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A periodic review replenishment model with a refined delivery scenario

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ABSTRACT

This paper considers a single-item replenishment problem where every n periods the buyer plans for the quantity delivered for each upcoming period. Assuming that demand not filled immediately is backlogged, we suggest that the quantity delivered in the immediately upcoming period be of any size and the quantity delivered in each of the subsequent $(n-1)$ periods be equal to or smaller than a fixed quantity Q. We derive the average cost per n periods and compute the optimal order-up-to level for the proposed replenishment policy. Next, assuming a fixed cost for reviewing inventory and planning deliveries, we obtain the optimal n by minimizing the expected cost per period. Finally, the optimal Q can also be determined. The proposed periodic review policy is easy to implement.

1. Introduction

One of the most widely used stochastic inventory models is an order-up-to periodic review policy. Unlike a continuous review model where the order quantity is fixed, an order-up-to periodic policy places an order in every period that will raise inventory to a target level. Since demand fluctuates period by period, the order quantity varies too. This may result in costs of adjustment in practice due to, for example, changes in capacity or production plans. These costs are incurred by the supplier but may be charged to the buyer (e.g., Urban, 2000). However, an ordinary order-up-to policy implicitly assumes that these costs do not exist (or simply ignores these costs).

It is possible that the order quantity is made (almost) fixed in periodic review systems. Such a scenario is attractive and desirable to both the supplier and the buyer. It simplifies the production, order picking, delivery, unloading (for the supplier), inspection process, and inventory record updating procedure (for the buyer). It avoids any extra costs that might be incurred due to variations in the order quantity. It also agrees with the JIT management philosophy. However, since demand is stochastic in the real world, a certain mechanism is needed to absorb variations of demand. In general, there are several ways to achieve this. One is through a standing order inventory system where provision is made for procuring extra units in the case of an emergency and selling off excess inventory. Standing order systems were first considered by Rosenshine and Obee (1976) and recently studied in Chiang (2007). Another is through a two-supplier inventory system (Janssen and de Kok, 1999) in which while one supplier delivers a fixed quantity in each period the other ships any quantity needed to raise inventory to a target level.

In this paper, we propose a third approach to facilitating the scenario of a (almost) fixed order quantity in periodic review systems. We suppose that every n periods the buyer, who employs an order-up-to policy, plans for the quantity delivered for each upcoming period. Assuming that demand not filled immediately is backlogged, we suggest that a fixed quantity Q be delivered by the supplier for each of the upcoming n periods *except* perhaps the first several periods. To be more specific, the

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immediately upcoming period's shipment size is adjusted first, and, if not sufficient, the upcoming second period's shipment size is adjusted next, and so on, such that the quantity shipped in each of the subsequent periods is exactly Q. Thus, the quantity to be delivered in the immediately upcoming period is of any size and the quantity delivered in each of the subsequent $(n-1)$ periods is equal to or smaller than Q. For example, suppose that $Q=5$ and $n=5$. If demand of the previous 5 periods is 28 (resp. 22), the quantity delivered in each of the upcoming 5 periods is 8 (resp. 2), 5, 5, 5, 5, respectively. On the other hand, if demand of the previous 5 periods is 17 (resp. 13), the quantity delivered in each of the upcoming 5 periods is 0, 2 (resp. 0), 5 (resp. 3), 5, 5, respectively. For a certain n, the optimal order-up-to level for the delivery scenario described is computed by minimizing the average n-periods' cost. Next, assuming that there is a fixed cost incurred every n periods for auditing the inventory level as well as planning and adjusting order quantities, the optimal n can be obtained by minimizing the average cost per period through a simple procedure. Finally, the optimal Q can also be determined. Hence, the proposed replenishment policy is easy to implement.

