_m}^m+\sum_{j=1}^{p}\frac{\partial L_f^{r_m-2}h_m}{\partial X}q_j^*(\theta_{\mathrm{jd}}+\theta_{\mathrm{ju}})\end{aligned} \tag{50}$$

$$\begin{aligned}\dot{\xi}_{r_m}^m&=\frac{\partial\phi_{r_m}^m}{\partial X}\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t}=\frac{\partial L_f^{r_m-1}h_m}{\partial X}\left[\boldsymbol{f}+\boldsymbol{g}\cdot\boldsymbol{u}+\sum_{j=1}^{p}q_j^*\theta_{\mathrm{jd}}+\Delta\boldsymbol{f}\right]\\&=L_f^{r_m}h_m+L_{g_1}L_f^{r_m-1}h_mu_1+\cdots+L_{g_m}L_f^{r_m-1}h_mu_m\\&\quad+\sum_{j=1}^{p}\frac{\partial L_f^{r_m-1}h_m}{\partial X}q_j^*(\theta_{\mathrm{jd}}+\theta_{\mathrm{ju}})\\&=c_m+d_{m1}u_1+\cdots+d_{mm}u_m\\&\quad+\sum_{j=1}^{p}\frac{\partial L_f^{r_m-1}h_m}{\partial X}q_j^*(\theta_{\mathrm{jd}}+\theta_{\mathrm{ju}})\end{aligned} \tag{51}$$

$$\dot{\eta}_k(t)=\frac{\partial\phi_k}{\partial X}\frac{\mathrm{d}X}{\mathrm{d}t}=\frac{\partial\phi_k}{\partial \boldsymbol{X}}\left[\boldsymbol{f}+\boldsymbol{g}\cdot\boldsymbol{u}+\sum_{j=1}^{p}q_j^*\theta_{\mathrm{jd}}+\Delta\boldsymbol{f}\right]$$

$$= L_f \phi_k + \sum_{j=1}^{p} \frac{\partial \phi_k}{\partial X} q_j^* (\theta_{jd} + \theta_{ju})$$

$$= q_k + \sum_{j=1}^{p} \frac{\partial \phi_k}{\partial X} q_j^* (\theta_{jd} + \theta_{ju})$$

$$k = r+1, r+2, \ldots, n \tag{52}$$

Since

$$c_i(\boldsymbol{\xi}(t), \boldsymbol{\eta}(t)) \equiv L_f^{r_i} h_i(\boldsymbol{X}(t)), \quad 1 \leqslant i \leqslant m \tag{53}$$

$$d_{ij} \equiv L_{g_j} L_f^{r_i - 1} h_i(\boldsymbol{X}), \quad 1 \leqslant i \leqslant m, \quad 1 \leqslant j \leqslant m \tag{54}$$

$$\boldsymbol{q}_k(\xi(t), \boldsymbol{\eta}(t)) = L_f \phi_k(\boldsymbol{X}), \quad k = r+1, r+2, \ldots, n \tag{55}$$

the dynamic equations of system (1) in the new coordinates are as follows:

$$\dot{\xi}_i^1(t) = \xi_{i+1}^1(t) + \sum_{j=1}^{p} \frac{\partial}{\partial \boldsymbol{X}} L_f^{i-1} h_1 q_j^* (\theta_{jd} + \theta_{ju})$$

$$i = 1, 2, \ldots, r_1 - 1 \tag{56}$$

$$\begin{aligned} \dot{\xi}_{r_1}^1(t) &= c_1(\xi(t), \eta(t)) + d_{11}(\xi(t), \eta(t)) u_1 + \cdots \\ &\quad + d_{1m}(\xi(t), \eta(t)) u_m \\ &\quad + \sum_{j=1}^{p} \frac{\partial}{\partial \boldsymbol{X}} L_f^{r_1 - 1} h_1 q_j^* (\theta_{jd} + \theta_{ju}) \end{aligned} \tag{57}$$

$$\vdots$$

$$\dot{\xi}_i^m(t) = \xi_{i+1}^m(t) + \sum_{j=1}^{p} \frac{\partial}{\partial \boldsymbol{X}} L_f^{i-1} h_m q_j^* (\theta_{jd} + \theta_{ju})$$

$$i = 1, 2, \ldots, r_m - 1 \tag{58}$$

$$\begin{aligned} \dot{\xi}_{r_m}^m(t) &= c_m(\xi(t), \eta(t)) + d_{m1}(\xi(t), \eta(t)) u_1 + \cdots \\ &\quad + d_{mm}(\xi(t), \eta(t)) u_m \\ &\quad + \sum_{j=1}^{p} \frac{\partial}{\partial \boldsymbol{X}} L_f^{r_m - 1} h_m q_j^* (\theta_{jd} + \theta_{ju}) \end{aligned} \tag{59}$$

