行動無線隨意網路之可適性拓撲控制

Adaptive Topology Control in Mobile Ad Hoc Networks

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國 立 交 通 大 學

資 訊 工 程 學 系

博 士 論 文

A dissertation
Submitted to Department of Computer Science
College of Computer Science
National Chiao Tung University
in Partial
Fulfillment of the Requirements for the Degree of
Doctor of Philosophy
in computer Science
July 2007
Hsinchu, Taiwan, Republic of China

中 華 民 國　九十六年七月

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摘要

在無線環境中，網路的效能會高度受到底層的拓撲所影響。而一個稀疏的拓撲具有減少多餘流量的特性，因此可提升網路擴增性。然而一個稀疏拓撲經常會犧牲許多重要的網路線段，這些線段有可能是型成要能源有效路由的必經路徑。因此，在拓撲的能源有效性和稀疏度之間存在了相互牽制的議題。

在這篇論文中，我們將提出以幾合圖型為基礎的拓撲控制方法，這個方法可透過參數的設定達到在能源有效性和稀疏度之間調整的彈性，理論結果證明此方法可保證拓撲的連通性、平行性，和對稱性。更重要的是，每一個節點利用區域內所收集的資訊即可建構出所需的拓撲。

為了解決節點的移動性，我們以前面的圖型方法為基礎，提出了一個可適性的拓撲控制協定，此協定具有在保有節點能源和改進整體耗能之間動態調整的能力。數據及摸擬結果提出，我們的方法可有效減少能源消耗，特別是對高度行動的網路有明顯的改進。

Adaptive Topology Control in Mobile Ad Hoc Networks

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Abstract

The wireless ad hoc network is convenient to many applications, such as conferences, hospitals, battlefields, and etc. In these environments, the network performance heavily relies on the underlying topology. Especially, keeping the topology sparser enhances network scalability. However, a sparse topology may sacrifice some routes that consume less power. Therefore, a tradeoff is between the sparseness and the energy efficiency of the topology.

In this dissertation, we propose a geometric structure, named the *r*-neighborhood graph, to control the topology. The structure allows the flexibility to be adjusted between energy efficiency and node's degree through a parameter r, $0 \le r \le 1$. Theoretic results show that it can always result in a connected planar topology with symmetric edges. More importantly, the structure can be constructed in localized fashion using only 1-hop information.

To cope with node's mobility, we investigate an adaptive protocol, based on a generalized version of the *r*-neighborhood graph. In this protocol, the parameter r can be adjusted distributively by each node according to the overall energy efficiency. To reduce the construction power, we further incorporate the protocol with a shrinking power mechanism for the topology maintenance. Simulation and numeric results show that the proposed approaches can significantly improve the energy consumption, especially in high mobility environment.

Acknowledgements

Special thank goes to my advisor Professor Rong-Hong Jan for his guidance in my dissertation work. Thank also to all member of Computer Network Lab for their assistance and kindly helping both in the research and the daily life during these years. Finally, I will dedicate this dissertation to my families for their love and support.

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Chapter 1

Introduction

The continuing growing of techniques in mobile ad hoc network (MANETs) have led to many available applications in such as commercial, hospitals, military, search and rescue teams, education, etc. In MANETs, all transmissions are carried on wireless links without any wired connection, which enhances the conventional deployment of communicating environments. However, unlike a wired network, mobile devices are usually powered by limited energy supplies, where a continuing replacing or recharging could be hardly attainable. Hence, a substantial body of research has been devoted to improve the energy efficiency [34].

Due to the severe path loss in wireless links, the power required to transmit from one end to another will be exponentially grown by their distance. Thus, instead of a single long-distance transmission, relaying message through multiple hops with shorter distances usually consume less energy [24]. During the relaying process, each participating node has to consume energy to transmit or/and receive messages. Thus the total power required for a communication will be crucially influenced by the choice of relaying path. This motivates the recent research efforts on designing the energy-efficient communication protocols [35].

To compute the energy-efficient route, the global view of the network topology is required. However, the information is typically invisible to an individual node in wireless environments. Thus, if without addition information, such as the position of destination, enormous control packets have to be flooded all over the entire network to find out the route. The incurred overhead will quickly drain out node's energy.

In order to achieve the energy-efficient routing with less overhead, one promising way is by controlling the topology. Generally speaking, the basic idea is to keep the underlying topology as sparse as possible, while still preserve the energy-efficient route that consume less power for communications. A sparser topology can significantly mitigate the excessive flow flooded by nodes.

To reduce the communication overhead, one promising way is to control the underlying topology as sparse as possible to avoid excessive messages, while still preserve the energy-efficient route for any nodes pair. This is the so called *energy-efficient communication topology control problem*. The topology control problem in wireless ad hoc networks has been widely studied in recent years [3, 15, 18, 19, 20, 23, 29, 32]. Generally speaking, the core of this problem is to determine set of wireless links such that the composed topology is able to achieve certain goals [23]. These goals would be variant depending upon the circumstances and could be either qualitative features or quantitative objectives.

In general, the current effort on mobile topology control is mainly focused on reducing the transmission power required for each node to maintain the network connectivity. This objective is most appealing when the energy consumption of an individual node is crucial. However, to support an energy-efficient communication, the quality of routes preserved in the underlying topology is also important. Overall, the two goals are equally important in regard to deign an energy-efficient topology control: The former avoids exhausting individual node that in turn causes network partition and the later declines the per-packet energy consumption.

However, there is usually a tradeoff between the two desires: In order to constitute an energy-efficient route, a node may connect itself with a neighbor that is farther than the least requirement for connectivity. Contrarily, lowering down a node' transmission power may instead increase the total relaying power. See the example in Figure 1.1 (a), the communicate power between *u* and *v* is 5, while the least power of *u* to achieve connectivity is 3. In contrast, in Figure 1.1 (b), *u*'s transmission power is minimal, while the total relaying power (4 + 4 = 8) is now worse.

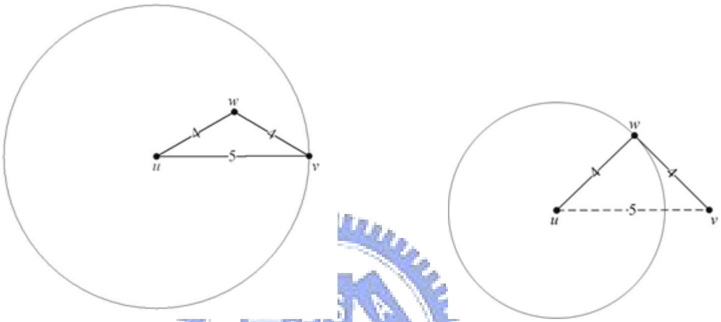

Figure 1.1: (a) Preserving energy-efficient route; (b) Reducing transmission power.

In this dissertation, our ultimate goal is to design an *adaptive topology control protocol* for mobile nodes. In this protocol, each node can adaptively change its way to contribute to the overall energy efficiency: if a node has sufficient energy, it will aggressively participate in supporting the energy-efficient communication; and when the deposited energy downs to a relatively low level, the node will turn to conserve its own energy.

The main idea is based on a geometric structure, named the *r-neighborhood graph*. The structure consists of several theoretic properties that can be exploited for designing the mobile topology control with the adaptive goal. Most importantly, based on such structure, each node can decide its neighbors in a *fully distributive and localized* way. We will also extend the structure to several generalized versions. These extensions enable an elegant self-configuration process on each node

On the other hand, to keep the design clean and compatible with the IEEE 802.11 DCF, we let each node periodically announce its current position using beacon message. However, such maintenance power could be considerable, especially when the broadcasting range is large. For this reason, we incorporate the protocol with a *shrinking power mechanism*. It can reduce the topology maintenance power significantly.

Furthermore, our protocol can simultaneously achieve the following desirable features without additional control message.

1. *Symmetric*: A topology is symmetric if the presence of an edge *uv* implies that its inverse *vu* exists. If without the symmetricity, the implementations of many network primitives, such as ACK in link-layer, will be much complicated [21]. Our protocol ensures this property for any resulted topology.

2. *Connected*: Connectivity is unquestionably the most essential prerequisite in any communicable topology [23]. Two nodes *u* and *v* are strongly connected if there is a directed path from *u* to *v* and vice versa. A directed topology is strongly connected if all pairs of nodes are strongly connected. If the links are symmetric, we should aim at the *connectivity* of an undirected topology.

3. *Sparse*: Numerous distributed and localized routing protocols are based on flooding [13]; however this may burden networks with unavoidable redundant messages. Thus, keeping a *sparse* topology, consisting of linear number of links [15], would be an ingenious way to shrink the expenditure from network operations.

4. *Bounded Maximum node degree*: For some nodes with overly-large degrees, the network flows will concentrate on them and rapidly draw out their energy. Besides, a larger node degree means tighter dependency among nodes, which is not expected when wireless nodes move frequently. Therefore, the maximum node degree over a topology should be bounded from above by some constant.

5. *Planar*: A graph is *planar* if it has no crossed links inside. It is helpful for many geometric problems: The shortest path (least energy unicast route) can be quickly found in linear time when the underlying topology is planar [12]; Besides, in many position-based routing algorithms, the successful delivery can be guaranteed only if the underlying topology is a planar [2, 11].

In addition, in wireless ad hoc networks, due to the absence of a central arbitrator and the limited sensing range, a centralized approach for controlling the topology is rarely attainable [3, 30]. Therefore, a variety of distributed approaches were proposed [17, 19, 29]. A distributed protocol passes messages hop-by-hop. This however may cause considerable overhead through the entire network. So, a localized approach is more preferred. According to the definition given by Stojmenovic and Lin [27], a localized topology control approach allows each node to determine its neighbors using only constant hop information. However, in some localized approaches [15, 16, 18, 27], the operations should *recursively* depend upon the computed status or partial results from nearby nodes, which may hurt their practicability. Therefore, in the following we define a new type of mythology for more practicability.

DEFINITION 1.1: An algorithm *L* is *purely localized* if it is localized and all operations depend upon only the information *inherent*[1] in nodes, available before any execution of *L*.

A purely localized topology control algorithm is more useful to large scale and high mobility environments, since the operation of a node is completely isolated from any

[1]The node's position and *id* are usually assumed to be inherited in nodes. See Chapter 2 for more explanation.

execution of other nodes. Further, we say that a structure is *purely localizable* if we can construct it by a purely localized algorithm. One of our goals is to investigate a purely localizable structure so that all desired features mentioned above can achieve.

The rest of this dissertation is organized as follows. Chapter 2 specifies the network model and formally describes the problem under study. In Chapter 3, we review and summarize the related works. The main geometric structures, components, and their theoretical results are presented in Chapter 4. In Chapter 5, we suggest a localized algorithm for stationery nodes and a shrinking power mechanism to constitute the skeleton of the desired protocol. In Chapter 6, we present the theoretic definition, properties, and algorithms of the self-configured process in mobile environment. Extensive simulation and numeric studies are conduced in Chapter 7. Finally, concluding remarks and some worth directions for the further research are given in the last Chapter. Detailed derivations are given in Appendix.

Chapter 2

Background and Related Works

In this chapter, the network model studied in this dissertation will be formally described. Then, we will review some related works in the literature. According to the assumption of node mobility, existing works for stationary and mobile nodes will be discussed, respectively, in Chapter 2.2 and Chapter 2.3.

2.1 Network Model

The wireless ad hoc network concerned in this paper consists of a set V of n wireless nodes distributed on a deployment region $\aleph$, which is a subset of the two-dimension plane $\Re^2$. We assume that each node is equipped with an omnidirectional antenna and can change its transmission range by adjusting the transmitting power at any level. The maximum transmission ranges are equal among all nodes. In other words, we can normalize the maximum transmission ranges of all nodes to be 1 for simplicity. In addition, each node u can obtain its location $Loc(u)$ through a lower-power GPS or some other ways [14], and an unique $id(u)$ is also available to each node u.

This network can be modeled as a *unit disk graph*, $UDG(V)$. In this graph, an edge uv exists if and only if the Euclidean distance between u and v, denoted as $||uv||$, is at most 1.

The least power required to transmit immediately between u and v is modeled as $p(u, v) = \|uv\|^{\alpha}$, where α is typically taken on a value between 2 and 4, depending on the attenuation strength of the communication environment [5]. To measure the power efficiency of a topology, Li *et al*. [15] defined a well-formed measure, named *power stretch factor*. We reintroduce it as below.

Let $\pi(u, v) = v_0 v_1 \ldots v_{h-1} v_h$ be a unicast path connecting nodes u and v, where $v_0 = u$ and $v_h = v$. The *total transmission power* consumed by path $\pi(u, v)$ is defined as

$$p(\pi(u,v)) = \sum_{i=1}^{h} p(v_{i-1}, v_i).$$

Let $\pi^*_{G(V)}(u,v)$ be the *least-energy path* connecting u and v in graph $G(V)$. Given a controlled topology $S(V)$ of $UDG(V)$, tthe *power stretch factor* of $S(V)$ with respect to $UDG(V)$ is defined as,

$$\rho(S(V)) = \max_{u,v \in V} \frac{p\left(\pi^*_{S(V)}(u,v)\right)}{p\left(\pi^*_{UDG(V)}(u,v)\right)}.$$

This factor indicates the worst ratio of the least energy required to relay on $S(V)$ in compared to that of a uncontrolled topology for all possible communication pairs. Clearly, a smaller ratio is preferable. On the other hand, the maximum node degree of the topology $S(V)$ is defined as

$$d_{\max}(S(V)) = \max_{u \in V} d_u(G(V)),$$

where $d_u(S(V))$ is the degree of node u in $S(V)$.

In addition, the following symbols will be used throughout this article.

- $D(u, d)$: the closed disk centered at $Loc(u)$ with radius d.
- $C(u, d)$: the circled centered at $Loc(u)$ with radius d.
- $N_u(G(V))$: the set of neighbor of u in a graph $G(V)$.

2.2 Stationery Topology Control

In the field of topology control for stationary nodes, a majority of researches were conducted by designing the *proximate graph*. A proximate graph is a geometric structure in which each node determines its neighbors based on the positions of nodes in its province. In other words, a topology approach based on such structure can be carried out in a fully distributed and localized way. A number of instances can be found in the literature [15, 16, 18, 26]. These works are diverse in their sparseness and the energy efficiency of preserved routes. We discuss the most well-know structures below. Most of them or their extensions are purely localizable:

- The *constrained Relative Neighborhood Graph* [28], denoted by *RNG*(*V*), has an edge *uv* if and only if $\|uv\| \le 1$ and the intersection of two *open* disks[1] centered at *u*, *v* with radius $\|uv\|$ contains no node $w \in V$, see Figure 2.1 (a),
- The *constrained Gabriel Graph* [6], denoted by *GG*(*V*), has an edge *uv* if and only if $\|uv\| \le 1$ and the open disk using $\|uv\|$ as diameter contains no node $w \in V$, see Figure 2.1 (b).
- The *constrained Yao Graph* [33] with a parameter $k \ge 6$, denoted by $\overrightarrow{YG_k}(V)$ is constructed as follows. For each node *u*, define *k* equal cones by *k* equal-separated rays originated at *u*. At each cone, a directed edge *uv* exists, if $\|uv\| \le 1$ and the cone contains no vertex $w \in V$ such that $\|uw\| < \|uv\|$. Ties are broken arbitrarily. $YG_k(V)$ is denoted as the underlying undirected graph of $\overrightarrow{YG_k}(V)$, see Figure 2.1 (c).
- A *Delaunay Triangulation*, denoted by *Del*(*V*), is a triangulation of *V* in which the interior of the circumcircle of each Δuvw contains no node $w \in V$. The *unit*

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[1] An open disk centered at point *x* with radius *d* is the collection of points with distance *less* than *d* from *Loc*(*x*).

Delaunay Triangulation, denoted by *UDel*(*V*), has all edges of *Del*(*V*) except those longer than 1 [8, 18], see Figure 2.1 (d).

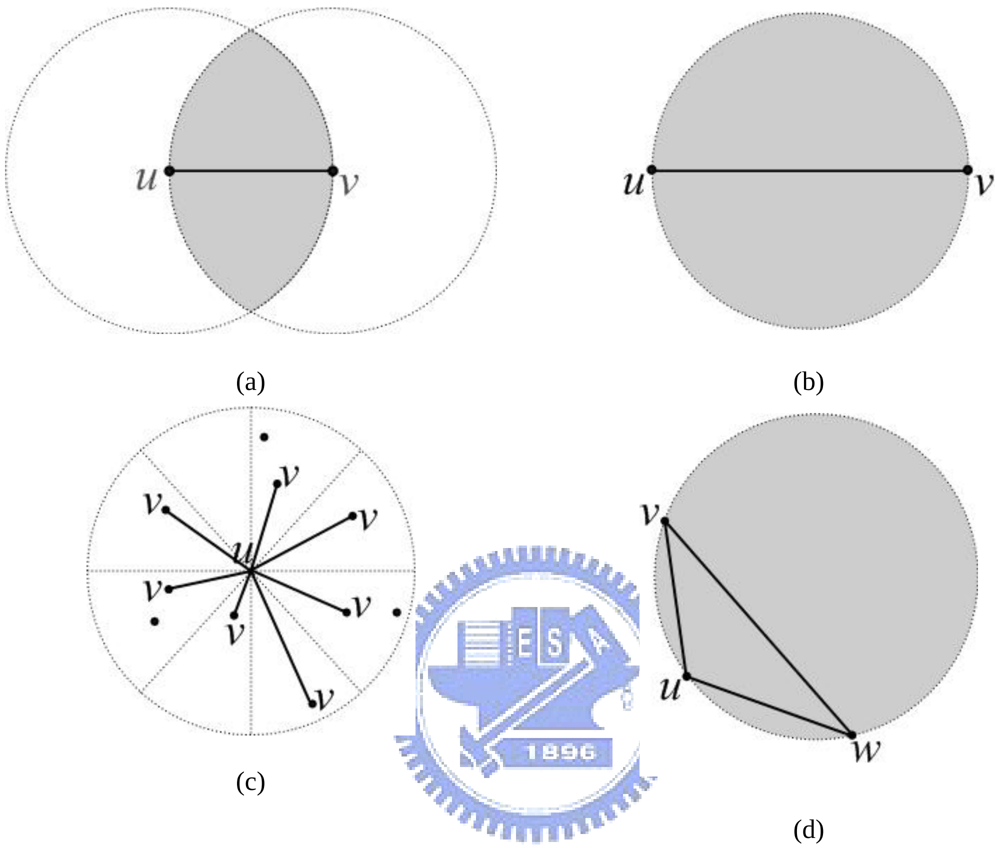


Figure 2.1: (a) *RNG*(*V*) (b) *GG*(*V*) (c) $YG_k(V)$, $k = 8$ (d) *UDel*(*V*).

Let us discuss the properties of these structures and their extensions. We say a objective *f*(.) of a structure *S*(*V*) is *bounded* if there is a constant *C* such that $f(S(V)) \leq C$, for any set *V* of *n* nodes. Li *et al*. [15] showed that $d_{\max}(RNG(V))$ is unbounded if there is a node $u \in V$ having an unbounded number of neighbors adjacent to *u* at exactly the same distance in the underlying *UDG*(*V*). To overcome this problem, Wattenhofer and Zollinger [32] proposed an algorithm to find a structure, denoted by *XTC*(*V*). They showed that that *XTC*(*V*) is a subgraph of *RNG*(*V*) and the $d_{\max}(XTC(V))$ is at most 6. Especially, if there is no node having two or more neighbors at exactly the same distance in V, $XTC(V)$ is identical to $RNG(V)$ [24]. Their results infer the following theorem.

THEOREM 2.1: Given a set V of nodes on $\Re^2$, if there is no node having two or more neighbors at exactly the same distance, then $d_{\max}(RNG(V)) \leq 6$.

We denote the condition in Theorem 2.1 as assumption *AS*. That is,

AS : There is no node in V having two or more neighbors at exactly the same distance.

This theorem reveals that even $RNG(V)$ has no constant bound on its node degree, it is still useful since the distances of nodes in real world are rarely exactly the same. The constrained Gabriel Graph $GG(V)$ has the least power stretch factor 1, in comparison with the unbounded power stretch factor $n - 1$ of $RNG(V)$ [15]. However, $d_{\max}(GG(V))$ could be as large as $n - 1$. An extended structure, *Enclosure graph* [16, 14, 24], denoted by $EG(V)$ is generalized from $GG(V)$. It can always result in a subgraph of $GG(V)$ [16]. Even so, its maximum node degree is still unbounded [20, 24].

