Using Node Diagnosability to Determine t-Diagnosability under the Comparison Diagnosis Model

Chieh-Feng Chiang and Jimmy J.M. Tan

Abstract—Diagnosis is an essential subject for the reliability of a multiprocessor system. Under the comparison diagnosis model, Sengupta and Dahbura proposed a polynomial-time algorithm with time complexity $O(N^5)$ to identify all the faulty processors for a given syndrome in a system with N processors. In this paper, we present a novel idea on system diagnosis called *node diagnosability*. The node diagnosability can be viewed as a local strategy toward system diagnosability. There is a strong relationship between the node diagnosability and the traditional diagnosability. For this local sense, we focus more on a single processor and require only identifying the status of this particular processor correctly. Under the comparison diagnosis model, we propose a sufficient condition to determine the node diagnosability of a given processor. Furthermore, we propose a useful local structure called an *extended star* to guarantee the node diagnosability and provide an efficient algorithm to determine the faulty or fault-free status of each processor based on this structure. For a multiprocessor system with total number of processors N, the time complexity of our algorithm to diagnose a given processor is $O(\log N)$ and that to diagnose all the faulty processors is $O(N \log N)$ under the comparison model, provided that there is an extended star structure at each processor and that the time for looking up the testing result of a comparator in the syndrome table is constant.

Index Terms—Fault diagnosis, comparison diagnosis model, MM* diagnosis model, node diagnosability, extended star structure, diagnosis algorithm.

1 Introduction

Recently, high-speed multiprocessor systems have become more and more popular in computer technology. The *reliability* of the processors in a system is significant since even a few faulty processors may cause the system failure. Whenever processors are found faulty, we should replace the faulty ones with fault-free ones to maintain the reliability of the system. Identifying all the faulty processors of a system is called diagnosis of the system. The maximum number of faulty processors that can be ensured to be identified is called the *diagnosability* of the system. A system G is *t-diagnosable* if all the faulty processors can be precisely pointed out given that the number of faulty processors is at most t. The maximum number t for which G is t-diagnosable is called the *diagnosability* of G.

Multiprocessor systems consist of processors and communication links between the processors. Practically, most multiprocessor systems are based on an underlying bus structure or fabric and perfectly feasible for a central test controller (an independent processor acting as a controller) to check each processor in the system. In such a scheme, the central controller itself can be tested externally. Several relevant papers are selected in the following, concerning the network-on-chip (NoC) issue: Pande et al. [14] developed an evaluation methodology to compare the performance and characteristics of a variety of NoC topologies; Bartic et al. [2] presented an NoC design, which is suitable for building networks with irregular topologies.

Throughout this paper, each processor in a system is presented as a node, and a single edge between two arbitrary nodes represents the communication bus or fabric. A diagnosis testing signal is supposed to be delivered from one node to another node through the communication bus at one time. A system performs system-level diagnosis by making each processor act as a tester to test each of the directly connected ones, and such a scheme contains no central test controller instead. All assumptions are given in order to be consistent with the classic comparison diagnosis model proposed by Maeng and Malek [12].

Several well-known approaches on diagnosis have been developed. One major approach, called the PMC diagnosis model, was first proposed by Preparata et al. [15]. It performs diagnosis by sending a test signal from a processor to another linked one and getting a returning response in the reverse direction. According to the collective testing results, the faulty or fault-free status of all processors in a system can be identified. Following the PMC model, Dahbura and Masson [4] proposed a diagnosis algorithm with time complexity $O(N^{2.5})$ to identify all the faulty processors in a system with N processors. Another major diagnosis approach is called the comparison model, which was proposed by Maeng and Malek [12], [13]. In this model, the diagnosis is performed by simultaneously sending two identical signals from a processor to two other linked ones and comparing the responses. Under the comparison model, Sengupta and Dahbura [16] discussed some characterizations of a t-diagnosable system

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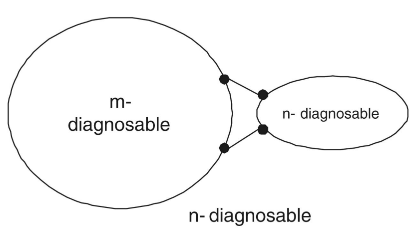

Fig. 1. An n-diagnosable system generated by integrating an n-diagnosable subsystem and an m-diagnosable subsystem.

and gave a polynomial-time algorithm with time complexity $O(N^5)$ to diagnose a system of N processors.

Following the diagnosis models above, most previous studies focused on the diagnosis ability of a system in a global sense but ignored some local systematic details. A system is *t-diagnosable* if all the faulty processors can be identified whenever the number of faulty processors is at most t. However, it is possible to correctly point out all the faulty processors in a t-diagnosable system when the number of faulty processors is greater than t. For example, consider two hypercube systems Q_m and Q_n, which are m-diagnosable and n-diagnosable [17], respectively, where m and n are integers, and $m \gg n$. A new system can be generated by integrating these two systems with few communication links in some way, and such a way may cause the diagnosability of the new system to become n. However, the strong diagnosis ability of some part of the entire system, the part of the original m-diagnosable subsystem Q_m, is ignored. Thus, if only considering the global status, we lose some local details of the system. See Fig. 1 for an illustration.

In some circumstances, however, we are only concerned about some substructure of a multiprocessor system, which is implementable in very large-scale integration (VLSI). For example, such a substructure can be a ring structure or a path structure. If all processors in such a substructure can be guaranteed to be fault free, the procedure is still workable even though a few processors in some other part of the system are faulty. Thus, the local substructure of a system is more critical than the global status of the entire system.

In this paper, we present a novel idea on system diagnosis, which we call *node diagnosability*. The node diagnosability can be viewed as a local strategy toward system diagnosability. There is a strong relationship between the node diagnosability and the traditional one. For this local sense, we focus more on a single processor and require only identifying the status of this particular processor correctly. More specifically, every processor in a system has its own node diagnosability. Under the comparison diagnosis model, we propose a sufficient condition to determine the node diagnosability of a given processor x. On the basis of this sufficient condition, we propose a useful local structure called an *extended star* at processor x to guarantee its node diagnosability. Along this way, we have an efficient algorithm to determine the faulty or fault-free status of each processor based on the extended star structure. For most practical multiprocessor systems, the number of links connecting to each processor is in the order of $\log N$, where N is the total number of processors. The time complexity of our algorithm to diagnose a given processor is bounded by $O(\log N)$ and that to diagnose all the faulty processors in a system with N processors is bounded by $O(N \log N)$ under the comparison model, provided that there is an extended star structure at each processor and that the time for looking up the testing result of a comparator in the syndrome table is constant. In general, the time complexity of our algorithm can be represented as $O(N\Delta)$, where Δ is the maximum degree of a processor in the N-processor system.

The rest of this paper is organized as follows: Section 2 provides preliminaries and necessary background for diagnosing a system. Section 3 introduces the concepts of *node diagnosability* and provides a sufficient condition to check whether a system is *t-diagnosable* at a given processor. The extended star local structure for guaranteeing a processor's node diagnosability is also introduced in this section. In Section 4, we propose an efficient algorithm to determine the faulty or fault-free status of a given processor. Finally, some applications are discussed in Section 5, and our conclusions are given in Section 6.

