Chapter 2

Principle

2.1 Introduction

The SiN thin film membrane is used as the optical pattern of the PBS. The characteristics of the thin film such as transmittance, reflectance and absorptance can be derived from the electromagnetic equations. The Brewster angle used to determine the incident angle will be then described. In addition, the proper thickness of the thin film micro-PBS will be decided by the matrix which is a popular method when the number of layers is more than one.

2.2 Principle

2.2.1 Transmittance, reflectance, and absorptance

If we consider a single homogeneous and isotropic layer bounded by isotropic and homogeneous layers, the structure can be described by [12]

$$n(x) = \begin{cases} n1, & x < 0, \\ n2, & 0 < x < d, \\ n3, & d < x. \end{cases} \qquad (2.1\text{-}1)$$

where *n1*,*n2*,and *n3* are the indices of refraction of medium 1,2,and 3.

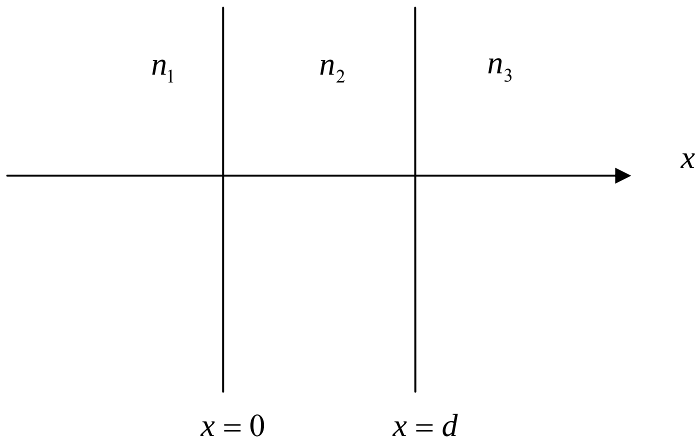

Fig.2.1 A thin homogenous layer of dielectric material

As shown in Fig. 2-1, if the plane wave is incident from the left, the electric field vector E(x) can be expressed using the form:

$$E(x) = \begin{cases} Me^{-ik_{1x}x} + Ne^{ik_{1x}x}, & x < 0, \\ Ce^{-ik_{2x}x} + De^{ik_{2x}x}, & 0 < x < d, \\ Fe^{-ik_{3x}(x-d)}, & d < x, \end{cases} \tag{2.1-2}$$

where *M*,*N*,*C*,*D*, and *F* are the complex amplitudes.

Assume the electric field vector is TE(s) polarized (the electric field is perpendicular to the plane of incidence), and then the complex amplitudes of the incident wave and the reflected and transmitted waves *M*, *N*, *F* are constant. k_{ix} is the x components of the wave vectors:

$$\mathrm{k}_{ix} = \left[\left(\frac{n_i w}{c}\right)^2 - \beta^2\right]^{1/2} = \left(\frac{w}{c}\right) n_i \cos\theta_i, \quad \text{i=1, 2, 3,} \tag{2.1-3}$$

where θ_i is the ray angle measured from the x axis. *w* is the angular frequency and c is the speed of light in vacuum.

The magnetic field can be obtained from the equation:

$$\mathrm{H} = \frac{i}{wu} \nabla \times \mathrm{E} . \qquad (2.1\text{-}4)$$

From the boundary conditions of Maxwell's equations, the tangential component of the electric and magnetic vectors are continuous across a discontinuity surface [13], so that the electric vector has the same value in each dielectric layer. Using the conditions and Snell's law, the Fresnel reflection and transmission coefficients of the dielectric interfaces for TE wave can be written as [14-16]:

$$r_{12} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} , \qquad (2.1\text{-}5)$$

$$\mathrm{t}_{12} = \frac{2 n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2} , \qquad (2.1\text{-}6)$$

$$\mathrm{r}_{23} = \frac{n_2 \cos\theta_2 - n_3 \cos\theta_3}{n_2 \cos\theta_2 + n_3 \cos\theta_3} , \qquad (2.1\text{-}7)$$

$$t_{23} = \frac{2 n_2 \cos\theta_2}{n_2 \cos\theta_2 + n_3 \cos\theta_3} , \qquad (2.1\text{-}8)$$

respectively.

And the transmission and reflection coefficients can be written as

$$\mathrm{t} = \frac{t_{12} t_{23} e^{-i\phi}}{1 + r_{12} r_{23} e^{-2i\phi}} , \qquad (2.1\text{-}9)$$

$$\mathrm{r} = \frac{r_{12} + r_{23} e^{-2i\phi}}{1 + r_{12} r_{23} e^{-2i\phi}} , \qquad (2.1\text{-}10)$$

respectively.