To overcome the tradeoff between the maximum node degree and the power stretch factor, an adjustable structure, having the flexibility to be adjusted between the two objectives, becomes more attractive. $\overrightarrow{YG}_k(V)$ is an adjustable structure. It can be adjusted through a parameter k such that for any given k, the maximum *out*-degree is at most k, and the power stretch factor is at most $1/\left(1-(2\sin\pi/k)^{\alpha}\right)$ [15]. We say an objective $f(.)$ of an adjustable structure $S_k(V)$ with parameter k is *partially bounded* if there is *at least one* k_0 such that $f(S_{k_0}(V))$ is *bounded*. According this definition, the maximum *out*-degree and power stretch factor of $\overrightarrow{YG}_k(V)$ are partially bounded since for some ranges of k, k and $1/\left(1-(2\sin\pi/k)^{\alpha}\right)$ are constants. However, the asymmetric edges of $\overrightarrow{YG}_k(V)$ may lead to large *in*-degrees even when k is very small [15]. So, $d_{\max}(YG_k(V))$ can be neither bounded nor partially bounded. To improve this, an extension of $\overrightarrow{YG}_k(V)$, named *Yao and Sink*, was proposed [15, 17, 29]. It can limit the maximum node degree in $(k+1)^2 - 1$ and result symmetric edges. Unfortunately, in this structure the neighbors of some node should be recursively determined by one another so that it can not be purely localizable. The unit Delaunay triangulation *UDel*(*V*) has bounded power stretch factor. However, neither *Del*(*V*) nor *UDel*(*V*) can be computed locally. So, Li *et al*. [18] suggested a localized version of the Delaunay graph, denoted by $LDel^{(h)}(V)$, where h means that each node uses at most k-hop information. The power stretch factor of $LDel^{(k)}(V)$ is bounded for all $k \geq 1$. Even so, its maximum node degree is not bounded for any h.

The relations among these structures were studied in several papers [7, 10, 16, 22, 24, 33]. We summarize them on Figure 2.2, where *EMST*(*V*) is the Euclidean minimum spanning tree of *UDG*(*V*). With these relations, their connectivity can planarity can be easily inferred.

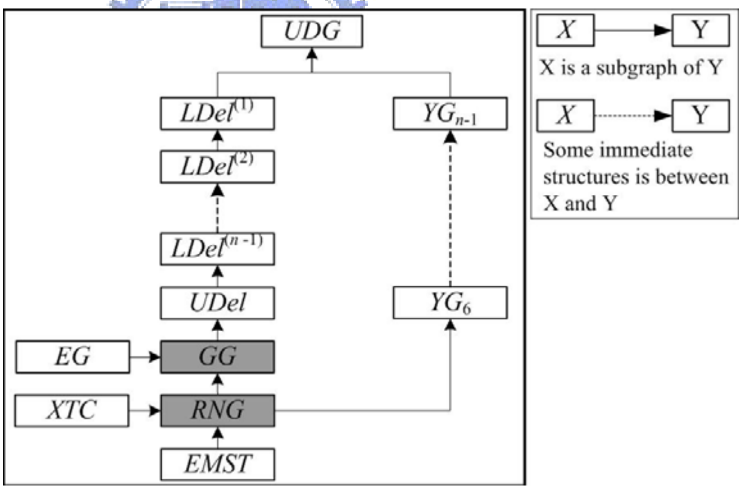


Figure 2.2: The relations of the pure localizable structures and their extensions.

Regarding the connectivity: we know that *EMST*(*V*) is connected if *UDG*(*V*) is itself a connected component of *V*. Therefore, when *UDG*(*V*) is connected, all graph containing *EMST*(*V*) are connected. That is, *RNG*(*V*), *GG*(*V*), *EG*(*V*), *UDel*(*V*),

$LDe^{(k)}l(V)$, $YG_k(V)$ are all connected. The connectivity of $XTC(X)$ was proven by different way [24].

Regarding the planarity: $LDel^{(k)}(V)$ is planar for any $k \geq 2$ [18]. Therefore, all subgraphs of $LDel^{(2)}(V)$ are planar. That is, $UDel(V)$, $GG(V)$, $EG(V)$, $RNG(V)$, $XTC(V)$, $EMST(V)$ are all planar. On the contrary, $\overrightarrow{YG}_k(V)$, and $LDe^{(1)}l(V)$ can not avoid producing the crossed link, so they are not planar [15, 18]. Table 2.1 summarizes above discussion.

From above table, we can see that no presented structure can bound or even partially bound the two objectives. Besides to the best of our knowledge, no other structure can be purely localizable and achieve this goal. Therefore, we will propose the first purely localizable structure, named *r-Neighborhood Graph*, to fill this gap. This structure is adjustable and can always result in a connected planar with symmetric edges. In addition, we can show that our structure is a generation of both $GG(V)$ and $RNG(V)$.

Table 2.1: The properties of the four main purely localizable structures.

	Power stretch factor	Maximum node degree	Planar	Symmetric	Connected
$RNG(V)$	Unbounded	Bounded (with *AS*) Unbounded (without *AS*)	Yes	Yes	Yes
$GG(V)$	Bounded	Unbounded	Yes	Yes	Yes
$YG_k(V)$	Partially bounded	Unbounded	No	No	Yes
$LDel^{(k)}(V)$	Partially bounded	Unbounded	No ($k=1$) Yes ($k \geq 2$)	Yes	Yes

Apart from the purely localizable structures, several composite methods, based on combining two or more existent structures, were investigated in the last few years [17, 19, 25, 31]. Conceptually, the main idea is to use the virtue of one structure to patch up the fault in the other structures. For examples, the *ordered Yao structure*, denoted as *OrdYao*(*V*) [1], is a variation of $YG_k^*(V)$. It has the partially bounded maximum node degree and length stretch factor. However, the planarity can not be guaranteed. Therefore, Wang and Li [19, 31] applied *OrdYao*(*V*) onto $LDel^{(2)}(V)$ to avoid the crossed edges produced by *OrdYao*(*V*); Song *et al*. [25] improves it by applying the *OrdYao*(*V*) on *GG*(*V*), using only one-hop information. Their Result are summary in Table 2.2. However, the construction of *OrdYao*(*Y*) requires exchanging the computed status as well as partial results between nodes. Consequently, none of them is purely localized or purely localizable.

Table 2.2: The properties of representative adjustable structures.

	Parameter	Power stretch factor	Maximum node degree
YG_k^*	$k = 6, \ldots, n\text{-}1$	$\frac{1}{1-2\sin\frac{\pi}{k}}$	$\frac{1}{1-\left(2\sin\frac{\pi}{k}\right)^{\alpha}}$
OrYaoGG	$k = 7, \ldots, n\text{-}1$	$\frac{1}{1-(2\sin\pi/k)^{\beta}}$	$k + 5$
SYaoGG	$k = 9, \ldots, n\text{-}1$	$\frac{\sqrt{2}^{\beta}}{1-(2\sqrt{2}\sin\pi/k)^{\beta}}$	k

2.3 Mobile Topology Control Protocols

Distributed protocols for proximate graphs can be also found in the literature [34]. However, existent results are all applied to stationary network only. There is no approach explicitly designed for mobile nodes based on such structure. The reason is probably that the construction depends on node positions, so that even a slight change in nodes placement could trigger a reconstruction process to handle the broken link or deteriorated link quality.

There are relatively fewer works considering nodes mobility. The LINT (and its extension LILT) is perhaps the first topology control protocol explicitly designed for mobile network [36, 37]. In this protocol, each node continually adjusts its transmission power such that the number of covered neighbors is within a lower and high threshold. Accordingly, the energy can be saved by declining the high threshold, and the network connectivity can be achieved by uplifting the low threshold. It however has no guarantee on connectivity if the low threshold is underestimated. To improve that, Blough *et al* [38] proposed a similar approach, named the K-NEIGH. The protocol connects each node with its k-closest neighbors and removes all asymmetric links, where k is a predefined parameter. The most interesting result is that if n nodes are uniformly distributed at random and k is taken as $\Theta(\log n)$, then the connectivity can be held with high probability. These protocols are called the *neighbor-based* approach, since a node's construction relays on the ability of ordering or measuring distances of nodes in its province [34]. The *direction-based* approach is another stem. It uses the angles among nodes for the construction. An example is the Cone Based Topology Control (CBTC) [39]. The basic idea is to let each node transmits with the minimum power that covers at least one neighbor in every cone of an angle ρ centered at it. The authors show that $\rho \leq 2\pi/3$ is a sufficient condition to ensure connectivity. Li *et al.* [40] proposed a reconfiguration procedure to deal with node mobility by detecting changing events from received beacons.

The most important features of these protocols are that their constructions are based on either nodes distance or nodes directions. Compared with the proximate graph, both the neighbor-based and direction-based approaches can be more accommodating to nodes movement. The reason is that the changing on nodes distances or directions will be relatively small with respect to nodes positions. Therefore, by using either of the two less precise information, a fewer number of topology reconstruction will be required when nodes move.

Even thought our protocols are based on a proximate graph. We will show that such disadvantage can be easily mitigated in an elegant way. In addition, Compared with K-NEIGH, LINT (LILT) and CBTC, our protocol guarantees the network connectivity in any stabilized status, without any assumption on nodes distribution, or parameter setting. Furthermore, both CBTC and K-NEIGH attend symmetricity by exchanging linking status among nodes. This will incur additional control overhead. Our protocol ensures that any established link is inherently bidirectional.

Chapter 3

Graphic Structures

In this chapter, we will introduce a new adjustable structure, called the *r-neighbor graph*. It can be adjusted between the maximum node degree and power stretch factor through the parameter *r*. The structure can also produce connected planar with symmetric edges. However, its maximum node degree will be unbounded in certain cases. To comprehend the theoretic property, we will then propose an enhanced version, called the *extended r-neighborhood graph* to deal with the special circumstance.

To apply the proposed structure to our mobile protocol, extensive investigations on the *r*-neighborhood graph will be given. First of all, we define a generalized structure, called the (*r*, α)-*neighborhood graph*. The generalization can gain better quantitative results. Next, an equivalent structure, called the (*r*, α)-*Enclosed graph* will be given. Its diverse representation enables the design of a shrinking power mechanism in Chapter 4. Then, we further generalized the structure such that each node having its own *r*, named the (f_r, α)-*neighborhood Graph*. This graph provides essential properties for the self-configuration process in Chapter 6.

3.1 *r*-Neighborhood Graph

In this section, we introduce the adjustable structure. First, we define a region on

$\Re^2$. It will be used to compose our structure.

DEFINITION 3.1: Given a nodes pair (u, v) on $\aleph$, the *r-neighborhood region* of (u, v), denoted as $NR_r(u, v)$, is defined as:

$$NR_r(u,v) = D(u,\|uv\|) \cap D(v,\|uv\|) \cap D(m_{uv}, l_{uv}),$$

where m_{uv} is the middle point on uv, $l_{uv} = (\|uv\|/2)(1 + 2r^2)^{1/2}$, and $0 \le r \le 1$.

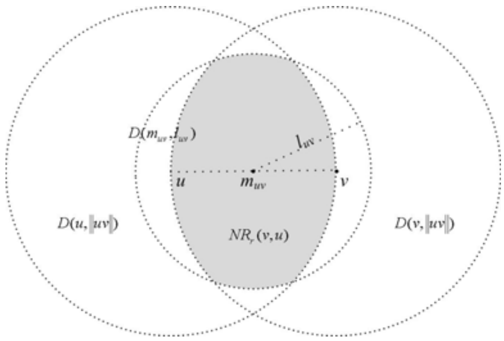


Figure 3.1: The r-neighborhood region of nodes u and v.

When no confused, we use m and l instead of m_{uv} and l_{uv} respectively. In Figure 3.1, the shaded region intersected by the three open disks sketches an example of the r-neighborhood region. This region is obviously equivalent to the following point set:

$$NR_r(u,v) = \{Loc(x) \in \Re^2 \mid \|ux\| < \|uv\|, \|vx\| < \|uv\|, \|mx\| < l\} \qquad (3.1)$$

For any node w located on $NR_r(u, v)$, this region limits the power consumed by path uwv. This property is shown in Lemma 3.1 and derived in Appendix.

LEMMA 3.1: Given two nodes u and v on $\aleph$, for any node w such that $Loc(w) \in NR_r(u, v)$, $p(uwv) < \|uv\|^{\alpha}(2 + r^{\alpha})$, for all $\alpha \ge 2$.

This lemma explains why we call such plane a *neighborhood region*: For any node w located in the region $NR_r(u, v)$, it should be an alternative neighbor for u with respect to v, in the sense that the power required for relaying from u to v through w is no greater than $1 + r^{\alpha}$ times of the immediate transmission. Based on this region, we structure is defined below.

DEFINITION 3.2: Given a set V of nodes on $\aleph$, the *r-neighborhood graph* of V, denoted as $NG_r(V)$, has of an edge uv if and only if $||uv|| \le 1$ and $NR_r(u, v)$ contains no node $w \in V$, where $0 \le r \le 1$.

By Definition 3.2, if edge uv is not in $UDG(V)$ or a node w is inside $NR_r(u, v)$, there is no direct link connecting u and v in $NG_r(V)$, which mean that all transmissions between u and v should be relied through some other node(s) in $NG_r(V)$. Now, we explore the desired properties in our structure. Before this, we shall discussion the following relations.

LEMMA 3.2: For any set V of nodes on $\aleph$, $RNG(V) \subseteq NG_r(V) \subseteq GG(V)$, for all $0\le r\le 1$.

Proof. Consider the open disk $D(m, ||uv||/2)$, defining $GG(V)$. Suppose $uv \in NG_r(V)$, the region $NR_r(u, v)$ has no node inside. Since $D(m, ||uv||/2)$ is obviously a subregion of $NR_r(u, v)$, for any $0 \le r \le 1$, there is also no node in $D(m, ||uv||/2)$. Therefore, according to the definition of $GG(V)$, we get $uv \in GG(V)$. On the other hand, consider the two open disks $D(u, ||uv||)$ and $D(v, ||uv||)$, defining $RNG(V)$. Suppose $uv \in RNG(V)$, no node is inside the intersection of $D(u, ||uv||)$ and $D(v, ||uv||)$, which obviously covers the region $NR_r(u, v)$, for any $0 \le r \le 1$. Therefore, no node can be inside $NR_r(u, v)$ and we get $uv \in NG_r(V)$. □

Specifically, as $r = 0$, $NR_0(u, v) \equiv D(m, ||uv||/2)$, which is the disk defining $GG(V)$. On the contrary, as $r = 1$, $NR_1(u, v) \equiv D(m, ||uv||)$, which is the disk defining $RNG(V)$. Therefore, $GG(V) \equiv NG_0(V)$ and $RNG(V) \equiv NG_1(V)$. So, we can conclude the following theorem.

THEOREM 3.1: The r-neighborhood graph is a generalized structure of both the restricted Gabriel graph and the restricted relative neighborhood graph.

Since a subgraph of a planar graph is always planar, and a supergraph of a connected graph is always connected, with the planarity of $GG(V)$ and connectivity of $RNG(V)$, we can infer the following two theorems.

THEOREM 3.2: For any set V of nodes on $\aleph$, $NG_r(V)$ is planar, for all $0 \le r \le 1$.

THEOREM 3.3: For any set V of nodes on $\aleph$, if the underlying $UDG(V)$ is connected, $NG_r(V)$ is *connected*, for all $0 \le r \le 1$.

Now we consider the energy efficiency and node degree of $NG_r(V)$. We will show that the upper bound of $\rho(NG_r(V))$ is increased by r and contrarily the upper bound of $d_{\max}(NG_r(\mathrm{V}))$ is decreased by r. In other words, the r-neighborhood graph is adjustable to the two objectives through the parameter r. With these results, we can further show that the power stretch factor and maximum node degree are partially bounded in our structure. Before these, a property proposed by Li *et al.*[15] shall be mentioned first. It can be used to simplify our proof.

LEMMA 3.3 [15]: Given a subgraph $G'(V) \subseteq UDG(V)$ and a constant C, $\rho(G'(V)) \le C$ if and only if for any edge uv in $G(V)$, there is a path $\pi(u, v)$ in $G'(V)$ such that $p_{G'(V)}(u,v) \le C\|uv\|^{\alpha}$.

This lemma indicates that to derive an upper bound for $\rho(NG_r(V))$, it is sufficient to the consider only those nodes pairs having direct links in $UDG(V)$. So, we aim to derive a strictly decreasing function $F(r)$, such that for any uv in $UDG(V)$, a path $\pi(u,v)$ is in $NG_r(V)$ such that $p(\pi(u,v)) \le F(r)\|uv\|^{\alpha}$. To achieve this, we investigate an algorithm **EXPANSION** with an input of any two nodes (u, v) and outputs subgraph S of $NG_r(V)$ related to (u, v). Let $P(S)$ be the total transmission power of edges in S. i.e. $P(S) = \sum_{st \in S} p(s, t)$. We can show that there is some path in S connecting (u, v) and $P(S) \le F(r)\|uv\|^{\alpha}$.

In this algorithm, S' is a set of nodes pairs, in which an edge st in $NG_r(V)$ can be a part of S only if its two ends (s, t) are in S' as described at step 3. So, to determine S, we have discuss the S' first. Initially, S' contains only (u, v). Then, it will be recursively expanded as follows: for each (s, t) in S', if a node w is in $NR_r(s, t)$ and not considered before, replace (s, t) with (s, w) and (w, t); if a node w is in $NR_r(s, t)$ but considered before, replace (s, t) with (s, w); Otherwise, keep (s, t) unchanged. We use the set Q to record the considered nodes.

ALGORITHM EXPANSION

Input: A nodes pair (u, v) in V

Output: A subgraph S and a positive value P.

Step 1: $S = \{\}$, $S' = \{(u, v)\}$, $Q = \{u, v\}$, $P = \|uv\|^{\alpha}$;

Step 2: When some node pair (s, t) is in S such that a node $w \in NR_r(s, t)$

  $S' = S' - (s, t)$;

  If $w \notin Q$ then

    $S' = S' \cup (s, w) \cup (w, t)$;

    $Q = Q \cup \{w\}$;

    $P = P + (\|st\|r)^{\alpha}$;

  Otherwise,

    $S' = S' \cup (s, w)$;

Step 3: $S = \{xy \in NG_r(V) \mid (x, y) \in S'\}$;

Step 4: Stop and output E and P.

When some (s, t) is in S' such that a node $w \in NR_r(s, t)$, no matter w is considered or not, by (4.1), the replaced nodes pair(s) must be shorter than $\|st\|$. i.e. $\|sw\| < \|st\|$ and $\|wt\| < \|st\|$. Thus after finite iterations, each node pair in S' can be replaced by another node pair with shortest distance. So, the algorithm is terminable. Now we show that (u, v) is connected by some path in the subgraph S when termination.

LEMMA 3.4: Given any set V of node on $\aleph$, for any two nodes u and v in V, if edge uv is in $UDG(V)$ and $UDG(V)$ is connected, there is some path in S connecting (u, v).

Proof: Since Q includes u and v, we can prove this lemma by showing that all nodes in the Q are connected in S. For each expansion of S', we define a dummy graph S'' in which an edge st exists if and only if (s, t) is in S' (Note that any edge in S'' is not necessarily in either $UDG(V)$ or $NG_r(V)$). First, we show that at any iteration, all considered nodes in Q are connected by S''. Initially, Q is connected by S'', since $S' = \{(u, v)\}$ and $Q = \{u, v\}$. We assume for induction that all nodes in Q are connected by S'' at k-th iteration. Then, we show that it is true for the next iteration. At k+1-th iteration, if there is no pair in S' satisfies the entrance condition of step 2, the claim is correct, since Q and S'' are unchanged; Otherwise, a node pair $(s, t) \in S'$ is expended. In this case, if the chosen $w \notin Q$, w is connected with all nodes in Q via dummy edges sw and wt; otherwise, $w \in Q$, which implies all nodes in Q are still connected by S'' as the previous iteration. As described above, the distance of any expended nodes pair is no longer than the previous one. So, if uv is in $UDG(V)$, all edges in S'' are also in $UDG(V)$. Then, as the algorithm processes to step 3, no nodes can be in the r-neighborhood region of any nodes pair in S'. With these two facts, all dummy edges in S'' are also in $NG_r(V)$ when termination. So S is equivalent to the last S''. Consequently, if $UDG(V)$ is connected, by Theorem 4.4, all nodes in the last Q are connected S. □

Then we derive a strictly decreasing function $F(r)$ using the value P in this algorithm.

LEMMA 3.4: Given any set V of n nodes on $\aleph$, for any two nodes u and v in V,

$$P(S) \le F(r)\|uv\|^{\alpha} \quad \text{and} \quad F(r) = 1 + (n-2)r^{\alpha}$$

for all $0 \le r \le 1$ and $\alpha \ge 2$.

Proof: Let $P(S') = \sum_{(s,t)\in S'} p(s, t)$. We show that $P(S') \le P$ at each iteration of step 2.