2 Preliminaries

The topology of a multiprocessor system can be modeled as an undirected graph $G = (V, E)$, where V represents the set of all processors, and E represents the set of all connecting links between the processors.

Under the *comparison model* [12], [13], also called the *MM model*, the system diagnosis is performed by a specific testing procedure. For each processor w, which has two distinct links to two other processors u and v, the diagnosis can be performed by simultaneously sending two identical signals from w to u and from w to v and then comparing their returning responses in the reverse direction. Furthermore, in the *MM* model* [16], it is assumed that a comparison is performed by each processor for each pair of distinct connected neighbors. Throughout this paper, all discussions are based on the MM* model, the complete version of the MM model.

This diagnosis-by-comparison strategy can be modeled as a labeled multigraph $M = (V, C)$, called a comparison graph, where V represents the set of all processors same as that in G and C represents the set of labeled edges. For each labeled edge $(u, v)_w \in C$, w is a label on the edge, which means that processors u and v are being compared by a *comparator*, the processor w.

In order to be consistent with the MM model, several assumptions are needed [16]:

1. All faults are permanent.
2. A faulty processor produces incorrect output for each of its given testing tasks.
3. The output of a comparison performed by a faulty processor is unreliable.
4. Two faulty processors with the same input do not produce the same output.

The output on a labeled edge $(u, v)_w \in C$ is denoted by $r((u, v)_w)$, which represents the comparison result of w for the two responses from u and v. An agreement is denoted by $r((u, v)_w) = 0$, whereas a disagreement is denoted by $r((u, v)_w) = 1$. Since the comparator processor itself might be faulty or not, the testing result might be unreliable. For this reason, some conclusions are made: if $r((u, v)_w) = 1$, at least one member of $\{u, v, w\}$ is faulty, or if $r((u, v)_w) = 0$ and w is known to be fault free, both u and v are fault free.

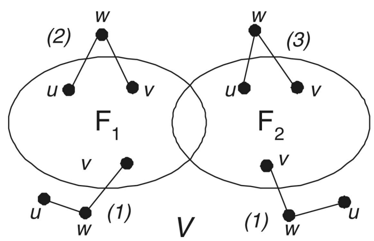

Fig. 2. Illustration of Lemma 1—the distinguishability of two distinct subsets of nodes.

After the completion of testing on each comparator in a system, the collection of all testing results can be defined as a function $\sigma : C \rightarrow \{0, 1\}$ and referred to be a *syndrome* of the diagnosis. For a given syndrome σ, a subset of nodes $F \subset V(G)$ is *consistent* with σ if the syndrome σ can be produced from the situation that all nodes in F are faulty and all nodes in $V - F$ are fault free. Let σ_F denote the set of syndromes that are consistent with F, that is, the collection of all possible syndromes that can be produced if F is the faulty set.

Notice that for a syndrome σ, there might be more than one faulty subset of V that are consistent with σ. A system is defined to be *diagnosable* if, for every syndrome σ, a unique set of nodes $F \subseteq V$ is consistent with it. In addition, we call a system *t-diagnosable* if the system is diagnosable as long as the number of faulty processors is at most t. The maximum number t for a system to be t-diagnosable is called the *diagnosability* of the system. Two distinct subsets of nodes F_1, $F_2 \subset V$ are *distinguishable* if and only if $\sigma_{F_1} \cap \sigma_{F_1} = \phi$; otherwise, F_1 and F_2 are *indistinguishable*.

The following is a useful characterization, proposed by Sengupta and Dahbura [16], for the distinguishability of two sets of nodes under the comparison model. The *symmetric difference* of two sets A and B is defined as the set $A \Delta B = (A \cup B) - (A \cap B)$.

Lemma 1 [16]. *For every two distinct subsets of nodes F_1 and F_2, that is, $F_1 \neq F_2$ and F_1, $F_2 \subset V$, (F_1, F_2) is a distinguishable pair if and only if at least one of the following conditions is satisfied (as illustrated in Fig. 2):*

1. $\exists\, u,\ w \in V - F_1 - F_2$ *and* $\exists\, v \in F_1 \Delta F_2$ *such that* $(u, v)_w \in C$.
2. $\exists\, u,\ v \in F_1 - F_2$ *and* $\exists\, w \in V - F_1 - F_2$ *such that* $(u, v)_w \in C$.
3. $\exists\, u,\ v \in F_2 - F_1$ *and* $\exists\, w \in V - F_1 - F_2$ *such that* $(u, v)_w \in C$.

The detailed proof of this lemma was demonstrated by Sengupta and Dahbura [16]. For the completeness of this paper, we sketch the proof briefly. If one of the three conditions holds, the distinguishability is absolutely determined:

i. Suppose condition 1 is satisfied. If $v \in F_1 - F_2$, then $r((u, v)_w) = 0$ for each $\sigma(F_2)$, and $r((u, v)_w) = 1$ for each $\sigma(F_1)$. Similarly, if $v \in F_2 - F_1$, then $r((u, v)_w) = 0$ for each $\sigma(F_1)$, and $r((u, v)_w) = 1$ for each $\sigma(F_2)$. Either case implies that $\sigma(F_1) \cap \sigma(F_2) = \phi$.
ii. Suppose condition 2 is satisfied. Then, $r((u, v)_w) = 0$ for each $\sigma(F_2)$, and $r((u, v)_w) = 1$ for each $\sigma(F_1)$, which lead to $\sigma(F_1) \cap \sigma(F_2) = \phi$.
iii. Suppose condition 3 is satisfied, a similar argument is used as in condition 2.

On the contrary, if none of the three conditions holds. We consider a syndrome such that for each $(u, v)_w \in C$, the comparison result can be classified to the following nine situations [16]:

i. If $u,\ v,\ w \in V - F_1 - F_2$, then $r((u, v)_w) = 0$.
ii. If $w \in V - F_1 - F_2$ and $u,\ v \in F_1$, then $r((u, v)_w) = 1$.
iii. If $w \in V - F_1 - F_2$ and $u,\ v \in F_2$, then $r((u, v)_w) = 1$.
iv. If $w \in V - F_1 - F_2$, $u \in F_1$, and $v \in F_2$, then $r((u, v)_w) = 1$.
v. If $w \in F_1 - F_2$, $v \in V - F_2$ and $u \in V - F_1 - F_2$, then $r((u, v)_w) = 0$.
vi. If $w \in F_2 - F_1$, $v \in V - F_1$, and $u \in V - F_1 - F_2$, then $r((u, v)_w) = 0$.
vii. If $w \in F_1 - F_2$ and $u \in F_2$, then for all v, $r((u, v)_w) = 1$.
viii. If $w \in F_2 - F_1$ and $u \in F_1$, then for all v, $r((u, v)_w) = 1$.
ix. Other arbitrary comparison results.

Then, the syndrome above belongs to $\sigma(F_1) \cap \sigma(F_2)$, and therefore, F_1 and F_2 are indistinguishable. For example, if $w \in V - F_1 - F_2$, $u \in F_1 \cap F_2$, and $v \in F_1 - F_2$, then $r((u, v)_w) = 1$ whenever the faulty set of nodes is either F_1 or F_2. In such a circumstance, pair (F_1, F_2) cannot be distinguished only with such few information.

