國 立 交 通 大 學

電信工程研究所

碩 士 論 文

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細胞系統的性能分析

Performance Analysis of Small Cells in
General Fading Channels Using Stochastic
Geometry Approach

研究生：陳佳佳

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摘　　要

隨著小細胞在蜂窩式網路拓撲結構占的比重越來越大，小細胞網路覆蓋幾率的分析日趨重要。最近一種新的蜂窩式網路的拓撲結構被提出，即利用隨即幾何理論，建模以及研究小細胞網路。為了避免分析上的複雜度，之前大多數的對於泊松分佈的細胞網路研究都只是基於一種簡單的瑞利衰弱。在這篇文章中，我們研究基於更多普遍通道影響中泊松分佈的細胞網路的覆蓋幾率問題。首先本文得到了一個整潔的在瑞利陰影衰弱的通道下覆蓋幾率的運算式。其次本文得到一個低複雜的在雙斜線路徑衰減通道中覆蓋幾率運算式，最後本文得到了一個在一般陰影衰弱的通道下的覆蓋幾率運算式。我們發現陰影衰弱對小細胞網路的性能起著很大的影響。

Abstract

Coverage analysis of a small cell network is very crucial since small cell deployment will dominates the topology of a cellular in the future. The majority of prior work on the coverage probability is studied based on a simple and consistent Rayleigh fading models in a Poisson-distributed cellular network in order to avoid analytical intractability. In this paper, we study the coverage probability problem in a Poisson small cell network with much more general channel impairments. First a neat expression of the coverage probability with compose Rayleigh fading and log-normal shadowing is derived and it discloses two important facts – the coverage performance is not improved by deploying more base stations and it is significantly weakened by shadowing. Then we find the coverage probability with low complexity for the case that a dual-slop path loss is used, the desired signal experiences Nakagami-m fading and interference signals undergo Rayleigh fading. It is able to more practically reflect the coverage performance of a user.

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I would like to thank my parents. They always give me endless supports. I especially thank Professor Li-Chun Wang who gave me many valuable suggestions in my research during these two years. I would not finish this work without his guidance and comments.

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Contents

Chapter 1

Introduction

Small cells are viewed as one of the key technologies to the success of the next generation cellular networks since they aim at boosting network capacity and saving energy. They are essentially defined as the low-powered radio access nodes operating in licensed or unlicensed bands, which are also called femtocells, picocells and microcells in different environments [1].

Small cells are viewed as one of the key technologies to the success of the next generation wireless systems, So the performance analysis of small cell becomes extensive discussions. For system operator aspect, they want to make performance analysis to determine the number of small cells within an area to optimize the trade-off among system throughput, coverage, energy consumption and deployment cost. For cellular phone company aspect, they analyze performance of small cell to have a knowledge that What data rates can be transmitted to their cellular phones and Which services can be delivered and leverage this insightful information in designing applications for smart phone.

One thing that influences the performance of small cell most is the electromagnetic propagation in the wireless environment. When we focus on the

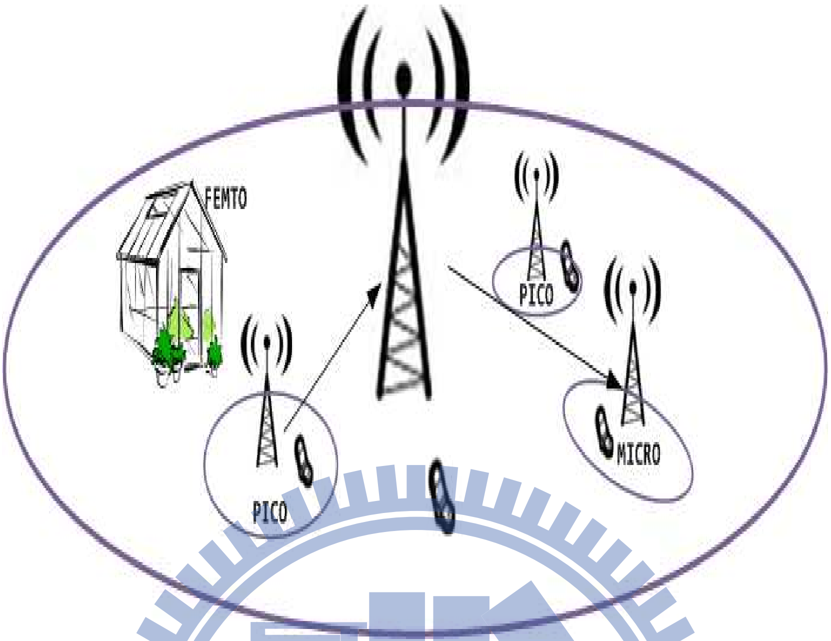


Figure 1.1: Small Cell System

downlink, base station(BS) to mobile case, em energy radiated from the BS of a given cell, it encounters a lots of obstacles, not just like trees, buildings, but the particles in the air. Then it reflected, diffracted and scattered by these various obstacles. So the signals received in the receiver's antenna will probably change drastically. We will introduce a propagation model to model these environment. [2]

What's more, as the increasing numbers of small cells are built, the interference received in mobile will increase too. So we should attach importance to the interference analysis of small cells.

1.1 Problem and Solution

One thing that influences the performance of small cell most is the electromagnetic propagation in the wireless environment. When we focus on the downlink, base station(BS) to mobile case, em energy radiated from the BS of a given cell, it encounters a lots of obstacles, not just like trees, buildings, but the particles in the air. Then it reflected, diffracted and scattered by these various obstacles. So the signals received in the receiver's antenna will probably change drastically. We will introduce a propagation model to model these environment. [2]

What's more, as the increasing numbers of small cells are built, the interference received in mobile will increase too. So we should attach importance to the interference analysis of small cells.

However, there exist lots of problems in tractably analyzing the performance of a small cell network because the radio propagation model of signals and the spatial distribution model of base stations (BSs) jointly make the analysis with high complexity. In order to make the analysis much more tractable, most of the existing work usually uses a fairly simple channel model to characterize some channel impairments such as path loss and fading. Shadowing is intentionally overlooked due to the intractability it induces in mathematical analysis.

Using oversimplified channel models is the main drawback in the previous work on PPP-based cellular networks. Therefore the accuracy of the previous results of the coverage probability or outage probability is doubtable. In this paper, we consider a more general channel impairment model that includes small-scale fading, shadowing, single-slop path loss and/or dual-slop path loss. In particular, first we consider a Poisson-distributed small cell network with Rayleigh fading and log-normal shadowing. The coverage probability is obtained in a near closed form and it is severely reduced by shadowing. This observation certainly contradicts the opinion in some previous work that shadowing has a modest impact on the system performance. We also show that the coverage probability can be largely improved only when the network is not interference-limited. Then we study the coverage probability with a dual-slop path loss, Nakagami-m fading in the desired signal channel and Rayleigh fading in the interference channels. A low-complexity expression of the coverage probability is derived, which provides a more realistic result for the coverage probability and thus the coverage probability with simple Rayleigh fading and path loss obtained in the previous work could be seriously underestimated.

1.2 Thesis Outline

The research in this thesis develop several channel modeling techniques to evaluate the performance of small cell system using stochastic geometry - log-normal shadowed rayleigh fading channel, nakagami fading channel, log-normal shadowed nakagami fading channel. We organize the remaining chapters of this dissertation as follows. Chapter 2 introduces the background of smal cell systems, 3GPP sec used in this paper and point process theory in stochastic geometry. Chapter3 develops a system model for small cell system. Chapter 4 introduce the previous framework and result. Chapter 5, chapter 6 demonstrate a framework to analysis the performance of the proposed system model. Chapter 7 shows the analysis result and set up a simulator to verify the analysis result. Chapter 8 is the conclusion.

Chapter 2

Background

In this chapter, we firstly survey related works to performance analysis of small cells. Then, the core method, "stochastic geometry," for analyse the coverage probability is introduced in depth. At last, we will demonstrate the propagation model.