Note that Flynn and Garstka (1990) consider a related but more complex problem where one schedules delivery quantities for the next n periods that are generally not equal to one another. Flynn and Garstka (1997) further extend the analysis to determine the optimal review period. Chiang (2001) also studies a delivery splitting periodic model where n shipments are scheduled in future time points that are evenly separated. However, three major shortcomings limit the applicability of Chiang's model. First, these n shipments are of different sizes. Second, the costs of adjustment due to changes in the shipment size are not included. Third, the interval between delivery epochs may not be an integral multiple of a basic time unit (e.g., a day). Thus, the suggested model may not be easily implemented in practice. Other related periodic review models are investigated in Ehrhardt (1997) and Urban (2000). The former considers the problem of selecting a fixed replenishment quantity to be delivered in each of n consecutive periods in the future, while the latter describes a multi-period "recurrent" newsvendor problem where changes in the order quantity result in an additional cost to the buyer.

The rest of this paper is organized as follows. In Section 2, we propose a periodic review replenishment model with the delivery scenario described above. In Section 3, we consider a simplified version of the proposed policy. Section 4 reports some computational results. Finally, Section 5 concludes this paper.

2. A periodic review replenishment model

Consider the following replenishment problem: every n periods (one period is 1 day, for example) the buyer reviews an item and plans its shipment size for each upcoming period to raise the inventory position (i.e., inventory on hand minus backorders plus inventory on order) to a target level (i.e., an order-up-to policy is used). Assume that the demand of each period is independently and identically distributed. As explained in Section 1, it benefits the supplier as well as the buyer if a fixed quantity Q is shipped in each period. Such a situation occurs only when demand in the previous n periods is exactly nQ. If demand of the previous n periods is not nQ, the buyer would like to adjust first the immediately upcoming period's shipment size so that the quantity shipped in each of the subsequent $(n-1)$ periods is exactly Q; if adjustment to the immediately upcoming period's shipment size is not sufficient, the buyer would adjust next the upcoming second period's shipment size so that the quantity shipped in each of the subsequent $(n-2)$ periods is Q; and so on. Notice that we have implicitly assumed in the above scenario that the immediately upcoming period's shipment size can be adjusted if requested by the buyer. If it takes a positive lead time (an integral multiple of the period length) to adjust the shipment size, our replenishment model can be modified by appropriately defining $G_i(Y|n, Q)$ introduced below, as in an ordinary periodic review model (e.g., Porteus, 1990).

For the proposed replenishment policy to be clearer, let $f_k(\cdot)$ be the probability density function of k periods' demand, $k=1, \ldots, n$, and D the demand during the previous n periods (its probability density function is thus $f_n(\cdot)$). If $D \geqslant (n-1)Q$, the quantity delivered for each of the upcoming n periods is $D-(n-1)Q, Q, \ldots, Q$, respectively; if $(n-1)Q > D \geqslant (n-2)Q$, the quantity delivered for each upcoming period is $0, D-(n-2)Q, Q, \ldots, Q$, respectively; if $(n-2)Q > D \geqslant (n-3)Q$, the quantity delivered for each upcoming period is $0, 0, D-(n-3)Q, Q, \ldots, Q$, respectively; and so on. Let Q_i be the expected quantity shipped in the upcoming ith period. Then

$$Q_1 = \int_{(n-1)Q}^{\infty} (\zeta - (n-1)Q) f_n(\zeta)\,\mathrm{d}\zeta, \tag{1}$$

$$Q_i = \int_{(n-i+1)Q}^{\infty} Q f_n(\zeta)\,\mathrm{d}\zeta + \int_{(n-i)Q}^{(n-i+1)Q} (\zeta - (n-i)Q) f_n(\zeta)\,\mathrm{d}\zeta \quad \text{for } 2 \leqslant i \leqslant n. \tag{2}$$

The following theorem is immediate from (1).

Theorem 1. *Q_1 is decreasing in Q.*

Letting μ be the mean demand of a period, we have the following result.

Theorem 2. *If $Q < \mu$, then $Q_1 > \mu$.*

Proof. The expected demand of n periods is $n\mu$, i.e., $E[D] = n\mu$ and thus $Q_1+Q_2+\cdots+Q_n = n\mu$. Since $Q_i \leqslant Q$ for $2 \leqslant i \leqslant n$, it follows that if $Q < \mu$, then $Q_1 > \mu$.