The parameter ψ in (2.1-9) and (2.1-10) is given by

$$\phi = \mathrm{k}_{2x}\,\mathrm{d} = \frac{2\pi d}{\lambda} n_2 \cos\theta_2 \qquad (2.1\text{-}11)$$

and is proportional to the thickness d and index n_2 of the layer.

The expression for the transmission and reflection coefficients of the TM(p) wave are the same, except the coefficients t_{12}, t_{23} and r_{12}, r_{23} must be those associated with the TM waves.

If media 1 and 3 are no absorbing, reflectance (R) defined as the energy reflected from the dielectric structure and transmittance (T) are given by

$$\mathrm{R}=|r|^2, \qquad (2.1\text{-}12)$$

$$\mathrm{T}=\frac{n_3\cos\theta_3}{n_1\cos\theta_1}|t|^2. \qquad (2.1\text{-}13)$$

Regardless of whether the layer (medium 2) is absorbive, both Eqs. (2.1-12) and (2.1-13) can be used.

Absorptance (**A**)defined as the fraction of energy dissipated is given by

$$\mathbf{A} = 1 - \mathrm{R} - \mathrm{T}. \qquad (2.1\text{-}14)$$

2.2.2 Brewster angle

TE and TM states do not behave in the same way, depending on the angle of incidence. In particular, if the incident angle is at a specific angle, so-called Brewster angle, the reflection coefficient of the TM mode will vanish completely. Therefore, the TM mode will totally transmit, leaving the reflected light to be TE mode. The Brewster angle, noted as θ_B, is given by

$$\tan\theta_B = n_t / n_i \qquad (2.1\text{-}15)$$

where n_t is the refractive index of the thin film and n_i is the material/air index from the incident side.

2.2.3 2 x 2 Matrix formulation for a thin film

The proper thickness of TE mode can be determined by the matrix formulation. As shown in Fig. 2-2, the electric field E(x) consists of a right-propagating and left-propagating waves can be expressed as the form:

$$\mathrm{E(X)} \equiv \mathrm{A(x)} + \mathrm{B(x)}, \qquad (2.2\text{-}1)$$

Let A(x) represent the amplitude of the right right-traveling wave and B(x) be that of the left-traveling one. To illustrate the matrix method, we define

$A_1 = A(0^-)$,

$B_1 = B(0^-)$,

$A_{2'} = A(0^+)$,

$B_{2'} = B(0^+)$,

$A_2 = A(d^-)$,

$B_2 = B(d^-)$,

$A_{3'} = A(d^+)$,

$B_{3'} = B(d^+)$,

Where 0^- represents the left side of the interface, x=0, and 0^+ represents the right side of the same interface. Similarly, d^- and d^+ are defined for the interface at x=d.

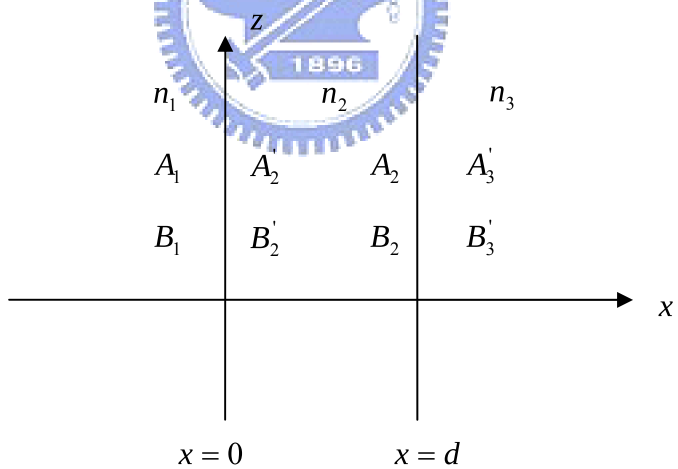

Fig.2.2 A thin layer of dielectric material

The transmission matrices that link the amplitudes of the waves on the two sides of the interfaces, noted D_{12} and D_{23}, can be expressed by

$$
\mathrm{D}_{12} = \begin{pmatrix} \frac{1}{2}\left(1+\frac{k_{2x}}{k_{1x}}\right) & \frac{1}{2}\left(1-\frac{k_{2x}}{k_{1x}}\right) \\ \frac{1}{2}\left(1-\frac{k_{2x}}{k_{1x}}\right) & \frac{1}{2}\left(1+\frac{k_{2x}}{k_{1x}}\right) \end{pmatrix} \quad \text{for TE wave} \qquad (2.2\text{-}2)
$$

And

$$
\mathrm{D}_{12} = \begin{pmatrix} \frac{1}{2}\left(1+\frac{n_2^2 k_{1x}}{n_1^2 k_{2x}}\right) & \frac{1}{2}\left(1-\frac{n_2^2 k_{1x}}{n_1^2 k_{2x}}\right) \\ \frac{1}{2}\left(1-\frac{n_2^2 k_{1x}}{n_1^2 k_{2x}}\right) & \frac{1}{2}\left(1+\frac{n_2^2 k_{1x}}{n_1^2 k_{2x}}\right) \end{pmatrix} \quad \text{for TM wave.} \qquad (2.2\text{-}3)
$$