2.1 Literature Survey

The coverage probability of a cellular network is an important index of the system performance, which is defined as the probability that the singal-to-interference plus noise (SINR) of a user in a cell is higher than some predesignated threshold. Prior work pertaining to the coverage probability analysis in a wireless network focuses on the analysis of outage probability or success probability. For instance, reference [3] provides an overview of how a Poisson point process (PPP) is successfully applied to do the tractable analysis of the outage probability under some random access protocols. In [4] a PPP-based cellular network is proposed to characterize a more realistic distribution of BSs in the network. The authors claim to acquire some tractable results of the coverage probability and achievable rate of a user by using a simple path loss and Rayleigh fading models. However, they only obtain the "worse-case" results that would detach far away from the reality. Reference [5] found the average rate of downlink heterogeneous cellular networks with a general fading assumption. It lacks of the analysis of the coverage probability and no shadowing is in its model.

2.2 Point Process Theory in Stochastic Geometry

Stochastic geometry is a mathematical tool that analyses the characterization of the interference. Also, instead of obtaining the results that are only valid for a deterministic network (hexagon modeled network), it allows averaging of the performance, in other words, general results, over likely network realizations. What's more, the spatial model in wireless networks is emphasized by stochastic geometry.

2.2.1 The Poisson Point Process

In this paper, we use the homogeneous poisson point process as a reference model for the distribution of base stations in the wireless network.

A point process $\Phi = x_1, x_2, ...x_i, ...$ is point measures mapping on a locally finite space A. The x_i is random variables. Most often, the dimension of the space A is larger than 1. In this paper, we use the dimension $d = 2$, that is the space A is the euclidean space $\mathbb{R}^2$. We define the intensity measure Λ of Φ as $\Lambda(B) = \mathbb{E}\Phi(B)$. $\Phi(A_i)$ means the counting measure of $\Phi \bigcap$ Borel B.For a stationary PPP of intensity λ,$\Lambda(B) = \lambda|B|$.

Poisson point processes: A point processes Φ is Poisson on A if

- The random variables $\Phi(A_i)$ are independent and $A_i \in a$ are non-overlapping areas.
- The random variables $\Phi(A_i)$ are Poisson, satisfying

$$\mathbb{P}[\Phi(A_i) = n] = \exp(-\lambda|A_i|)\frac{(\lambda|A_i|)^n}{n!} \tag{2.1}$$

  where $|\,.\,|$ the Lebesque measure(area), and λ is the density of the homogeneous PPP.

For convenience, we use palm probability P^O which means it always has a point at the origin. Later, we will use Campbell's theorem for PPPs. Campbell's theorem: Let f be non-negative function. Then

$$\mathbb{E}\sum_{x\in\Phi} f(x) = \int_{\mathbb{R}^2} f(x)\Lambda(dx) \tag{2.2}$$

2.2.2 Path Loss Law Concern of Interference

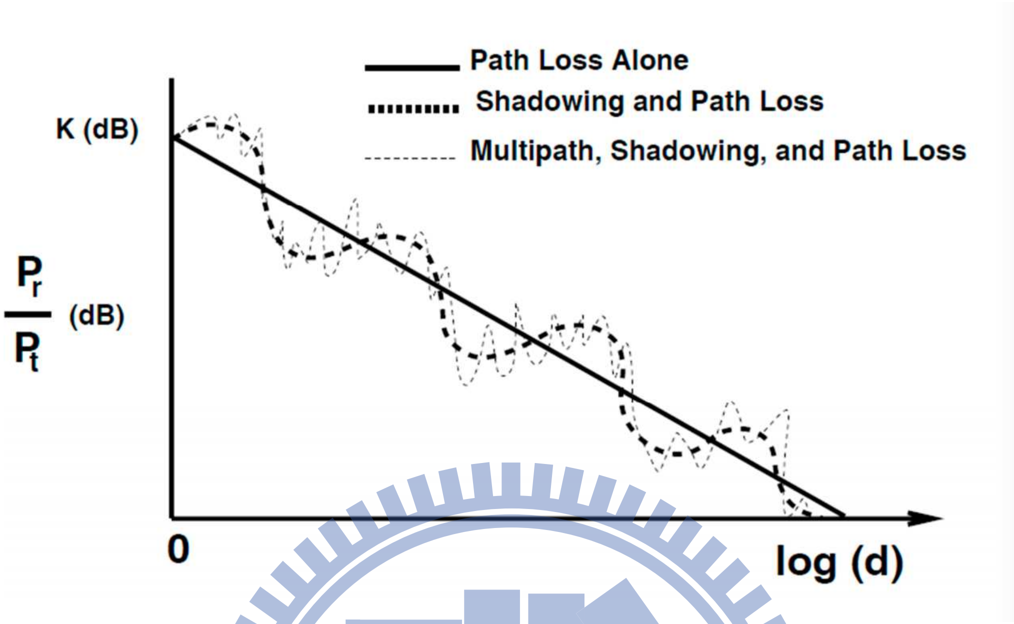
We assume that the path loss is $g(r) = r^{-\alpha}$,h_x is the power fading coefficient. Interference received at origin can be written:

$$I = \sum_{x\in\Phi} h_x\|x\|^{-\alpha} \tag{2.3}$$

Since $\mathbb{E}h_x = 1$,

$$\mathbb{E}(I) = \mathbb{E}\sum_{r\in\Phi^*} r^-\alpha \tag{2.4}$$

Where $\Phi^* = \|x_1\|, \|x_2\|, \ldots$ Using Campbell's theorem, we find that if $\alpha < 2$,there is too much interference from the nodes and will be infinite. So, if we want to apply stochastic geometry to analysis of propagation model of small cell, we should assume that $\alpha > 2$.



2.3 Propagation Model

Here, we considers on the downlink, base station to user case. The power emitted from the Base station will often encounter a lot of obstacles, such as buildings, trees, animals, etc., The radio will be reflected, diffracted and scattered by these obstacles. What's more, the received power varies as the distance between transmitter and receiver changes. One finds several effects appearing: path-loss, shadowing and fading.

Composite these three phenomena together, the received signal can be modeled and given by the following equation.

$$P_r = G\ell(d)P_T \tag{2.5}$$

Where P_r is the received power, G is is used to characterize the fading and/or shadowing effect of the desired signal channel and the path loss law of the desired signal channel is described by $\ell(d)$

2.3.1 Path Loss

Path loss is a large-scale fading, varying as they do at relatively long distances caused by complicated interactions of three large-scale propagation mechanisms: free space loss, reflection, and diffraction [6]. We define area-mean power as the average signal power in the area where the receiver is.And it can be estimated through path loss prediction model.

2.3.2 Shadowing

Shadowing is a large-scale fading, varying as they do at relatively long distances caused by diffraction and scattering when em encounters buildings, terrain, and trees. The shadow loss represents the actual local-mean power received, statistically fluctuating about the area-mean power. We use a log-normal distribution to characterize the changing of local mean power.

2.3.3 Multipath Fading

Multipath fading is a small-scale fading. At much smaller distances, there is a large variation of the sigmals. The received signal is usually a summation of multiple reflected waves from different path. Because of the different phase of each reflected waves, the signal will be distorted. There is several distribution to model the statistic of the multipath fading: Rayleigh fading, Rician fading, m-Nakagami fading, etc. In this paper, we use m-Nakagami fading, take the advantages of that The Nakagami pdf bdcomes the Rayleigh density function when m is equal to unity and the Nakagami distribution matches some empirical data better than other models.