Let $G_i(Y|n, Q)$ be the average cost of the upcoming ith period for the proposed delivery scenario given an order-up-to level $Y \geqslant 0$ and $G(Y|n, Q)$ the average total n-periods' cost, i.e.,

$$G(Y|n,Q) \equiv G_1(Y|n,Q) + G_2(Y|n,Q) + \cdots + G_n(Y|n,Q). \tag{3}$$

Only inventory holding and shortage costs (which are charged based on the ending inventory of a period) are included above. The procurement cost $cE[D]$ (where c is the unit cost) for n periods is excluded since it is constant. Let h and p be the holding and shortage cost per unit per period, respectively. To derive $G_i(Y|n, Q)$, for $i \leqslant (n-1)$, we consider two cases: $Y \geqslant (n-i)Q$ and $Y<(n-i)Q$. In the former case, $G_i(Y|n, Q)$ is evaluated according to whether $D \geqslant (n-i)Q$ or $D<(n-i)Q$. In the latter case, $G_i(Y|n, Q)$ is evaluated according to whether $D \geqslant (n-i)Q$, $(n-i)Q>D \geqslant Y$, or $Y>D$. It follows that for $i \leqslant (n-1)$,

$$
\begin{aligned}
G_i(Y|n,Q) = {} & \int_{(n-i)Q}^{\infty} f_n(\zeta)\,\mathrm{d}\zeta \left\{ \int_0^{Y-(n-i)Q} h(Y-(n-i)Q-\xi) \right. \\
& \left. \times f_i(\xi)\,\mathrm{d}\xi + \int_{Y-(n-i)Q}^{\infty} p(\xi - Y + (n-i)Q) f_i(\xi)\,\mathrm{d}\xi \right\} \\
& + \int_0^{(n-i)Q} f_n(\zeta) \left\{ \int_0^{Y-\zeta} h(Y-\zeta-\xi) f_i(\xi)\,\mathrm{d}\xi \right. \\
& \left. + \int_{Y-\zeta}^{\infty} p(\xi - Y + \zeta) f_i(\xi)\,\mathrm{d}\xi \right\} \mathrm{d}\zeta \quad \text{if } Y \geqslant (n-i)Q,
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
G_i(Y|n,Q) = {} & p(i\mu + (n-i)Q - Y) \int_{(n-i)Q}^{\infty} f_n(\zeta)\,\mathrm{d}\zeta \\
& + \int_Y^{(n-i)Q} f_n(\zeta) p(i\mu + \zeta - Y)\,\mathrm{d}\zeta + \int_0^Y f_n(\zeta) \\
& \times \left\{ \int_0^{Y-\zeta} h(Y-\zeta-\xi) f_i(\xi)\,\mathrm{d}\xi + \int_{Y-\zeta}^{\infty} p(\xi - Y + \zeta) \right. \\
& \left. \times f_i(\xi)\,\mathrm{d}\xi \right\} \mathrm{d}\zeta \quad \text{if } Y<(n-i)Q.
\end{aligned} \tag{5}
$$

Also,

$$
G_n(Y|n,Q) = \int_0^Y h(Y-\xi) f_n(\xi)\,\mathrm{d}\xi + \int_Y^{\infty} p(\xi - Y) f_n(\xi)\,\mathrm{d}\xi. \tag{6}
$$

Let DG_i be the first derivative of the function G_i with respect to Y. Then, for $i \leqslant (n-1)$,

$$
\begin{aligned}
DG_i(Y|n,Q) = {} & -p + (h+p) \left(\int_{(n-i)Q}^{\infty} f_n(\zeta)\,\mathrm{d}\zeta \int_0^{Y-(n-i)Q} f_i(\xi)\,\mathrm{d}\xi \right. \\
& \left. + \int_0^{(n-i)Q} f_n(\zeta) \left(\int_0^{Y-\zeta} f_i(\xi)\,\mathrm{d}\xi \right) \mathrm{d}\zeta \right) \\
& \text{if } Y \geqslant (n-i)Q,
\end{aligned} \tag{7}
$$

$$
DG_i(Y|n,Q) = -p + (h+p) \int_0^Y f_n(\zeta) \left(\int_0^{Y-\zeta} f_i(\xi)\,\mathrm{d}\xi \right) \mathrm{d}\zeta \quad \text{if } Y<(n-i)Q \tag{8}
$$

and

$$
DG_n(Y|n,Q) = -p + (h+p) \int_0^Y f_n(\zeta)\,\mathrm{d}\zeta. \tag{9}
$$

The following lemma can be verified by examining the second derivative of $G_i(Y|n, Q)$.