The expression for D_{23} is similar to those of D_{12}, except that the subscript indices have to be replaced with 2 and 3. Equations (2.2-2) and (2.2-3) can be written formally as

$$
\mathrm{D}_{12} = \frac{1}{t_{12}} \begin{pmatrix} 1 & r_{12} \\ r_{12} & 1 \end{pmatrix} \equiv D_1^{-1} D_2 \qquad (2.2\text{-}4)
$$

where t_{12} and r_{12} are the Fresnel transmission and reflection coefficients, respectively, and are given by

$$
r_{12} = \begin{cases} \dfrac{k_{1x} - k_{2x}}{k_{1x} + k_{2x}} & \text{for TE wave} \\ \\ \dfrac{n_1^2 k_{2x} - n_2^2 k_{1x}}{n_1^2 k_{2x} + n_2^2 k_{1x}} & \text{for TM wave} \end{cases} \qquad (2.2\text{-}5)
$$

And

$$
t_{12} = \begin{cases} \dfrac{2k_{1x}}{k_{1x} + k_{2x}} & \text{for TE wave} \\ \\ \dfrac{2n_1^2 k_{2x}}{n_1^2 k_{2x} + n_2^2 k_{1x}} & \text{for TM wave} \end{cases} \qquad (2.2\text{-}6)
$$

Respectively.

where $k_{\alpha x} = n_\alpha \dfrac{w}{c} \cos\theta_\alpha$. (2.2-7)

Then the amplitudes A_1, B_1 and A_3', B_3' are related by

$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = D_1^{-1} D_2 P_2 D_2^{-1} D_3 \begin{pmatrix} A_3' \\ B_3' \end{pmatrix} \qquad (2.2\text{-}8)$$

where D_1, D_2, and D_3 are the dynamical matrices given by

$$D_\alpha = \begin{cases} \begin{pmatrix} 1 & 1 \\ n_\alpha \cos\theta_\alpha & -n_\alpha \cos\theta_\alpha \end{pmatrix} & \text{for TE wave,} \\ \begin{pmatrix} \cos\theta_\alpha & \cos\theta_\alpha \\ n_\alpha & -n_\alpha \end{pmatrix} & \text{for TM wave,} \end{cases} \qquad (2.2\text{-}9)$$

where θ_α is the ray angle in each layer and is related to β and $k_{\alpha x}$ by

$$\beta = n_\alpha \frac{w}{c} \sin\theta_\alpha \,, \qquad (2.2\text{-}10)$$

And P_2 is the so-called propagation matrix, which accounts for propagation through the bulk of the layer

$$P_2 = \begin{pmatrix} e^{i\phi_2} & 0 \\ 0 & e^{-i\phi_2} \end{pmatrix} \qquad (2.2\text{-}11)$$

And ϕ_2 is given by $\phi_2 = k_{2x} d$. (2.2-12)

The column vectors representing the plane-wave amplitudes in each layer are related by a product of 2×2 matrices in sequence. A dynamical and propagation matrix can represent each side of an interface and the bulk of each layer, respectively.

We now recall the scheme of Fig. 2-2, with a collimated incident light of wavelength λ , according to Equation (2.2-8), the characteristic matrix of a thin dielectric film of thickness *x* is given by

$$M[x] = \begin{bmatrix} \cos\left(\frac{2\pi}{\lambda} n_f x \cos(\theta_f)\right) & -\frac{i}{p_f} \sin\left(\frac{2\pi}{\lambda} n_f x \cos(\theta_f)\right) \\ -ip_f \sin\left(\frac{2\pi}{\lambda} n_f x \cos(\theta_f)\right) & \cos\left(\frac{2\pi}{\lambda} n_f x \cos(\theta_f)\right) \end{bmatrix} \qquad (2.2\text{-}13)$$

Where θ_f is the incident angle inside the film, $p_f = \sqrt{(\varepsilon_f / \mu_f)} \cos(\theta_f)$ for TE, $p_f = \sqrt{\mu_f / \varepsilon_f} \cos(\theta_f)$ for TM, and μ_f , ε_f are the permeability and permittivity of the thin film, respectively.

The reflectance R and transmittance T can be determined from M(x). From (2.2-13), R and T are periodical function of x with the period

$$\Delta x = \frac{\lambda}{2n_f \cos\theta_f} \qquad (2.2\text{-}14)$$

2.3 Summary

The characteristics of the thin film and the related formulation are described. The transmittance, reflectance and absorption for evaluating the thin film are defined. The Brewster angle implies the transmittance of TM mode depending on the incident angle. The reflectance of TE mode can be determined by the characteristic matrix described by the matrix method.