Initially, $S' = \{(u,v)\}$. We can get $P(S') = \|uv\|^{\alpha} = P$. Then at the first iteration, if no node w is in $NR_r(u, v)$, the claim remains true since neither P nor S is changed; Otherwise, a node w is in $NR_r(u, v)$. Besides, any chosen w can not be in Q, since no nodes except u and v are in Q so far. So, uv is replaced by vw and wv. By Lemma 4.1, $P(vw)+P(wu) \leq P(uv)(1+r^{\alpha}) = P + (\|uv\|r)^{\alpha}$. Consequently the new P remains a upper bound of $P(S')$. We assume for induction that $P(S') \leq P$ at k-th iteration. Then we prove the claim is true at the next iteration. If the entrance condition of step 2 is not satisfied or the chosen $w \notin Q$, it can be proved by the same reasons as in the first iteration. Otherwise, assume (s, t) is taken, st is replaced by only sw. By (4.1), $P(vw) \leq P(uv)$, which implies that the unchanged P is still an upper bound of $P(S')$. Besides, (4.1) further implies that all distance of two nodes in E are no greater than $\|uv\|$. So, another upper bound P' can be get by replacing $P = P + (\|st\|r)^{\alpha}$ by $P' = P' + (\|uv\|r)^{\alpha}$. Moreover, we can observe that the situation that as a w is chosen from some $NG_r(s, t)$ is not in Q never happens over $n - 2$ times, since in this case the size of Q must be increased 1. Consequently, $P(S') \leq P \leq P' \leq P(uv) + P(uv)r^{\alpha}(n-2)$. Finally, we get $F(r) = (1+r^{\alpha})(n-2)$. □

With lemmas 3.3, 3.4 and 3.5, we can conclude the following theorem.

THEOREM 3.4: For any set V of n nodes on $\Re$, for all $0 \leq r \leq 1$ and $\alpha \geq 2$,

$$\rho(NG_r(V)) \leq 1 + r^{\alpha}(n-2) = F(r).$$

Although this bound is related to the node size n so that $\rho(NG_r(V))$ can not be bounded, it can still be constant when r is 0 or some sufficiently small. i.e. $\rho(NG_r(V))$ is bounded in some range of r. So, we can make the following conclusion.

COROLLARY 3.1: The power stretch factor of the r-neighborhood graph is partially bounded.

Consider the maximum node degree of the r-neighborhood graph. Since $NG_r(V)$ consists of all edges in $RNG(V)$, the maximum node degree of $NG_r(V)$ is no less than that of *RNG*(*V*). In Chapter 2, we know that $d_{\max}(RNG(V))$ is not always bounded in any case of *V*. Thus, $d_{\max}(NG_r(V))$ is also unbounded. Fortunately, Theorem 2.1 indicates that $d_{\max}(RNG(V))$ is bounded in most cases of *V*, where *AS* is assumed. Therefore, in the following theorem, we analyze the maximum node degree of the *r*-neighborhood graph under assumption *AS*.

THEOREM 3.5: For any set *V* of nodes on $\aleph$ with assumption *AS*, for all $0 \le r \le 1$,

$$d_{\max}(NG_r(V)) \le \left| \pi / \sin^{-1}(r/2) \right|.$$

Proof. To prove this statement, it is sufficient to show that in $NG_r(V)$, there are no adjacent edges enclosing an angle less than $2\sin^{-1}(r/2)$. Assume for contradiction that two edges *uv* and *uw* in $NG_r(V)$ enclose an angle $\theta < 2\sin^{-1}(r/2)$ at node *u*, where *w*, $v \in V$. Without a loss of generality, we assume that $\|uw\| < \|uv\|$. With assumption *AS*, all nodes are placed on different positions. i.e. $Loc(x) \neq Loc(y)$, for any two nodes *x*, $y \in V$; Consider the length of *vw*: If $\angle uwv$ is obtuse, it is clear that $\|vw\| < \|uv\|$ (note that $\|vw\|$ can not be equal to $\|uv\|$, since $Loc(u) \neq Loc(w)$), see Figure 3.2 (b); Otherwise, if $\angle uwv$ is not obtuse, $\|vw\|$ is less $\|vw'\|$, where $\|uw'\| = \|uv\|$, see Figure 3.2 (a). By the law of cosine, we have

$$\begin{aligned} \|vw'\|^2 &= \|uw'\|^2 + \|uv\|^2 - 2\|uw'\|\|uv\|\cos\theta \\ &= 2\|uv\|^2 - 2\|uv\|^2\cos\theta \\ &< 2\|uv\|^2 - 2\|uv\|^2\cos\left(2\sin^{-1}(r/2)\right) \end{aligned} \qquad (3.2)$$

Let $\theta' = 2\sin^{-1}(r/2)$, we get $\sin(\theta'/2) = r/2$. Then one of the corresponding right-angled triangles is as shown in Figure 4.2 (c). In this case, $\cos\theta' = (2 - r^2)/2$. Thus we can get that $2\sin^{-1}(r/2) = \theta' = \cos^{-}\left((2 - r^2)/2\right)$. Consequently,

$$\begin{aligned} (3.2) &= 2\|uv\|^2 - 2\|uv\|^2\cos\left(\cos^{-}\left(\left(2 - r^2\right)/2\right)\right) \\ &= 2\|uv\|^2 - 2\|uv\|^2\left(\left(2 - r^2\right)/2\right) = \|uv\|^2 r^2 \end{aligned} \qquad (3.3)$$

Consequently, we have that for any case of $\angle uwv$,

$$\|vw\| < \max\{\|uv\|r, \|uv\|\} = \|uv\| \tag{3.4}$$

Consider the length of *um*: if ∠*uwm* is obtuse, ||*wm*|| < ||*uv*||/2 see Figure 3.2 (b); Otherwise, ||*mw*|| is less ||*mw*’||, see Figure 3.2 (b). By the law of cosine, we have

$$\|mw'\|^2 = \|uw'\|^2 + \|um'\|^2 - 2\|uw'\|\|um'\|\cos\theta .$$

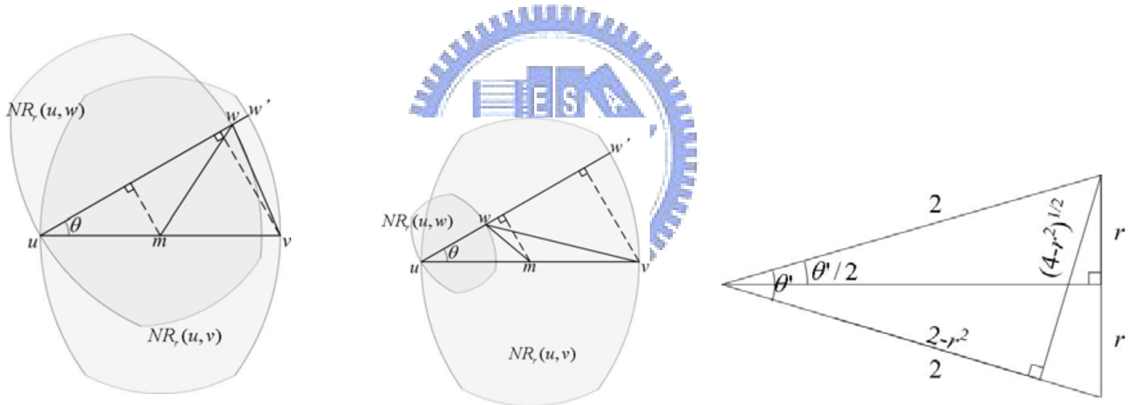
$$< \|uv\|^2 + \|uv\|^2/4 - \|uv\|^2\cos\theta$$

$$< 5\|uv\|^2/4 - \|uv\|^2\left(\left(2-r^2\right)/2\right) = \|uv\|^2\left(\left(1+2r^2\right)/4\right) \tag{3.6}$$

Similarly, we have for any case of ∠*uwm*

$$\|mw\| < \max\left\{\|uv\|\sqrt{1+2r^2}/2, \|uv\|/2\right\} = l . \tag{3.6}$$

By (3.4), (3.6) and the assumption of ||*uw*|| < ||*uv*||, *w* is included in the set of points specified in (3.1). Therefore, $P(w) \in NR_r(u, v)$. It however contradicts the assumption that *uv* is in $NG_r(V)$. Thus we conclude this theorem. □



(a) (b) (c)

Figure 3.2: (a) ∠*uwv* and ∠*uwm* are not obtuse; (b) ∠*uwv* and ∠*uwm* are obtuse; (c) a right-angled triangle with angle $\theta = 2\sin^{-}(r/2)$.

However, for those instances of *V* without *AS*, Theorem 3.5 can not hold anymore. See the instance in Figure 3.3, all nodes except v_i are placed on the outlier of $NR_r(v_i, v_1)$. This will result *n* – 1 neighbors adjacent to v_i in $NG_r(V)$. So, in the next section, we propose an extended version the *r*-neighborhood graph. As the readers will see, the extended structure has the partially bounded maximum node degree for all cases of V and inherits almost all desired features in $NG_r(V)$.

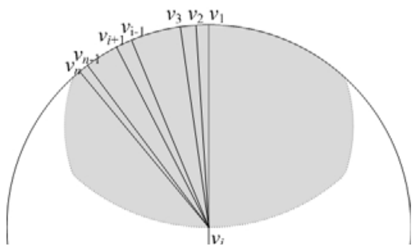


Figure 3.3: $d_{\max}(NG_r(V))$ is not bounded if assumption AS does not hold.

3.2 Extended *r*-Neighborhood Graph

In this section, an extended structure of the r-neighborhood graph is given. The main goal is to avoid the unbounded maximum node degree in $NG_r(V)$. In this extension, assumption AS is not required anymore. Instead, a unique identifier $id(u)$ is available to each node u in V. The structure is defined as follows.

DEFINITION 3.3: Given a set V of nodes $\aleph$, the *extended r-neighborhood graph* of V, denoted as $NG_r^*(V)$, has an edge uv if and only if $\|uv\| \leq 1$ and there exists no node $w \in V$ satisfying one of the following three conditions:

D_1: $Loc(w) \in NR_r(u,v)$;

D_2: $Loc(w) \in D(m_{uv}, l_{uv}) \cap C(v, \|uv\|)$ and $id(u) > id(w)$;

D_3: $Loc(w) \in D(m_{uv}, l_{uv}) \cap C(u, \|uv\|)$ and $id(v) > id(w)$.

Without D_2 and D_3, $NG_r^*(V)$ is clearly equivalent to the original r-neighborhood graph. In conditions D_2 and D_3, the two sub-regions of $D(m_{uv}, l_{uv})$ intersected by $C(v, \|uv\|)$ and $C(u, \|uv\|)$ are, as depicted in Figure 3.4, the solid left arc and right arc along the outlier of $NR_r(u, v)$, respectively. When a node w is located in these two arcs, the existence of edge uv should be further determined by their identifiers.

Hereafter, we say that a node $w \in V$ *blocks* an edge *uv* in *UDG*(*V*) if and only if *w* satisfies one of the three conditions in Definition 3.3.

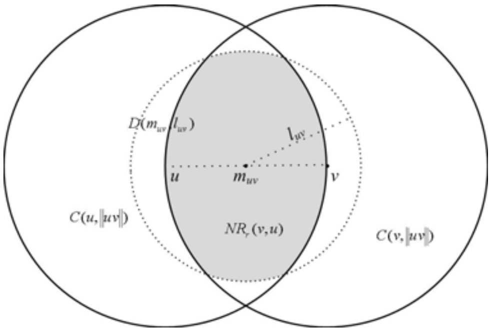


Figure 3.4: The *r*-neighbor region of nodes *u* and *v*, and the two intersections defined in D_2 and D_3.

In $NG_r^*(V)$, an edge *uv* of *UDG*(*V*) will not only be blocked by some node *w* in $NR_r(u,v)$, but may also be blocked when either D_2 or D_3 happens. Therefore, $NG_r^*(V)$ constitutes a subgraph of $NG_r(V)$, which means that the maximum node degree of $NG_r^*(V)$ is no worse than its original version. In the following theorem, we show that the upper bound of $d_{\max}(NG_r(V))$ in Theorem 3.5 remains correct in $d_{\max}(NG_r^*(V))$, and the correctness is for any case of *V*, not subject to assumption *AS*.

THEOREM 3.6: For any set *V* of nodes on ℵ, for all $0 \le r \le 1$,

$$d_{\max}(NG_r^*(V)) \le \left\lceil \frac{\pi}{\sin^{-1}(r/2)} \right\rceil.$$

Proof. Using the same argument as Theorem 3.5, we assume for contradiction that two edges *uv* and *uw* in $NG_r^*(V)$ enclose an angle $\theta' < 2\sin^{-1}(r/2)$ at node *u*. Without loss of generality, we assume that $\|uw\| \le \|uv\|$. If $\|uw\| < \|uv\|$, the argument of Theorem 3.5 has proved the contradiction. Consider $\|uw\| = \|uv\|$: Let w' be a point crossed by $C(u, \|uv\|)$ and the outlier of $D(m_{uv}, l_{uv})$, as shown in Figure 3.5. The two edges $w'u$ and *uv* enclose an angle θ'. By the law of cosine, we have

$$\cos\theta' = \frac{\|uw'\|^2 + (\|uv\|/2)^2 - \|m_{uv}w'\|^2}{\|uw'\|\|uv\|} = \frac{\|uv\|^2 + (\|uv\|/2)^2 - l_{uv}^{2\;2}}{\|uv\|\|uv\|} = 1 + r^2/2$$

Then one corresponding right-angle triangulation is as Figure 3.2 (c). In this case, $\sin(\theta'/2) = r/2$. Thus, we can get that $\theta < \theta' = 2\sin^{-1}(r/2)$. Since $\|uw\| = \|uv\|$, both *Loc*(*w*) and *Loc*(*v*) are on *C*(*u*, ||*uv*||). The fact that $\theta < \theta'$ further limits *Loc*(*w*) on the arc intersected by $D(m_{uv}, l_{uv})$. Similarly, *Loc*(*v*) is limited on the arc intersected by $D(m_{uw}, l_{uw})$ for the same reason. Therefore, *Loc*(*w*) and *Loc*(*v*) are on the regions defined in D_2, with respect to edges *uw* and *uv*, respectively.

Next, the existence of *uv* and *uw* should be determined by their identifiers. If *id*(*v*) > *id*(*w*), *uv* is blocked by *w*. Otherwise, if *id*(*v*) < *id*(*w*), *uw* is blocked by *v*. As a sequel, no matter what the values of *id*(*v*) and *id*(*w*) are, at least one of the edges enclosing *θ'* can not be in $NG_r^*(V)$. Thus we proved this theorem. □

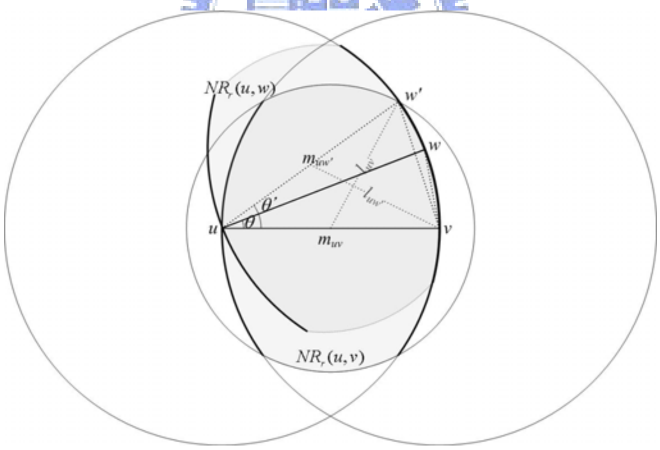


Figure 3.5: If $\theta < 2\sin^{-1}(r/2)$ and ||*uw*|| = ||*uv*||, either *uw* or *uv* can not be in $NG_r^*(V)$

From Theorem 3.6, we can see that $d_{\max}(NG_r^*(V))$ is constant when *r* is sufficiently large. Therefore, there has some setting of *r* such that $d_{\max}(NG_r^*(V))$ is bounded by some constant, for any set *V* of *n* nodes. So, we reach the following conclusion.

COROLLARY 3.2: The maximum node degree of the extended *r*-neighborhood graph is partially bounded.

In the rest part, we show that $NG_r^*(V)$ inherits all desired properties achieved by $NG_r(V)$, except the generality for *RNG*(*V*). The fact that $NG_r^*(V) \subseteq NG_r(V)$ confirms the planarity of $NG_r^*(V)$, since $NG_r(V)$ is planar for any *r*. Moreover, when $r = 0$, the two arcs defined in D_2 and D_3 are empty. Thus whether an edge is in $NG_r^*(V)$ is solely depending on D_1, which means that $NG_0^*(V) \equiv NG_0(V) \equiv GG(V)$. Therefore, $NG_r^*(V)$ remains a general structure of *GG*(*V*).

However, as shown Theorem 3.6, some adjacent edges having the same length in *RNG*(*V*) would be avoided in $NG_r^*(V)$. Thus *RNG*(*V*) is not always a subgraph of $NG_r^*(V)$. This means that $NG_1^*(V)$ is not essentially equivalent to *RNG*(*V*). Even more, $NG_1^*(V)$ could be a subgraph of *RNG*(*V*). Therefore, $NG_r^*(V)$ is no longer a general structure of *RNG*(*V*).

About the connectivity, because *RNG*(*V*) is not always a subgraph of $NG_r^*(V)$, we cannot ensure the connectivity of $NG_r^*(V)$ directly from that of *RNG*(*V*). Therefore, we apply an entirely different logic to prove this property. The idea is based on comparing the lexicography orders of nodes pairs. This idea has been successfully used to prove the connectivity of *XTC*(*V*) [32], another subgraph of *RNG*(*V*).

We define a three-field tuple ($\|uv\|$, $id(u)$, $id(v)$) for each nodes pair (u, v). The lexicographic order of (u, v) is smaller than that of another nodes pair (s, t) if one of the following three cases happens: 1) $\|uv\| < \|st\|$; 2) $\|uv\| = \|st\|$ and $id(u) < id(s)$; 3) $\|uv\| = \|st\|$, $id(u) = \mathrm{id}(s)$ and $id(v) < id(t)$. Now, we prove the connectivity of $NG_r^*(V)$ in Theorem 3.7.

THEOREM 3.7: For any set *V* of nodes on $\aleph$, if the underlying *UDG*(*V*) is connected,

$NG_r^*(V)$ is *connected*, for all $0 \le r \le 1$.

Proof. Suppose $UDG(V)$ is connected. Let $U(V)$ be the set of unconnected nodes pairs in $NG_r^*(V)$. We assume for contradiction that some nodes pairs in $NG_r^*(V)$ are not connected. i.e., $U(V)$ is not empty. Let (u, v) be the node pair with smallest lexicographic order in $U(V)$.

Assume that edge uv is not in $UDG(V)$, i.e. $\|uv\| > 1$. Since $UDG(V)$ is connected, there must be some path longer than one hop connecting u and v. Let $\pi(u, v)$ be such path in $UDG(V)$. Since $\|uv\| > 1$, the lengths of each edge on $\pi(u, v)$ is less than $\|uv\|$. When this path is mapped to $NG_r^*(V)$, there is some nodes pairs on $\pi(u, v)$ unconnected in $NG_r^*(V)$. Thus some unconnected node pair on $\pi(u, v)$ has length shorter than $\|uv\|$, which however contradicts that (u, v) has the smallest lexicographic order in $U(V)$. Therefore, edge uv must be in $UDG(V)$.

Since edge uv is in $UDG(V)$ and not in $NG_r^*(V)$, there must be some node w satisfying one of the three conditions in Definition 3.3. Besides, either (u, w) or (w, v) is in $U(V)$, otherwise (u, v) can be connected by path uwv. We consider the three cases:

1) If D_1 happens, $Loc(w) \in NR_r(u, v)$. So, we has $\|uw\| < \|uv\|$ and $\|wv\| < \|uv\|$, which means that the lexicographic orders of (u, w) and (w, v) are less than that of (u, v).
2) If D_2 happens, we have $\|wv\| = \|uv\|$ and $id(u) > id(w)$, which means that the lexicographic order of (w, v) is less than that of (u, v);
3) If D_3 happens, we have $\|uw\| = \|uv\|$ and $id(v) > id(w)$, which means that the lexicographic order of (u, w) is less than that of (u, v).

Therefore, we cannot find any nodes pair in $U(V)$ having the smallest lexicographic order. In other words, $U(V)$ is empty, which however is a contradiction. Thus, we proved this. □

Due to the fact that $NG_r^*(V) \subseteq NG_r(V)$, there may has some paths in $NG_r(V)$

not in $NG_r^*(V)$. Therefore, $\rho(NG_r^*(V))$ is no better or even worse than $\rho(NG_r(V))$. Even so, the upper bound of $\rho(NG_r^*(V))$ can be as good as that proved in Theorem 3.4; we briefly explain this: All arguments in Theorem 3.4 are not related to the two additional conditions D_2 and D_3, except those referred from Lemma 3.1. Whatever D_1, D_2 or D_3 happens, $||uw|| \le ||uv||$, $||vw|| = ||uv||$ and $||mv|| < l$, which means that all inequalities in the proof of Lemma 3.1 are unchanged. Consequently, Theorem 3.4 is still correct, even if all conditions of Definition 3.3 are considered. So, $\rho(NG_r^*(V))$ is also partially bounded.