Lemma 3. *$G_i(Y|n, Q)$ for each i is convex on Y.*

Hence, the optimal order-up-to level, denoted by Y^*, can be obtained by equating $DG(Y|n, Q)$ to zero and solving for Y. For discrete demand distribution, we find the smallest value of Y such that $DG(Y|n, Q) \geqslant 0$. The minimum n-periods' cost is $G(Y^*|n, Q)$.

Next, we investigate the issue of how often one should schedule for deliveries. Let K be the fixed cost for reviewing the inventory level as well as planning and adjusting order quantities (particularly the order quantity of the immediately upcoming period, since it can be of any size). Part of this cost is incurred by the buyer (for reviewing inventory); the remaining part that is incurred by the supplier due to changes in the shipment size is charged to the buyer (as mentioned in Section 1). It is assumed that the entire fixed cost K is incurred even if the quantity to be delivered in each of the upcoming n periods is exactly Q. The average cost per period, denoted by $AC(Q, n)$, is given by

$$
AC(Q,n) = [G(Y^*|n,Q) + K]/n. \tag{10}
$$

Minimizing $AC(Q, n)$ requires a certain procedure of determining the optimal n, denoted by n^*. Since the computation shows that $AC(Q, n)$ is quasi-convex (but not necessarily convex) in n given Q (though we cannot prove this), we suggest that one simply tabulates $AC(Q, n)$ as a function of n to obtain n^*, as an approximate periodic review model determines the optimal review period (Hadley and Whitin, 1963, Section 5-2). Let $AC^*(Q) \equiv AC(Q, n^*(Q))$.

Finally, it may be desirable to determine the optimal Q, denoted by Q^*, such that $AC^*(Q)$ is minimized. A simple method of finding Q^* is to enumerate feasible (possibly integral) values of Q and choose the lowest $AC^*(Q)$.

3. A simplified policy

In the above delivery scenario, the quantity shipped in the immediately upcoming period is of any size and the quantity shipped in each of the subsequent $(n-1)$ periods is equal to or smaller than Q. Suppose that excess inventory in the immediately upcoming period can be salvaged or returned to the supplier at c per unit. Then, only the quantity shipped in this period is variable and the quantity shipped in each of the subsequent $(n-1)$ periods is exactly Q. This greatly simplifies the replenishment policy. Referring to the second paragraph of Section 2, if $D<(n-1)Q$, the quantity shipped for each upcoming period is 0, Q, $Q, \ldots, Q$, respectively, and the excess units $(n-1)Q-D$ in the immediately upcoming period are salvaged or returned to the supplier. If $Q=\mu$ and these excess units are regarded as a negative order quantity from the supplier, then $Q_1 = Q$ and thus $Q_1 = Q_2 = \cdots = Q_{n-1} = Q_n$. For this simplified policy, $G_i(Y|n, Q)$, for $i \leqslant (n-1)$, in expressions (4) and (5) reduce to

$$
\begin{aligned}
G_i(Y|n,Q) = {} & \int_0^{Y-(n-i)Q} h(Y-(n-i)Q-\xi) f_i(\xi)\,\mathrm{d}\xi \\
& + \int_{Y-(n-i)Q}^{\infty} p(\xi - Y + (n-i)Q) f_i(\xi)\,\mathrm{d}\xi \\
& \text{if } Y \geqslant (n-i)Q,
\end{aligned} \tag{11}
$$

$$
G_i(Y|n,Q) = p(i\mu + (n-i)Q - Y) \quad \text{if } Y<(n-i)Q, \tag{12}
$$

and expression (6) remains unchanged for $G_n(Y|n, Q)$. Also, $DG_i(Y|n, Q)$, for $i \leqslant (n-1)$, reduce to

$$DG_i(Y|n,Q) = -p + (h+p)\int_0^{Y-(n-i)Q} f_i(\xi)\,d\xi \quad \text{if } Y \geqslant (n-i)Q, \tag{13}$$

$$DG_i(Y|n,Q) = -p \quad \text{if } Y < (n-i)Q. \tag{14}$$

Let Y^s be the optimal order-up-to level for this simplified model. By comparing (13) to (7) and (14) to (8), one can easily verify that $Y^s \geqslant Y^*$.