$$\begin{aligned} \dot{\eta}_k(t) &= q_k(\xi(t), \eta(t)) \\ &\quad + \sum_{j=1}^{p} \frac{\partial}{\partial \boldsymbol{X}} \phi_k(X) q_j^* (\theta_{jd} + \theta_{ju}), \quad k = r+1, \ldots, n \end{aligned} \tag{60}$$

$$y_i(t) = \xi_1^i(t), \quad 1 \leqslant i \leqslant m \tag{61}$$

According to equations (18), (44), (53), and (54), the tracking controller can be rewritten as

$$\boldsymbol{u} = \mathbf{A}^{-1}[-\boldsymbol{b} + \boldsymbol{v}] \tag{62}$$

Substituting equation (62) into equations (57) and (59), the dynamic equations of system (1) can be shown as follows

$$\begin{bmatrix} \dot{\xi}_1^i(t) \\ \dot{\xi}_2^i(t) \\ \vdots \\ \dot{\xi}_{r_i-1}^i(t) \\ \dot{\xi}_{r_i}^i(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 \cdots & 0 \\ & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \xi_1^i(t) \\ \xi_2^i(t) \\ \vdots \\ \xi_{r_i-1}^i(t) \\ \xi_{r_i}^i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} v_i + \begin{bmatrix} \sum_{j=1}^{p} \frac{\partial}{\partial \boldsymbol{X}} h_i q_j^* (\theta_{jd} + \theta_{ju}) \\ \sum_{j=1}^{p} \frac{\partial}{\partial \boldsymbol{X}} L_f^1 h_i q_j^* (\theta_{jd} + \theta_{ju}) \\ \vdots \\ \sum_{j=1}^{p} \frac{\partial}{\partial \boldsymbol{X}} L_f^{r_i - 1} h_i q_j^* (\theta_{jd} + \theta_{ju}) \end{bmatrix} \tag{63}$$

$$\begin{bmatrix} \dot{\eta}_{r+1}(t) \\ \dot{\eta}_{r+2}(t) \\ \vdots \\ \dot{\eta}_{n-1}(t) \\ \dot{\eta}_n(t) \end{bmatrix} = \begin{bmatrix} q_{r+1}(t) \\ q_{r+2}(t) \\ \vdots \\ q_{n-1}(t) \\ q_n(t) \end{bmatrix} + \begin{bmatrix} \sum_{j=1}^{p} \frac{\partial}{\partial \boldsymbol{X}} \phi_{r+1} q_j^* (\theta_{jd} + \theta_{ju}) \\ \sum_{j=1}^{p} \frac{\partial}{\partial \boldsymbol{X}} \phi_{r+2} q_j^* (\theta_{jd} + \theta_{ju}) \\ \vdots \\ \sum_{j=1}^{p} \frac{\partial}{\partial \boldsymbol{X}} \phi_{n-1} q_j^* (\theta_{jd} + \theta_{ju}) \\ \sum_{j=1}^{p} \frac{\partial}{\partial \boldsymbol{X}} \phi_n q_j^* (\theta_{jd} + \theta_{ju}) \end{bmatrix} \tag{64}$$

$$\boldsymbol{y}_i = [1 \quad 0 \quad \cdots \quad 0 \quad 0]_{r\times 1} \begin{bmatrix} \xi_1^i(t) \\ \xi_2^i(t) \\ \vdots \\ \xi_{r_i-1}^i(t) \\ \xi_{r_i}^i(t) \end{bmatrix}_{r\times 1}$$

$$= \xi_1^i(t), \quad 1 \leqslant i \leqslant m \tag{65}$$

Combining equations (18), (20), (21), (26), and (44), it can be easily verified that equations (63) to (65) can be transformed into the following form

$$\dot{\boldsymbol{\eta}}(t) = \boldsymbol{q}(\xi(t), \boldsymbol{\eta}(t)) + \phi_\eta(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u}) \\ \equiv \boldsymbol{q}_{22}(t, \boldsymbol{\eta}(t), \bar{\boldsymbol{e}}) + \phi_\eta(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u}) \tag{66a}$$

$$\varepsilon \dot{\overline{\boldsymbol{e}^i}}(t) = \mathbf{A}_\mathrm{c}^i \overline{\boldsymbol{e}^i} + \phi_\xi^i(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u}), \quad 1 \leqslant i \leqslant m \tag{66b}$$

$$\boldsymbol{y}_i(t) = \xi_1^i(t), \quad 1 \leqslant i \leqslant m \tag{67}$$

It is considered that $L(\bar{\boldsymbol{e}}, \boldsymbol{\eta})$ defined by a weighted sum of $V(\boldsymbol{\eta})$ and $W(\bar{\boldsymbol{e}})$

$$L(\bar{\boldsymbol{e}}, \boldsymbol{\eta}) \equiv V(\boldsymbol{\eta}) + k(\varepsilon) W(\bar{\boldsymbol{e}}) \\ \equiv V(\boldsymbol{\eta}) + k(\varepsilon)\left(W^1\left(\overline{\boldsymbol{e}^1}\right) + \cdots + W^m\left(\overline{\boldsymbol{e}^m}\right)\right) \tag{68}$$

where

$$W(\bar{\boldsymbol{e}}) \equiv W^1\left(\overline{\boldsymbol{e}^1}\right) + \cdots + W^m\left(\overline{\boldsymbol{e}^m}\right) \tag{69}$$

is a composite Lyapunov function of the subsystems (66a) and (66b) [**33**, **34**], where $W\left(\overline{e^i}\right)$ satisfies

$$W^i\left(\overline{\boldsymbol{e}^i}\right) \equiv \frac{1}{2} \overline{\boldsymbol{e}^i}^\mathrm{T} \mathbf{P}^i \overline{\boldsymbol{e}^i} \tag{70}$$