Let $G = (V, E)$ be a graph and let $M = (V, C)$ be the comparison graph of G. Define the *order graph* [16] of a node $u \in V$ to be a digraph $G_u = (X_u, Y_u)$, where $X_u = \{v \mid \text{either } (u, v) \in E \text{ or } (u, v)_w \in C \text{ for some } w\}$, and $Y_u = \{(v, w) \mid v, w \in X_u \text{ and } (u, v)_w \in C\}$.

A *node cover* of G is a subset of nodes $Q \subseteq V$ such that every edge of E has at least one end node in Q. A node cover with the minimum cardinality is called a *minimum node cover*. For a given node $u \in V$, the *order* of u is defined as the cardinality of a minimum node cover of G_u. For a subset of nodes $U \subset V$, define $T(G, U)$ to be the set $\{v \mid (u, v)_w \in C$ *and* $u, w \in U$ *and* $v \in V - U\}$. Next is a characterization proposed by Sengupta and Dahbura, which gives a sufficient condition for a system being t-diagnosable.

Lemma 2 [16]. *A system $G(V, E)$ with N nodes is t-diagnosable if*

1. $N \geq 2t + 1$,
2. *each node has order at least* t, *and*
3. *for each* $U \subset V$ *such that* $\mid U \mid = N - 2t + p$ *and* $0 \leq p \leq t - 1$, $\mid T(G, U) \mid\ > p$.

In the rest of this paper, we present our novel concept of node diagnosability under the comparison diagnosis model and discuss some properties of it.

3 Node Diagnosability

There were some studies on system diagnosability of some well-known networks under the comparison model. For example, Wang [17], [18] presented that the diagnosability of an n-dimensional hypercube Q_n is n for $n \geq 5$ and the diagnosability of an n-dimensional enhanced hypercube is $n + 1$ for $n \geq 6$. Fan [8], [9] showed that the diagnosability of an n-dimensional crossed cube is n and the diagnosability of an n-dimensional Möbius cube is n for $n \geq 4$. Lai et al. [11] proposed that the diagnosability of the matching composition network is n for $n \geq 4$.

As we observe, the traditional system diagnosability describes the global status of a system. The purpose of this paper for considering the *node diagnosability* is to keep the local connective detail of a system that we might neglect. For example, for any two integers m and n with $m >> n \geq 4$, the diagnosability of two hypercube systems Q_m and Q_n is m and n [17], [11], respectively. Combining these two systems with few communication links in some way may cause the diagnosability of the new topology to become n. In this situation, the strong diagnosis ability of some part of the entire system, the substructure Q_m, is ignored. Therefore, the need of keeping local information of each node is concerned.

In the previous studies on diagnosis, most results focused on the diagnosis ability of a system in a global sense: a system is t-diagnosable if all the faulty nodes can be identified given that there are at most t faulty nodes. In contrast to the global sense, we emphasize more on a single node x in a local sense: we require only identifying the status of one particular node correctly. More specifically, if x belongs to a set of faulty nodes, we must correctly identify x to be faulty, or x is identified to be fault free if x is indeed fault free. In other words, we are only concerned about the status of the node x.

We now introduce the concept of a system being *t-diagnosable at a given node*.

Definition 1. *A system $G(V, E)$ is t-diagnosable at node $x \in V(G)$ if given a test syndrome σ_F produced by the system under the presence of a set of faulty nodes F containing node x with $|F| \leq t$, every set of faulty nodes F' consistent with σ_F and $|F'| \leq t$ must also contain node x.*

An equivalent way of stating the above definition is given below.

Proposition 1. *A system $G(V, E)$ is t-diagnosable at node $x \in V(G)$ if for each pair of distinct sets, F_1, $F_2 \subset V(G)$ such that $F_1 \neq F_2$, $|F_1|$, $|F_2| \leq t$, and $x \in F_1 \Delta F_2$, (F_1, F_2) is a distinguishable pair.*

Then, we define the *node diagnosability* of a given node as follows:

Definition 2. *The node diagnosability $t_l(x)$ of a node $x \in V(G)$ in a system $G(V, E)$ is defined to be the maximum number of t for G being t-diagnosable at x, that is, $t_l(x) = max\{t \mid G \text{ is t-diagnosable at } x\}$.*

The concept of a system being *t-diagnosable* at a node is consistent with the traditional concept of a system being *t-diagnosable* in the global sense. The relationship between these two is given as follows:

Proposition 2. *A system $G(V, E)$ is t-diagnosable if and only if G is t-diagnosable at every node.*

Proof. We prove the necessary condition first. Suppose that there exists a node $y \in V(G)$ such that G is not t-diagnosable at y. By Proposition 1, there exists an indistinguishable pair (F_1, F_2) with $|F_i| \leq t$, $i = 1$ and 2, and $y \in F_1 \Delta F_2$. This contradicts that G is t-diagnosable. Next, we prove the sufficiency. Suppose G is not t-diagnosable. Then, there exists an indistinguishable pair (F_1, F_2) with $|F_i| \leq t, i = 1$ and 2. Pick any node y in $F_1 \Delta F_2$; the system is not t-diagnosable at y by Proposition 1, which is a contradiction. □

Proposition 3. *The diagnosability $t(G)$ of a system $G(V, E)$ is equal to the minimum value among the node diagnosability of every node in G, that is, $t(G) = min\{t_l(x) \mid x \in V(G)\}$.*

Proof. The result follows trivially from Definition 2 and Proposition 2. □

From Propositions 2 and 3, the relationship between the traditional diagnosability and the node diagnosability was pointed out. Through this concept, the system diagnosability can be determined by testing the node diagnosability of each node. Especially in some well-known regular networks, the diagnosability can be easily identified because of the system symmetry. For example, in some graphs like hypercubes, cube-connected cycles, or complete graphs, the system diagnosability and the node diagnosability of each node in the system are the same, and such a result can be applied in other applications.

Now, we need some definitions for further discussion. For any set of nodes $U \subseteq V(G)$, $G[U]$ denotes the subgraph of G induced by the node subset U. Let H be a subgraph of G and v be a node in H; $deg_H(v)$ denotes the degree of v in subgraph H. For a given set of nodes $S \subseteq V(G)$, we use $G - S$ to denote the induced subgraph $G[V(G) - S]$. Let S be a set of nodes and x be a node *not* in S; we use $C_{x,S}$ to denote the connected component that x belongs to in $G - S$.

In the following, we propose a sufficient condition for verifying whether a system G is t-diagnosable at a given node x.