4. Computational results

To illustrate, consider the base case: $\mu = 4$/period, $h = \$1$, and $p = \$100$. Demand is assumed to follow a Poisson process. If $Q = 4$ and $n = 5$, then $Y^* = 29$ and $G(Y^*|n, Q)/n = \$11.06$. We vary μ and Q as well as n and p in the base case (specifically, $\mu = 2$, 4, and 6, $Q = \mu-1$, μ, $\mu+1$, $\mu+2$, and $\mu+3$, $n = 1, 2, \ldots, 20$, and $p = \$10$, \$100, and \$1000) and solve 900 problems. It is found that $G(Y^*|n, Q)/n$ is increasing in n. This implies that if $K = 0$, $n^* = 1$, i.e., the proposed model reduces to an ordinary periodic policy where an order is placed in every period.

Suppose that K is positive in the base case. If $K = \$100$ and $Q = 4$, $n^* = 13$, $Y^* = 66$, and $AC^*(Q) = \$24.46$. Tables 1–3 report $AC^*(Q)$ for different values of Q given K and p (while holding h fixed). Tables 1–3 also report the percentage savings if the simplified policy in Section 3 is used (which is obtained by comparing the lowest $AC^*(Q)$ of the proposed model and the simplified policy). As we see, Q^* approaches μ and n^* becomes larger as p decreases (other things being equal), i.e., one can afford to plan for the order quantities for more periods and Q^* may equal μ when the unit shortage cost is low. In addition, n^* becomes larger as the fixed cost K increases, which intuitively makes sense. In Tables 4 and 5, we vary μ in the base case and observe similar results. If the ordinary order-up-to policy is used, the total one-period cost is K plus the one-period holding and shortage cost (which equals, for example, \$6.24 in Table 2). Thus, as K is larger, the proposed model becomes more attractive relative to the ordinary order-up-to policy.

In general, we observe throughout the computation that $Q^* \geqslant \mu$. This is probably because for $Q < \mu$, Q_1 is greater than μ by Theorem 2, implying that the safety stock for the immediately upcoming period will be larger than that for the upcoming n periods, which is not reasonable. For example, in Table 2, if $K = \$100$ and $Q = 3$, then $n^* = 10$ and $Y^* = 51$; thus, the safety stock for the upcoming 10 periods is equal to 11. But the on-hand inventory after the first delivery is at least $Y^* - (n^*-1)Q = 24$ and thus the safety stock for the immediately upcoming period is at least 20, which is apparently too high! Related to this result is that in Tables 1–5, if Q is very large or small (relative to μ), n^* is small, i.e., the inventory system cannot plan for the order quantities for too many periods; hence, K is spread over less periods and a large or small Q seems not optimal.

Note that for a given Q the simplified policy usually gives a lower one-period cost, as it allows the system to salvage or return excess units (in the immediately upcoming period) to the supplier. However, if Q is smaller than μ the simplified policy yields almost the same one-period cost as the proposed model, since the probability of salvaging excess units is very low, and if Q is very large the simplified policy may give a higher one-period cost than

Table 1
$AC^*(Q)$ for various values of Q (data: $\mu = 4$, $p = \$1000$, and $h = \$1$)