Chapter 3

System Models

We consider an infinitely large small cell network in which all base stations are assumed to be independently and randomly scattered by following a homogeneous PPP Φ_B of intensity λ_B on the plane $\mathbb{R}^2$, and they all use the same transmit power P. Each user in the network is associated with his nearest BS. Without loss of generality, our analysis in this paper is based on the reference user located at the origin, as shown in Fig. 3.1. All transmitted signals in the downlink suffer from the following channel impairments – path loss, (small-scale) Rayleigh fading and/or (large-scale) shadowing. [7]

Let X_0 and $\{X_j, j \in \mathbb{N}_+\}$ denote the serving BS and interfering BSs of the reference user, respectively. Then the interference received by the reference user is expressed as

$$I_0(P) \triangleq P \sum_{X_j \in \Phi_B \setminus X_0} \tilde{G}_j \tilde{\ell}(\|X_j\|), \tag{3.1}$$

where $\tilde{G}_j$'s stand for the i.i.d. interference channel (power) gain due to fading and/or shadowing from BS X_j to the reference user, $\|X_j\|$ denotes the Euclidean distance between BS X_j and the origin, and $\tilde{\ell}(\|X_j\|)$ represents the path loss law of the interference channel between node X_j and the origin.

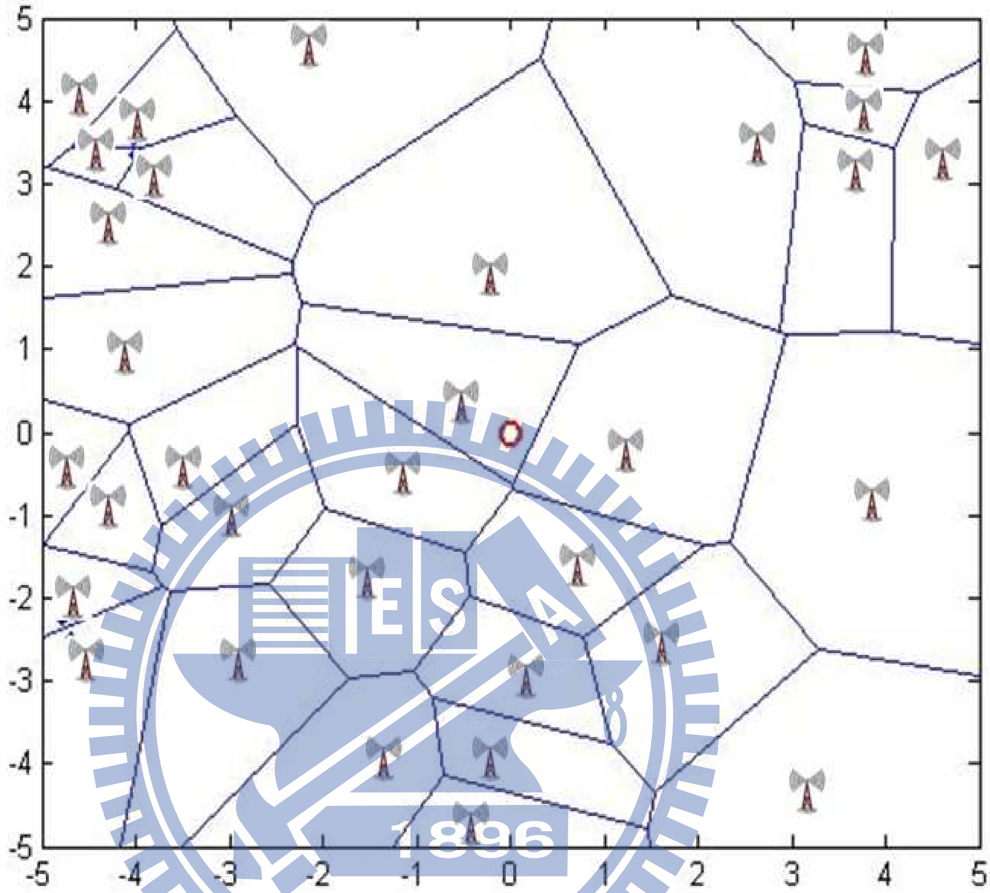


Figure 3.1: A schematic representation of a PPP-based small cell network with Voronoi tessellation. The reference user is located at the origin.

With the interference given in (3.1), the signal-to-interference plus noise ratio (SINR) of the reference user is given by

$$\mathtt{SINR}_0 \triangleq \frac{G_0 \ell(R)}{I_0(1) + \sigma_0^2/P}, \tag{3.2}$$

where G_0 is used to characterize the fading and/or shadowing effect of the desired signal channel from BS X_0 to the reference user, σ_0^2 is the ambient noise power, and the path loss law of the desired signal channel is described by $\ell(R)$ in which R denotes the distance from BS X_0 to the reference user, i.e. $R = \|X_0\|$. Since X_0 is the nearest BS of the reference user, the probability density function (pdf) of distance R is

$$f_R(r) = 2\pi r \lambda_B e^{-\pi r^2 \lambda_B},$$

which can be obtained by differentiating $\mathbb{P}[R \le r] = 1 - \exp(-\pi r^2 \lambda_B)$ with respect to r [8].

For successful decoding, the received SINR of each user should be higher than some predesignated value. In particular, we use the SINR in (3.2) to define the coverage probability of the reference user in the network as follows

$$p_{\mathsf{cov}} \triangleq \mathbb{P}\left[\mathtt{SINR}_0 \ge \theta\right] = \mathbb{P}\left[\frac{G_0 \ell(R)}{I_0(1) + \sigma_0^2/P} \ge \theta\right] \tag{3.3}$$

assuming the SINR threshold is θ.

Chapter 4

Coverage Probability with Rayleigh Fading and Log-Normal Shadowing

In order to analyze the coverage probability in (3.3), first the statistic property of the interference $I_0(P)$ needs to be characterized since (3.3) can be rewritten with $\gamma_0 \triangleq \frac{P}{\sigma_0^2}$ as

$$\begin{aligned} p_{\mathsf{cov}} &= \mathbb{P}\left[G_0 \geq \frac{(I_0(1)+1/\gamma_0)}{\ell(R)}\theta\right] \\ &= \int_0^{\infty} \mathbb{E}_{I_0}\left[F^{\mathsf{c}}_{G_0}\left(\frac{(I_0(1)+1/\gamma_0)\theta}{\ell(r)}\right)\right] f_R(r)\mathrm{d}r, \end{aligned} \tag{4.1}$$

where $f_Z(\cdot)$ and $F^{\mathsf{c}}_Z(\cdot)$ denote the pdf and complementary cumulative distribution function (CCDF) of random variable Z, respectively. Thus, the distributions of random variables I_0 and G_0 are needed to simplify the result in (4.1) to a much neat form.

4.1 Channel Environment

4.1.1 Path-loss

Note that $\mathcal{L}_{I_0}(s)$ in (4.12) could be significantly simplified if $\tilde{\ell}(r)$ has an appropriate function of r. The path loss law between two nodes in the network can be formulated in different ways according to different environmental situations. For simplicity, in this section we consider the case that the path loss models of all channels have the same simple single-slope form, i.e.

$$\ell(r) = \tilde{\ell}(r) = Kr^{-\alpha} \tag{4.2}$$

in which $\alpha > 2$ is the path loss exponent and K is a small constant with unit meter$^{\alpha}$ capable of making the received power smaller than transmit power as $r \leq 1$.

4.1.2 Log-normal shadowing and Rayleigh fading

For the interference, the channel environment is To characterize both log-normal shadowing and Rayleigh fading effects on p_{cov}, the following composite shadowing-fading pdf of G_0 obtained from [9] is adopted:

$$f_{G_0}(g) = \int_{0^+}^{\infty} \frac{1}{\sqrt{2\pi}\sigma x^2} \exp\left(-\frac{g}{x} - \frac{(\ln(x) - \mu)^2}{2\sigma^2}\right) \mathrm{d}x, \tag{4.3}$$

where σ_s^2 is the variance of the log-normal shadowing. The closed-form distribution function of I_0 is impossibly obtained. However, its Laplace functional is useful to further simplify p_{cov} in (4.1), which is given in the following theorem.