Theorem 1. *A system $G(V, E)$ is t-diagnosable at a given node $x \in V(G)$ if for every set of nodes $S \subset V(G)$, $|S| = p$, $0 \leq p \leq t - 1$, and $x \notin S$, the cardinality of every node cover including x of the component $C_{x,S}$ is at least $2(t - p) + 1$.*

Proof. We prove it using contradiction. Suppose system G is *not* t-diagnosable at node x. According to Proposition 1, there exists an indistinguishable pair of distinct node set (F_1, F_2) with $|F_1| \leq t$, $|F_2| \leq t$, and $x \in F_1 \Delta F_2$. Let S be the intersection of node sets F_1 and F_2; then, the cardinality of S is less than or equal to $t - 1$. (Otherwise, if $|S| = t$, then $F_1 = F_2$.) According to the condition that $x \notin S$ and $0 \leq |S| \leq t - 1$, the cardinality of every node cover including x of the component $C_{x,S}$ is at least $2(t - p) + 1$. Comparing this number with $|F_1 \Delta F_2| \leq 2(t - p)$ and $x \in F_1 \Delta F_2$, we get the fact that $F_1 \Delta F_2$ cannot be a node cover of $C_{x,S}$. In other words, at least one member (a node) of the node cover of $C_{x,S}$ is outside $F_1 \Delta F_2$ (and also outside S according to the definition of component $C_{x,S}$). Consequently, by the property of node cover, there exists an edge $e = (u, v)$ in $C_{x,S}$ but outside $F_1 \Delta F_2$. Since edge e, nodes u and v, and node x belong to the same connected component $C_{x,S}$, there is a path leading from edge e to node x through set F_1 (as shown in Fig. 3a) or F_2 (as shown in Fig. 3b). Then, by condition 1 of Lemma 1, (F_1, F_2) is a distinguishable pair. This is a contradiction, and the result follows. □

Under the comparison model, Sengupta and Dahbura [16] proposed a polynomial-time algorithm with time complexity $O(N^5)$ to identify all the faulty nodes from a given syndrome, where N is the total number of nodes. In this paper, we present another algorithm using the concept of node diagnosability and a specific systematic structure, called the extended star structure, to diagnose all the faulty nodes. Our algorithm has time complexity $O(N \log N)$ in some well-known multiprocessor systems or interconnection networks.

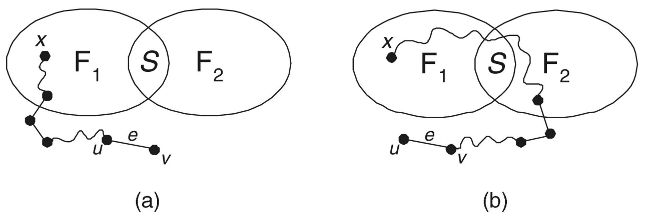

Fig. 3. Illustration of the proof of Theorem 1—at least one edge lies in $C_{x,S} - F_1 \Delta F_2$.

Let us introduce a structure first.

Definition 3. *Let x be a node in a graph $G(V, E)$ with $deg_G(x) \geq n$. Define $H(x; n)$ to be a subgraph of G of order n at node x, where the set of nodes is $\{x\} \cup \{v_{i1}, v_{i2} \mid 1 \leq i \leq n\}$ and the set of edges is $\{(x, v_{i1}), (v_{i1}, v_{i2}) \mid 1 \leq i \leq n\}$. (Fig. 4 depicts the structure.)*

We notice that the term "order" is used in two different places: one is the order of a node x, $order(x)$, and the other is the order of the substructure defined here.

Proposition 4. *Let $G(V, E)$ be a graph and x be a node in G. The order of x is at least n if G contains a subgraph $H(x; n)$ of order n at node x.*

Proposition 5. *Let $G(V, E)$ be a graph without cycles of length three and x be a node in G. G contains a subgraph $H(x; n)$ of order n at node x if the order of x is at least n.*

Proof. Let S_1 and S_2 be two sets of nodes with a distance of one and two to the node x, respectively. Since G contains no cycles of length three, there is no edge with both ends in S_1. Therefore, the order graph of x forms a bipartite graph with the partition (S_1, S_2). Because the node x has order at least n, which means that the cardinality of a minimum node cover of the order graph of x is at least n. By a classical theorem of König [5] and Egerváry [6], the cardinality of a minimum node cover of a bipartite graph equals the maximum size of a matching in the bipartite graph. Then, there is a matching between S_1 and S_2 with the maximum size n. Consequently, G contains a subgraph $H(x; n)$ of order n at node x. □

The above two propositions state that the order of node x is at least n if and only if the system contains a subgraph $H(x; n)$ of order n at x. It implies that if the node diagnosability of node x is n, then G contains a subgraph $H(x; n)$ at x, provided that G has no cycles of length three. However, having the substructure $H(x; n)$ at x, it does not necessarily guarantee that the node diagnosability of node x is at least n.

We now propose a substructure at node x, called an extended star, which can guarantee the node diagnosability of node x.

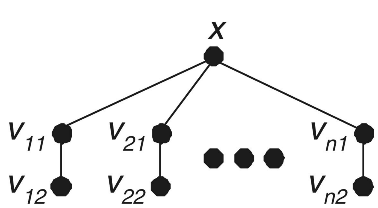

Fig. 4. Subgraph $H(x; n)$ of G of order n at node x.

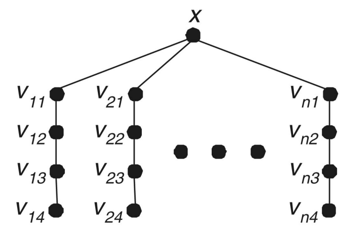

Fig. 5. Extended star structure $ES(x; n)$.

Definition 4. *Let x be a node in a graph $G(V, E)$. For $n \leq deg_G(x)$, an extended star $ES(x; n)$ of order n at node x is defined as $ES(x; n) = (V(x; n), E(x; n))$, where the set of nodes $V(x; n) = \{x\} \cup \{v_{ij} \mid 1 \leq i \leq n, 1 \leq j \leq 4\}$, and the set of edges $E(x; n) = \{(x, v_{k1}), (v_{k1}, v_{k2}), (v_{k2}, v_{k3}), (v_{k3}, v_{k4}) \mid 1 \leq k \leq n\}$. (See Fig. 5 for an illustration.)*

We say that there is an *extended star* structure $ES(x; n) \subseteq G$ at node x if G contains an extended star $ES(x; n)$ of order n at node x as a subgraph. Note that in the definition of the extended star, each node and each edge can occur only once in this structure. In other words, the problem of setting up the extended star structure turns into the problem of finding n node-disjoint paths of length four (3 hops) with dedicated starting nodes. In addition, such a problem can be done offline by the systematic structure of most well-known multiprocessor systems.

Theorem 2. *Let x be a node in a system $G(V, E)$. The node diagnosability of x is at least n if there exists an extended star $ES(x; n) \subseteq G$ at x.*

Proof. We use Theorem 1 to prove this result. First, we define $l_k = (v_{k1}, v_{k2}, v_{k3}, v_{k4})$ to be a quadruple of four consecutive nodes for any k, $1 \leq k \leq n$, with respect to $ES(x; n)$. We note that l_k is a path of length three. Accordingly, the cardinality of a node cover of each l_k is at least two. Let $S \subset V(G)$ be a set of nodes in G with $|S| = p$, $0 \leq p \leq n - 1$, and $x \notin S$. After deleting S from $V(G)$, there are at least $(n - p)$ complete l_k's still remaining in $ES(x; n)$, where the word "complete" means that all v_{k1}, v_{k2}, v_{k3}, and v_{k4} of an l_k have not been deleted in $G - S$. Thus, the cardinality of a node cover including x of the connected component $C_{x,S}$ is at least $1 + 2(n - p)$. Therefore, the system G with an extended star $ES(x; n)$ is n-diagnosable at x by Theorem 1. By Definition 2, the node diagnosability of x is at least n, that is, $t_l(x) \geq n$. □

Proposition 6. *Let x be a node in a system $G(V, E)$ with $deg_G(x) = n$. The node diagnosability of x is at most n.*

By Theorem 2 and Proposition 6, we have the following result.