Below, we show that the bound $1 + r^{\alpha}(n-2)$ in Theorem 3.4 is not only correct, but also *asymptotically tight* to the worst possible value of $\rho(NG_r^*(V))$. In other words, it is very hard to find another upper bound of $\rho(NG_r^*(V))$ better than ours. We apply the same argument as that used to verify the tightness of the length stretch factor [3] and the power stretch factor [15] of $RNG(V)$

THEOREM 3.8: For any $n \ge 2$ and $0 \le r \le 1$, there is a set V of n nodes such that

$$\sup_{|V|=n} \rho(NG_r^*(V)) > 1 + r^{\alpha}(n-2) - \varepsilon,$$

for any sufficient small $\varepsilon > 0$.

Proof. Let $\theta_1 = 2\sin^{-1}(r/2) - 2\lambda$ and $\theta_2 = \pi/2 - \sin^{-1}(r/2) + \lambda$, where $\lambda > 0$. We construct a set $V = \{v_1, v_2, \ldots, v_{2m-1}, v_{2m}, \ldots, v_n\}$ of n nodes, where $n \ge 2$ is even and $m = n/2$, as follows:

1) $\|v_1v_2\| \le 1$ and $\|v_iv_{i+1}\| = \|v_1v_2\|$, for $i = 2, 3, \ldots, 2m-1$;
2) $\angle v_iv_{i+1}v_{i+2} = \theta_1$, for $i = 1, 2, \ldots, 2m-2$;
3) $\angle v_{i+2}v_iv_{i+1} = \angle v_iv_{i+2}v_{i+1} = \theta_2$, for $i = 1, 2, \ldots, 2m-2$;
4) $id(v_i) = n - i + 1$, for $i = 1, 2, \ldots, n$;

One corresponding $UDG(V)$ is as shown in Figure 3.6. For $i = 1, 2, \ldots, 2m-2$, since $\angle v_iv_{i+1}v_{i+2} = \theta_1 < 2\sin^{-1}(r/2)$ and $\|v_iv_{i+1}\| = \|v_{i+1}v_{i+2}\|$, by the argument in Theorem 7, we get $P(v_{i+2}) \in D(v_i, v_{i+1}) \cap C(m_{v_iv_{i+1}}, l_{v_iv_{i+1}})$. That is, $P(v_{i+2})$ is in the regions with respect to edge v_iv_{i+1}, defined in D_2. Moreover, $id(v_i) > id(v_{i+2})$. Thus, edge v_iv_{i+1} is not in $NG_r^*(V)$. Then, the remaining edges are exactly a path (spanning tree) $v_1v_3v_5 \ldots v_{2m-3}v_{2m-1}\ v_{2m}v_{2m-2} \ldots v_6v_4v_2$ of V, connecting all nodes, as the bold links in Figure 3.6 (a). Therefore, we can get that

$$p\left(\pi^*_{NG_r^*(V)}(v_1, v_2)\right) = \sum_{i=1}^{2m-2} r^\alpha \|v_iv_{i+2}\|^\alpha + \|v_{2m-1}v_{2m}\|^\alpha$$

As $\lambda \to 0$, $\theta_1 \to 2\sin^{-1}(r/2)$, which implies that $\|v_iv_{i+2}\| \to r\|v_iv_{i+2}\| = r\|v_1v_2\|$, according to (3.3). Consequently, as $\lambda \to 0$, we get that

$$\sum_{i=1}^{2m-2} r^\alpha \|v_iv_{i+2}\|^\alpha + \|v_{2m-1}v_{2m}\|^\alpha$$

$$\to \sum_{i=2}^{2h-2} r^\alpha \|v_1v_2\|^\alpha + \|v_1v_2\|^\alpha$$

$$= \|v_1v_2\|^\alpha \left((n-2)r^\alpha + 1\right)$$

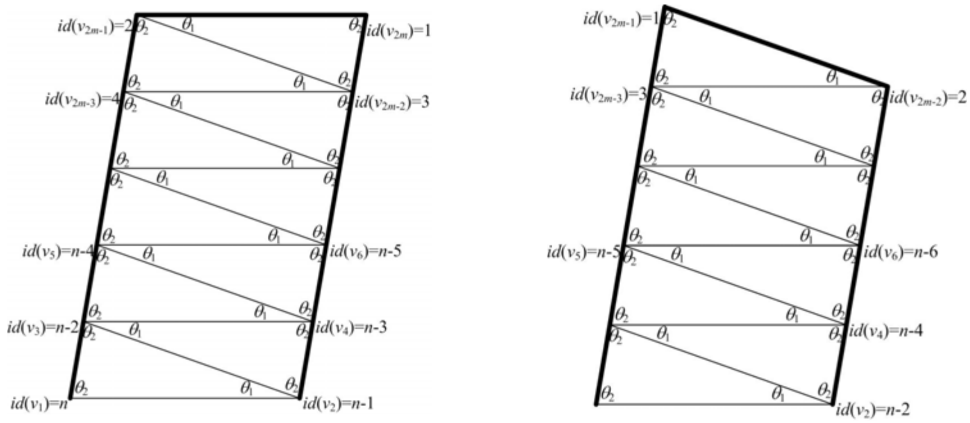
On the other than, since $\|v_1v_2\| \le 1$, we get $p\left(\pi^*_{UDG(V)}(u, v)\right) = \|uv\|^\alpha$. Therefore, as $\lambda \to 0$, $\rho_{NG_r(V)}(UDG(V)) \to 1 + r^\alpha(n-2)$. That is,

$$\sup_{|V|=n} \rho(NG_r^*(V)) > 1 + r^\alpha(n-2) - \varepsilon,$$

for any sufficient $\varepsilon > 0$. For any odd $n \ge 2$, the result can be obtained by applying the same argument to the instance as shown in Figure 3.6 (b). So, we proved this it. □

Actually, an equivalent structure of $NG_r^*(V)$, without a original version like $NG_r(V)$, was mentioned in our previous paper[2] [9]. In that preliminary work, however only qualitative results were given. To prove the quantitative results, we separate $NG_r(V)$ from $NG_r^*(V)$ in this paper, because $NG_r(V)$ has a clearer form in definition that can be used to highlight the main tricky in our derivations. Besides, all qualitative results in [9] are re-evaluated here using different arguments.

[2] The term "r-neighborhood graph" in [9], is not refereed to the original version in Definition 3, but the extended version in Definition 4. In this paper, we reuse the same term to name the original version and rename the previous structure in [9] the extended version



(a) (b)

Figure 4.6: A worst-case instance V of n nodes in $NG_r^*(V)$: (a) n is even; (b) n is odd.

3.3 (r, α)-Neighborhood Graph

In [7], we showed that for any point $x \in NR_r(u, v)$, $\|ux\|^\alpha + \|xv\|^\alpha < \|uv\|^\alpha (1 + r^\alpha)$. This result combined with Definition 3.2 indicates that if an edge uv of $UDG(V)$ is not in $NG_r(V)$, there must be a node w located in $NG_r(u, v)$, such that $p(u, w) + p(w, v) < p(u, v)(1 + r^\alpha)$. In other words, for any $uv \in UDG(V)$, if there is no node w such that $p(u, w) + p(w, v) < p(u, v)(1 + r^\alpha)$, then $uv \in NG_r(V)$. The upper bound of the power stretch factor in Theorem 3.4 is then an inductive consequence of this fact. This argument implies the following lemma,

LEMMA 3.6: For any graph $S(V)$, if it contains an edge uv, whenever there is no other node w such that $p(u, w) + p(w, v) < p(u, v)(1 + r^\alpha)$, then $\rho(S(V)) \leq 1 + r^\alpha(n - 2)$.

According to this lemma, a structure that has the same upper bound on the power stretch factor of $NG_r(V)$ is defined as follows.

DEFINITION 3.3: Given two nodes u and v on $\aleph$, a parameter r, $0 \leq r \leq 1$, and a constant $\alpha \geq 2$, the (r, α)-*neighborhood region*, is defined by

$$NR_r^\alpha(u,v) = \{x \in \aleph \mid \|ux\| < \|uv\|, \|vx\| < \|uv\|, p(u,x) + p(x,v) < p(u,v)(1 + r^\alpha)\}.$$

DEFINITION 3.5: Given a set V of nodes on $\aleph$, the (r, α)-*neighborhood graph*, denote as $NG_r^{\alpha}(V)$, has an edge uv if and only if $\|uv\| \leq 1$ and there is no other node w located in $NR_r^{\alpha}(u,v)$.

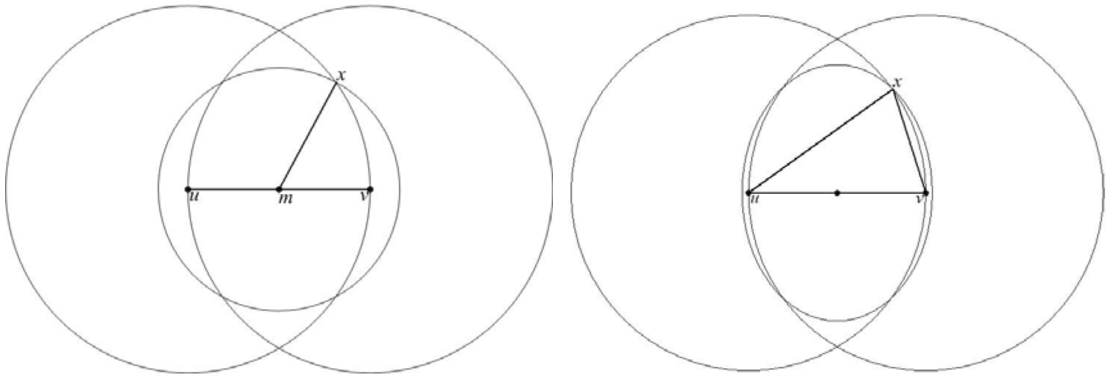


Figure 3.7: $NR_r(u, v)$ vs. $NR_r^{\alpha}(u, v)$.

Their relationships are as follows. We can see that $NG_r^{\alpha}(V)$ *is* actually a general structure of $NG_r(V)$. Notably, it achieves the same upper bound on the power stretch using equal or less edges.

PROPERTY 3.1: For any two nodes u and v on $\aleph$, $0 \leq r \leq 1$, and $\alpha \geq 2$

(i) $NG_r^{\alpha}(V) \subseteq NG_r(V)$;

(ii) $NG_r^{2}(V) \equiv NG_r(V)$.

Proof: Consider an edge $uv \in NG_r^{\alpha}(V)$. By definition, there is no node w such that $p(u,w) + p(w,v) < p(u,v)(1+r^{\alpha})$, which implies that $uv \in NG_r(V)$.

Consider an edge $uv \in NG_r(V)$. There is no nodes w such that $\|mw\| < (\|uv\| / 2)(1 + 2r^2)^{1/2}$. Consider a point x. From simple derivation, we get

$$\|ux\|^2 + \|vx\|^2 = \|uv\|^2 \backslash 2 + 2\|xm\|^2 .$$

Combining these two facts, $\|ux\|^2 + \|vx\|^2 < \|uv\|^2(1+r^{\alpha})$. So, $uv \in NG_r^{\alpha}(V)$. □

Furthermore, the properties below indicate that the number of links could be even lower as the environmental attenuation factor α is strengthened. Both properties can be easily verified by inspecting the last condition in $NR_r^{\alpha}(u,v)$. We omit the proofs.

PROPERTY 3.2: For any set of n nodes on $\aleph$,

(i) $NG_r^{\alpha_1}(V) \supseteq NG_r^{\alpha_2}(V)$, $2 \le \alpha_1 \le \alpha_2$;

(ii) $NG_{r_1}^{\alpha}(V) \supseteq NG_{r_2}^{\alpha}(V)$, $0 \le r_1 \le r_2 \le 1$;

3.4 (*r*, α)-Enclosed Graph

In this part, we present an equivalent structure of the (*r*, α)-neighborhood graph, called the (*r*, α)-Enclosed Graph. The idea is mainly borrowed from the *Enclosed Graph*, proposed by Rodoplu *et al.* [24]. We briefly reintroduce it below. For any two nodes *u* and *v*, there is space, named the *relaying region, RR*(*u, w*), in which any node fall in the region can be reached from *u* with less power by relaying through *v*. The *enclosed region*, $ER(u)$, is a subspace merged from of the complement of relaying regions of all surrounding *v*'s of *u*. The *enclosed graph*, *EG*(*V*), is a graph in which a node *v* is adjacent to *u* if and only if *v* is in the enclosed region of *u*. This graph has the optimal power stretch factor 1. Actually, as the receiving cost is neglected, it is an equivalent representation of *GG*(*V*).

The following, we generalize this idea to the (*r*, α)-neighborhood graph (recall that $NG_r^{\alpha}(V)$ is a generalized from NG_r which is further a general structure of *GG*(*V*)).

DEFINITION 3.6: Given two nodes *u* and *w* on $\aleph$, the (*r*, α)-*relaying region*, denoted as $RR_r^{\alpha}(u,w)$, is defined as

$$RR_r^{\alpha}(u,w) = \left\{x \in \Re^2 \mid \|uw\| < \|ux\|, \|wx\| < \|ux\|, p(u,w) + p(w,x) < p(u,x)(1+r^{\alpha})\right\}.$$

Consider three nodes *u*, *v* and *w*, according definitions 3.4 and 3.6, we can observe that no matter $v \in RR_r^{\alpha}(u,w)$ or $w \in NR_r^{\alpha}(u,v)$, the following three conditions are satisfied:

$$\|uw\| < \|uv\|,\ \ \|wv\| < \|uv\|, \text{ and } \ p(u,w) + p(w,v) < p(u,v)(1 + r^{\alpha}). \qquad (3.6)$$

In other words, for any three nodes *u*, *v* and *w*,

$$v \in RR_r^{\alpha}(u,w) \Leftrightarrow w \in NR_r^{\alpha}(u,v). \qquad (3.7)$$

The different is in their representation. $NR_r^{\alpha}(u,v)$ consists of all points where may have a *relaying node w* satisfying (3.6) for the two fixed nodes *u* and *v*, while $RR_r^{\alpha}(u,w)$ is composed by all points on which a node satisfies (3.7) if it receives relaying from *u* through *w*.

Now compare the two regions on the same node pair (*s*, *t*)[3]. If the two regions are overlapped, then there will be some point *x* such that the two facts

$$p(s,x) + p(x,t) < p(s,t)(1 + r^{\alpha}) \ \text{ and } \ p(s,t) + p(t,x) < p(s,x)(1 + r^{\alpha})$$

are satisfied at the same time, which is conflicted. Therefore, the two regions must be *disjoint*, i.e. (see Figure 3.8)

$$RR_r^{\alpha}(s,t) \cap NR_r^{\alpha}(s,t) = \phi \qquad (3.8)$$

Compared to different parameters setting, see Figure 3.8, we can get that

$$RR_r^{\alpha_1}(u,w) \subseteq RR_r^{\alpha_2}(u,w),\ \ 2 \le \alpha_1 \le \alpha_2; \qquad (3.9)$$

$$RR_{r_1}^{\alpha}(u,w) \subseteq RR_{r_2}^{\alpha}(u,w),\ \ 0 \le r_1 \le r_2 \le 1; \qquad (3.10)$$

These observations can be easily validated by inspecting their definitions. We omit the proof.

[3] to avoid the confusion if reusing *u*, *v*, *w*.

The region enclosed by the complements of the relaying regions of nodes surrounding to *u* is defined below.

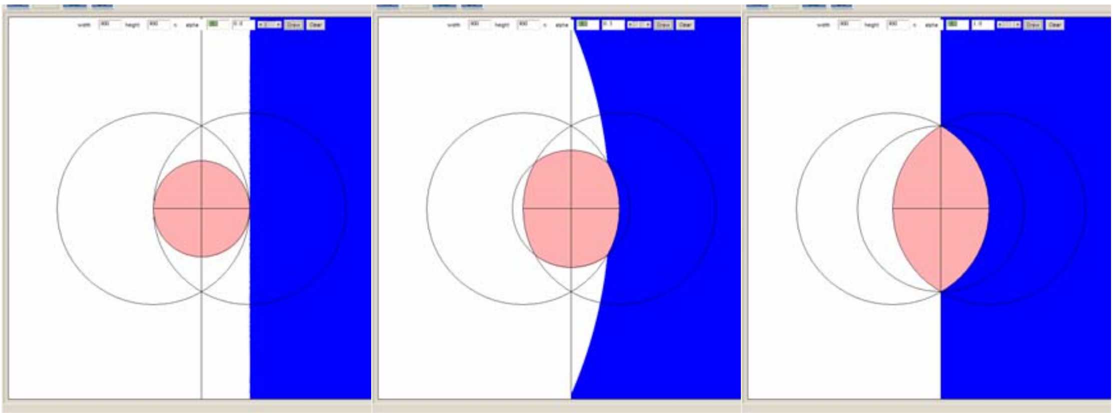
DEFINITION 3.7: Given a node *u* on $\aleph$, the (*r*, α)-*enclosed region* of *u*, denoted as $ER_r^{\alpha}(u)$, is defined as

$$ER_r^{\alpha}(u) = \bigcap_{uw \in UDG(V)} \left(\aleph \cap D(u,1) - RR_r^{c}(u,w) \right).$$

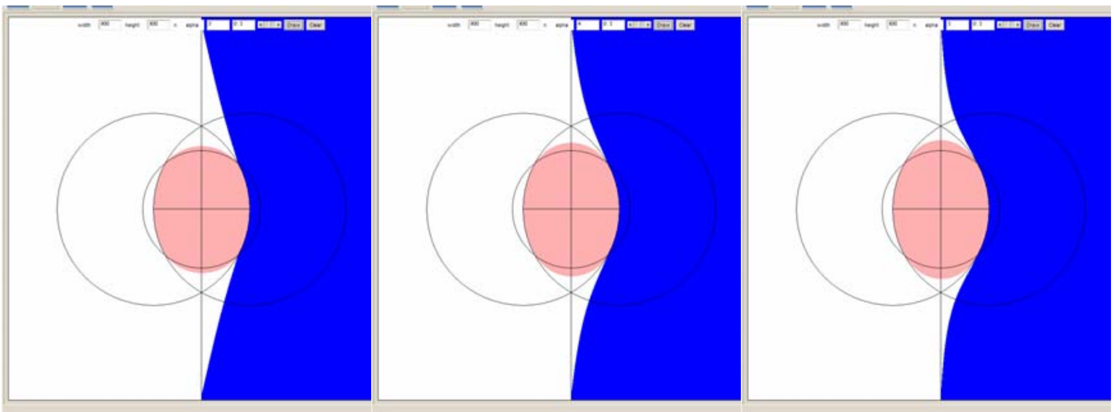
According to (3.9) and (3.10), see Figure 3.9, we have

$$ER_r^{\alpha_1}(u) \supseteq ER_r^{\alpha_2}(u), \quad 2 \le \alpha_1 \le \alpha_2; \tag{3.11}$$

$$ER_{r_1}^{\alpha}(u) \supseteq ER_{r_2}^{\alpha}(u), \quad 0 \le r_1 \le r_2 \le 1; \tag{3.12}$$



(a) $\alpha = 2$, $r = 0.0$, (b) $\alpha = 2$, , $r = 0.5$ (c) $\alpha = 2$, $r = 1.0$

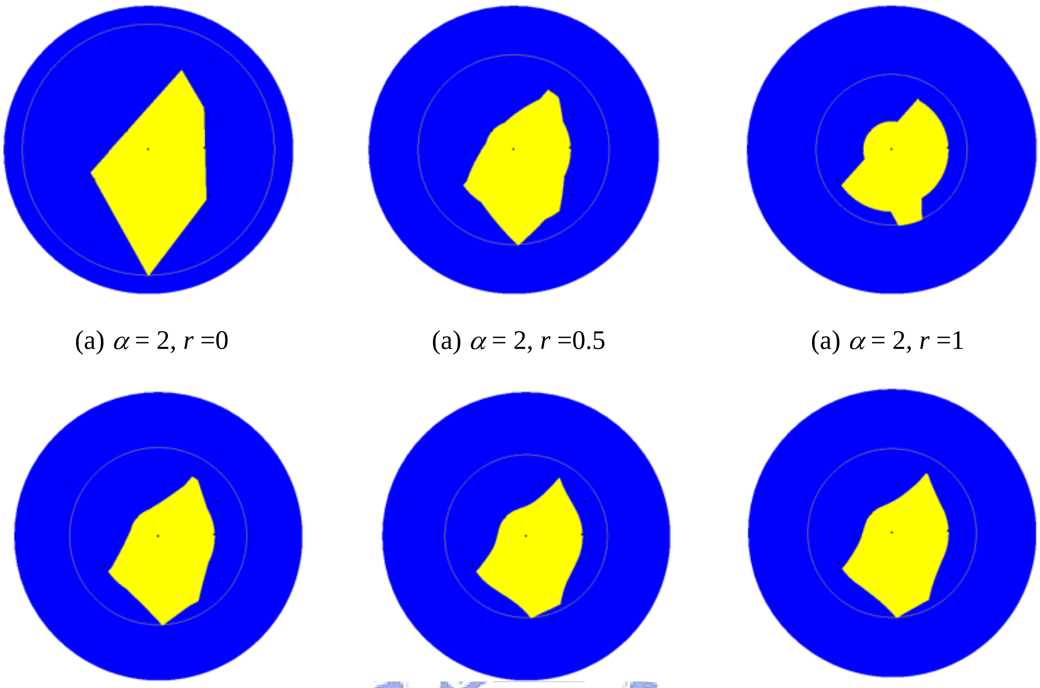


(d) $\alpha = 3$, $c = 0$, $r = 0.5$ (e) $\alpha = 4$, $r = 0.5$ (f) $\alpha = 5$, $r = 0.5$

Figure 3.8: (*r*, α)-relaying region vs. (*r*, α)-neighborhood region.