In view of equations (18), (33), and (40), the derivative of L along the trajectories of equations (66a) and (66b) is given by

$$\begin{aligned}
\dot{L} &= \left[\nabla_t \boldsymbol{V} + (\nabla_\eta \boldsymbol{V})^\mathrm{T} \dot{\boldsymbol{\eta}}\right] + \frac{k}{2}\left[\left(\dot{\overline{\boldsymbol{e}^1}}\right)^\mathrm{T} \mathbf{P}^1 \overline{\boldsymbol{e}^1} + \left(\overline{\boldsymbol{e}^1}\right)^\mathrm{T} \mathbf{P}^1 \left(\dot{\overline{\boldsymbol{e}^1}}\right) + \cdots + \left(\dot{\overline{\boldsymbol{e}^m}}\right)^\mathrm{T} \mathbf{P}^m \overline{\boldsymbol{e}^m} + \left(\overline{\boldsymbol{e}^m}\right)^\mathrm{T} \mathbf{P}^m \left(\dot{\overline{\boldsymbol{e}^m}}\right)\right] \\
&= \left[\nabla_t \boldsymbol{V} + (\nabla_\eta \boldsymbol{V})^\mathrm{T} \dot{\boldsymbol{\eta}}\right] + \frac{k}{2}\left[\left(\frac{1}{\varepsilon}\mathbf{A}_\mathrm{c}^1 \overline{\boldsymbol{e}^1} + \frac{1}{\varepsilon}\phi_\xi^1(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u})\right)^\mathrm{T} \mathbf{P}^1 \overline{\boldsymbol{e}^1} + \left(\overline{\boldsymbol{e}^1}\right)^\mathrm{T} \mathbf{P}^1 \left(\frac{1}{\varepsilon}\mathbf{A}_\mathrm{c}^1 \overline{\boldsymbol{e}^1} + \frac{1}{\varepsilon}\phi_\xi^1(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u})\right) + \cdots\right. \\
&\quad \left. + \left(\frac{1}{\varepsilon}\mathbf{A}_\mathrm{c}^m \overline{\boldsymbol{e}^m} + \frac{1}{\varepsilon}\phi_\xi^m(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u})\right)^\mathrm{T} \mathbf{P}^m \overline{\boldsymbol{e}^m} + \left(\overline{\boldsymbol{e}^m}\right)^\mathrm{T} \mathbf{P}^m \left(\frac{1}{\varepsilon}\mathbf{A}_\mathrm{c}^m \overline{\boldsymbol{e}^m} + \frac{1}{\varepsilon}\phi_\xi^m(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u})\right)\right] \\
&\leqslant \left[\nabla_t \boldsymbol{V} + (\nabla_\eta \boldsymbol{V})^\mathrm{T} \boldsymbol{q}_{22}(t, \boldsymbol{\eta}(t), \bar{\boldsymbol{e}}) + (\nabla_\eta \boldsymbol{V})^\mathrm{T} \phi_\eta(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u})\right] - \frac{k}{2\varepsilon}\left[\left(\overline{\boldsymbol{e}^1}\right)^\mathrm{T} \overline{\boldsymbol{e}^1} + \cdots + \left(\overline{\boldsymbol{e}^m}\right)^\mathrm{T} \overline{\boldsymbol{e}^m}\right] \\
&\quad + \frac{k}{\varepsilon}\left[\|(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u})\| \|\phi_\xi^1\| \|\mathbf{P}^1\| \left\|\overline{\boldsymbol{e}^1}\right\| + \cdots + \|(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u})\| \|\phi_\xi^m\| \|\mathbf{P}^m\| \|\overline{\boldsymbol{e}^m}\|\right] \\
&\leqslant -2\alpha_x \|\boldsymbol{\eta}\|^2 + \omega_3 \|\boldsymbol{\eta}\| M \|\bar{\boldsymbol{e}}\| + \omega_3 \|\boldsymbol{\eta}\| \|\phi_\eta\| \|(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u})\| \\
&\quad - \frac{k}{2\varepsilon}\|\bar{\boldsymbol{e}}\|^2 + \frac{k^2}{\varepsilon^2}\|\phi_\xi^1\|^2 \|\mathbf{P}^1\|^2 \left\|\overline{\boldsymbol{e}^1}\right\|^2 + \frac{1}{4}\|(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u})\|^2 + \cdots + \frac{k^2}{\varepsilon^2}\|\phi_\xi^m\|^2 \|\mathbf{P}^m\|^2 \|\overline{\boldsymbol{e}^m}\|^2 + \frac{1}{4}\|(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u})\|^2 \\
&\leqslant -2\alpha_x \|\boldsymbol{\eta}\|^2 + \frac{1}{16}\omega_3^2 M^2 \|\boldsymbol{\eta}\|^2 + 4\|\bar{\boldsymbol{e}}\|^2 + \omega_3^2 \|\boldsymbol{\phi}_\eta\|^2 \|\boldsymbol{\eta}\|^2 + \frac{1}{4}\|(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u})\|^2 \\
&\quad - \frac{k}{2\varepsilon}\|\bar{\boldsymbol{e}}\|^2 + \frac{k^2}{\varepsilon^2}\|\phi_\xi^1\|^2 \|\mathbf{P}^1\|^2 \|\bar{\boldsymbol{e}}\|^2 + \frac{1}{4}\|(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u})\|^2 + \cdots + \frac{k^2}{\varepsilon^2}\|\boldsymbol{\phi}_\xi^m\|^2 \|\mathbf{P}^m\|^2 \|\bar{\boldsymbol{e}}\|^2 + \frac{1}{4}\|(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u})\|^2 \\
&= -\|\boldsymbol{\eta}\|^2 \left[2\alpha_x - \frac{1}{16}\omega_3^2 M^2 - \omega_3^2 \|\boldsymbol{\phi}_\eta\|^2\right] \\
&\quad - \|\bar{\boldsymbol{e}}\|^2 \left[\frac{k}{2\varepsilon} - \frac{k^2}{\varepsilon^2}\|\phi_\xi^1\|^2 \|\mathbf{P}^1\|^2 - \cdots \frac{k^2}{\varepsilon^2}\|\boldsymbol{\phi}_\xi^m\|^2 \|\mathbf{P}^m\|^2\right] + \frac{m+1}{4}\|(\boldsymbol{\theta}_\mathrm{d} + \boldsymbol{\theta}_\mathrm{u})\|^2
\end{aligned} \tag{71}$$