4.2 Performance Analysis

4.2.1 Laplace Transform of Interference

Lemma 1. *Suppose the interference channel gains* $\{\tilde{G}_j\}$ *have the same pdf as that of* G_0 *given in* (4.3)*. The Laplace functional of the interference in* (3.1) *is given by*

$$\mathcal{L}_{I_0}(s) = e^{-2\pi\lambda\mathbb{E}_N[\frac{sKr^{2-\alpha}}{\alpha=2}{}_2F_1(1,1-\frac{2}{\alpha};2-\frac{2}{\alpha};-\frac{\Omega sK}{r^{\alpha}})]e^{-\sigma N}}, \tag{4.4}$$

where $\mathcal{L}_Z(s) \triangleq \mathbb{E}[e^{-sZ}]$ *is the Laplace functional of random variable* Z *and* N *is the standard normal random variable.*

The lemma above is a form of Gaussian quadrature. In numerical analysis, We can use Gauss-Hermite quadrature to approximate the value of integrals of this.

Proof. According to the result in [10], $\mathcal{L}_{I_0}(s)$ has the following identity:

$$\begin{aligned} \mathcal{L}_{I_0}(s) &\triangleq \mathbb{E}_{I_0}[\exp(-sI_0)] \\ &= \mathbb{E}_{\phi^*,g_r}\left(\exp(-s\sum_{r\in\phi^*} g_r\ell(r))\right) \\ &= \mathbb{E}_{\phi^*}\left[\prod_{r\in\phi^*}\mathbb{E}_{g_r}[\exp(-sg_r\ell(r))]\right] \end{aligned} \tag{4.5}$$

Where $\phi^* = \|x_1\|, \|x_2\|, ...$ is the PPP ϕ of the distances.
Using Probability generating functional, we can get that,

$$\begin{aligned} \mathcal{L}_{I_0}(s) &= \exp(-2\pi\lambda\int_R^{\infty}(1-\mathbb{E}_{G_0}[\exp(-sg\ell(r))])r\mathrm{d}r) \\ &= \exp(-2\pi\lambda\int_0^{\infty}(\int_R^{\infty}[1-\exp(-sgl(r)]r\mathrm{d}v)f_{G_0}(g)\mathrm{d}g \end{aligned} \tag{4.6}$$

Using hypergeometry function, (4.6) can be described as,

$$\begin{aligned}\mathcal{L}_{I_0}\{s\} &= \exp(-2\pi\lambda\int_R^\infty [1-\exp(-sgl(r))]r\mathrm{d}r \int_0^\infty f_{G_0}(g)\mathrm{d}g) \\ &= \exp(-\frac{2\pi\lambda}{\sqrt{2\pi}\sigma}\int_0^\infty \exp(-\frac{(\ln\Omega-\mu)^2}{2\sigma^2})\frac{scr^{2-\alpha}}{\alpha-2} \\ &\quad {}_2F_1(1,1-\frac{2}{\alpha};2-\frac{2}{\alpha};-\frac{\Omega sc}{r^\alpha})\mathrm{d}\Omega)\end{aligned} \tag{4.7}$$

By changing the variables $\frac{\ln(\Omega)-\mu}{\sqrt{2}\sigma} = t_I$, that is $\Omega = \exp(\sqrt{2}\sigma t_I + \mu)$ we have

$$\mathcal{L}_{I_0}\{s\} = \exp(-\frac{2\pi\lambda}{\sqrt{\pi}}\int_{-\infty}^\infty f_{inter}(t_I)\exp(-t_I^2)\mathrm{d}\Omega) \tag{4.8}$$

where $f_{inter}(t_I) = \frac{scr^{2-\alpha}\exp(\sqrt{2}\sigma t_I+\mu)}{\alpha-2}{}_2F_1(1,1-\frac{2}{\alpha};2-\frac{2}{\alpha};-\frac{\exp(\sqrt{2}\sigma t_I+\mu)sc}{r^\alpha})$.

$\square$

4.2.2 Coverage Probability

Theorem 1. *Suppose all channel fading power gains are i.i.d. and have the same pdf and the path loss of all channels follows the single-slope power law specified . Then the coverage probability can be shown as follows*

$$p_{\mathsf{cov}} = \int_{-\infty}^\infty f_{desired}(s)\exp(-t_d^2)\,dt_d \tag{4.9}$$

Where, $s = \frac{\theta r^\alpha}{C\exp(\sqrt{2}\sigma t_d+\mu)}$,
$f_{desired}(s) = \int_0^\infty \frac{1}{\sqrt{\pi}}\mathcal{L}_{I_0+\sigma_0^2}(s) f_R(r)\,dr_d$,
$\mathcal{L}_{\sigma_0^2}(s) = \exp(-\frac{Ps}{\sigma_0^2})$ *and* $\mathcal{L}_{I_0}(s) = \exp(-sI_0(1))$

Proof. The term $\mathbb{E}_{I_0}$ in (4.1) can be rewritten as follows

$$\begin{aligned}
p_{\mathsf{cov}} &= \int_0^\infty \mathbb{E}_{I_0}[F^c_{G_0}\left(\frac{(I_0(1)+1/\gamma_0)\theta}{\ell(r)}\right)]f_R(r)\mathrm{d}r \\
&= \int_0^\infty \int_0^\infty \frac{1}{\Omega_d\sqrt{2\pi}\sigma}\mathbb{E}_I[F^c_{G_0|\Omega_d}\left(\frac{(I_0(1)+\frac{1}{\gamma_0})\theta}{\ell(r)}\right)]\exp(-\frac{\ln\Omega_d-\mu}{2\sigma^2})f_R(r)\mathrm{d}r\mathrm{d}\Omega_d \\
&= \int_0^\infty \int_0^\infty \frac{1}{\Omega_d\sqrt{2\pi}\sigma}\exp(-\frac{\ln\Omega_d-\mu}{2\sigma^2})\mathcal{L}_{I_0}(s)\mathcal{L}_{\sigma_0^2}(s)f_R(r)\mathtt{d}\Omega_d\mathtt{d}r \\
&= \int_{-\infty}^\infty f_{desired}(s)\exp(-t_d^2)\mathtt{d}t_d
\end{aligned} \tag{4.10}$$

$f_{desired}(s)$ in (4.1) can be rewritten as follows

$$\begin{aligned}
f_{desired}(s) &= \int_0^\infty \frac{1}{\sqrt{\pi}}\mathcal{L}_{I_0+\sigma_0^2}(s)f_R(r)\mathtt{d}r_d \\
&= \int_0^\infty \frac{1}{\sqrt{\pi}}\mathcal{L}_{I_0+\sigma_0^2}(s)(2\pi\lambda\exp(-\pi\lambda r^2))\mathtt{d}r_d \\
&= \int_0^\infty \frac{1}{\sqrt{\pi}}\mathcal{L}_{I_0+\sigma_0^2}(s')\exp(-t_r)\mathtt{d}t_r
\end{aligned} \tag{4.11}$$

Where$s' = \frac{\theta(\frac{t_r}{\pi\lambda})^{\alpha/2}}{C\exp(\sqrt{2}\sigma t_d+\mu)}$, $t_d = \frac{\ln\Omega_d-\mu}{\sqrt{2}\sigma}$.

Using Laguerre polynomials can be used to compute $f_{desired}$. $\square$

4.3 Lower-Bound Analysis of Coverage Probability

Lemma 2. *Suppose the interference channel gains $\{\tilde{G}_j\}$ have the same pdf as that of G_0 given in (4.3). The Laplace functional of the interference in (3.1) is given by*

$$\mathcal{L}_{I_0}(s) = e^{-2\pi\lambda\int_0^\infty \mathbb{E}_N\left[\frac{s\tilde{\ell}(r)e^{\sigma N}r}{1+s\tilde{\ell}(r)e^{\sigma N}}\right]dr}, \tag{4.12}$$

where $\mathcal{L}_Z(s) \triangleq \mathbb{E}[e^{-sZ}]$ is the Laplace functional of random variable Z and N is the standard normal random variable.