Based on this merged region, the graph is defined below for each node *u*.

DEFINITION 3.8: Given a set of nodes on $\aleph$, the (*r*, α)-*enclosed graph*, denoted as $EG_r^\alpha(V)$, has an edge *uv* if and only if $v \in ER_r^\alpha(u)$



(a) $\alpha = 2$, $r = 0$ (a) $\alpha = 2$, $r = 0.5$ (a) $\alpha = 2$, $r = 1$

(a) $\alpha = 3$, $r = 0.5$ (a) $\alpha = 4$, $r = 0.5$ (a) $\alpha = 5$, $r = 0.5$

Figure 3.9: (*r*, α)-enclosed region.

The equivalence between $NG_r^\alpha(V)$ and $EG_r^\alpha(V)$ is shown below.

PROPERTY 3.3: For any set *V* of nodes on $\aleph$, $\alpha \geq 2$, and $0 \leq r \leq 1$,

$$NG_r^\alpha(V) \equiv EG_r^\alpha(V).$$

Proof: Consider an edge $uv \in NG_r^\alpha(V)$. We know that there is no *w* such that $Loc(w) \in NR_r^\alpha(u,v)$. In other words, for any *w*, $\|uw\| \geq \|uv\|$, $\|vw\| \geq \|uv\|$, and $p(u, w) + p(w, v) \geq p(u, v)(1 + r^\alpha)$, which equivalent to say that for any $w \in N_u(UDG(V))$, $Loc(v) \notin RR_r^\alpha(u,w)$. So, we get $Loc(v) \in ER_r^\alpha(u)$, and thus $uv \in EG_r^\alpha(V)$. In opposition, if $uv \in EG_r^\alpha(V)$, by definition, $v \in ER_r^\alpha(u)$. Thus, for any *w* where $\|uw\|$

≤ $T_{\max}$, $||uw|| \geq ||uv||$, $||vw|| \geq ||uv||$ and $p(u, w) + p(w, v) \geq p(u, v)(1 + r^{\alpha})$, which implies that for any $w \in N_{UDG}(u)$, $w \notin NR_r^{\alpha}(u,v)$. So, we get $uv \in NG_r^{\alpha}(V)$ □

Cleary, similar to $NG_r^{\alpha}(V)$, each node *u* can distributively decide it neighbors in EG_r^{α} using local information.

3.5 (f_r, α)-Neighborhood Graph

In above structures, the parameters among nodes are all identical. This consideration simplifies the theoretical discussion. However, to be applied on a distributed environment, a more flexible structure is required. The following, we define an more general structure of the (r, α)-neighborhood graph. It allows each node possessing its own parameters, while still preserves to several desired features.

DEFINITION 3.9: Given a set *V* of nodes on $\aleph$, a parameter set f_r: $\{r_{v1}, r_{v2}, \ldots, r_{vn}\}$, and $\alpha \geq 2$, the *(f_r, α)-neighborhood graph*, denoted as $NG_{f_r}^{\alpha}(V)$, has an edge *uv* if and only if $||uv|| \leq 1$ and there is no $w \in V$ in $NR_{r_{uv}}^{\alpha}(u,v)$, where $r_{uv} = \max\{r_u, r_v\}$ and $\alpha_{uv} = \max\{\alpha_u, \alpha_v\}$

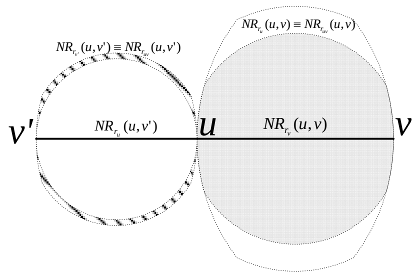


Figure 3.10: (f_r, f_α)- neighborhood graph

An example of $NG_{f_r}^{\alpha}(V)$ is illustrated in Figure 3.10. We can see that for any two nodes, their neighbor regions are determined by the smaller one. Let $r_{\min} = \min\{r_v \mid v \in V\}$ and $r_{\max} = \max\{r_v \mid v \in V\}$. The graph has the following quantitative results.

PROPERTY 3.4: For any set V of nodes on $\aleph$, parameter set f_r, and $\alpha \geq 2$,

(i) $d_u(NG_{f_r}^{\alpha}(V)) \leq \left\lceil \dfrac{\pi}{\cos^{-1}\left(1 - r_u^{\alpha}/2\right)} \right\rceil$, $\forall\ u \in V$;

(ii) $d_{\max}(NG_{f_r}^{\alpha}(V)) \leq d_{\max}(NG_{r_{\min}}^{\alpha}(V))$

(iii) $\rho(NG_{f_r}^{\alpha}(V)) \leq 1 + (n-2)r_{\max}^{\alpha}$

Proof: Consider (i). For any $uv \in UDG$, by definition, $r_{uv} \geq r_u$, which means that no matter what the value of r_v is, $NR_{r_{uv}}^{\alpha}(u,v) \subseteq NR_{r_u}^{\alpha}(u,v)$. So, if there is a node v' such that $\angle vuv' \leq 2\cos^{-1}\left(1 - r_u^{\alpha}/2\right)$, then either $uv \notin NG_{f_r}^{\alpha}(V)$ or $uv' \notin NG_{f_r}^{\alpha}(V)$. (ii) is a direct result of (i). (iii) follows from the fact that $NR_{r_{uv}}^{\alpha}(u,v) \subseteq NG_{r_{\max}}^{\alpha}(u,v)$, which means that $NG_{f_r}^{\alpha}(V) \supseteq NG_{r_{\max}}^{\alpha}(V)$. So

$$\rho(NG_{f_r}^{\alpha}(V)) \leq \rho(NG_{r_{\max}}^{\alpha}(V)) \leq 1 + (n-2)r_{\max}^{\alpha}.$$ □

Notably, the generalization still preserves all qualitative properties in NG_r, which is the most primitive version.

PROPERTY 3.5: The (f_r, α)-neighborhood graph is *connected planar* with *symmetric edges*.

Proof: The graph is symmetric since for any two nodes u and v, the presences of both uv and vu are based one the same r_{uv}. The connectivity and planarity are due to the facts that $NG_{r_{\max}}^{\alpha}(V) \subseteq NG_{f_r}^{\alpha}(V) \subseteq NG_{r_{\min}}^{\alpha}(V)$. □

Chapter 4

Energy-Efficient Construction

In this chapter, we design energy-efficient algorithms as well as protocol for the structures proposed in the preview chapter. First, a purely localized algorithm will be presented. In this algorithm, each node start its transmission power from a small level, and then incrementally increasing the power, until certain criteria are satisfied. It can avoid a long distance construction power if the increment can stop earlier before the maximum power is reached.

However, such incremental approach typically requires several iterations to complete. The incurred latency would not be tolerable to mobile nodes. In addition, it is not adapt to the periodic beacon. To construct and maintain the proximate graph in mobile environment, a node can periodically announce its current position to its neighbors. However, such periodic transmission would consume considerable power, especially when nodes are highly moved where an intensive update is required. Therefore, we will investigate a shrink power mechanism for the periodic beacon.

4.1 Localized Algorithm for Stationary Nodes

In this section, we propose an efficient purely localized algorithm, named *PLA*, to construct the *r*-neighborhood graph. This algorithm consists of two main procedures, GETINF and FINDNB. First, GETINF collects a set of nodes' information within one-hop distance, denoted as IN_u. Then, the collected information will be fed into FINDNB to determine a set of neighbors in $NG_r(V)$, denoted as NB_u.

ALGORITHM *PLA*

Input: A ratio $0 \le r \le 1$.

Output: A set of neighbors adjacent to u.

Step 1: IN_u := GETINF(u, r);

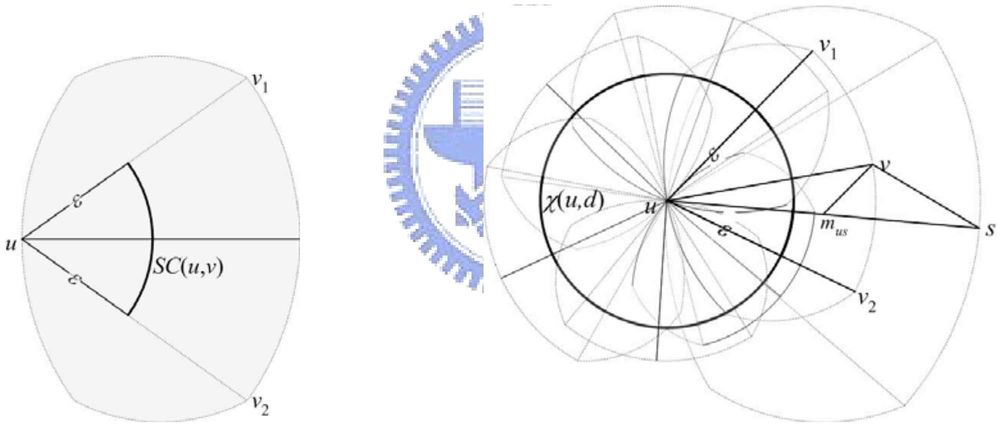
Step 2: NB_u := FINDNB(u, r, IN_u);

Step 3: Stop and output NB_u;

To collect the one-hop information, the simplest way is to let each node broadcast its information at the maximum transmission range 1 and gather the information from others. However, the severe path loss and the frequent change in topology may cause considerable power in such transmission. Therefore, in GETINF we aim to reduce the transmission range during construction. The main idea is to incrementally raise the transmission power from a small range and then use some rule to stop the increment earlier before the transmission range 1 is reached. The detail steps are explained as follows: the transmission range is initiated at a small distance d_0, and then it will be incrementally raised for several rounds. Let d_1 and d_2 be the previous and the current transmission ranges of a round respectively. In each round, a node broadcasts a request to distance d_2, and waits for the responses from receiving nodes to gather the nodes' information. To avoid replying to a node for the second times, the request of a node u contains the position $Loc(u)$ and the previous distance d_1. As a node v receives this request, it calculates the Euclidean distance $||uv||$. Then, if $||uv|| > d_1$, v responses its information, $Loc(v)$, to u at distance $||uv||$, otherwise, just neglects the request. In each round, the range is increased by multiplying $\sqrt[\alpha]{2}$, which means the transmission power is multiplied by 2 each time. The process is continued until the following stopping criterion is satisfied. Let v_1 and v_2 be two crossed points intersected by $C(u, \|uv\|)$ and $C(m, l)$, see Figure 4.1 (a). We define $SC(u, v)$ to be the semicircle enclosed by uv_1 and uv_2 with radius ε, where $\varepsilon > 0$ is a small value less than the distance between any pair of nodes in V. Then, given a distance d, a semicircle $\chi(u, d)$ is defined as follows

$$\chi(u,d) = \bigcup_{\|uv\| \le d} SC(u,v) .$$

We can prove that if $\chi(u, d)$ is exactly the circle $C(u, \varepsilon)$, like Figure 4.1 (b), then a disk centered at u with d radius can cover all neighbors of u in $NG_r(V)$. In other words, GETINF can be halted as $\chi(u, d_2) \equiv C(u, \varepsilon)$. Let $N_u(G(V))$ be the set of neighbors of node u in a graph $G(V)$. This property is proven in Lemma 4.1.



(a) (b)

Figure 4.1: (a) the semicircle $SC(u, v)$; (b) the $\chi(u, d)$ is the union of all $SC(u, v)$ where v is within distance d.

LEMMA 4.1: Given a node $u \in V$ and distance $d \in \Re$, if $\chi(u,d) \equiv C(u,\varepsilon)$,

$$N_u(NG_r(V)) \subseteq \{v \in V \mid Loc(v) \in D(u,d)\} .$$

Proof. We assume for contradiction that some node s in $N_u(NG_r(V))$ is not in $\{v \in V \mid Loc(v) \in D(u,d)\}$. Since ε is less than the distance between any pair of nodes in V, we get $||us|| > \varepsilon$. Thus, edge us intersects a point on the circle $C(u, \varepsilon)$. Do the fact that $\chi(u,d) \equiv C(u,\varepsilon)$, us must intersect at least one semicircle that composes $\chi(u, d)$, see Figure 4.1 (b). Let $SC(u, v)$ be one of the semicircles intersected by us. Then, us is enclosed by uv_1 and uv_2 in $SC(u, v)$. In other words, $\angle suv \le \angle v_1uv$ or $\angle suv \le \angle vuv_2$. According to the argument in Theorem 3.6, we can get that $\angle v_1uv = \angle vuv_2 = 2\sin^{-1}(r/2)$. Therefore, we have $\angle suv \le 2\sin^{-1}(r/2)$. Moreover, since s is not in $\{v \in V \mid Loc(v) \in D(u,d)\}$, s must be farer than v from u. So, $Loc(v) \in NR_r(u, s)$. According to Definition 3.3, us in not in $NG_r(V)$, which however contradicts that s is a neighbor of u in $NG_r(V)$. Thus, we concluded this lemma. □

The total transmission power used by GETINF could be as large as $d_0^{\alpha}(1+2^1+2^2+\cdots+2^I)$, where I is the number of rounds. This result could be worse than the maximum transmission power 1 as I is large. Fortunately, when n is large, nodes are closer to and evenly surrounded by each other so that $\chi(u, d)$ has more change to be quickly shaped as $C(u, \varepsilon)$. So we can gain benefit from GETINF in higher probability as the number of nodes increases.

The steps of GETINF are described below. Neglecting the communication overhead at step 2, the execution time of GETINF is dominated by the union operation at step 4. This step can be implemented by some search-and-merge algorithm. Thus, the time complexity of GETINF is $O(n\log n)$.

Now we discuss the communication cost of GETINF. As d_0 is multiplied by $\sqrt[\alpha]{2}$ over $\alpha\log_2(1/d_0)$ times, it is larger than 1. Therefore, the number of rounds to increase the transmission range d_2 is dominated by $\alpha\log_2(1/d_0) + 1$. Assume a node's position can be encoded by $\log_2 n$ bits. Each node has to broadcast at most $(\log_2 n)(\alpha\log_2(1/d_0) + 1)$ bits for the request messages. In addition, a node will reply to same node no more than once. Thus, a node needs at most $(\log_2 n)(n - 1)$ bit to reply all requests.

Combining these results, communication cost of a node is no more than $(\log_2 n)(\alpha \log_2(1/d_0) + n)$ bits.

<u>GETINF(u, r)</u>

Step 1: $d_1 := 0$, $d_2 := d_0$, $IN := \phi$, $\chi(u, d_2) := \phi$;

Step 2: Broadcast a request ($Loc(u)$, d_1) to distance d_2 and gather a set R of responses from nodes within d_1 and d_2;

Step 3: For each $Loc(v) \in R$ do

$$\chi(u, d_2) := \chi(u, d_2) \cup SC(u, v);$$

Step 4: $IN := IN \cup R$;

Step 5: If $d_2 \leq 1$ and $\chi(u, d_2)$ is not the circle $C(u, \varepsilon)$ do

$d_1 = d_2$;

$d_2 := d_2 \times 2^{1/\alpha}$;

Return to step 2;

Step 6: Stop and output IN;

Once the information IN_u is collected, node u can start to determine its neighbors in $NG_r(V)$. One institutive way is to apply Definition 2 on IN_u directly, as the follows procedure.

Step 1: $N := IN_u$;

Step 2: For each node v in N do

For each node $w \in IN_u$ do

If $P(w) \in NR_r(u, v)$ do

$N := N - \{v\}$;

Step 3: Output N and stop;

In this procedure, the existence of a neighbor v in IN_u is determined by checking whether some node w is located in $NR_r(u, v)$. The correctness is obvious, while in the worst case it should take $O(n^2)$ time on each node. This time is usually not tolerable when topology changes frequently. Therefore, we aim to reduce time complexity in this part. In FINDNB, the main idea is to reverse the original procedure. That is, instead of checking whether some node w can block an edge uv, for each uv, we check whether some edge uv can be blocked by a node w, for each w. The procedure is below.

This checking is begun from the farthest to the closet nodes in IN_u. So, we index all elements of IN_u in non-decreasing order of $||uw||$ in step 2. The set NB contains all candidates that could be a neighbor of u during the process. As a node w is given, we remove from NB all fail candidates that that are already blocked by w. After that, w is added into NB to be an new candidate of $N_u(NG_r(V))$. The process continues until all w's in IN_u were considered. Now, we prove the correctness of FINDNB.

FINDNB(u, r, IN$_u$)

Step 1: NB *:=* ϕ*;*

Step 2: Index the elements of IN_u *in non-increasing order of* $||uw||$*;*

Step 3: For each node $w \in IN_u$ *with* smallest *index do*

  For each node $Loc(v) \in NB$ *do*

    If $Loc(w) \in NR_r(u,v)$ *do*

      NB *:=* NB – *{*v*};*

  NB *:=* NB + *{*w*};*

Step 4: Stop and output NB*;*

THEOREM 4.1: For any set V of nodes on $\Re^2$, $NB_u = N_u(NG_r(V))$, for any $u \in V$.

Proof. We prove this by showing that for any $v \in V$, $v \in NB_u$ if and only if edge uv is in $NG_r(V)$. Suppose an edge uv is in $NG_r(V)$. By Definition 3.1, there is no $w \in N_u(UDG(V))$ such that $Loc(w) \in NR_r(u, v)$. This implies that once v is added in NB, there is also no $w \in IN_u$ such that v can be removed at step 3. Since $v \in N_u(NG_r(V)) \subseteq IN_u$ and each node in IN_u can be added to NB, v must be in NB at least one time. So, we can get that v is in the final output of NB_u. Contrarily, we suppose $uv \notin NG_r(V)$. Some node $w \in N_u(UDG(V))$ is located in $NR_r(u, v)$. If $v \notin IN_u$, the result clearly follows by Lemma 4.1. Otherwise, $v \in IN_u$. In this case, all nodes blocking uv are in IN_u. Besides, every node w blocking uv is always considered after v in GETNB. Therefore, even if v can be added to NB, there must be a node $w \in IN_u$ such that v can be removed from NB at the successive iteration. So we get $v \notin IN_u$. □

Lemma 4.1 also implies that if $uv \in NG_r(V)$, then $v \in N_u$ and $u \in N_v$ and that if $uv \in NG_r(V)$, then $v \notin N_u$ and $u \notin N_v$. So, the neighbors (links) determined by GETNB are symmetric.

COROLLARY 4.1: Any topology resulted by *PLA* is symmetric.

Consider the time complexity of FINDNB. Step 2 can be done by some sorting algorithm in $O(n\log n)$. Before a node $w \in IN_u$ is added to NB, any $v \in IN$ blocked by w is removed from NB. Therefore, for any two nodes in NB, none of them can be blocked by each other. Let s and t be two nodes in NB. The argument of Theorems 3.5 indicates that if $\angle sut < 2\sin^{-1}(r/2)$, then either s blocks t or t block s. Since neither s blocks t nor t blocks s, we get that $\angle sut \geq 2\sin^{-1}(r/2)$. Therefore, during the process, the size of NB can be never greater than $d_{\max}(NG_r(V))$. Consequently, FINDNB can be done in $O(n\max\{\log n, d_{\max}(NG_r)\})$ time. We can observe that this time complexity depends on the parameter r. When r equals or closes to 0 (the worst cases), the time complexity of FINDNB is still $O(n^2)$. However, when r is sufficiently large such that

$d_{max}(NG_r(V))$ is a constant, FINDNB can be done in O($n$log$n$).
With a slight modification, *PLA* can be easily applied on the extended r-neighbors graph and all results can be preserved. We omit the detail explanation here.

