Chapter 3

Suppression of phase and supermode noise in a harmonic mode-locked erbium-doped fiber laser with a semiconductor optical amplifier based high-pass filter

3.1 Introduction

Various techniques have recently been proposed to reduce the mode-beating induced supermode noise (SMN), environmental perturbations, pumping power fluctuations and single-sided-band (SSB) phase noise, etc. in mode-locked erbium-doped fiber lasers (EDFLs). Sanders *et al.* [3.1] reduced the SMN from an mode-locked EDFL by using intra-cavity spectral filtering in contrast to the regeneratively mode-locking [3.2]. With the use of an intra-cavity optical band-pass filter (OBPF) in an EDFL, the SMN suppression ratio can be as high as 60-70 dB [3.3, 3.4] and the relaxation oscillation components can be completely eliminated [3.5]. Seo *et al.* have minimized the SSB phase noise (or timing jitter) of EDFL with external injection-seeding [3.6]. The suppression in SMN and intensity noises of mode-locked EDFL was recently demonstrated by adding an intra-cavity semiconductor optical amplifier (SOA) based high-pass filter [3.7, 3.8]. The maximum SMN suppression ratio of up to 33 dB was reported [3.9], however, the SSB phase noise characteristics of the SOA-filtered EDFL has never been investigated. In this paper, we theoretically and experimentally demonstrate the effect of SOA biased condition on both the SSB phase noise and the SMN suppression ratio of a mode-locked EDFL. The reduction of SSB phase noise and the enhancement of SMN suppression ratio in the mode-locked EDFL with an SOA at high-gain condition are discussed. The degraded SSB phase noise due to the adding of SOA can be minimized by driving the SOA at nearly transparent mode and by adding an OBPF concurrently.

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3.2 Experimental Setup

The experimental setup is shown in Fig. 3.1.

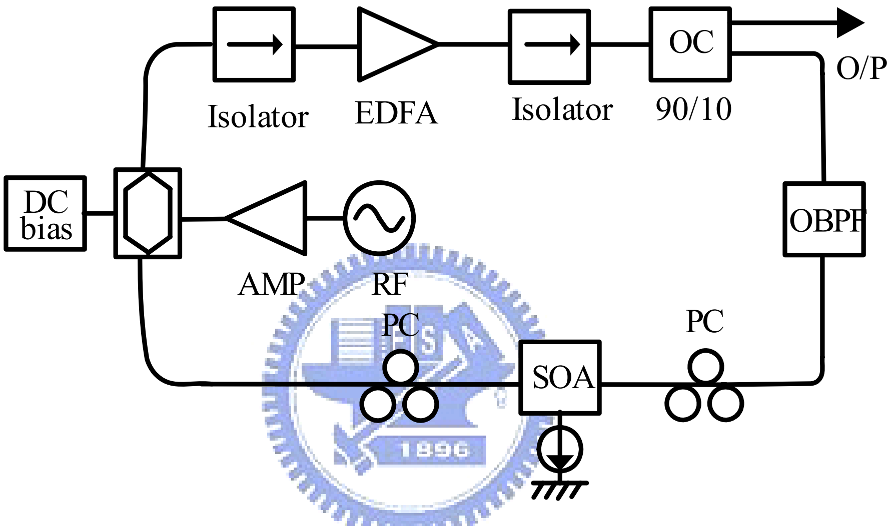

Fig. 3.1 The experimental setup of SOA filtered EDFL. MZM: Mach-Zehnder intensity modulator; PC: polarization controller; OC: optical coupler; OBPF: optical band-pass filter; EDFA: erbium-doped fiber amplifier; SOA: semiconductor optical amplifier.

The small-signal power gain of the erbium doped fiber amplifier (EDFA) can be as high as 31 dB, and the total cavity loss of the EDFL is about 23 dB. A commercial fiber-pigtailed SOA with small-signal gain and saturation output power of 25 dB and 8 dBm, respectively, is used as an SMN filter in the EDFL. A LiNbO3 Mach-Zehnder intensity modulator (MZM) biased at half-wave voltage ($V_\pi \cong 8$ V) is driven by a microwave synthesizer at 22 dBm and 977.64 MHz. A pair of polarization controllers (PCs) and Faraday optical isolators are employed to optimize the polarization orientation of the circulating pulses and ensure the unidirectional propagation. The output coupling ratio of the EDFL is 10%. The length of the EDFL ring cavity is 32.1 m (corresponding to a longitudinal mode spacing of 6.24 MHz). The OBPF (JDS, TB1500B) inserted between the EDFA and SOA exhibits a 3-dB bandwidth of 1.38 nm, which enhances the gain profile of the SOA at 1532 nm and reduces the ASE components over a wide wavelength range. The SSB phase noise spectral power density of the mode-locked EDFL pulse-train are measured by a high-speed photodetector (New Focus Model 1014) and an RF spectrum analyzer (HP8565E).