Proof:

According to the result in [10], $\mathcal{L}_{I_0}(s)$ has the following identity:

$$\mathcal{L}_{I_0}(s) = \exp\left\{-2\pi\lambda \int_{\mathbb{R}_+} \left[1 - \mathcal{L}_{G_0}\left(s\tilde{\ell}(r)\right)\right] r\mathrm{d}r\right\}. \tag{4.13}$$

Whereas the explicit result of $\mathcal{L}_{G_0}(s\tilde{\ell}(r))$ with the pdf of G_0 in (4.3) is carried out as

$$\begin{aligned}\mathcal{L}_{G_0}\left(s\tilde{\ell}(r)\right) &= \iint\limits_{\mathbb{R}_+} \frac{1}{\sqrt{2\pi}\sigma x^2} e^{-\left([\frac{1}{x}+s\tilde{\ell}(r)]g+\frac{(\ln x)^2}{2\sigma^2}\right)} \mathrm{d}g\mathrm{d}x \\ &= \frac{1}{\sqrt{2\pi}\sigma}\int_0^\infty \frac{1}{x(1+s\tilde{\ell}(r)x)} e^{-\frac{(\ln x)^2}{2\sigma^2}}\mathrm{d}x.\end{aligned}$$

Then doing the variable change of $y = \frac{\ln x}{\sigma}$ (i.e. $x = e^{\sigma y}$) gives rise to

$$\begin{aligned}\mathcal{L}_{G_0}\left(s\tilde{\ell}(r)\right) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty \frac{1}{1+s\tilde{\ell}(r)e^{\sigma y}} e^{-\frac{y^2}{2}}\mathrm{d}y \\ &= \mathbb{E}_N\left[\frac{1}{1+s\tilde{\ell}(r)e^{\sigma N}}\right].\end{aligned}$$

Substituting the above result into (4.13), we arrive at (4.12). Furthermore, by Jensen's inequality $\mathcal{L}_I(s)$ is lower bounded by

$$\mathcal{L}_{I_0}(s) \geq \exp\left(-2\pi\lambda_B \int_0^\infty \frac{1}{1+e^{-\frac{\sigma^2}{2}}/s\tilde{\ell}(r)}\mathrm{d}r\right), \tag{4.14}$$

which indicates much clearly that the shadowing power significantly affects the magnitude of $\mathcal{L}_0(s)$. Note that $\mathcal{L}_{I_0}(s)$ in (4.12) could be significantly simplified if $\tilde{\ell}(r)$ has an appropriate function of r. The path loss law between two nodes in the network can be formulated in different ways according to different environmental situations. For simplicity, in this section we consider the case that the path loss models of all channels have the same simple single-slope form, i.e.

$$\ell(r) = \tilde{\ell}(r) = Kr^{-\alpha} \tag{4.15}$$

in which $\alpha > 2$ is the path loss exponent and K is a small constant with unit meter$^{\alpha}$ capable of making the received power smaller than transmit power as $r \leq 1$. With the consistent path loss law in (6.1), the result in Lemma 2 can have a closed form as given in the following Lemma.

Lemma 3. *If the path loss law in* (6.1) *is in use, the Laplace functional of* I *in* (4.12) *is given by*

$$\mathcal{L}_{0I}(s) = \exp\left(-\pi\lambda_B (sK)^{\frac{2}{\alpha}}\phi_\alpha\right), \tag{4.16}$$

where $\phi_\alpha \triangleq \frac{2}{\alpha}\Gamma\left(\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)e^{2\left(\frac{\sigma}{\alpha}\right)^2}$ *and* $\Gamma(t) \triangleq \int_0^\infty x^{t-1}e^{-x}dx$ *is the standard Gamma function.*

Proof: Since $\tilde{\ell}(r) = Kr^{-\alpha}$, we rewrite (4.12) as

$$\mathcal{L}_{I_0}(s) = e^{-2\pi\lambda_B \mathbb{E}_N\left[\int_0^\infty \frac{sKe^{\sigma N}r}{r^\alpha + sKe^{\sigma N}}\mathrm{d}r\right]}.$$

and for constant ρ we know

$$\int_0^\infty \frac{2\rho r}{r^\alpha + \rho}\mathrm{d}r = \frac{2\rho^{\frac{2}{\alpha}}}{\alpha}\Gamma\left(\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right).$$

Therefore, it follows that

$$\mathcal{L}_{I_0}(s) = e^{-\frac{2\pi}{\alpha}\Gamma\left(\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)\lambda_B (sK)^{\frac{2}{\alpha}}\mathbb{E}_N\left[e^{\frac{2\sigma N}{\alpha}}\right]}.$$

Also, we know $\mathbb{E}_N[e^{\frac{2\sigma N}{\alpha}}] = e^{2(\frac{\sigma}{\alpha})^2}$ from the moment generating function (MGF) of N and thus we can use it to derive (4.16). This completes the proof. Apparently, Lemma 3 verifies the fact that shadowing significantly impacts the statistic property of the interference as well, and it is also very helpful for attaining a near-closed form of the coverage probability, as shown in the following theorem.

Theorem 2. *Suppose all channel fading power gains are i.i.d. and have the same pdf given in* (4.3) *and the path loss of all channels follows the single-slope power law specified in* (6.1)*. Then the coverage probability in* (4.1) *can be shown as follows*

$$p_{\mathsf{cov}} = \mathbb{E}_N\left[\int_0^\infty \frac{\pi\lambda_B dw}{e^{\left(\frac{\theta w^{\frac{\alpha}{2}}}{K\gamma_0 e^{\sigma N}}+\pi\lambda_B w(\theta^{\frac{2}{\alpha}}\phi_\alpha e^{\frac{-2\sigma N}{\alpha}}+1)\right)}}\right]. \tag{4.17}$$

For the interference-limited network case (i.e. $\gamma_0=\infty$*),* p_{cov} *in* (4.17) *reduces to*

$$p_{\mathsf{cov}} = \mathbb{E}_N\left[\frac{1}{1+\theta^{\frac{2}{\alpha}}\phi_\alpha e^{-\frac{2\sigma N}{\alpha}}}\right]. \tag{4.18}$$

Proof: The term $\mathbb{E}_{I_0}[\cdot]$ in (4.1) can be rewritten as follows

$$\begin{aligned}
&\mathbb{E}_{I_0}\left[F^c_{G_0}\left((I_0(1)+1/\gamma_0)\theta K^{-1}r^\alpha\right)\right]\\
&=\mathbb{E}_{I_0}\left[\int_{(I_0(1)+1/\gamma_0)\theta r^\alpha/K}^\infty\int_0^\infty \frac{e^{\left(-\frac{g}{x}-\frac{(\ln(x))^2}{2\sigma^2}\right)}}{\sqrt{2\pi}\sigma x^2}\mathrm{d}x\mathrm{d}g\right]\\
&=\int_0^\infty \frac{1}{\sqrt{2\pi}\sigma x}e^{-\left(\frac{(\ln(x))^2}{2\sigma^2}+\frac{\theta r^\alpha}{K\gamma_0 x}\right)}\mathbb{E}_{I_0}\left[e^{-\frac{\theta r^\alpha}{Kx}I_0}\right]\mathrm{d}x\\
&=\int_0^\infty \frac{1}{\sqrt{2\pi}\sigma x}e^{-\left(\frac{(\ln(x))^2}{2\sigma^2}+\frac{\theta r^\alpha}{K\gamma_0 x}\right)}\mathcal{L}_{I_0}\left(\frac{\theta r^\alpha}{Kx}\right)\mathrm{d}x.\\
&=\mathbb{E}_N\left[e^{-\left(\frac{\theta r^\alpha}{K\gamma_0}e^{-\sigma N}\right)}\mathcal{L}_{I_0}\left(\frac{\theta r^\alpha}{Ke^{\sigma N}}\right)\right]\\
&=\mathbb{E}_N\left[e^{-\left(\frac{\theta r^\alpha}{K\gamma_0}e^{-\sigma N}\right)}e^{-\pi\lambda_B r^2\phi_\alpha\theta^{\frac{2}{\alpha}}e^{-\frac{2\sigma N}{\alpha}}}\right].
\end{aligned}$$