that is

$$\dot{L} \leqslant -k_{11}\|\bar{\boldsymbol{e}}\|^2 - k_{22}\|\boldsymbol{\eta}\|^2 + \frac{m+1}{4}\|(\boldsymbol{\theta}_{\mathrm{d}}+\boldsymbol{\theta}_{\mathrm{u}})\|^2 \tag{72}$$

Using equation (45c) yields

$$\dot{L} \leqslant -N_2\left(\|\bar{\boldsymbol{e}}\|^2 + \|\boldsymbol{\eta}\|^2\right) + \frac{m+1}{4}\|(\boldsymbol{\theta}_{\mathrm{d}}+\boldsymbol{\theta}_{\mathrm{u}})\|^2 \tag{73}$$

Define

$$\bar{\boldsymbol{e}} \equiv \begin{bmatrix} \overline{\boldsymbol{e}^1} \\ \overline{\boldsymbol{e}^2} \\ \vdots \\ \overline{\boldsymbol{e}^m} \end{bmatrix} \equiv \begin{bmatrix} \overline{\boldsymbol{e}^1} \\ \overline{\boldsymbol{e}^1_{\mathrm{rem}}} \end{bmatrix}, \quad \overline{\boldsymbol{e}^1_{\mathrm{rem}}} \in \Re^{r-1} \tag{74}$$

Hence

$$\dot{L} \leqslant -N_2\left(\|\boldsymbol{\eta}\|^2 + \left\|\overline{\boldsymbol{e}^1_1}\right\|^2 + \left\|\overline{\boldsymbol{e}^1_{\mathrm{rem}}}\right\|^2\right) + \frac{m+1}{4}\|(\boldsymbol{\theta}_{\mathrm{d}}+\boldsymbol{\theta}_{\mathrm{u}})\|^2 \tag{75}$$

Utilizing equation (75) yields

$$\int_{t_0}^{t} \left(y_1(\tau) - y_{\mathrm{d}}^1(\tau)\right)^2 \mathrm{d}\tau \leqslant \frac{L(t_0)}{N_2} + \frac{m+1}{4N_2}\int_{t_0}^{t} \|(\boldsymbol{\theta}_{\mathrm{d}}(\tau)+\boldsymbol{\theta}_{\mathrm{u}}(\tau))\|^2 \mathrm{d}\tau \tag{76}$$