		Proposed model			Simplified policy			
K	Q	n^*	Y^*	$AC^*(Q)$	n^*	Y^*	$AC^*(Q)$	Savings (%)
\$0		1	11	\$8.29	1	11	\$8.29	
50	2	5	33	28.27	5	33	28.27	
	3	6	38	26.30	6	38	26.30	
	4	7	43	24.21	7	43	24.11	
	5	8	50	<u>22.83</u>	9	55	22.12	
	6	7	47	23.21	8	53	<u>21.88</u>	4.16
	7	6	44	24.55	6	45	23.30	
100	2	7	42	36.43	7	42	36.43	
	3	9	52	33.26	9	52	33.26	
	4	11	63	29.92	11	63	29.79	
	5	12	70	<u>27.87</u>	13	75	<u>26.73</u>	4.09
	6	10	64	29.29	11	71	27.35	
	7	8	56	31.66	9	66	30.10	
200	2	11	61	47.37	11	61	47.37	
	3	13	71	42.50	13	71	42.50	
	4	18	95	37.07	18	95	36.93	
	5	18	100	<u>34.44</u>	20	110	<u>32.67</u>	5.14
	6	14	87	37.72	15	95	35.28	
	7	12	81	41.39	12	87	39.92	

Note that the lowest $AC^*(Q)$ values given K are underlined.

Table 2
$AC^*(Q)$ for various values of Q (data: $\mu = 4$, $p = \$100$, and $h = \$1$)

		Proposed model			Simplified policy			
K	Q	n^*	Y^*	$AC^*(Q)$	n^*	Y^*	$AC^*(Q)$	Savings (%)
\$0		1	9	\$6.24	1	9	\$6.24	
50	2	6	33	24.12	6	33	24.12	
	3	7	38	21.98	7	38	21.97	
	4	8	43	19.77	8	43	19.65	
	5	9	51	<u>18.83</u>	10	56	<u>18.05</u>	4.14
	6	7	43	19.93	8	50	18.89	
	7	6	40	21.46	6	43	20.80	
100	2	8	41	31.38	8	41	31.38	
	3	10	51	28.00	10	51	28.00	
	4	13	66	24.46	13	66	24.32	
	5	14	75	<u>23.32</u>	15	80	<u>22.18</u>	4.89
	6	10	60	25.58	11	68	24.32	
	7	9	58	28.01	9	63	27.63	
200	2	12	59	41.31	12	59	41.31	
	3	15	73	36.14	15	73	36.14	
	4	20	97	30.43	20	97	30.27	
	5	19	100	<u>29.36</u>	21	110	<u>27.70</u>	5.65
	6	15	88	33.46	15	91	32.19	
	7	13	82	36.96	12	84	37.31	

Note that the lowest $AC^*(Q)$ values given K are underlined.

Table 3
$AC^*(Q)$ for various values of Q (data: $\mu = 4$, $p = \$10$, and $h = \$1$)

		Proposed model			Simplified policy			
K	Q	n^*	Y^*	$AC^*(Q)$	n^*	Y^*	$AC^*(Q)$	Savings (%)
\$0		1	7	\$3.93	1	7	\$3.93	
50	2	7	30	18.53	7	30	18.53	
	3	8	35	16.29	8	35	16.29	
	4	12	55	<u>14.12</u>	12	55	13.99	
	5	11	55	14.23	11	56	<u>13.62</u>	3.54
	6	8	44	15.89	8	47	15.63	
	7	7	41	17.34	6	39	17.94	
100	2	10	42	24.63	10	42	24.63	
	3	12	51	21.11	12	51	21.11	
	4	18	80	<u>17.53</u>	18	80	<u>17.38</u>	0.86
	5	16	79	18.13	16	80	17.39	
	6	12	65	20.85	11	64	20.91	
	7	11	64	22.90	9	60	24.51	
200	2	14	57	33.06	14	57	33.06	
	3	18	75	27.64	18	75	27.64	
	4	24	105	<u>22.04</u>	24	105	<u>21.86</u>	0.82
	5	22	108	23.49	22	110	22.68	
	6	18	97	27.70	15	87	28.57	
	7	15	86	30.62	12	80	34.00	

Note that the lowest $AC^*(Q)$ values given K are underlined.

the proposed model. For example, if $K = \$200$ and $Q = 7$ in Table 2, then $n^* = 13$, $Y^* = 82$ and $AC^*(Q) = \$36.96$ for the proposed model and $n^* = 12$, $Y^* = 84$ and $AC^*(Q) = \$37.31$ for the simplified policy. This is because a large Q yields a relatively high Y^* for the simplified policy, since the system needs some safety stock for the immediately upcoming period and still has Q to be delivered for each of the subsequent $(n-1)$ periods; in order for Y^* not to be too high, n^* may be smaller and K is thus spread over less periods.