4.2 Shrinking Power mechanism for Mobile Nodes

In this section, we propose an energy-efficient construction of the proposed proximate graph for mobile nodes. For simplicity, we first discussion the (r, α)-neighborhood graph for mobile nodes, where r is identical among nodes . This construction provides the skeleton of the mobile protocol in the next Chapter.

The basic idea is borrowed from a distributed protocol of the primitive enclosed graph [20, 24], though we confront more challenges when designing for our structure, discussed below. This mechanism is based on the characteristic the (r, α)-enclosed graph. Recall that this graph has been shown to be equivalent to the (r, α)-neighborhood graph. Hence the following the two structures will be used interchanged when necessary.

Consider a node u. Let S_u denote the set of information collected by u from its neighboring nodes during a period of time According to Definition 3.7, we define ER_u as the enclosed region of u based on the set of collected information S_u, i.e.

$$ER_u = \bigcap_{w \in S_u} \left(\aleph \cap D(u,1) - RR_r^{\alpha}(u,w) \right) \tag{4.1}$$

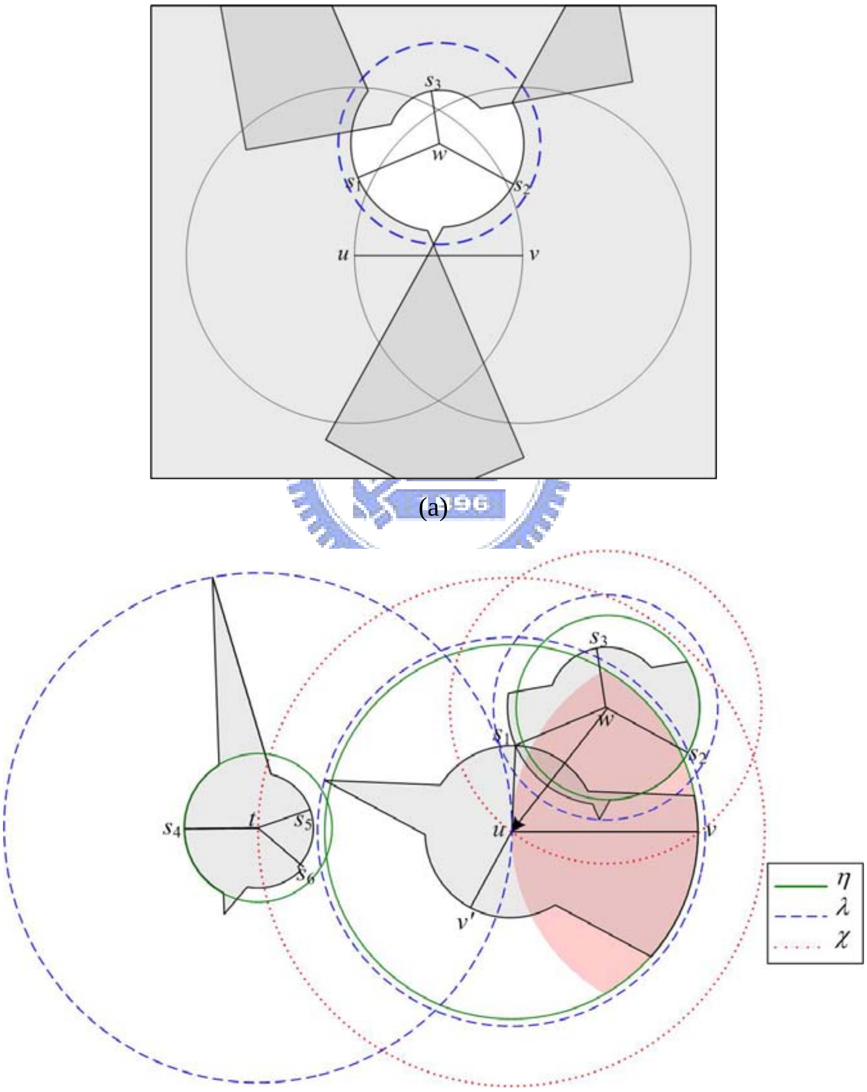
Since S_u must be a subset of $N_u(UDG(V))$, by Definition 3.7, we have

$$ER_r^{\alpha}(u) \subseteq ER_u . \tag{4.2}$$

So, if a node v is not in ER_u, it is also not $ER_r^{\alpha}(u)$. Consequently, by Definition 3.8, it is no possible that v is a neighbor of u in $NG_r^{\alpha}(V)$. This fact points out that to let all neighbors of u in $NG_r^{\alpha}(V)$ be aware of the existence of u, it is sufficient for u to broadcast using the least radius that covers the all points in ER_u. We denote such radius as λ_u, i.e.

$$\lambda_u = \max\{\|ux\| \mid x \in ER_u, x \in \aleph\}. \tag{4.3}$$

Note that in (4.3) only considers points in deployment region $\aleph$. The transmission radius in (4.3) ensures that each node will aware its neighbors in $NG_r^\alpha(V)$. Hence, all links in NG_r^α can be preserved using possibly less construction power.



(a)

(b)

Figure 4.2: (a) the shrunk power λ_w; (b) the enlarge power (r = 1, α = 2).

In the abovementioned, we discuss how reduce the power that is sufficient to include all necessary neighbors. To construct the desired graph, we need to further ensure that all non-neighbor nodes will be blocked. However, the shrunk power may prevent nodes from be aware of some non-neighbor nodes that are necessary to block other non-neighbor nodes. See the example in Figure 4.2, where r = 1 and α = 2. There is only one node $w \in ER_r^\alpha(u,v)$. By definition, the edge *uv* does not exist in $NG_r^\alpha(V)$ due to the presence of *w*. However, when the shrunken power of (4.3) is used, *w* will be enclosed by its surrounding nodes s_1, s_2 and s_3, which leads to the transmission radius of *w* shorter than its distances to *u* and *v*. Thus, both *u* and *v* will be no longer being able to find *w*. In other words,

$$w \notin S_u \text{ and } w \notin S_u .$$

This in turn causes that $v \in ER_u$ and $v \in ER_u$, since

$$Loc(w) \in NR_r^\alpha(u,v) \Leftrightarrow Loc(v) \in ER_r^\alpha(u,w) \quad (4.4)$$

Consequently, both *uv* and *vu* exists. It however are not allowed in the desired graph.

To fix this problem, a simplest way is to enlarge *w*'s power such that both *u* and *v* can be aware of *w*, while the prerequisite is that *w* should be able to be aware of both *u* and *v* first. Fortunately, this prerequisite can be self-contained, since if $w \in ER_r^\alpha(u,v)$, by definition 3.3,

$$||uw|| \leq ||uv|| \text{ and } ||uw|| \leq ||uv||,$$

which mean that the transmission radiuses of both *u* and *v* will cover *w*. So, *w* must be able to overhear the existences of *u* and *v*. In other word, there must be

$$u \in S_u \text{ and } v \in S_u .$$

From this observation, for each node *u*, to ensure that all links which should be blocked by *u* will not exist, it is sufficient for *u* to transmit to all nodes that have been received by *u*. We denote such transmission radius as $\chi_u^{'}$, i.e.

$$\chi_u^{'} = \max\{\|uv\|, v \in S_u\}. \tag{4.5}$$

The transmission radius in (4.5) ensures that all links that are belong to $NG_r^{\alpha}(V)$ will be removed. Nevertheless, the readers may node notice that the radius is now enlarged, which may counteract the original benefit from (4.3). Therefore, below we attempt to further shrink the radius of (4.5).

Let N_u denote the set of neighbors determined by u. Consider a node w. By Definition 3.4, a link uv will be blocked by w only if $\|uw\| \le \|uv\|$ and $v \in N_u$. In other words, let η_u denote the least radius cover all node in N_u, i.e.

$$\eta_u = \max\{\|uv\| \mid v \in N_u\}, \tag{4.6}$$

if the distance between u and v is larger than η_u, then there is no link adjacent to u should be blocked by w. So, if $\|uv\| > \eta_u$, it is not necessary for w to transmit to u. More generally, consider a node u, define B_u to be the set of nodes where $\|uv\| \le \eta_u$,

$$B_u = \max\{\|uv\| \le \eta_u \mid u \in S_v\}, \tag{4.7}$$

So, from (4.5), the following radius χ_u is sufficient to covering all nodes in B_u

$$\chi_u = \max\{\|uv\|, v \in B_u\}. \tag{4.8}$$

Compared with (4.5), since B_u is a subset of S_u, the radius required by $\chi_u^{'}$ must be equal to or even less than that of χ_u. Summarily, to construction the $EG_r^{\alpha}(V)$ correctly using less using less power, it is sufficient to broadcast using the following transmission radius

$$T_u = \max\{\lambda_u, \chi_u^{'}\}. \tag{4.9}$$

4.3 Neighborhood Graph based Topology Control Protocol

Based on above discussion, a distributed protocol that constructs the(r,

α)-neighborhood graph for mobile nodes is now presented here. In this protocol, each node will periodically broadcast a message in every T time interval using the shrunken radius T_u. Each message will consist of the current position and the radius η_u. As a message is received from a node v, it will include v in the collecting set S_u. In addition, if the node observes that the distance to v is shorter than the received radius η_u, it includes v into B_u such that χ_u will be sufficiently large whenever there are some links of v that should be block by u. On the other hand, if u do not received from v over an beacon interval, then u discard v's information. Upon a message is either received or expired, the reconstruction process will proceed. Then, before sending the next beacon, related variables mentioned above will be recalculated based upon the information collected in the previous interval. The protocol is summarized below. We named it the *Neighborhood Graph based Topology Control Protocol*, abbreviated as *NGTC*.

The correction is directly followed from the meaning of each variable. We omit the proof. Now we show an interest feature of this protocol below.

PROPERTY 4.1: For any set of nodes, the construction radius T_u of each node u is decreased by r.

Proof: As r increases, by (3.12), the region $ER_r^{\alpha}(u)$ will be smaller, which leads to a lower λ_u. The same observation is on χ_u. By Property 3.2 (ii), the node degree of each node v will be strictly decreased as r goes up. So, by (4.6), a large r leads to a smaller η_v, which in turn declines the size of B_u for any u received v. As a sequel, by (4.8), $\chi_u^{'}$ can be lower. Combining these two facts, we provide this. □

The rest of this subsection, we discuss additional observations on the *NGTC* protocol.

1) We can see that $T_u \geq \lambda_u$ and $T_u \geq \chi_u^{'}$ are the sufficient conditions for u to

preserve all necessary links and block all unnecessary links, respectively. However, we have to admit that there is a potential wasting on T_u. It occurs when $T_u > \lambda_u$ but there is no $v \in B_u$ such that $u \in NR(v, s)$ for some $s \in N_v$. In other words, u enlarges its r to cover B_u but there is no additional blocked by u, see Figure 4.2 (b). So, the radius T_u is not the minimum.

NGTC Protocol

$N_u = \{\}; S_u = \{\}; B_u = \{\};$
For every T time
 $\lambda_u = \max\{\|ux\| \mid x \in ER_u, x \in \aleph\};$
 $\chi_u = \max\{\|uv\| \mid v \in B_u\};$
 $T_u = \max\{\lambda_u, \chi_u\};$
 $\eta_u = \max\{\|uv\| \mid v \in N_u\};$
 Broadcast ($Loc(u)$, η_u) in radius T_u;
Upon **received** a message ($Loc(v)$, η_v) from a node v,
 $S_u = S_u + \{v\};$
 If $\|uv\| \leq \eta_v$, $B_u = B_u + \{v\}$, otherwise, $B_u = B_u - \{v\}$;
Upon a message received from some v in S_u is **expired** (over T time),
 $S_u = S_u - \{v\}; B_u = B_u - \{v\};$
Upon a message is received or expired
 $ER_u = \bigcap_{w \in S_u} \left(\aleph \cap D(u, T_{\max}) - RR_r^{\alpha}(u, w)\right);$
 $N_u = \{u \in S_u \mid u \in ER_u\};$

Since $\lambda_u \geq \eta_u$, by sending λ_u instead of η_u in the message, it is also sufficient to block all unnecessary neighbors. However, whether sending λ_u or η_u is better? See the following discussion: if u sends λ_u, since $\lambda_u \geq \eta_u$, a node v covered by u would include u into B_v even if $||uv|| > \eta_u$. So, the radius $\chi_u^{'}$ which supports blocking other links increases. But by receiving information from some node farther than the farthest neighbor in N_u, the coverage of $ER(u)$ is more possibly to be shrunk down and thus lead to a lower χ_u. In other words, if u sends λ_u, the radius T_u of u itself could be lower (at least one larger), while its neighbor's radiuses would be increased, and vice versa. So, there is a tradeoff between sending λ_u or η_u. We will give our suggestion in the later part.

4.4 Convergency

As nodes placement changes, each node u would maintain a new set of neighbor N_u and recalculate the radius T_u. However, due to the recursive dependency among nodes, such as χ_u depends on B_u and B_u further depends on η_v, some variable (radius or set) may require several iterations to recover from the change. In this section, we show that the topology as well as construction power of *NGTC* can converge in a constant time.

Consider a node u. Suppose the current timer of u is at t and some topological change occurs during $[t - T, t)$, i.e. $\exists v \in V$, $Loc^{t-1}(v) \neq Loc^{t}(v)$, and there is no change after t. i.e. $\forall v \in V$, $c \in Z^{+}$, $Loc^{t}(v) = Loc^{t+cT}(v)$ (note: the change may be caused by u itself). Assume the network is synchronous (time slot is aligned among each node), and the propagation delay and computation time are negligible with respect to the interval T.

Table 4.1: The sufficient status of each variable.

λ_u	$\|uv\| \le \lambda_u, \forall uv \in NG_r^{\alpha}(V)$	S_u	$v \in S_u, \forall uv \in NG_r^{\alpha}(V)$
χ_u	$\|uv\| \le \chi_u, \forall vs \notin NG_r^{\alpha}(V), Loc(u) \in NR(s,v)$	B_u	$v \in B_u, \forall v \in N_u, uv \notin NG_r^{\alpha}(V)$
η_u	$\|uv\| \le \eta_u, \forall v \in N_u, uv \notin NG_r^{\alpha}(V)$	ER_u	$Loc(v) \in ER_u, \forall v \in NG_r^{\alpha}(V)$
T_u	Both λ_u and χ_u' are sufficient	N_u	$v \in N_u, \forall v \in NG_r^{\alpha}(V)$

We define three statuses for each variable in *NGTC*:

- *Stale*: it is neither of the following two statuses;
- *Sufficient*: it is sufficient for its functionality, see Table 4.1;
- *Converge*: it will change any more (For T_u, it means the radius can not be shrunken any more and for N_u, it means $N_u(NG_r^{\alpha}(V)) \equiv N_u$).

For a variable X, we denote X^t as the status at time t. We have the following property.

PROPERTY 4.2: In a synchronous network, if each node u sends ($Loc(u)$, λ_u) every T time, the set N_u of neighbors can converge in **4*T*** and the radius T_u can converge in and **6*T***.

Proof: Consider a node u. Without loss of generality, we set T = 1. At time t: a set S_u^t of nodes with updated positions is gathered. By Definition 3.6, a point consist of a neighbor of u only if it is not blocked by any node in S_u^t. So, ER_u^t is sufficient to cover all logical neighbors. At time t+1: λ_u^{t+1} is now sufficient due to the sufficiency of ER_u^t. In turn, S_u^{t+1} is sufficient, since $T_v^{t+1} \ge \lambda_v^{t+1} \ge \|uv\|, \forall uv \in NG_r^{\alpha}(V)$. Because of the sufficiency of S_u^{t+1} and ER_u^t, N_u^{t+1} is now sufficient to include all neighbors. At time t+2: η_u^{t+2} covers all nodes that should be blocked due to the sufficiency of N_u^{t+1}. Accordingly, χ_u^{t+3} and B_u^{t+2} are sufficient since a node v should be blocked only if $\|uv\| \le \eta_v^{t+2}$. Thus, T_u^{t+3} is sufficient large to cover all neighbors as well as blocking nodes. Since S_u^{t+3} will be no longer expended, ER_u^{t+3} is converged, which in turn implies that N_u^{t+3} is now converged. At time t +4: λ_u^{t+4} and η_u^{t+4} are converged since ER_u^{t+3} and N_u^{t+3} are stable. So, B_u^{t+4} is will no longer change, since η_v^{t+4} is now fixed. At time t +5: χ_u^{t+5} converges to cover the fixed B_u^{t+4}. Therefore, T_u^{t+5} will no longer change. □

The statuses of these variables are summarized in Figure 4.3. We can see that N_u and T_u converge at the beginning of the fourth and fifth intervals after the change occurred.

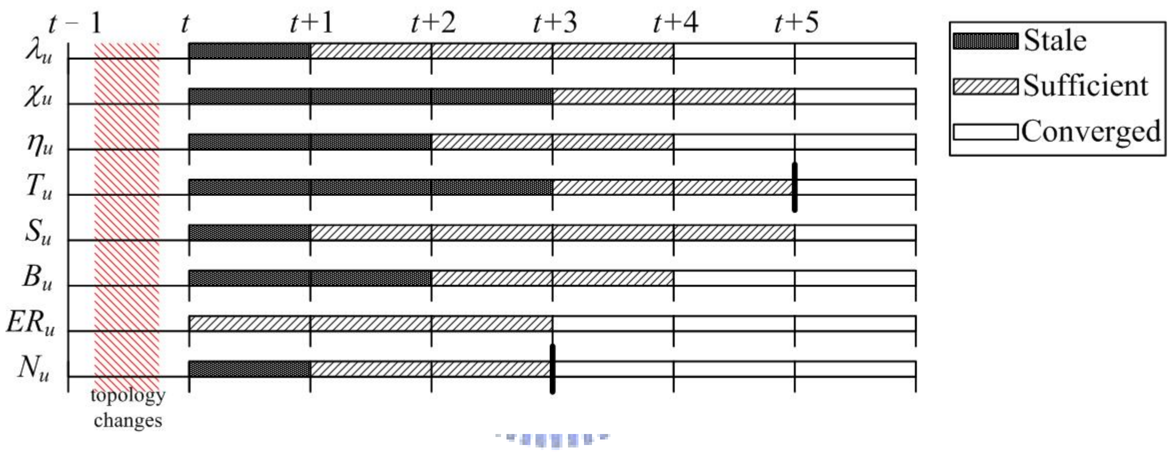


Figure 4.3: The statues of each variable over time intervals.

For the alternative where sending λ_u instead of η_u in the beacon message, the protocol can converge even faster.

Property 4.3: In a synchronous network, if each node u sends ($Loc(u)$, λ_u) every T time, the neighborhood set N_u can converge in **3*T*** and the radius T_u can converge in and **5*T***.

Chapter 5

Mobile Topology Control Protocol

In this Chapter, we present our mobile topology control protocol. The protocol is based the most general version, where each node is allowed having its own *r*. So, first we extend the shrink power mechanism to the (f_r, α)-neighborhood graph. Then, the protocol is presented. We will discuss how automatically configure parameter to adapt to changing. Lastly, we discuss how efficiently perform the protocol and the corresponding time complexity.

5.1 Extending on Shrinking Power mechanism

Now we extend the shrink power mechanism such that all links of $NG^{\alpha}_{r_{uv}}$ will be preserved and all links not belong to $NG^{\alpha}_{r_{uv}}$ will be blocked. Before that, we should extend the (r, α)-enclosed region to the (f_r, α)-neighborhood graph.

Definition 5.1: Given a set *V* of nodes on $\aleph$, the (f_r, α)-*enclosed graph*, denoted as $EG^{\alpha}_{f_r}(V)$, has an edge *uv* if and only if $v \in ER^{\alpha}_{r_{uv}}(u)$, where $r_{uv} = \max\{r_u, r_v\}$.

The equivalence between $EG^{\alpha}_{f_r}(V)$ and $NG^{\alpha}_{f_r}(V)$ is obvious. We can see that only different is the identical *r* which is not replaced by r_{uv}. Institutively, it seems that the mechanism can be applied to $NG^{\alpha}_{f_r}(V)$ based on $EG^{\alpha}_{f_r}(V)$ directly. However, there is one difficulty: The enclosed region of *u* is now depending on not only r_u but also r_v. It means that the least radius where λ_u covers a possible neighbor *v* would be variant by r_v, which however is uncertain before acquiring a message from v.