Similarly, it is easy to prove that

$$\int_{t_0}^{t} \left(y_i(\tau) - y_{\mathrm{d}}^i(\tau)\right)^2 \mathrm{d}\tau \leqslant \frac{L(t_0)}{N_2} + \frac{m+1}{4N_2}\int_{t_0}^{t} \|(\boldsymbol{\theta}_{\mathrm{d}}(\tau)+\boldsymbol{\theta}_{\mathrm{u}}(\tau))\|^2 \mathrm{d}\tau, \quad 2 \leqslant i \leqslant m \tag{77}$$

so that statement (39) is satisfied. From equation (73)

$$\dot{L} \leqslant -N_2\left(\|\boldsymbol{y}_{\mathrm{total}}\|^2\right) + \frac{m+1}{4}\|(\boldsymbol{\theta}_{\mathrm{d}}+\boldsymbol{\theta}_{\mathrm{u}})\|^2 \tag{78a}$$

where

$$\|\boldsymbol{y}_{\mathrm{total}}\|^2 \equiv \|\bar{\boldsymbol{e}}\|^2 + \|\boldsymbol{\eta}\|^2 \tag{78b}$$

By virtue of Theorem 5.2 of reference [**32**], equation (78a) implies the input-to-state stability for the closed-loop system. Furthermore, it is easy to see that

$$\Delta_{\min}\left(\|\bar{\boldsymbol{e}}\|^2 + \|\boldsymbol{\eta}\|^2\right) \leqslant L \leqslant \Delta_{\max}\left(\|\bar{\boldsymbol{e}}\|^2 + \|\boldsymbol{\eta}\|^2\right) \tag{79}$$

that is

$$\Delta_{\min}\left(\|\boldsymbol{y}_{\mathrm{total}}\|^2\right) \leqslant L \leqslant \Delta_{\max}\left(\|\boldsymbol{y}_{\mathrm{total}}\|^2\right) \tag{80}$$

where

$$\Delta_{\min} \equiv \min\left\{\omega_1, \frac{k}{2}\lambda^*_{\min}\right\}$$

and

$$\Delta_{\max} \equiv \max\left\{\omega_2, \frac{k}{2}\lambda^*_{\max}\right\}$$

Equations (73) and (80) yield, that

$$\dot{L} \leqslant -\frac{N_2}{\Delta_{\max}}L + \frac{m+1}{4}\left(\sup_{t_0 \leqslant \tau \leqslant t}\|(\boldsymbol{\theta}_{\mathrm{d}}(\tau)+\boldsymbol{\theta}_{\mathrm{u}}(\tau))\|\right)^2 \tag{81}$$

Hence,

$$L(t) \leqslant L(t_0)\exp\left(-\frac{N_2}{\Delta_{\max}}(t-t_0)\right) + \frac{\Delta_{\max}(m+1)}{4N_2}\left(\sup_{t_0 \leqslant \tau \leqslant t}\|(\boldsymbol{\theta}_{\mathrm{d}}(\tau)+\boldsymbol{\theta}_{\mathrm{u}}(\tau))\|\right)^2 \quad t \geqslant t_0 \tag{82}$$

which implies

$$|y_1(t) - y_{\mathrm{d}}^1(t)| \leqslant \sqrt{\frac{2L(t_0)}{k\lambda^*_{\min}}}\exp\left(-\frac{N_2}{2\Delta_{\max}}(t-t_0)\right) + \sqrt{\frac{\Delta_{\max}(m+1)}{2k\lambda^*_{\min}N_2}}\left(\sup_{t_0 \leqslant \tau \leqslant t}\|(\boldsymbol{\theta}_{\mathrm{d}}(\tau)+\boldsymbol{\theta}_{\mathrm{u}}(\tau))\|\right) \tag{83}$$

Similarly, it is easy to prove that

$$|y_i(t) - y_{\mathrm{d}}^i(t)| \leqslant \sqrt{\frac{2L(t_0)}{k\lambda^*_{\min}}}\exp\left(-\frac{N_2}{2\Delta_{\max}}(t-t_0)\right) + \sqrt{\frac{\Delta_{\max}(m+1)}{2k\lambda^*_{\min}N_2}}\left(\sup_{t_0 \leqslant \tau \leqslant t}\|(\boldsymbol{\theta}_{\mathrm{d}}(\tau)+\boldsymbol{\theta}_{\mathrm{u}}(\tau))\|\right) \quad 2 \leqslant i \leqslant m \tag{84}$$

so that equation (38) is proved and then the tracking problem with almost disturbance decoupling is globally solved. This completes the proof.

According to the previous theorems and discussions, an efficient algorithm for deriving the almost disturbance decoupling control is proposed as follows:

Step 1. Calculate the vector relative degree r_1, r_2, ..., r_m of the given control system.
Step 2. Choose the diffeomorphism ϕ such that the assumption 1 is satisfied.
Step 3. Adjust some parameters $\alpha_1^i, \alpha_2^i, \ldots, \alpha_{r_i}^i$ such that the matrices $\mathbf{A}_c^i$ are Hurwitz and calculate the positive definite matrices $\mathbf{P}^i$ of the Lyapunov equations (28) by some software package, such as Matlab.
Step 4. Based on the famous Lyapunov approach, design a Lyapunov function to solve the conditions (40a) to (40c). If the relative degree $r_1+r_2+\cdots+r_m$ is equal to the system dimension n, then this step should be omitted and immediately go to the next step.
Step 5. Appropriately tune the parameters k and ε such that $NN_2>1$ and go to the next step. Otherwise, go to step 3 and repeat the overall designing procedures.
Step 6. According to equation (41), the desired feedback linearization controller $\boldsymbol{u}_{\text{feedback}}$ can be constructed such that the uniform ultimate bounded stability is guaranteed. That is, the system dynamics enter a neighbourhood of zero state and remain within it thereafter.