Thus, it follows that

$$\begin{aligned}
p_{\mathsf{cov}} &= \iint_0^\infty \frac{\sqrt{2\pi}\lambda_B}{2\sigma x}e^{-\left(\frac{(\ln x)^2}{2\sigma^2}+\frac{\theta w^{\frac{\alpha}{2}}}{K\gamma_0 x}+\pi w\lambda_B\right)}\mathcal{L}_{I_0}\left(\frac{\theta w^{\frac{\alpha}{2}}}{Kx}\right)\mathrm{d}x\mathrm{d}w\\
&=\lambda_B\sqrt{\frac{\pi}{2\sigma^2}}\iint_0^\infty e^{-\left(\frac{(\ln x)^2}{2\sigma^2}+\frac{\theta w^{\frac{\alpha}{2}}}{K\gamma_0 x}+\pi\lambda_B w\left(\left(\frac{\theta}{x}\right)^{\frac{2}{\alpha}}\phi_\alpha+1\right)\right)}\frac{\mathrm{d}x}{x}\mathrm{d}w.
\end{aligned}$$

Then (4.17) is obtained by changing the variable x with $e^{\sigma N}$ and we can have (4.18) by substituting $1/\gamma_0 = 0$ into (4.17). There are two important observations that can be drawn from Theorem 2. First, we notice that p_{cov} in (4.18) does not depend on the BS intensity λ_B for the interference-limited case, i.e. $\gamma_0 = \infty$. This phenomenon discloses the fact that deploying more BSs cannot improve the coverage probability accordingly and large shadowing power σ_s^2 significantly reduces the coverage probability with high probability. Secondly, the upper bound on p_{cov} in the interference-limited case is

$$p_{\mathsf{cov}} \leq \frac{1}{1+\theta^{\frac{2}{\alpha}}\frac{2}{\alpha}\Gamma\left(\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)},$$

which is acquired by applying Jensen's inequality to the concave function of $e^{\frac{2\sigma}{\alpha}N}$ and using the MGF of N. This upper bound is exactly equal to the result in (4.17) for $\sigma = 0$ and $\gamma_0 = \infty$, i.e. the coverage probability without noise and shadowing. Thus this means that shadowing does not improve p_{cov} and could even weaken it significantly.

Chapter 5

Coverage Probability with Inconsistent Fading Impairments

5.1 Successful Transmission Probability in m-Nakagami Fading Desired Signal Channel With Rayleigh Fading Interfering Signal

In the previous section, the coverage probability analysis is completed under the assumption that desired and interference signals all experience the same channel impairment models, such as fading, shadowing and path loss. However, in general a user is usually associated with his nearest BS and interfering BSs are all far away from him very likely. Accordingly, this motivates us the analysis of the coverage probability with inconsistent fading models. Specifically, we assume the desired signal channel of a user undergoes Nakagami-m fading and all interference channels experiences Rayleigh fading.

The path loss for all interference channels is a dual-slop based model [12] [13] that is given by

$$\tilde{\ell}(r) = Kr^{-\alpha}\left(1+\frac{r}{\tau_d}\right)^{-\beta}, \tag{5.1}$$

where τ_d is the turning distance and β is called additional attenuation exponent. The path loss model for the desired signal channel is the regular single-slop model described in (6.1). In other words, $\tilde{\ell}(r) = (1+r/\tau_d)^{-\beta}\,\ell(r)$ and this inherently makes the interference channels have a larger path loss compared with the desired signal channel. According to these assumptions on channel models, the coverage probability can be explicitly reformulated in the following:

$$p_{\mathsf{cov}} = \int_0^\infty \mathbb{E}_{\tilde{I}_0}\left[F^c_{G_0}\left(\frac{r^\alpha\theta}{K}\left(\tilde{I}_0(1)+\frac{1}{\gamma_0}\right)\right)\right] f_R(r)\mathrm{d}r, \tag{5.2}$$

where $\tilde{I}_0(0) \triangleq \sum_{X_j\in\Phi_B\setminus X_0}\tilde{G}_j\|X_j\|^{-\alpha}\left(1+\frac{\|X_j\|}{\tau_d}\right)^{-\beta}$, $\{\tilde{G}_j\}$ are i.i.d. exponential random variables with unit mean and variance, and now G_0 has a Gamma distribution with unit mean and variance $1/m$ given by

$$f_{\tilde{G}}(g) = m^m\frac{g^{m-1}}{\Gamma(m)}\exp(-mg). \tag{5.3}$$

The coverage probability in (5.2) can be simplified as a low-complexity form as shown in the following theorem.

Theorem 3. *If the desired signal channel is under Nakagami-m fading and the interference channels are in Rayleigh fading. The coverage probability given in* (5.2) *is*

$$p_{\mathsf{cov}} = \pi\lambda_B\sum_{k=0}^{m-1}\frac{(-m\theta)^k}{k!}\int_0^\infty \mathcal{L}^{(k)}_{J_0}\left(m\theta w^{\frac{\alpha}{2}}\right)e^{-2\pi\lambda_B w}w^{\frac{\alpha k}{2}}\,dw, \tag{5.4}$$

where $J_0 \triangleq \tilde{I}_0(1)+\frac{1}{\gamma_0}$, $\mathcal{L}^{(k)}_{J_0}(s) = \frac{d^k}{ds^k}\mathcal{L}_{J_0}(s)$ *and* $\mathcal{L}_{J_0}(s)$ *is given by*

$$\mathcal{L}_{J_0}(s) = e^{\left(-2\pi\lambda_B\int_0^\infty \frac{sKr}{sK+r^\alpha(1+\frac{r}{\tau_d})^\beta}dr-\frac{s}{\gamma_0}\right)}. \tag{5.5}$$

Proof: The Laplace functional of $\tilde{I}_0(1)$ can be derived by using the result in (4.12) for $\sigma_s = 0$ and $\tilde{\ell}(r)$ in (5.1). For $J_0 = \tilde{I}_0(1) + \frac{1}{\gamma_0}$, the Laplace functional of J_0 is $\mathcal{L}_{J_0}(s) = \mathcal{L}_{\tilde{I}_0}(s)e^{-\frac{s}{\gamma_0}}$. The CCDF of G_0 with the distribution (5.3) is

$$F^c_{G_0}(g) = 1 - \frac{\Gamma_l(m, mg)}{\Gamma(m)} = e^{-mg}\sum_{k=0}^{m-1}\frac{(mg)^k}{k!}, \tag{5.6}$$

where $\Gamma_l(s,t) = \int_0^t x^{s-1}e^{-x}\mathrm{d}x$ is the lower incomplete Gamma function. Also we have the following identity of $\mathcal{L}_{\tilde{I}_0}(s)$:

$$\mathcal{L}^{(k)}_{J_0}(s) \triangleq \frac{\mathrm{d}^k\mathcal{L}_{J_0}(s)}{\mathrm{d}s^k} = (-1)^k\mathbb{E}\left[J_0^k e^{-sJ_0}\right],$$

which generates

$$\mathbb{E}_{\tilde{I}_0}\left[F^c_{G_0}\left(\frac{r^\alpha\theta}{K}J_0\right)\right] = \sum_{k=0}^{m-1}\frac{(-mr^\alpha\theta)^k}{k!}\mathcal{L}^{(k)}_{J_0}(m\theta r^\alpha).$$

Substituting the above result into (5.2) gives p_{cov} in (5.4).