Theorem 3. *Let x be a node in a system $G(V, E)$ with $deg_G(x) = n$. The node diagnosability of x is n if there exists an extended star $ES(x; n) \subseteq G$ at x.*

We observe that for an extended star structure, if the set of nodes is of the form $V(x; n) = \{x\} \cup \{v_{ij} \mid 1 \leq i \leq n, 1 \leq j \leq 3\}$ and the set of edges is of the form $E(x; n) = \{(x, v_{k1}), (v_{k1}, v_{k2}), (v_{k2}, v_{k3}) \mid 1 \leq k \leq n\}$, the node diagnosability n of node x cannot be guaranteed simply by this kind of substructure. For example, let F_1 be the set of nodes $\{x, v_{11}, v_{12}, v_{13}\}$ with $|F_1| = 4$ and F_2 be the set of

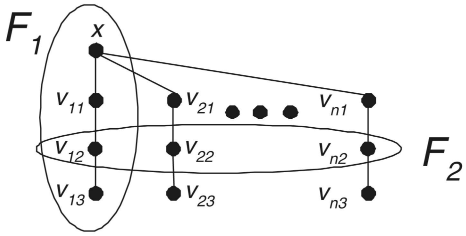

Fig. 6. An example of an indistinguishable pair in an incomplete extended star structure with only the set of nodes $\{x\} \cup \{v_{ij} \mid 1 \leq i \leq n, 1 \leq j \leq 3\}$ and the set of edges $\{(x, v_{k1}), (v_{k1}, v_{k2}), (v_{k2}, v_{k3}) \mid 1 \leq k \leq n\}$.

nodes $\{v_{k2} \mid 1 \leq k \leq n\}$ with $|F_2| = n$ (as shown in Fig. 6); (F_1, F_2) is not a distinguishable pair according to Lemma 1 unless there are other edges or nodes. Thus, the node diagnosability of x cannot be guaranteed to be n.

In most multiprocessor systems or interconnection networks, an extended star substructure at a given processor does exist. For example, in the well-known multiprocessor systems such as the hypercube, the crossed cube [7], the twisted cube [10], the Möbius cube [3], the star [1], the mesh, and other hypercubelike graphs, an extended star at a given processor can be carefully found because of the regular recursive construction, as long as the dimension n is suitably large. Based on this specific structure, a fault diagnosis algorithm is introduced in the next section.

4 A Diagnosis Algorithm

Given an extended star structure at a node, we shall present a diagnosis algorithm to determine whether this node is faulty or not for a given syndrome under the comparison model. As stated in Theorem 3, the node diagnosability of a node can be determined by the neighboring nodes (processors) around it. Intuitively, a node's faulty/fault-free status should also be determined by the comparison outputs of the nodes surrounding it, and Theorem 4 provides an algorithm for performing such procedure.

Let $ES(x;n)$ be an extended star at a given node x in $V(G)$; the diagnosing signals are sent back and forth inside $ES(x;n)$. Since there are communication links between x and v_{k1}, v_{k1} and v_{k2}, v_{k2} and v_{k3}, and v_{k3} and v_{k4}, for all $1 \leq k \leq n$, v_{k1}, v_{k2}, and v_{k3} can be the comparators of the comparison model. After the comparison test, each comparator has a testing result denoted by 0 (1, respectively) representing the agreement (disagreement, respectively). Given an extended star $ES(x;n)$ at a node x, we define $r_k = (r^1, r^2, r^3)$ to be the testing result of an ordered triple (v_{k1}, v_{k2}, v_{k3}) with respect to $ES(x;n)$, where r^1 is the comparison result of v_{k1} for the two responses from x and v_{k2}, r^2 is the comparison result of v_{k2} for the two responses from v_{k1} and v_{k3}, and r^3 is the comparison result of v_{k3} for the two responses from v_{k2} and v_{k4}. Then, r_k can be in one of the eight different states, which are $r(0) = (0,0,0)$, $r(1) = (0,0,1)$, $r(2) = (0,1,0)$, $r(3) = (0,1,1)$, $r(4) = (1,0,0)$, $r(5) = (1,0,1)$, $r(6) = (1,1,0)$, and $r(7) = (1,1,1)$. Let $R(i)$ be the set of the collection of all $r(i)$, for all $0 \leq i \leq 7$. Obviously, the summation of the cardinality of $R(0)$ to $R(7)$ is n, that is, $\sum_{i=0}^{7} |R(i)| = n$.

Let x be a node in a system. Suppose that the degree of x is n and suppose that there is an extended star $ES(x;n)$ at x. Then, the node diagnosability of x is n, which means that we may not be able to identify all the faulty nodes if the number of faulty nodes in $ES(x;n)$ is $n+1$ or more. Therefore, we assume that the number of faulty nodes is at most n. Under this assumption, we have an efficient algorithm to determine whether node x is faulty or not.

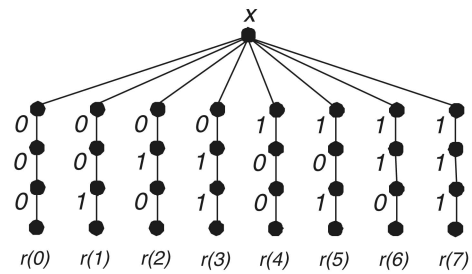

Fig. 7. Eight different output states for Theorem 4.

Theorem 4. *Let x be a node with degree n in a system $G(V,E)$. Suppose that there is an extended star $ES(x;n) \subseteq G$ at x. Define $r_k = (r^1, r^2, r^3)$ to be the testing result of (v_{k1}, v_{k2}, v_{k3}) with respect to $ES(x;n)$. Then, r_k can be in one of the eight states (as illustrated in Fig. 7): $r(0) = (0,0,0)$, $r(1) = (0,0,1)$, $r(2) = (0,1,0)$, $r(3) = (0,1,1)$, $r(4) = (1,0,0)$, $r(5) = (1,0,1)$, $r(6) = (1,1,0)$, and $r(7) = (1,1,1)$.*

Let $R(i)$ be the set of the collection of all $r(i)$ and $|R(i)|$ be the cardinality of $R(i)$. Then, under the assumption that the number of faulty nodes is at most n

i. *x is fault free if $|R(0)| \geq |R(4)|$, or*
ii. *x is faulty if $|R(0)| < |R(4)|$.*

Proof. Let $l_k = (v_{k1}, v_{k2}, v_{k3}, v_{k4})$ be an ordered quadruple, $1 \leq k \leq n$, with respect to $ES(x;n)$. We prove the first part of this theorem by contradiction. Suppose that the number of faulty nodes in $ES(x;n)$ is at most n and suppose that x is faulty; the counting of all the other faulty nodes is given as follows:

- For those l_k with result $r(0)$, there are at least three faulty nodes, which are v_{k1}, v_{k2}, and v_{k3}.
- For those l_k with result $r(1)$, there are at least two faulty nodes, which are v_{k1} and v_{k2}.
- For those l_k with result $r(2)$, there is at least one faulty node, which is v_{k1}.
- For those l_k with result $r(3)$, there are at least two faulty nodes, which are v_{k1} and one of v_{k2}, v_{k3}, and v_{k4} since the output of v_{k3} is a disagreement.
- For those l_k with result $r(4)$, the number of faulty nodes is uncertain.
- For those l_k with result $r(5)$, there is at least one faulty node, which is one of v_{k2}, v_{k3}, and v_{k4} since the output of v_{k3} is a disagreement.
- For those l_k with result $r(6)$, there is at least one faulty node, which is one of v_{k1}, v_{k2}, and v_{k3} since the output of v_{k2} is a disagreement.
- For those l_k with result $r(7)$, there is at least one faulty node, which is one of v_{k2}, v_{k3}, and v_{k4} since the output of v_{k3} is a disagreement.

Thus, the number of faulty nodes is at least $1 + 3|R(0)| + 2|R(1)| + |R(2)| + 2|R(3)| + |R(5)| + |R(6)| + |R(7)| = \Sigma_{i=0}^{7}|R(i)| + (1 + 2|R(0)| + |R(1)| + |R(3)| - |R(4)|)$.

By the assumption that $|R(0)| \geq |R(4)|$, the number of faulty nodes is strictly more than $\Sigma_{i=0}^{7}|R(i)|$, which is equal to n. This contradicts to the assumption that the number of faulty nodes in $ES(x;n)$ is at most n. Therefore, x is fault free.

Now, we prove the second part of this theorem. Suppose that the number of faulty nodes in $ES(x;n)$ is at most n and suppose that x is fault free; the counting of all the other faulty nodes given is as follows:

- For those l_k with result $r(0)$, the number of faulty nodes is uncertain.
- For those l_k with result $r(1)$, there is at least one faulty node, which is one of v_{k2}, v_{k3}, and v_{k4} since the output of v_{k3} is a disagreement.
- For those l_k with result $r(2)$, there is at least one faulty node, which is one of v_{k1}, v_{k2}, and v_{k3} since the output of v_{k2} is a disagreement.
- For those l_k with result $r(3)$, there is at least one faulty node, which is one of v_{k1}, v_{k2}, and v_{k3} since the output of v_{k2} is a disagreement.
- For those l_k with result $r(4)$, there are at least two faulty nodes for the reasons that 1) if v_{k1} is faulty, v_{k2} must be faulty since the comparison result of v_{k2} is wrong, or 2) if v_{k1} is fault free, v_{k2} must be faulty, and v_{k3} must be faulty too.
- For those l_k with result $r(5)$, there is at least one faulty node, which is one of v_{k1} and v_{k2} since the output of v_{k1} is a disagreement.
- For those l_k with result $r(6)$, there is at least one faulty node, which is one of v_{k1} and v_{k2} since the output of v_{k1} is a disagreement.
- For those l_k with result $r(7)$, there is at least one faulty node, which is one of v_{k1} and v_{k2} since the output of v_{k1} is a disagreement.

Thus, the number of faulty nodes is at least $|R(1)| + |R(2)| + |R(3)| + 2|R(4)| + |R(5)| + |R(6)| + |R(7)| = \Sigma_{i=0}^{7}|R(i)| + (|R(4)| - |R(0)|)$.

By the assumption that $|R(0)| < |R(4)|$, the number of faulty nodes is larger than $\Sigma_{n=i}^{7}|R(i)|$, which is equal to n. This contradicts to the assumption that the number of faulty nodes in $ES(x;n)$ is at most n. Therefore, x is faulty. □

Roughly speaking, the collections of testing results $R(0)$ and $R(4)$, with respect to the extended star $ES(x;n)$ found at node x, dominate the faulty/fault-free status of x. We can determine the faulty or fault-free status of a node by just comparing the number of the testing results $r(0)$'s and $r(4)$'s on an arbitrary extended star we found.

5 APPLICATIONS

In this section, we apply the concept of node diagnosability and the proposed diagnosis algorithm to several well-known multiprocessor systems and interconnection networks.

Among all well-known interconnection networks, the hypercube is one of the most popular ones. Following the structure of the hypercube, lots of similar networks had been proposed, such as the crossed cube [7], the twisted cube [10], and the Möbius cube [3]. We call the category of these systems a cube family. For each cube in the cube family, an

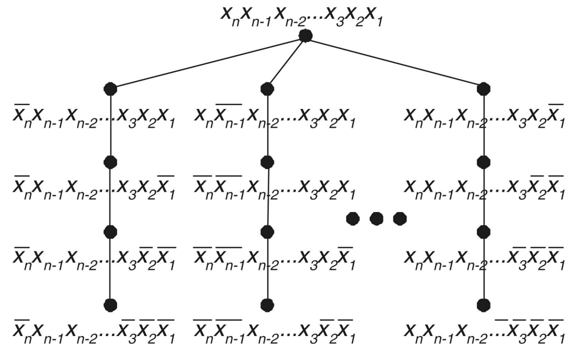

Fig. 8. An extended star structure in an n-dimensional hypercube with $n \geq 5$.

n-dimensional cube can be constructed in recurrence from two identical $(n-1)$-dimensional subcubes by adding a perfect matching between the two subcubes. A different perfect matching leads to a different cube. Because of the recursive construction, an n-dimensional cube has 2^n nodes in it. Each node in the cube is usually represented by an n-bit binary string. A binary string x of length n can be written as $x = x_n x_{n-1}, \ldots, x_2 x_1$, where x_i is 0 or 1, $1 \leq i \leq n$.

For each node x in an n-dimensional hypercube, there are n distinct nodes adjacent to it and with a 1-bit complement to it. It is easy to find an extended star structure $ES(x;n)$ at x in an n-dimensional hypercube with $n \geq 5$ as follows:

For each node $x = x_n x_{n-1}, \ldots, x_2 x_1$, there are n nodes adjacent to it, namely, $\overline{x_n} x_{n-1}, \ldots, x_2 x_1$, $x_n \overline{x_{n-1}}, \ldots, x_2 x_1, \ldots$, and $x_n x_{n-1}, \ldots, x_2 \overline{x_1}$, where the overline denotes the complement bit. Let $v_{n,1}, v_{n-1,1}, \ldots$, and $v_{1,1}$ be these nodes, respectively. For each $v_{k,1}$, $v_{k,1} = x_n x_{n-1}, \ldots, \overline{x_k}, \ldots, x_2 x_1$, there are n nodes adjacent to it also. We can find one of these nodes with the $(k+1)(mod\, n)\text{th}$ bit complement to $v_{k,1}$, for all $1 \leq k \leq n$, and name it $v_{k,2}$. Then, $v_{k,2} = x_n x_{n-1}, \ldots, \overline{x_{k+1} x_k}, \ldots, x_2 x_1$. Moreover, we can find $v_{k,3} = x_n x_{n-1}, \ldots, \overline{x_{k+2} x_{k+1} x_k}, \ldots, x_2 x_1$ and $v_{k,4} = x_n x_{n-1}, \ldots, \overline{x_{k+3} x_{k+2} x_{k+1} x_k}, \ldots, x_2 x_1$ in the same way, where the indices are modulo n (Fig. 8).