3 ILLUSTRATIVE EXAMPLE

Consider the half-car active suspension system with disturbances shown in Fig. 1. From Huang and Lin [**35**], the dynamic equations are given as follows

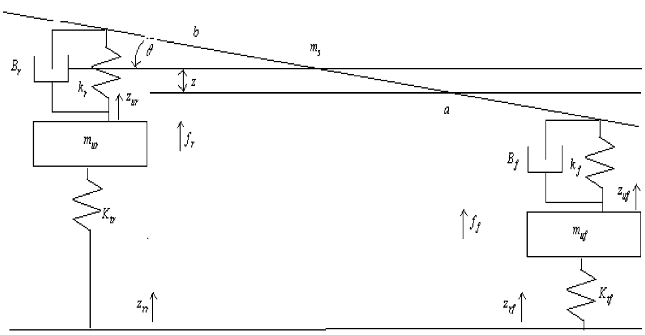

Fig. 1 The half-car active suspension system

$$
\begin{aligned}
\dot{x}_1 &= x_2 + \Delta f_1 \\
\dot{x}_2 &= \frac{1}{m_s}[-(B_f+B_r)x_2 + (aB_f - bB_r)x_4\cos x_3 \\
&\quad - k_f x_5 + B_f x_6 - k_r x_7 + B_r x_8 + (f_f + f_r)] \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{1}{J_y}[(aB_f - bB_r)x_2\cos x_3 \\
&\quad - (a^2B_f + b^2B_r)x_4\cos^2 x_3 + ak_f x_5 \cos x_3 \\
&\quad - aB_f x_6 \cos x_3 - bk_r x_7 \cos x_3 \\
&\quad + bB_r x_8 \cos x_3 + (-af_f + bf_r)\cos x_3] \\
\dot{x}_5 &= x_2 - ax_4\cos x_3 - x_6 \\
\dot{x}_6 &= \frac{1}{m_{uf}}[-K_{tf}x_1 + B_f x_2 + aK_{tf}\sin x_3 \\
&\quad - aB_f x_4 \cos x_3 + (k_f + K_{tf})x_5 \\
&\quad - B_f x_6 + K_{tf} z_{rf} - f_f] \\
\dot{x}_7 &= x_2 + bx_4\cos x_3 - x_8 \\
\dot{x}_8 &= \frac{1}{m_{ur}}[-K_{tr}x_1 + B_r x_2 - bK_{tr}\sin x_3 \\
&\quad + bB_r x_4 \cos x_3 + (k_r + K_{tr})x_7 \\
&\quad - B_r x_8 + K_{tr} z_{rr} - f_r]
\end{aligned}
\tag{85a}
$$

$$
\begin{aligned}
y_1 &= x_1 + x_2 := h_1 \\
y_2 &= x_3 + x_4 := h_2
\end{aligned}
\tag{85b}
$$

where $x_1 = z$ is the displacement of the centre of gravity, $x_2 = \dot{z}$ is the payload velocity, m_s is the mass of the car body, B_f and B_r are the front and rear damping coefficients, a is the distance between front axle and the centre of gravity, b is the distance between rear axle and the centre of gravity, $x_3 = \theta$ is the pitch angle, $x_4 = \dot{\theta}$ is the pitch velocity, k_f and k_r are the front and rear spring coefficients, z_{sf} and z_{sr} are the front and rear body displacement, z_{uf} and z_{ur} are the front and rear wheel displacements, $x_5 = z_{sf} - z_{uf}$ is the front wheel suspension travel, $x_6 = \dot{z}_{uf}$ is the front unsprung mass velocity, $x_7 = z_{sr} - z_{ur}$ is rear wheel suspension travel, $f_f = u_1$ and $f_r = u_2$ are the front and rear force inputs, J_y is its centroidal moment of inertia, m_{uf} and m_{ur} are the unsprung masses on the front and rear wheels, K_{tf} and K_{tr} are the front and rear tyre spring coefficients, z_{rf} and z_{rr} are the front and rear terrain height disturbances, Δf_1 is the system uncertainty, and $x_8 = \dot{z}_{ur}$ is the rear unsprung mass velocity. The following physical parameters are chosen in the present simulation: $m_s = 575\,\text{kg}$, $B_f = B_r = 1000\,\text{N m/s}$, $a = 1.38\,\text{m}$, $b = 1.36\,\text{m}$, $J_y = 769\,\text{kg/m}^2$, $m_{uf} = m_{ur} = 60\,\text{kg}$, $K_{tf} = K_{tr} = 190\,000\,\text{N/m}$, $k_f = k_r = 16\,812\,\text{N/m}$, $z_{rf} = z_{rr} = \mu_r(1 - \cos 8\pi t)$, $\Delta f_1 = 0.1\sin\ t(\cos\ x_5)$, and $\mu_r = 0.05\,\text{m}$. Hence the mathematical model can be rewritten as