5. Conclusion

This paper considers a single-item replenishment problem where every n periods the buyer plans for the

Table 4
AC*(Q) for various values of Q (data: $\mu = 2$, $p = \$100$, and $h = \$1$)

		Proposed model			Simplified policy			
K	Q	n*	Y*	AC*(Q)	n*	Y*	AC*(Q)	Savings (%)
$0		1	6	$4.60	1	6	$4.60	
50	1	8	23	18.51	8	23	18.51	
	2	10	29	15.81	10	29	15.66	
	3	10	32	<u>15.63</u>	11	36	<u>14.55</u>	6.91
	4	8	30	17.48	7	29	16.79	
	5	7	28	18.83	6	30	19.30	
100	1	11	30	23.91	11	30	23.91	
	2	16	43	<u>19.57</u>	16	43	19.40	
	3	15	46	19.83	16	50	<u>18.42</u>	5.88
	4	11	39	22.69	11	44	22.33	
	5	11	42	24.58	9	44	26.12	
200	1	16	41	31.24	16	41	31.24	
	2	26	66	<u>24.32</u>	26	66	24.13	
	3	21	63	25.53	21	65	<u>23.83</u>	2.01
	4	17	58	29.81	15	60	30.24	
	5	16	59	32.37	12	59	35.92	

Note that the lowest AC*(Q) values given K are underlined.

Table 5
AC*(Q) for various values of Q (data: $\mu = 6$, $p = \$100$, and $h = \$1$)

		Proposed model			Simplified policy			
K	Q	n*	Y*	AC*(Q)	n*	Y*	AC*(Q)	Savings (%)
$0		1	12	$7.48	1	12	$7.48	
50	4	5	40	26.37	5	40	26.37	
	5	6	47	24.46	6	47	24.45	
	6	7	55	22.54	7	55	22.45	
	7	8	64	<u>21.32</u>	9	71	<u>20.82</u>	2.35
	8	7	60	21.81	8	68	20.86	
	9	6	55	23.10	6	56	22.24	
100	4	8	60	34.10	8	60	34.10	
	5	9	67	31.04	9	67	31.03	
	6	11	82	27.93	11	82	27.81	
	7	13	99	<u>26.22</u>	13	99	<u>25.39</u>	3.17
	8	10	83	27.64	11	91	26.33	
	9	9	79	29.97	9	82	29.01	
200	4	11	79	44.53	11	79	44.53	
	5	14	99	39.78	14	99	39.78	
	6	18	128	34.69	18	128	34.55	
	7	19	141	<u>32.54</u>	20	148	<u>31.30</u>	3.81
	8	15	121	35.72	15	123	34.03	
	9	12	104	39.42	12	109	38.59	

Note that the lowest AC*(Q) values given K are underlined.

quantity delivered for each upcoming period. Depending on the demand of the previous n periods, the quantity delivered in the immediately upcoming period may be of any size and the quantity delivered in each of the subsequent $(n-1)$ periods is equal to or smaller than a fixed quantity Q. If excess inventory in the immediately upcoming period can be salvaged or retuned to the supplier at the original purchase cost, the quantity shipped in each of the subsequent $(n-1)$ periods is exactly Q.

Computation shows that the optimal Q is greater than or equal to the mean period demand. However, as the ratio p/h decreases, the optimal n increases and the optimal Q may approach the mean period demand. In addition, as K increases the optimal n increases, and if $K = 0$ the optimal n is equal to 1. In this sense, the ordinary order-up-to policy where an order is placed in every period could be regarded as a special case of the proposed replenishment model. More importantly, as K is larger, the proposed model becomes more attractive relative to the ordinary order-up-to policy.

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