Fortunately, there is an upper bound that can be calculated using a node's own r. Recall that in (3.11), for any $0 \le r_1 \le r_2 \le 1$, $ER_{r_1}^{\alpha}(u) \supseteq ER_{r_2}^{\alpha}(u)$. Since $r_{uv} \le r_u$, we get

$$ER_{r_{uv}}^{\alpha}(u) \subseteq ER_{r_u}^{\alpha}(u).$$

Further, similar to (4.3), we redefine

$$ER_{r_u} = \bigcap\nolimits_{w \in S_u} \left(\aleph \cap D(u,1) - RR_{r_u}^{c,\alpha}(u,w) \right) \tag{5.1}$$

Using the same argument of (11), we have

$$ER_r^{\alpha}(u) \subseteq ER_{r_u}$$

So the following radius λ_{r_u}, redefined from (4.3), is sufficient to cover all possible neighbors.

$$\lambda_{r_u} = \max\left\{ \|ux\| \mid x \in ER_{r_u}, x \in \aleph \right\}. \tag{5.2}$$

On the other hand, let

$$ER_{r_{uv}} = \bigcap\nolimits_{w \in S_u} \left(\aleph \cap D(u, T_{\max}) - RR_{r_{uv}}^{c,\alpha}(u,w) \right). \tag{5.3}$$

As a message is received from v, we have to now check whether v is in $ER_{r_{uv}}$, instead of ER_{r_u}, such that all unnecessary uv can be blocked. Finally, for any $v \in N_u$, λ_u is sufficiently large to cover all points in $NR_{r_{uv}}^{\alpha}(u,v)$, which means that u can still receive from all w's that block uv. So, η_u, B_u, χ_u, T_u are still corrected here.

5.2 Adaptive Mobile Topology Control Protocol

The main idea of this protocol is based on adjusting the parameter r_u for each u. Thus we start with a series of analyses how the parameter r_u of each node u influence the overall energy-efficiency from the following three dimensions:

1) *Energy efficiency of routes vs. operation time of individual node*:

Consider a node u. Given a ratio r_0, $0 \le r_0 \le 1$, we denote $NG_{f_r|r_u=r_0}^{\alpha}(V)$ to be the (f_r,

α)-neighborhood graph where r_u is fixed on r_0. We have the following observations. For any $0 \le r_1 \le r_2 \le 1$, by (3), we have $NR^{\alpha}_{\max\{r_1,r_v\}}(u,v) \subseteq NR_{\max\{r_2,r_v\}}(u,v)$. So, for any node w, where $Loc(w) \notin NR_{\max\{r_2,r_v\}}(u,v)$, it must be $Loc(w) \notin NR^{\alpha}_{\max\{r_1,r_v\}}(u,v)$. This implies that an edge $uv \in NG^{\alpha}_{f_r|r_u=r_2}$ only if $uv \in NG^{\alpha}_{f_r|r_u=r_1}$. In other words,

$$NG^{\alpha}_{f_r|r_u=r_2}(V) \subseteq NG^{\alpha}_{f_r|r_u=r_1}(V). \tag{5.4}$$

Therefore, we can get that

$$\rho(NG^{\alpha}_{f_r|r_u=r_2}(V)) \ge \rho(NG^{\alpha}_{f_r|r_u=r_1}(V)), \tag{5.5}$$

and

$$d_u(NG^{\alpha}_{f_r|r_u=r_2}(V)) \le d_u(NG^{\alpha}_{f_r|r_u=r_1}(V)). \tag{5.6}$$

Based pm these properties, we can observe that for each node u, no matter what the parameters of other nodes are taken, a smaller r_u will strictly lead to an overall better energy efficiency communication routes (at least on worse), and on the other hands, a smaller r_u can reduce the adjacency of u to its neighboring nodes. A smaller node degree can help prolong the operation time of an individual node in two reasons:

- *Broadcasting power*: Since the relationship in (5.4), a smaller degree implies that the farther selected neighbor is closer. So, for broadcasting operation, the node can spend less power to cover all neighbors.
- *Traffic Load*: A node with more links will let more traffic flow (both flooding and unicasting) pass through it, which may draw out its energy rapidly for those transmissions. Thus a smeller degree can help release the node' traffic.

2) *High mobility vs. low mobility*:

Consider how node mobility effects the energy consumption. As a node has high mobility. It will cause its surrounding nodes changing the links status (establish or remove a link) to itself frequently, which will in turn triggers more route reconstruction at the upper layer. More reconstruction implies extra energy wasting on flooding route discovery packets. To alleviate such undesirable circumstance, a highly moving node can reduce the adjacency to its neighboring. In other words, a large r_u which leads to a lower node degree on u is preferable as u is in high mobility.

3) *Topology maintenance Power*:

Addition consideration is from the topology maintenance power. Recall in Property 3.4, a larger r will cause a smaller T_u. It means the energy consumption of u can be reduced as a large r_u is used, which is surprisingly consist with the tendency of r_u toward the residual energy in the first consideration.

Combining the above considerations, a configuration rule for the parameter r_u is characterized as follows.

$$r_u = \left(1 - \frac{Energy_u}{Energy_{Full}}\right) \times \left(\frac{Mobility_u}{Mobility_{Max}}\right), \tag{5.7}$$

where $Energy_u$ and $Mobility_u$ are the current residual energy and mobility level of node u, and $Energy_{Full}$ and $Mobility_{Max}$ are the full power level and the maximum node mobility. The formulation in (5.7) can completely consist with all observation and anticipation in above considerations. We can see that the rule is extremely simple and can be carried out automatically by each node relied on only inherent statuses of itself. In addition, the configuration can be conducted independently by each node without additional control message to negotiate the symmetric, connectivity and planarity, since theoretically all these properties are preserved, see properties 3.4 and 3.5. For these reasons, the protocol will be very practical. More importantly, by reducing node dependency according mobility, the drawback of using nodes position in proximate graph can elegantly alleviated, since a node with lower degree will now trigger less reconstruction. The overall conceptions are depicted in Figure 5.1.

In practice, each node u can set its $r_u = 0$ at the initial stage, and then configure r_u

periodically according to several distinct energy levels and mobility. The node mobility can be measured by *node speed* or *remaining pause time*, i.e. as a node stops moving and anticipates that it will stay on the place for a relatively long period of time, it can turn up its r_u to allow more neighbors accessing to it.

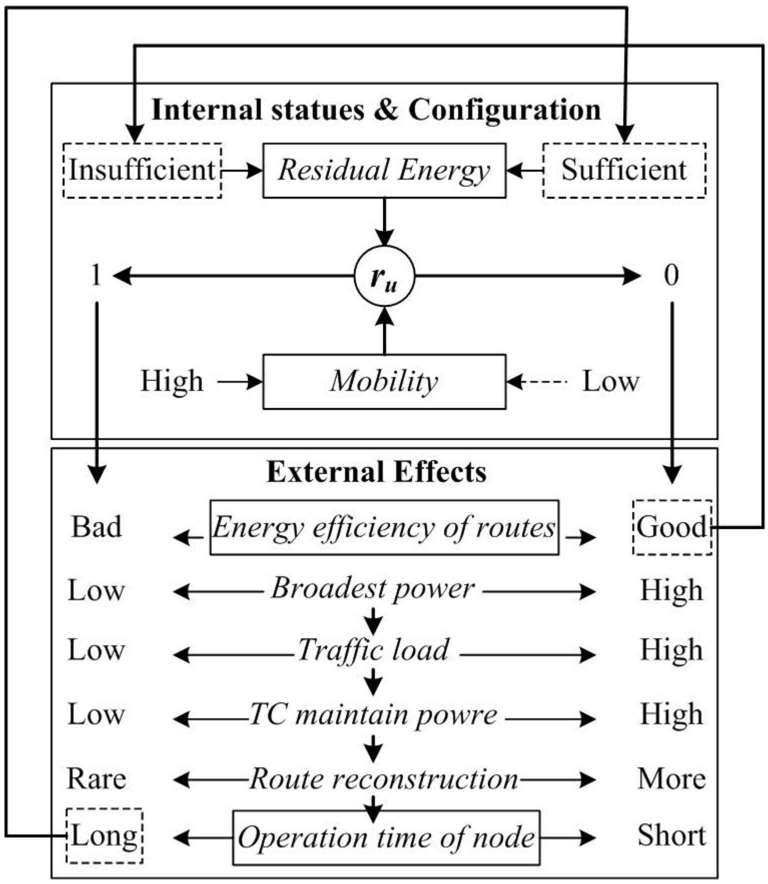


Figure 5.1: The relationship among the considerations, effects, and the configure process.

The final version of the mobile topology control protocol is given below. We named it the *Adaptive NGTC Protocol*, abbreviated as *ANGTC*. To save page space, we only highlight the different parts, in comparison with *NGTC*. The other part encapsulated from line 1 to 13 here is the same except that ER_u is no replaced by ER_{r_u} .

ANGTC Protocol

1 $N_u = \{\}$, $S_u = \{\}$; $B_u = \{\}$, $r_u = 0$.

The procedure from lines 2 to 13 are the same of the NGTC protocol;

14 $$r_u = \left(1 - \frac{Energy_u}{Energy_{Full}}\right) \times \left(\frac{Mobility_u}{Mobility_{Max}}\right)$$

15 $$ER_{r_u} = \bigcap\nolimits_{w \in S_u} \left(\aleph \cap D(u,1) - RR^{\alpha}_{r_u}(u,w)\right);$$

16 $$ER_{r_{uv}} = \bigcap\nolimits_{w \in S_u} \left(\aleph \cap D(u,1) - RR^{\alpha}_{r_{uv}}(u,w)\right)$$

17 $$N_u = \{u \in S_u \mid u \in ER_{r_{uv}}\};$$

5.3 Efficient Calculation and Time Complexity

In the rest part of this section, we discuss the complexity issues and suggest some efficient way for calculation the related variables.

1) *Calculation on N_u:*

If there are relatively smaller number of point on $\aleph$, each node requires only $O(|V|)$ to compute its neighbors in $EG_{r_{uv}}$ by set operation. However, if the there are infinite number of points on $\aleph$, each node can turn to determine its neighbors in $NG^{\alpha}_{f_r}(V)$ in $O(|V|^2)$.

2) 2) *Calculation on λ_u:*

The radius λ_u covers ER_u. Let x be a point on boundary of ER_u with longest distance to u. Clearly, $||ux|| \geq ||uy||$, for any point $y \in ER_u$. So, we will derive the distance of $||ux||$. In the following, we just consider $\alpha = 2$ for two reasons: First, there is no simple root function for $||ux||$ when $\alpha > 2$. Second, the radius λ_u covering $ER^2_r(u)$ is sufficiently large for any $\alpha > 2$, since $ER_r^2(u) \supseteq ER_r^\alpha(u)$.

Obviously, when $|S_u| \leq 2$, $ER_r^2(u)$ can not be enclosed. When $|S_u| > 2$, There are two cases: In the first case, x is crossed by the outers of $RR_r^\alpha(u,w)$ and $RR_r^\alpha(u,w')$, for some w, $w' \in S_u$. In this case, x is crossed by one of the equations from

(i) $\|ux\| = \|uv\|$, (ii) $\|ux\| = \|vx\|$, or (iii) $\|ux\|^\alpha (1+r^\alpha) = \|uv\|^\alpha + \|vx\|^\varepsilon$,

and another one of the equations from

(iv) $\|ux\| = \|uv'\|$, (v) $\|ux\| = \|v'x\|$, or (vi) $\|ux\|^\alpha (1+r^\alpha) = \|uv'\|^\alpha + \|v'x\|^\varepsilon$.

Let θ and y denote the angles of $\angle wuw'$ and $\angle wux$, respectively. By the law of cosine,

$$\begin{aligned} \|ux\|^2 &= \|wx\|^2 - \|uw\|^2 - 2\|uw\|\|ux\|\cos y \\ \|ux\|^2 &= \|w'x\|^2 - \|uw'\|^2 - 2\|uw'\|\|ux\|\cos(\theta - y) \end{aligned} \tag{5.8}$$

Consider four subcases:

If x satisfies either (i) or (iv), then $\|ux\| = \|uv\|$ or $\|ux\| = \|uv'\|$, respectively.

If x satisfies (ii) and (v), we get

$$\|ux\| = \frac{\|vv'\|}{2\sin\theta}. \tag{5.9}$$

If x satisfies (ii) and (vi), or (iii) and (v), we get respectively

$$\|ux\|^2 = \frac{\|uv\|^2\|uv'\| - 2\|uv\|^3\cos\theta - 4\|uv'\| + 8\|uv\|}{4\|uv'\|\sin^2\theta} \tag{5.10}$$

and

$$\|ux\|^2 = \frac{\|uv'\|^2\|uv\| - 2\|uv'\|^3\cos\theta - 4\|uv\| + 8\|uv'\|}{4\|uv\|\sin^2\theta} \tag{5.11}$$

Otherwise, if x satisfies (iii) and (vi), we get

$$\|ux\|^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \tag{5.12}$$

where $a = B^2 - r^4C^2$, $b = 2AB - 4C^2\|uv\|^2(1-r^2)$, $c = A^2 - 4C^2\|uv\|^4$,

and $B = (\|uv\| - \|uv'\|\cos\theta)r^2$, $A = 2\|uv\|^2\|uv'\|\cos\theta + 2\|uv\|\|uv'\|^2 - 4\|ux\|^3$, $C = \|uv'\|\sin\theta$.

In the second case, x is on the outer of some $RR_r^{\alpha}(u,w)$, where $w \in N_u$. Again, if x satisfies (i), then $\|ux\| = \|uv\|$; otherwise, if x satisfies either (ii) and (iii), the farthest point will finally be crossed by another $w' \in S_u$. As a sequel, it is sufficient to consider the first case.

Let d_1, d_2, d_3, d_4, denote the distances obtained by (5.9) to (5.12). We get

$$\|ux\| = \max\{d_{uv}, d_{uv'}, f_1, f_2, f_3, f_4\}. \tag{5.13}$$

In addition, in the second and third subcases,

$$\angle wux = \cos^{-1}\left(\frac{\|uv\|}{2\|ux\|}\right), \tag{5.14}$$

and in the fourth subcase

$$\angle wux = \cos^{-1}\left(\frac{\|ux\|^2 r^2 - 2\|uv\|^2}{2\|ux\|\|uv\|}\right). \tag{5.15}$$

Let $p_{w,w'}$ denote the point crossed by $RR_2(u, w)$ and $RR_2(u, w')$; We can get

$$\angle xuv = \angle wuv \pm \angle wu \quad \text{and} \quad \|vx\|^2 = \|wx\|^2 + \|uv\|^2 - 2\|wx\|\|uv\|\cos\angle xuv$$

Since x is itself a point in $ER(u)$, the longest distance can be obtained by

$$\lambda_u = \max\{\|ux\| \mid w \in S_u, \text{and } v \notin NR(u,w), \forall v \in S_u\}. \tag{5.16}$$

Chapter 6

Experiments

In this chapter, a series experiments will be conducted to evaluate the observations on theoretic results as well as the protocol designs.

6.1 Evaluations on Graph Structures

First of all, we evaluate the theoretic properties shown in Chapter 4. Recall that for any $\alpha \geq 2$, the power stretch of the r-neighborhood graph is bounded from above by an increasing function of r and conversely, the upper bound of the maximum node degree is decreased by r. Figure 6.1 draws the two theoretic functions for $n = 100$ and $\alpha = 2$. We can see that $NG_r(V)$ indeed has the flexibility to be adjusted between the two metrics through the parameter r.

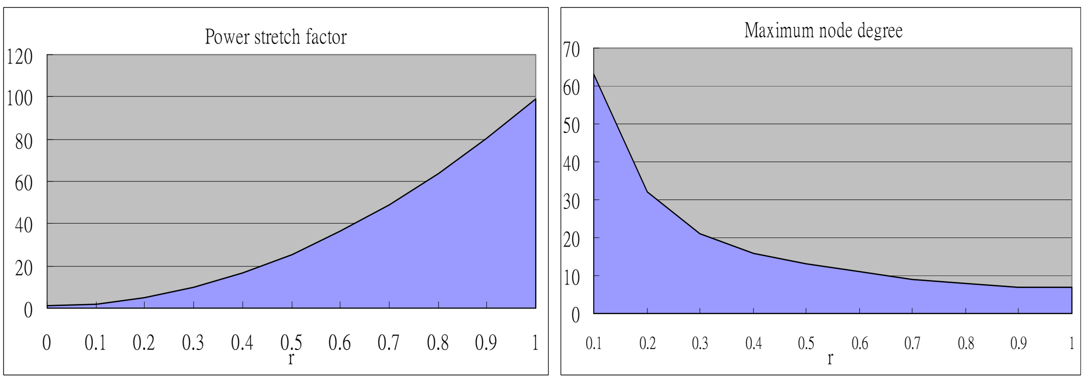
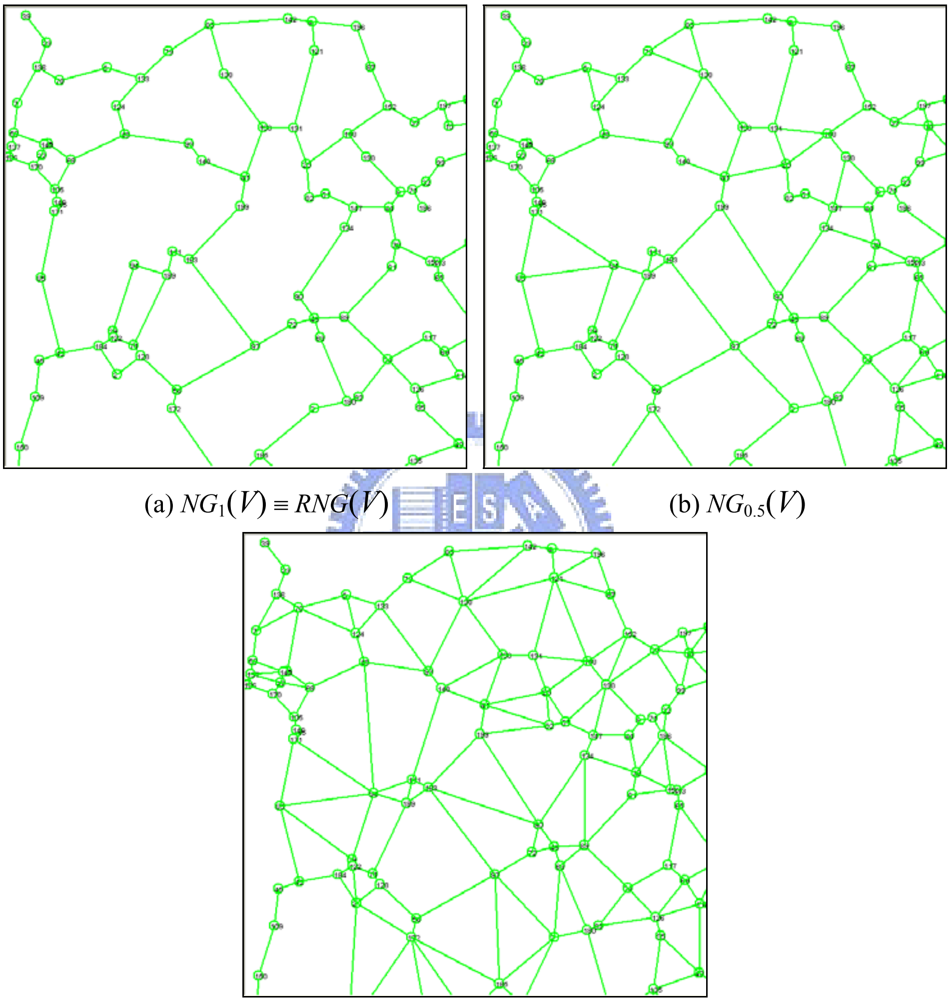


Figure 6.1: The upper bounds on the power stretch factor and maximum node degree of NG_r.

The topologies of $NG_r(V)$ of 3 different levels r are depicted in Figure 6.2. We can see that $NG_r(V)$ can construct any immediate structure between *RNG* and *GG*. A sparser topology can be constructed using a larger r, and contrarily, more routes will be preserved as a smaller r is applied.



(a) $NG_1(V) \equiv RNG(V)$ (b) $NG_{0.5}(V)$

(c) $NG_0 \equiv GG(V)$

Figure 6.2: The topologies for 3 different levels of r.

The results shown in Theorem 3.4 and Theorem 3.5 are the worst upper bound. Actually, the average values will be much better. See figures 6.3 and 6.4. The results are averaged from in 100 test cases for each parameter setting. Note that for a density ration d, $0 \le d \le 1$ it has the following means: As the nodes placements are created, we sort every nodes pair (u, v) according to their distance $\|uv\|$ in non-decreasing order. Then we set the maximum transmission range T_{max} as the d×100% percent shortest distance. It means that given a density ratio d, in the underlying $UDG(V)$, there will be at least d×100% of nodes can transmit to their neighbor using the directly a directly transmission.