$$\begin{aligned}
\dot{x}_1 &= x_2 + 0.1\sin t(\cos x_5)\\
\dot{x}_2 &= -3.478x_2 + 0.035x_4\cos x_3 - 29.238x_5\\
&\quad + 1.739x_6 - 29.238x_7 + 1.739x_8\\
&\quad + (0.0017u_1 + 0.0017u_2)\\
\dot{x}_3 &= x_4\\
\dot{x}_4 &= 3.563x_2\cos x_3 - 4.881x_4\cos^2 x_3\\
&\quad + 30.17x_5\cos x_3 - 1.794x_6\cos x_3\\
&\quad - 29.732x_7\cos x_3\\
&\quad + 1.768x_8\cos x_3\\
&\quad + (-0.0018u_1 + 0.00176u_2)\cos x_3\\
\dot{x}_5 &= x_2 - 1.38x_4\cos x_3 - x_6\\
\dot{x}_6 &= -3166.667x_1 + 16.667x_2 + 4370\sin x_3\\
&\quad - 23x_4\cos x_3 + 3446.867x_5 - 16.667x_6\\
&\quad + 158.33(1 - \cos 8\pi t) - 0.0167u_1\\
\dot{x}_7 &= x_2 + 1.36x_4\cos x_3 - x_8\\
\dot{x}_8 &= -3166.667x_1 + 16.667x_2\\
&\quad - 4306.667\sin x_3 + 22.667x_4\cos x_3\\
&\quad + 3446.067x_7\\
&\quad - 16.667x_8 + 158.33(1 - \cos 8\pi t)\\
&\quad - 0.0167u_2
\end{aligned} \tag{86a}$$

$$\begin{aligned}
y_1 &= x_1 + x_2 := h_1\\
y_2 &= x_3 + x_4 := h_2
\end{aligned} \tag{86b}$$

Now it is shown how to explicitly construct a controller that tracks the desired signals $y_d^1 = y_d^2 = 0$ and attenuates the disturbance's effect on the output terminal to an arbitrary degree of accuracy. Let us arbitrarily choose $\alpha_1^1 = \alpha_1^2 = 0.06$, $A_c^1 = A_c^2 = -0.06$, $P^1 = P^2 = 25/3$, and $\lambda^*_{\min} = \lambda^*_{\max} = 25/3$. From equation (41), the desired tracking controllers are obtained

$$\begin{aligned}
u_1 &= -1381.25x_4\cos x_3 + 16\,977.16x_5\\
&\quad - 1009.63x_6 + 150.69x_7\\
&\quad - 9.07x_8 + 1198.00x_2 - 523.44x_1\\
&\quad + 505.62x_3(\cos x_3)^{-1} + 786.52x_4(\cos x_3)^{-1}
\end{aligned} \tag{87}$$

$$\begin{aligned}
u_2 &= 1360.66x_4\cos x_3 + 220.63x_5\\
&\quad - 13.244x_6 + 17\,047.1x_7\\
&\quad - 1013.81x_8 - 799.21x_2 - 535.32x_1\\
&\quad - 505.62x_3(\cos x_3)^{-1} - 786.52x_4(\cos x_3)^{-1}
\end{aligned} \tag{88}$$

It can be verified that the relative conditions of Theorem 1 are satisfied with $\varepsilon = 0.03$, $B_d^1 = B_d^2 = 0$, $M = \sqrt{3}$, $\omega_1 = \omega_2 = 1$, $\alpha_x = 1$, $\omega_3 = 2$, $k_{11} = 24.86$, $k_{22} = 1.25$, $N_1 = 79$, $N_2 = 1.25$, and $k = 10\sqrt{\varepsilon}$. Hence the tracking controllers will steer the output tracking errors of the closed-loop system, starting from any initial value, to be asymptotically attenuated to zero by virtue of Theorem 1. The complete trajectories of the outputs are depicted in Fig. 2 and Fig. 3.