All these nodes do not have the same address (string bits) since the bit length is at least five. Thus, the procedure described above provides an extended star $ES(x;n)$ for every node x in $V(Q_n)$, for $n \geq 5$. Consequently, the node diagnosability of each node $x \in V(Q_n)$ is n, and the diagnosability of Q_n is n, for $n \geq 5$, which is the same conclusion as that proposed by Wang [17]. Note that there are more than one way for searching an extended star in a hypercube.

As another example, we show that the star graph [1] with a dimension of four or more contains an extended star structure as a subgraph at each node . Let n be a positive integer. The star graph of dimension n, denoted by S_n, is a graph whose set of nodes consists of all permutations of $\{1, 2, \ldots, n\}$. Each node is uniquely assigned a label $x_1 x_2, \ldots, x_n$ and is adjacent to the nodes $x_i x_2, \ldots, x_{i-1} x_1 x_{i+1}, \ldots, x_n$, for $2 \leq i \leq n$, that is, nodes obtained by a transposition of the first symbol with the ith symbol of the node. Consequently, there are $n!$ nodes in an n-dimensional star graph, and each node has degree $n-1$. We can find an extended star structure $ES(x;n-1)$ at a given node x in S_n with $n \geq 5$ as follows:

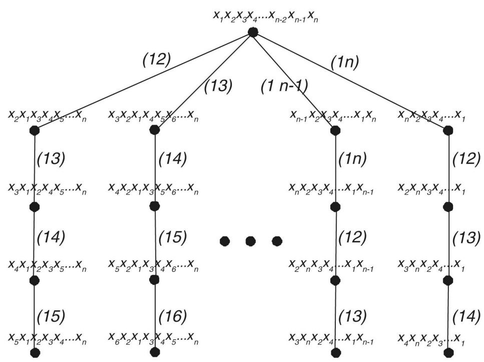

Fig. 9. An extended star structure in an n-dimensional star graph with $n \geq 5$.

For each node $x = x_1x_2, \ldots, x_n$, there are $n-1$ nodes adjacent to it, namely, $x_2x_1x_3x_4, \ldots, x_n$, $x_3x_2x_1x_4, \ldots, x_n$, $\ldots, x_ix_2x_3x_4, \ldots, x_{i-1}x_1x_{i+1}, \ldots, x_n, \ldots,$ and $x_nx_2x_3x_4, \ldots, x_{n-1}x_1$. Let $v_{2,1}, v_{3,1}, \ldots, v_{i,1}, \ldots,$ and $v_{n,1}$ be these nodes, respectively. For convenience of description, we say that two nodes are adjacent to each other with a $(1\ i)$ edge if one node can be obtained by a transposition of the first symbol with the ith symbol of the other node. Accordingly, x is adjacent to $v_{k,1}$ with a $(1\ k)$ edge, for all $2 \leq k \leq n$. For each $v_{k,1}$, there are $(n-2)$ more nodes adjacent to it except for x. We can choose one of these adjacent nodes of $v_{k,1}$ with a $(1\ k+1)$ edge if $2 \leq k \leq n-1$ and with a $(1\ ((k+2) mod\ n))$ edge if $k = n$. Let $v_{k,2}$ be these nodes, for all $2 \leq k \leq n$, respectively. We then find $v_{k,3}$ as one of the adjacent nodes of $v_{k,2}$ with a $(1\ k{+}2)$ edge if $2 \leq k \leq n-2$ and with a $(1\ ((k+3)\ mod\ n))$ edge if $n-1 \leq k \leq n$. Finally, we find $v_{k,4}$ as one of the adjacent nodes of $v_{k,3}$ with a $(1\ k+3)$ edge if $2 \leq k \leq n-3$ and with a $(1\ ((k+4) mod\ n))$ edge if $n-2 \leq k \leq n$ (Fig. 9).

Therefore, an extended star $ES(x; n-1)$ at every node $x \in V(S_n)$ can be retrieved for $n \geq 4$. We note, however, that for $n = 4$, the construction strategy described above has to be modified a little bit, since the construction strategy in the last paragraph will cause all $v_{k,4}$'s to be the same node, for all $2 \leq k \leq n$. We can choose $v_{k,4}$ as one of the adjacent nodes of $v_{k,3}$ with a $(1\ 3)$ edge for $k = 2$, a $(1\ 4)$ edge for $k = 3$, and a $(1\ 2)$ edge for $k = 4$ as a modified strategy. Therefore, for $n \geq 4$, the node diagnosability of each node $x \in V(S_n)$ is $n-1$, and the diagnosability of S_n is $n-1$, which is the same conclusion as that proposed by Zheng et al. [20].

For most multiprocessor systems or interconnection networks, an extended star at a given node can be carefully found, as long as the dimension n is suitably large. This explains the fact that the node diagnosability of a given node matches its degree in many cases.

As one more example, consider an m-dimensional hypercube system Q_m and an n-dimensional hypercube system Q_n, for $m \geq n \geq 5$. The node diagnosability of each node in Q_m (Q_n, respectively) is m (n, respectively). Let u be a node in Q_m and v be a node in Q_n. A new system can be formed by adding an edge (u, v) between Q_m and Q_n. Applying the extended star structure, the node diagnosability of each node in Q_m (Q_n, respectively) remains m (n, respectively) except for u (v, respectively), while the node diagnosability of node u (v, respectively) increases to $m+1$ ($n+1$, respectively). Overall, the diagnosability of this new system is n.

We now measure the time complexity to diagnose all the faulty nodes in a system. For most of the practical systems with N nodes, the degree of each node is in the order of $\log N$. For example, the n-dimensional hypercube Q_n has $N = 2^n$ nodes, and the degree of each node is n, $n = \log N$; the n-dimensional star S_n has $N = n!$ nodes, and the degree of each node is $n - 1 = O(n) = O(\frac{\log N}{\log n}) = O(\frac{\log N}{\log \log N})$. We assume that a testing result of each comparator for each pair of distinct neighbors with which it can communicate directly is stored in a syndrome table. Given an extended star structure $ES(x; n)$ at a node x, assume that the time for looking up the testing result of a comparator in the syndrome table is constant c. Then, the time needed for determining the faulty or fault-free status of node x is $3c \log N = O(\log N)$. Consequently, the total time for diagnosing all the faulty nodes is $O(N \log N)$.

As a result, for most practical multiprocessor systems, especially some well-known symmetric and regular topologies like hypercube systems, the time for self-diagnosis is $O(N \log N)$, where N is the total number of processors in it. On the other hand, the presented diagnosis algorithm is not restricted to symmetric systems only. We can apply such a method to diagnose a system node by node and, consequently, to diagnose the whole system. In general, the time complexity is $O(N\Delta)$, where Δ is the maximum degree of a node in this system.