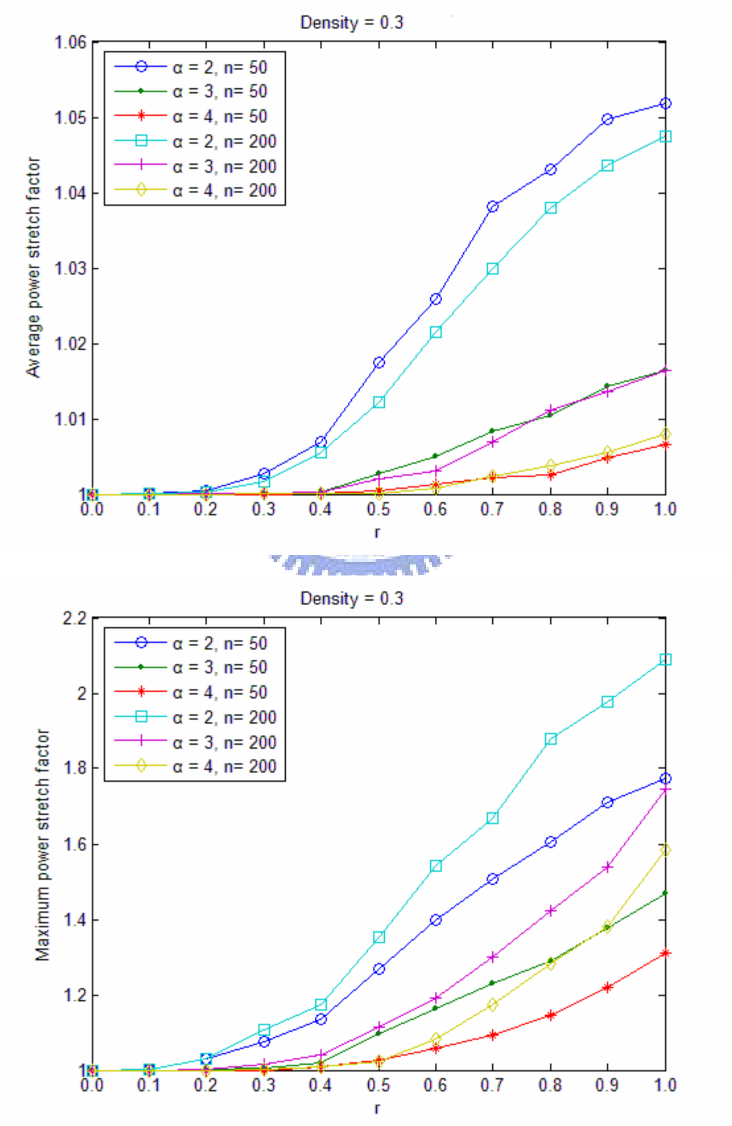


Figure 6.3: The power stretch factor of the

We can observe that when n is 100, the maximum value of $\rho(NG_r(V))$ is still within two times to the optimal value 1 in the most case, and the minimal relaying power among any nodes pair is almost closed to the optimal. The same observation is also on the node degree. The average and maximum node degrees are all limited within 4 and 8 respectively.

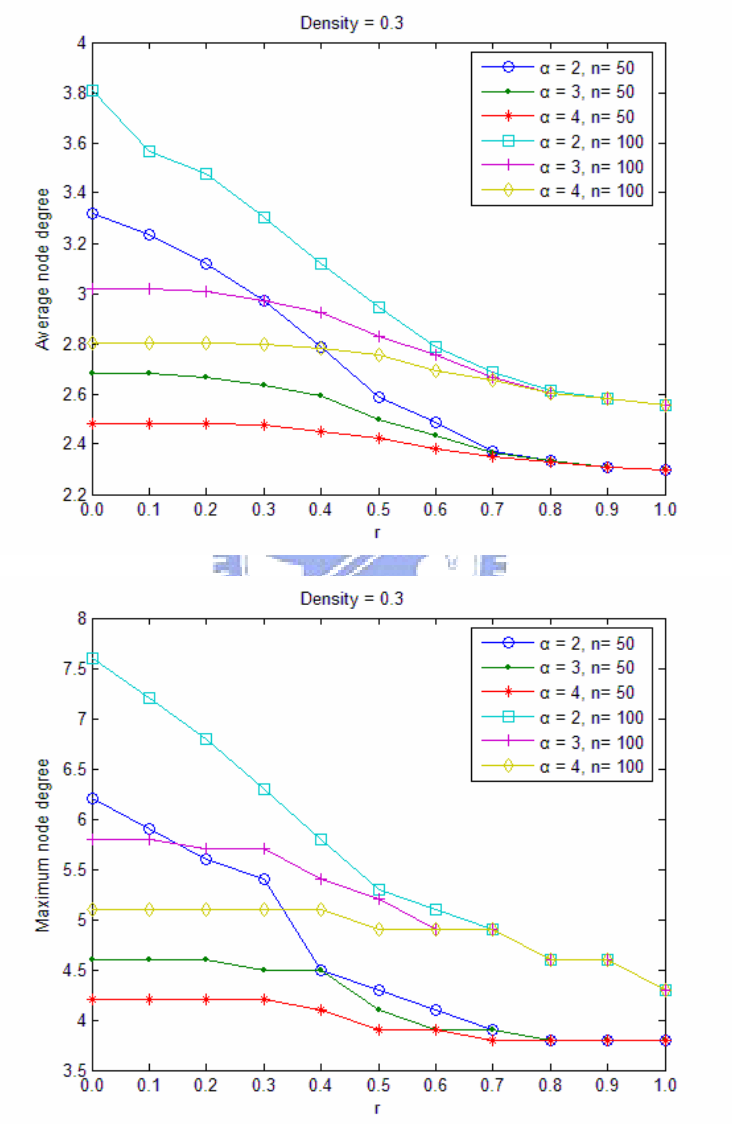


Figure 6.3: The power stretch factor and maximum node degree

Moreover, figures 6.3 and 6.4 also provide the results for the (r, α)-neighbor graph. The curves indicate the when the environment antennae factor α become worse (larger), both matrices decline significantly. This confirms our argument that a generalized structure of the r-neighborhood can gain better quantity results. The observation also tells use that our structure can adaptive well in a highly interference or obstacle environment. Therefore, the generalization is worth.

6.2 Evaluations on Shrinking Power Mechanisms

Next, we evaluate the shrink power mechanism for the (r, α)-neighborhood graph. The results of 100 test cases for 50 and 100 nodes are summarized below, where *dist* and *pwr* denote the remaining percent of radius and power of T_u in compared with the maximum radius $T_{\max}$. We can see the both *dist* and *pwr* can be strictly declined as r goes large. Such tendency does consist with the results proven in Property 5.1.

Table 6.1: The shrunken radius and power. (n = 50)

		$r = 0$		$r = 0.25$		$r = 0.5$		$r = 0.75$		$r = 1.0$	
α	*density*	*dist*	*pwr*	*dist*	*pwr*	*dist*	*Pwr*	*dist*	*pwr*	*dist*	*pwr*
2	0.1	93.61%	87.68%	92.37%	85.37%	89.08%	79.42%	85.90%	73.87%	84.69%	71.80%
	0.2	80.42%	64.76%	77.11%	59.55%	70.63%	49.95%	65.46%	42.91%	63.90%	40.89%
	0.3	67.15%	45.18%	63.55%	40.45%	57.31%	32.90%	52.71%	27.84%	51.38%	26.46%
3	0.1	89.86%	72.71%	89.60%	72.10%	88.11%	68.59%	85.78%	63.34%	84.69%	60.95%
	0.2	72.06%	37.57%	71.59%	36.84%	68.88%	32.81%	65.29%	27.96%	63.90%	26.20%
	0.3	58.62%	20.25%	58.20%	19.82%	55.73%	17.41%	52.57%	14.62%	51.38%	13.66%
4	0.1	87.96%	60.19%	87.93%	60.10%	87.33%	58.51%	85.66%	54.23%	84.69%	51.81%
	0.2	68.62%	22.36%	68.55%	22.26%	67.53%	20.96%	65.13%	18.15%	63.90%	16.82%
	0.3	55.51%	9.60%	55.45%	9.56%	54.52%	8.94%	52.44%	7.65%	51.38%	7.07%

On the other hand, as the network density, network size, or attenuate factor increase, this mechanism can perform even better. This phenomenon is due the fact that both influences will cause each node u confronting to more neighboring nodes, which in turn means that the (α, r)-enclosed region of u will be smaller. A smaller $ER_r^{\alpha}(u)$ implies a smaller λ_u. As a result, the node can transmit using a smaller T_u.

For this reason, the shrink power mechanism can perform well in a large scale as well as worse condition network.

Table 6.2: The shrunken radius and power. ($n = 200$)

		$r = 0$		$r = 0.25$		$r = 0.5$		$r = 0.75$		$r = 1.0$	
α	*density*	*dist*	*pwr*	*dist*	*pwr*	*dist*	*pwr*	*dist*	*pwr*	*dist*	*pwr*
2	0.1	61.84%	38.27%	58.07%	33.75%	52.47%	27.55%	48.56%	23.59%	47.42%	22.50%
	0.2	42.43%	18.02%	39.77%	15.83%	35.87%	12.87%	33.17%	11.01%	32.38%	10.49%
	0.3	33.40%	11.17%	31.31%	9.81%	28.24%	7.98%	26.12%	6.83%	25.49%	6.50%
3	0.1	53.49%	15.33%	53.17%	15.06%	51.00%	13.29%	48.41%	11.37%	47.42%	10.68%
	0.2	36.57%	4.90%	36.35%	4.81%	34.85%	4.24%	33.07%	3.62%	32.38%	3.40%
	0.3	28.79%	2.39%	28.62%	2.35%	27.44%	2.07%	26.04%	1.77%	25.49%	1.66%
4	0.1	50.71%	6.63%	50.65%	6.60%	49.99%	6.26%	48.32%	5.47%	47.42%	5.07%
	0.2	34.64%	1.45%	34.61%	1.44%	34.15%	1.37%	33.00%	1.19%	32.38%	1.10%
	0.3	27.27%	0.56%	27.24%	0.55%	26.89%	0.52%	25.98%	0.46%	25.49%	0.42%

6.3 Evaluations on the Mobile Protocol

In the last section, we conduct simulation study to emulate the really performance. This experiment was conducted by ns2 simulator [41]. The IEEE 802.11 distributed coordination function has been implemented in ns2 kernel. It uses RTS/CTS/DATA/ACK pattern for all unicast packets and simply sends out DATA for all broadcast packets. The implementation uses both physical and virtual carrier sense. The two-ray ground reflection model is chosen as radio propagation model. The initial energy of each node is 0.5 joules. Each node can choose a power level to transmit a packet according to distance to the next hop. We modified the route protocol DSDV [42] to find the least-energy path instead of the shortest path. That is, the transmission ranges are allowed to be adjusted: For unicasting traffic, the range is adjusted exactly to the next hop, and for broadcasting, the range is adjusted to the farthest neighbors determined by the underlying topology. Received packets will be dropped if there is no edge from the sender. This consideration ensures that packets are always transported on the constructed topology. There are 100 nodes uniformly distributed in a 1000 square meters field. The CBR traffics will be generated from 20% of nodes.

For other non-source or no-destination nodes, they will be responsible for relaying traffic. Each node will transmit in the best effort on 802.11b, i.e. data rate is 11Mbype. The maximum transmission radius is taken as 250 meters for all nodes. The mobility pattern is according to the Random Waypoint model. If not specific, the default node speed and pause time will be randomly taken from the intervals of [0, 20] m/s and [0, 10] s, respectively. For each test case, the observed results are averaged from 10 instances (a set of nodes). Each instance will be simulated over 200 sections.

First we evaluate the shrink power mechanism for variant *r*'s. To simulate the real circumstance, we allow each node configure its own *r* according to the adjusting rule in Chapter 6. The result is given in Figure 5. It shows that the construction power for the period beacon in a fully distributed circumstance can be reduced at a range from 20 % to 35% according the parameter. The conserved energy will be considerable especially when the beacon interval is intensive.

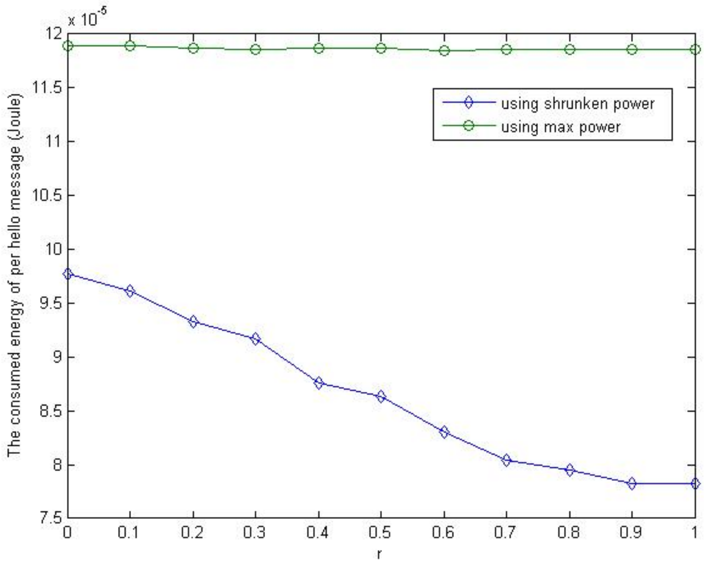


Figure 6.5: The shrinking power mechanism for variant *r's*.

The simulation is for our mobile topology control protocol. As we mentioned before, the overall energy-consumption would be influenced by many factors. However, for any communication network, like the MANET, the ultimate graph is to support transmission between send and receiver. Therefore, the overall energy efficient here is measured by the total energy required for each communication. In other words, we hope the average consumed power for each successful packet to be as low as possible. In Figure 6.6, we compare *ANGTC* protocol, the traditional proximate graphs *GG* and *RNG*, and an identical-*r* version, according to this measurement. The results show that the mobile protocol can imprecisely imrove in overall energy efficiency in comparison with other structures, especially when node mobility increases. The improvement is mainly due to the self-configure process in Chapter 6.

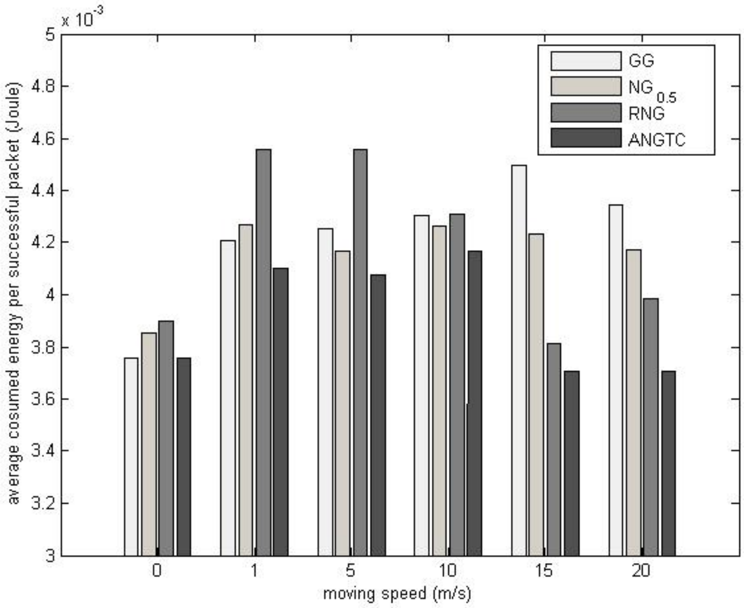


Figure 6.6: The comparison of the overall energy-efficient.

Chapter 7

Conclusions

In this dissertation, we proposed a purely localized structure to control the topology in wireless networks. We showed the worst case of the power stretch factor is an increasing function of r and the worst cast of the maximum node degree is contrarily a decreasing function of r. So, the two objectives can be adjusted in our structure. Although the power stretch factor is related to n so that our structure is not really a spanner, $\rho(NG_r(V))$ can still be bounded for some range of r. Therefore, the power stretch is partially bounded in our structure. About the maximum node degree, we proposed an upper bound derived for $d_{\max}(NG_r(V))$. However, this result is correct only no node having two or more neighbors at exactly distance. For this reason, an extended structure $NG_r^*(V)$ was given to comprehend this theorem.

Besides, the proposed structure can always result connected topology with symmetric edges. Any resulting topology is always a planar. The relations between the r-neighborhood graph and existent structures are summarized as follows. Specially, $NGr(V)$ is a general structure of both $GG(V)$ and $RNG(V)$.

To construct our structure, we proposed a 1-hop purely localized algorithm, *PLA*. It can avoid long-distance transmission when collecting information and can be efficiently done in O($n$log$n$) time when $d_{\max}(NG_r(V))$ is constant.

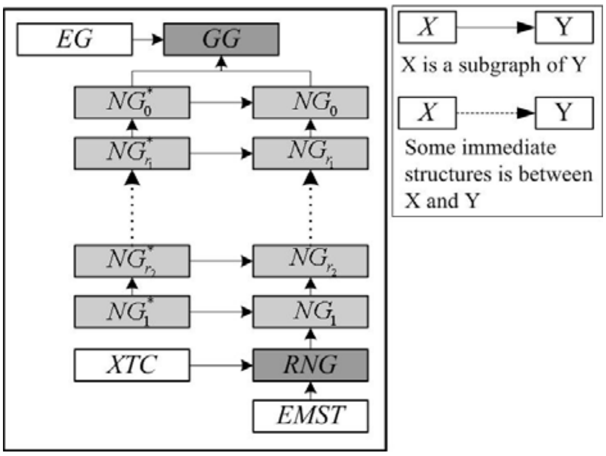


Figure 7.1: The relationships of $NG_r(V)$, $NG_r^*(V)$, $GG(V)$ and $RNG(V)$.

To cope with the mobile environment, we further proposed an adaptive topology control protocol, based on a generalized version. In this protocol, each node can self-configure it parameter to improve the overall energy efficiency, using only inherent status. We also incorporated the protocol with shrink power mechanism to reduce the topology construction power for periodic beacons.

For the further research, a localized topology control approach enables the design of localized routing protocols. For instance, the greedy route discovery in CFG [26] and GPSR [11] are based on *GG*. We anticipate that *r*-neighborhood graph could provide a concrete basis for many interesting extensions due to the sound theoretical results. Moreover, the parameter *r* can be turned to find the best settings for different scenarios. Another interesting issue for the possible further work is to evaluate the stability of the proposed structure when perfect position (range) information is not available or when the accuracy of position information differs from node to nodes.

In addition, implementation issues for the mobile protocol, such as mobility prediction, fault tolerance, using imprecise information, are the worth directions for the further research.

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Appendix

The proof of Lemma 4.2: Without a loss of generality, we assume that ||*uw*|| ≤ ||*vw*||. Let *y* be the projection of *w* on *uv* so that *yw* is perpendicular to *uv*. We can derive that

$$\|ym\| = \frac{\|vw\|^2}{2\|mv\|} - \frac{\|wm\|^2}{2\|mv\|} - \frac{\|mv\|}{2} \quad \text{and} \quad \|yx\| = \sqrt{\|wv\|^2 - \left(\frac{\|vw\|^2}{2\|mv\|} - \frac{\|wm\|^2}{2\|mv\|} + \frac{\|mv\|}{2}\right)^2}.$$

Thus,

$$\|uw\|^2 = \|wy\|^2 + (\|um\| - \|ym\|)^2 = \|wy\|^2 + (\|mv\| - \|ym\|)^2$$
$$= \|vw\|^2 - \left(\frac{\|wv\|^2}{2\|mv\|} - \frac{\|wm\|^2}{2\|mv\|} + \frac{\|mv\|}{2}\right)^2 + \left(\frac{3\|mv\|}{2} - \frac{\|vw\|^2}{2\|mv\|} + \frac{\|wm\|^2}{2\|mv\|}\right)^2$$
$$= 2\|mv\|^2 - \|vw\|^2 + 2\|wm\|^2$$

Then, power consumed by path *uwv* is as follows

$$p(uwv) = \|uw\|^\alpha + \|vw\|^\alpha = \left(2\|mv\|^2 - \|vw\|^2 + 2\|wm\|^2\right)^{\frac{\alpha}{2}} + \|vw\|^\alpha$$

From (4.1) we get $\|wm\| < l = \|uv\|\sqrt{1+2r^2}/2 = \|mv\|\sqrt{1+2r^2}$ and $\|vw\| < \|uv\|$, so

$$\left(2\|mv\|^2 - \|vw\|^2 + 2\|wm\|^2\right)^{\frac{\alpha}{2}} + \|vw\|^\alpha$$
$$\leq \left((4+4r^2)\|mv\|^2 - \|vw\|^2\right)^{\frac{\alpha}{2}} + \|vw\|^\alpha$$
$$\leq \left(\frac{(4+4r^2)}{4}\|uv\|^2 - \|uv\|^2\right)^{\frac{\alpha}{2}} + \|uv\|$$
$$= \left(r^2\|uv\|^2\right)^{\frac{\alpha}{2}} + \|uv\|^\alpha = \|uv\|^\alpha(1+r^\alpha)$$

Thus, we have that $p(uwv) \leq \|uv\|^\alpha(1+r^\alpha)$ □

Vita

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