4 COMPARATIVE EXAMPLE TO EXISTING APPROACH

Marino *et al.* [**27**] exploits the fact that for a nonlinear SISO system the almost disturbance decoupling problem can not be solved, as the following example shows:

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}\tan^{-1}(x_2)\\ 0\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u + \begin{bmatrix}1\\ 0\end{bmatrix}\theta(t) \tag{89a}$$

$$y(t) = x_1(t) \tag{89b}$$

where u and y denote the input and output respectively, $\theta(t) := 0.5\sin\ t$ is the disturbance. The feedback control algorithm proposed in this paper will solve it perfectly. Applying the same design

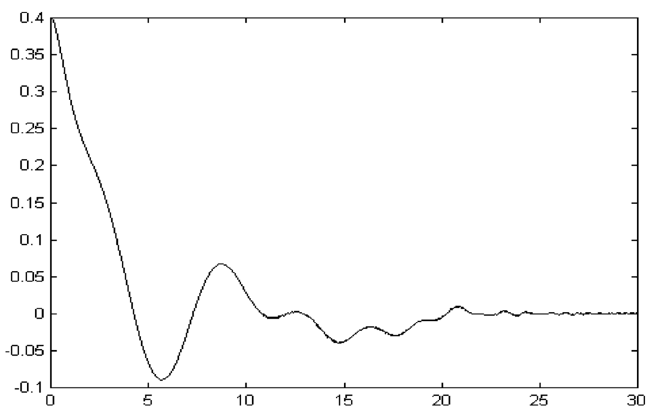

Fig. 2 The output trajectory x_1 of the half-car active suspension system

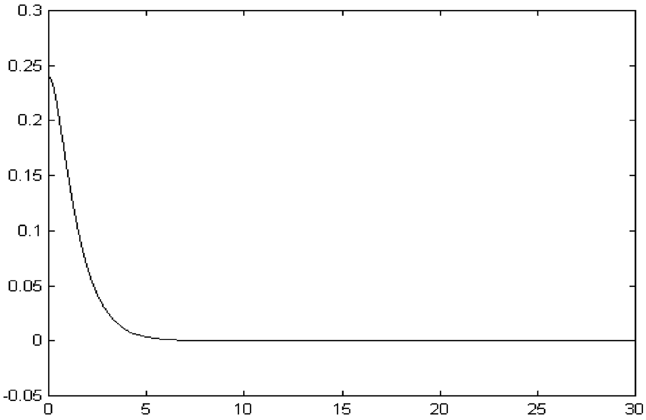

Fig. 3 The output trajectory x_3 of the half-car active suspension system

procedures of Theorem 1 yields the desired tracking and almost disturbance decoupling controller as follows

$$u=(1+x_2^2)\Big[-\sin t-(0.03)^{-2}(x_1-\sin t) \\ -(0.03)^{-1}(\tan^{-1}x_2-\cos t)\Big] \quad (90)$$

The output trajectory of the feedback-controlled system for (89) is depicted in Fig. 4. From Fig. 4, it is obvious to see that the desired tracking and almost disturbance decoupling performance are achieved.

It is worth noting that the sufficient conditions given in Marino *et al.* [**27**] (in particular the structural conditions on non-linearities multiplying disturbances) are not necessary in this study where a non-linear state feedback control is explicitly designed which solves the almost disturbance decoupling problem. For instance, the almost disturbance decoupling problem is solvable for the system (89) by a non-linear state feedback control, according to the current proposed approach, while the sufficient conditions given in Marino *et al.* [**27**] fail when applied to the system (89). The design techniques in this study are also entirely different than those in Marino *et al.* [**27**] since the singular perturbation tools are not used.

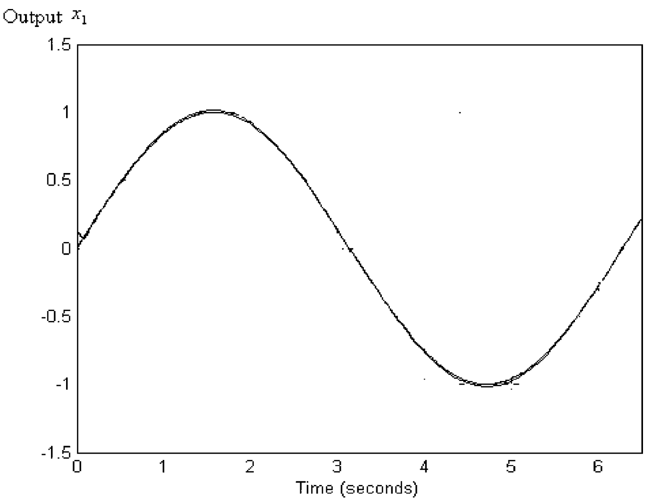

Fig. 4 The output trajectory of the feedback-controlled system for system (89)

5 CONCLUSION

A novel feedback control to globally solve the tracking problem with almost disturbance decoupling for multi-input multi-output non-linear uncertain system has been proposed. A discussion and a practical application of feedback linearization of non-linear control systems using a parameterized coordinate transformation have been presented. One comparative example is proposed to show the significant contribution of this paper with respect to existing approaches. A practical example of a half-car active suspension system has been used to demonstrate the applicability of the proposed feedback linearization approach and the composite Lyapunov approach. Simulation results have been presented to show that the proposed methodology can be successfully applied to the feedback linearization problem and is able to achieve the desired tracking and almost disturbance decoupling performances of the controlled system.

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