Theorem 3 assists us to learn that p_{cov} monotonically increases along m since larger m gives rise to less fading impact on the desired signal. Moreover, for the case of $\gamma_0 = \infty$ $m = 1$ (the interference-limited and Rayleigh fading case) p_{cov} in (5.4) lower bounded by

$$p_{\mathsf{cov}} \geq \frac{1}{1 + \frac{2}{\alpha+\beta}\theta^{\frac{2}{\alpha}}\Gamma\left(\frac{2}{\alpha+\beta}\right)\Gamma\left(1 - \frac{2}{\alpha+\beta}\right)}, \tag{5.7}$$

which shows that the coverage probability is able to considerably augment under the dual-slop path loss law for the interference channels. Numerical results in Section 7.1 will illustrate the observations here.

Chapter 6

Coverage Probability with Inconsistent Fading Impairments and Shadowing

In the previous section, the coverage probability analysis is completed under the assumption that desired and interference signals all experience the same channel impairment models, such as fading, shadowing and path loss. However, in general a user is usually associated with his nearest BS and interfering BSs are all far away from him very likely. Accordingly, this motivates us the analysis of the coverage probability with inconsistent fading models. Specifically, we assume the desired signal channel of a user undergoes Nakagami-m fading with log-normal shadowing and all interference channels experiences Rayleigh fading with log-normal shadowing.

6.1 Path-loss

Note that $\mathcal{L}_{I_0}(s)$ in (4.12) could be significantly simplified if $\tilde{\ell}(r)$ has an appropriate function of r. The path loss law between two nodes in the network can be formulated in different ways according to different environmental situations. For simplicity, in this section we consider the case that the path loss models of all channels have the same simple single-slope form, i.e. [12] [13]

$$\ell(r) = \tilde{\ell}(r) = Kr^{-\alpha} \tag{6.1}$$

in which $\alpha > 2$ is the path loss exponent and K is a small constant with unit meter$^\alpha$ capable of making the received power smaller than transmit power as $r \leq 1$.

6.1.1 Log-normal shadowing and m-Nakagami fading

For the interferers is farer than the serving base station, the environment channel for interference can be modeled by the composite of log-normal shadowed fading and Rayleigh fading channel. The pdf of the received interference is given by a log-normal distribution superimposed on a Rayleigh distributed random variable,

$$f_{G_0}(g) = \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma\Omega^2} \exp(-\frac{g}{\Omega} - \frac{(\ln\Omega - \mu)^2}{2\sigma^2})\mathtt{d}\Omega \tag{6.2}$$

where σ is the shadowing standard derivation of channel and Ω is the log-normal shadowing random variable. For the desired signal's channel, we use the composite of log-normal shadowed fading and m-Nakagami fading channel to model it. The pdf of the received interference is given by a log-normal distribution superimposed on a Rayleigh distributed random variable,

$$f_{G_0}(g) = \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma\Omega^2} m^m \frac{g^{m-1}}{\Gamma(m)} \exp(-\frac{mg}{\Omega} - \frac{(\ln\Omega - \mu)^2}{2\sigma^2})\mathtt{d}\Omega \tag{6.3}$$

where σ is the shadowing standard derivation of channel and Ω is the log-normal shadowing random variable.

The CCDF of the received power is [14]

$$F_{G_0}(g) = \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma\Omega^2} \sum_{k=0}^{m-1} \frac{(mg)^k}{k!} \exp(-\frac{mg}{\Omega} - \frac{(\ln\Omega - \mu)^2}{2\sigma^2}) \mathrm{d}\Omega \tag{6.4}$$

Theorem 4. *Suppose all channel fading power gains are i.i.d. and have the same pdf and the path loss of all channels follows the single-slope power law specified . Then the coverage probability can be shown as follows,*

$$p_s = \int_{-\infty}^{\infty} f_{desired}(s) \exp(-t_d^2) \mathtt{d} t_d \tag{6.5}$$

Where $s = \frac{\theta r^\alpha}{C\Omega_d}$
$\mathcal{L}_I^{(k)}(s) = \frac{d^k \mathcal{L}_I(s)}{ds^k} = (-1)^k \mathbb{E}[I^k e^{-sI}]$
and $f_{desired}(s) = \int_0^\infty \frac{1}{\sqrt{\pi}} \mathcal{L}_{I+\sigma_0^2}(s) f_R(r) dr$

Proof.

$$\begin{aligned}
p_{\mathsf{cov}} &= \int_0^\infty \mathbb{E}_I[F^c_{G_0}\left(\frac{(I+1/\gamma_0)\theta}{\ell(r)}\right)] f_R(r)\mathrm{d}r \\
&= \int_0^\infty \int_0^\infty \frac{1}{\Omega_d\sqrt{2\pi}\sigma} \mathbb{E}_I[F^c_{G_0|\Omega_d}\left(\frac{(I+\frac{1}{\gamma_0})\theta}{\ell(r)}\right)] \exp(-\frac{\ln\Omega_d - \mu}{2\sigma^2}) f_R(r) \mathrm{d}r\mathrm{d}\Omega_d \\
&= \int_0^\infty \int_0^\infty \frac{1}{\Omega_d\sqrt{2\pi}} \exp(-\frac{\ln\Omega_d - \mu}{2\sigma^2}) \sum_{k=0}^{m-1} \frac{\mathcal{L}^{(k)}_{I+\sigma_0^2}}{k!}(s) f_R(r) \mathtt{d}\Omega_d \mathtt{d}r \\
&= \sum_{k=0}^{m-1} \int_{-\infty}^{\infty} f_{desired}(s) \exp(-t_d^2) \mathtt{d} t_d
\end{aligned} \tag{6.6}$$

Where $s = \frac{\theta r^\alpha}{C\Omega_d}$
$\mathcal{L}_I^{(k)}(s) = \frac{d^k \mathcal{L}_I(s)}{ds^k} = (-1)^k \mathbb{E}[I^k e^{-sI}]$
and $f_{desired}(s) = \int_0^\infty \frac{1}{k!\sqrt{\pi}} \mathcal{L}^{(k)}_{I+\sigma_0^2}(s) f_R(r) \mathrm{d}r$

We can get Hermit integration form for coverage probability.

$f_{desired}(s)$ in (4) can be rewritten as follows

$$
\begin{aligned}
f_{desired}(s) &= \int_0^{\infty} \frac{1}{k!\sqrt{\pi}} \mathcal{L}^{(k)}_{I_0+\sigma_0^2}(s) f_R(r) \mathtt{d} r_d \\
&= \int_0^{\infty} \frac{1}{k!\sqrt{\pi}} \mathcal{L}^{(k)}_{I_0+\sigma_0^2}(s) (2\pi\lambda \exp(-\pi\lambda r^2)) \mathtt{d} r_d \\
&= \int_0^{\infty} \frac{1}{k!\sqrt{\pi}} \mathcal{L}^{(k)}_{I_0+\sigma_0^2}(s') \exp(-t_r) \mathtt{d} t_r
\end{aligned}
\tag{6.7}
$$

Where$s' = \frac{\theta(\frac{t_r}{\pi\lambda})^{\alpha/2}}{C\exp(\sqrt{2}\sigma t_d+\mu)}$, $t_d = \frac{\ln\Omega_d - \mu}{\sqrt{2}\sigma}$.

Using Laguerre polynomials can be used to compute $f_{desired}$. $\square$

Chapter 7

Numerical and Simulation Result

7.1 Simulation Results

To verify the accuracy and correction of the analytical results in the previous sections, simulation results of the coverage probability are provided in this section. The simulation setup is first specified in Section 7.1.1 and the numerical results are presented in Section 7.1.2.

7.1.1 Simulation setup

(i) According to [15], we can define a simulation area that has a center at the origin and radius $1/\sqrt{\epsilon}$ to approximate an infinitely large area so that the mean interference in the simulation area (outside radius 1) should match the theoretical mean in an infinite network up to a factor $1-\epsilon$. In the following simulation, $\epsilon = 0.01$ is used.