The time complexity $O(N \log N)$ obtained here is based on the symmetry of most recently practical multiprocessor systems. Applying the traditional approach by Sengupta and Dahbura [16] results in an initiate result of time complexity $O(N^5)$. However, under some constraints like symmetry or regularity of the systems, using the classical approach may result in a better computational complexity than $O(N^5)$, especially on some special cases of hypercubes or other well-known topologies. A recent paper can be referred on this issue; Yang and Tang [19] address the fault identification of diagnosable multiprocessor systems under the MM* comparison model and present an $O(N\Delta^3\delta)$ time diagnosis algorithm for an N-node system, where Δ and δ are the maximum and minimum degrees of a node, respectively.

6 Conclusions

The issue of identifying all the faulty processors is important in the design of interconnection networks or multiprocessor systems, which is implementable in VLSI. The process of identifying all the faulty processors is called diagnosis of a system. Under the asymmetric comparison diagnosis model, each processor acts as a comparator to test each pair of adjacent two processors. Further, Sengupta and Dahbura [16] proposed a polynomial-time algorithm with time complexity $O(N^5)$ to diagnose a system with total number N of processors. In some circumstances, it is not necessary to judge the status of all processors but several ones in some substructure of the system such as a ring structure or a path structure.

In this paper, we proposed a novel idea on system diagnosis called *node diagnosability*. Opposite to that of the traditional *diagnosability*, the concept of node diagnosability puts more focus on a single processor and requires only identifying the status of this particular processor correctly. Estimating the node diagnosability of each processor in a system also provides a new viewpoint for checking the diagnosability of the whole system. Under the comparison diagnosis model, we proposed a sufficient condition to determine a given processor's node diagnosability and an efficient algorithm to determine whether a processor is faulty based on the local syndrome of a given *extended star* structure. All these concepts can be applied to many well-known interconnection networks. For most practical multiprocessor systems, the number of links connecting to each processor is in the order of $\log N$, where N is the total number of processors. The time complexity of our algorithm to diagnose a given processor is $O(\log N)$, and that to diagnose all the faulty processors in a system is $O(N \log N)$.

Finally, we propose a research topic worth studying at the end of this paper, which is the issue of the underlying assumptions consistent with the comparison diagnosis model. As referred to those assumptions, all faults are permanent, and the comparison output performed by a faulty processor is unreliable. However, in future technologies, it is likely that many faults will be transient or nonpermanent, making fixed diagnosis strategies more complex and violating the comparison diagnosis strategy we are based on. Furthermore, a faulty processor may be able to perform self-diagnosis and identify itself as faulty. Therefore, violating each assumption of the comparison model may lead to a different situation, and each of the modifications will be an interesting problem for further research.

Acknowledgments

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References

[1] S.B. Akers and B. Krishnamurthy, "A Group-Theoretic Model for Symmetric Interconnection Networks," *IEEE Trans. Computers,* vol. 38, no. 4, pp. 555-566, Apr. 1989.

[2] T.A. Bartic, J.Y. Mignolet, V. Nollet, T. Marescaux, D. Verkest, S. Vernalde, and R. Lauwereins, "Topology Adaptive Network-on-Chip Design and Implementation," *IEE Proc. Computers and Digital Techniques,* vol. 152, no. 4, July 2005.

[3] P. Cull and S. Larson, "The Möbius Cubes," *IEEE Trans. Computers,* vol. 44, no. 5, pp. 647-659, May 1995.

[4] A. Dahbura and G. Masson, "An $O(N^{2.5})$ Fault Identification Algorithm for Diagnosable Systems," *IEEE Trans. Computers,* vol. 33, no. 6, pp. 486-492, June 1984.

[5] D. König, "Graphen und Matrizen," *Math. Lapok. 38,* pp. 116-119, 1931.

[6] E. Egerváry, "On Combinatorial Properties of Matrices (Hungarian with German Summary)," *Math. Lapok. 38,* pp. 16-28, 1931.

[7] K. Efe, "A Variation on the Hypercube with Lower Diameter," *IEEE Trans. Computers,* vol. 40, no. 11, pp. 1312-1316, Nov. 1991.

[8] J. Fan, "Diagnosability of Crossed Cubes under the Comparison Diagnosis Model," *IEEE Trans. Parallel and Distributed Systems,* vol. 13, no. 7, pp. 687-692, July 2002.

[9] J. Fan, "Diagnosability of the Möbius Cubes," *IEEE Trans. Parallel and Distributed Systems,* vol. 9, no. 9, pp. 923-928, Sept. 1998.

[10] P. Hilbers, M. Koopman, and J. Snepscheut, "The Twisted Cube," *Proc. Int'l Conf. Parallel Architecture and Languages Europe (PARLE '87),* pp. 152-159, June 1987.

[11] P.L. Lai, J.J.M. Tan, C.H. Tsai, and L.H. Hsu, "The Diagnosability of the Matching Composition Network under the Comparison Diagnosis Model," *IEEE Trans. Computers,* vol. 53, no. 8, Aug. 2004.

[12] J. Maeng and M. Malek, "A Comparison Connection Assignment for Self-Diagnosis of Multiprocessors Systems," *Proc. 11th Int'l Symp. Fault-Tolerant Computing (FTCS '81),* pp. 173-175, 1981.

[13] M. Malek, "A Comparison Connection Assignment for Diagnosis of Multiprocessors Systems," *Proc. Seventh Int'l Symp. Computer Architecture (ISCA '80),* pp. 31-36, 1980.

[14] P.P. Pande, C. Grecu, M. Jones, A. Ivonov, and R. Saleh, "Performance Evaluation and Design Trade-Offs for Network-on-Chip Interconnect Architectures," *IEEE Trans. Computers,* vol. 54, no. 8, Aug. 2005.

[15] F.P. Preparata, G. Metze, and R.T. Chien, "On the Connection Assignment Problem of Diagnosis Systems," *IEEE Trans. Electronic Computers,* vol. 16, no. 12, pp. 848-854, Dec. 1967.

[16] A. Sengupta and A. Dahbura, "On Self-Diagnosable Multiprocessor Systems: Diagnosis by the Comparison Approach," *IEEE Trans. Computers,* vol. 41, no. 11, pp. 1386-1396, Nov. 1992.

[17] D. Wang, "Diagnosability of Hypercubes and Enhanced Hypercubes under the Comparison Diagnosis Model," *IEEE Trans. Computers,* vol. 48, no. 12, pp. 1369-1374, Dec. 1999.

[18] D. Wang, "Diagnosability of Enhanced Hypercubes," *IEEE Trans. Computers,* vol. 43, no. 9, pp. 1054-1061, Sept. 1994.

[19] X. Yang and Y.Y. Tang, "Efficient Fault Identification of Diagnosable Systems under the Comparison Model," *IEEE Trans. Computers,* vol. 56, no. 12, Dec. 2007.

[20] J. Zheng, S. Latifi, E. Regentova, K. Luo, and X. Wu, "Diagnosability of Star Graphs under the Comparison Diagnosis Model," *Information Processing Letters,* vol. 93, pp. 29-36, 2005.

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