(ii) The number of base stations in this simulation area is generated by a

Poisson distribution with intensity $\lambda_B = 0.4\lambda_u$, and $\lambda_u = 370$ users/km^2 is the intensity of users.

(iii) The reference user is located at origin and he is associated with the nearest base station from the origin in the simulation area. Other base stations in the simulation area except the associated base station are generating interference to the reference user.

(iv) By referring to [16], the network parameters for simulation here are listed in the following: $\gamma_0 = \frac{P}{\sigma_0^2} = 140.7dB$, $C = 8.7 \times 10^{-4} m^{-\alpha}, \mu = -ln(10)\sigma^2/20, \sigma = 8dB$, the intensity of user is $370/km^2$

7.1.2 Numerical Results and Discussions

The numerical results in Fig. 7.1 show the coverage probability versus the base station density in three different fading environments. The three combinations of the fading models for the desired and interference channels are: (1) Nakagami-2 fading in the desired channel and Rayleigh fading in the interference channels; (2) Rayleigh fading in all channels (3) Rayleigh fading and log-normal shadowing in the desired channel and Rayleigh fading in the interference channels. As can be seen, the analytical results perfectly match with the simulated results, which indicates our analysis is correct and accurate. The coverage probability initially increases along the base station intensity and then gradually becomes constant in the large intensity regime. This phenomenon has been point out in the previous analysis, that is, deploying many base stations in a given area is not an effective method to improve the coverage probability. Also, we can observe the fact that shadowing indeed plays a pivotal role that weakens the coverage probability. However, shadowing effects are neglected in most of prior work on the stochastic-geometry-based

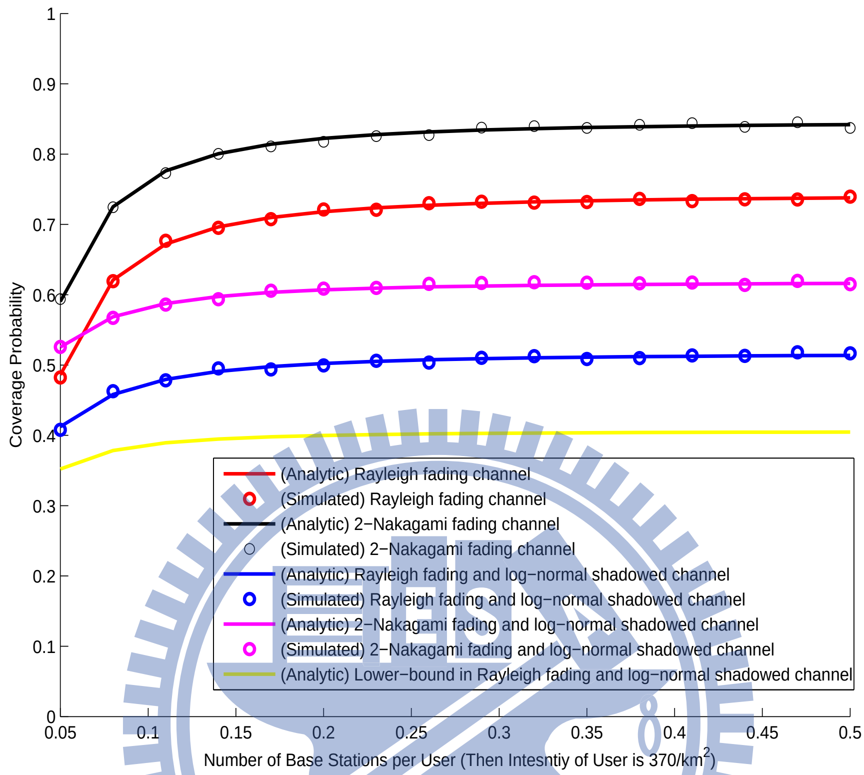


Figure 7.1: Coverage probability versus the number of the base stations per user. All the solid lines represent the analytical results and all the lines with circles stand for the simulation results: The red lines are for the case of all channels with Rayleigh fading. The back lines are for the case of the desired channel with Nakgami-2 fading and the dual-slope path loss law and the interference channels with Rayleigh fading. Whereas the blue lines are for the case of all channels with Rayleigh fading and log-normal shadowing.

analysis in cellular network.

Fig. 7.2 presents the numerical results for the coverage probability versus the SINR threshold in the three different fading environments, as the same combinations used in Fig. 7.1. In this figure, we also verify the accuracy of

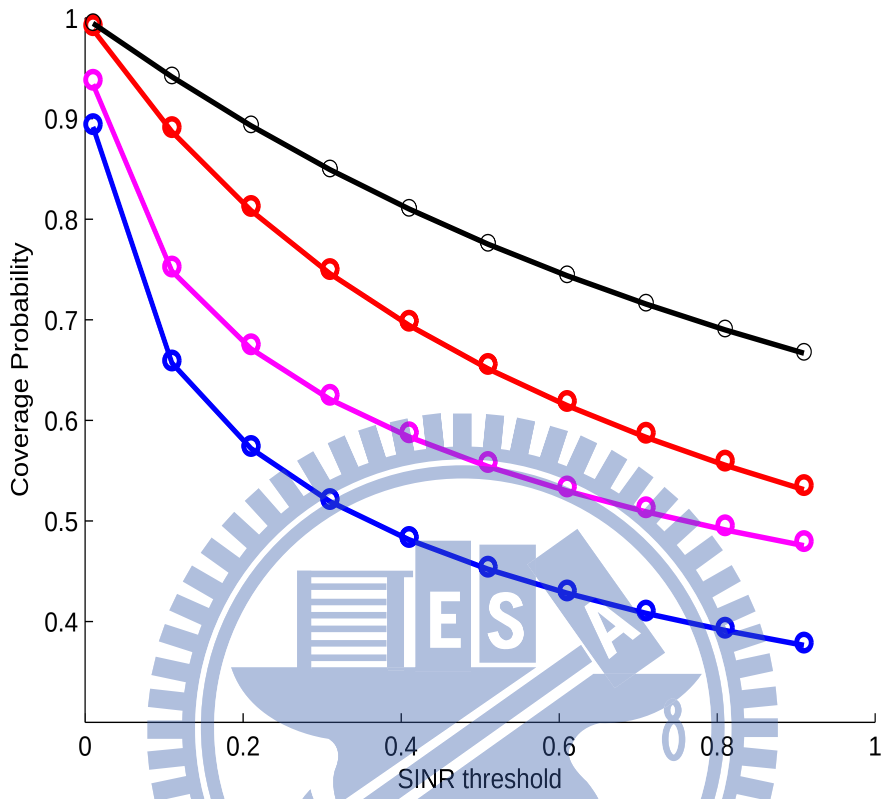


Figure 7.2: Coverage probability versus SINR threshold. All colorful lines have the same representative meaning as those in Fig. 7.1.

our previous analysis again, and we also can see that shadowing significantly impacts on the coverage performance especially when the SINR thresholds are small. Hence, showing effects should not be neglected in the cellular networks under the stochastic geometry framework.

Chapter 8

Conclusion

In this paper, first we investigate how the desired channel and interference channels impact the coverage performance when they experience inconsistent fading and shadowing in a Poisson small cell network. A very neat formula of the coverage probability with Rayleigh fading and log-normal shadowing is found and it intuitively shows the severe impact of shadowing on the coverage performance of a user. We also show that coverage performance cannot be improved in a dense (interference-limited) network. A low-complexity expression of the coverage probability is found for the case that all channels follow the dual-slop path loss law, the desired signal channel has Nakagami-m fading and Rayleigh fading exists in all interference channels. This expression reflects a more practical coverage performance in a cellular network. Finally, some simulation results are provided to support the correctness and accuracy of our analysis.

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Vita

Vita

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Publication List

Publication

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