3.3 Theoretical model

3.3.1 Spontaneous emission induced phase noise

The electric field of the amplifier output as whiten as [3.10]

$$E(t) = E_s \sin[2\pi\nu t + \phi_n(t)] + a_n(t)\sin(2\pi\nu t) + b_n \cos(2\pi\nu t) \quad (1)$$

where E_s denotes the amplitude of the output signal. ν is the frequency of the signal light, $\phi_n(t)$ is the indirect phase noise induced by the carrier density fluctuation, and $a_n(t)$ and $b_n(t)$ are in-phase and out-of-phase noise due to amplified spontaneous emission, respectively. Let Fourier transforms of $a_n(t)$ and $b_n(t)$ be $A_n(f)$ and $B_n(f)$,respectively. These noises are not correlated:

$$< A_n(f) * B_n(f) >= 0 \quad (2)$$

but have the same spectral density

$$S_A(f) = S_B(f) = (2/\varepsilon_0 cA) h\nu n_{sp}(G-1) \quad (3)$$

where c denotes the velocity of light, A is the amplifier cross section, n_{sp} is the spontaneous emission factor, and G represents the amplifier gain.

When the bandwidth of the spontaneous emission is sufficiently reduced by an optical bandpass filter, the beat noise between the signal and the spontaneous emission is the main cause of the intensity noise. In such a case, the output power fluctuation in the time domain is given as

$$\Delta P_{out}(t) = (\varepsilon_0 cA/2)2a_n(t)E_s \tag{4}$$

The power spectrum of the fluctuation is then written as

$$S_p(f) = (\varepsilon_0 cA/2)4GP_{in}S_A(f) = 4h\nu n_{sp}G(G-1)P_{in} \tag{5}$$

where P_{in} is the average input power.

The total phase fluctuation is given, when $\phi_n(t)$ is small, as

$$\theta(t) = \phi_n(t) + b_n(t)/E_s \tag{6}$$

The second term is the direct phase noise induced by the spontaneous emission indicated as the process 1 in Fig. 3.2. From (3), the power spectrum of this term is white and is given as

$$S_{\phi 1}(f) = h\nu(G-1)n_{sp}/GP_{in} \tag{7}$$

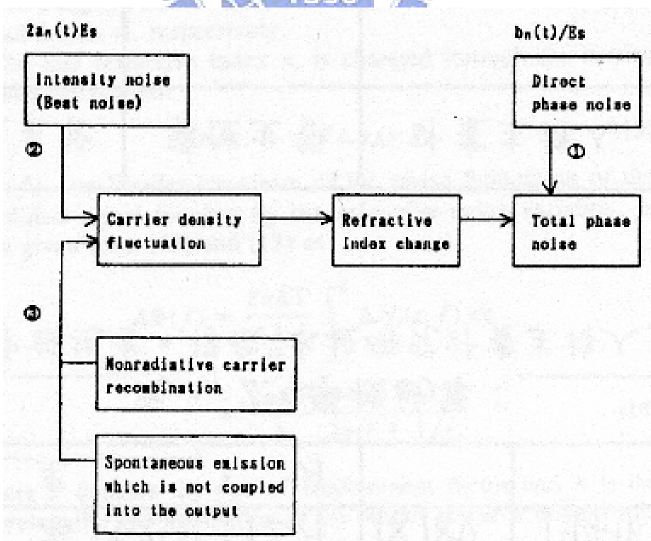

Fig. 3.2 Generation process of the phase noise from semiconductor optical amplifiers (from IEEE J. Quantum Electron., vol. 27, pp. 416-422, 1991).

3.3.2 Stimulated emission induced phase noise

In what follows, we discuss the origin of the first term of (6), which is induced by indirect processes through the carrier density fluctuation indicated as processes 2 and 3 in Fig. 3.2. The carrier density fluctuation modulates the real refractive index, which in term generates the phase noise. The carrier density fluctuation is associated with the intensity fluctuation through the induced emission process (process 2). The carrier recombination due to spontaneous emission and the nonradiative carrier recombination (process 3) are also responsible for the carrier density fluctuation.

From the rate equations for the carrier density n_e(z, t) and the photon density n_{ph}(z, t), we have

$$\frac{\partial n_e}{\partial t} - \frac{J}{ed} + \frac{n_e}{\tau_e} = -\frac{\partial n_{ph}}{\partial z}\upsilon_g \tag{8}$$

where J denotes the injection current density, d is the active layer thickness, and v_g, is the group velocity of the light in the amplifier. The carrier lifetime τ_e is determined from the spontaneous emission and the nonradiative carrier recombination in the linear operation mode of the amplifier. Since these carrier recombination processes are stochastic, the carrier density is inevitably fluctuating; however, for the moment, we ignore this effect and consider only the process 2. Equation (8) shows that the carrier density fluctuation is induced by the photon number fluctuation.

By integrating (8) over the entire length L of the amplifier, we have

$$\int_0^L \left(\frac{\partial n_e}{\partial t} - \frac{J}{ed} + \frac{n_e}{\tau_e} \right) dz = -\upsilon_g n_{ph}(L,t) \tag{9}$$

where we assume that the amplifier gain is sufficiently high. In the stationary state, (9) reduces to

$$\int_0^L \left(-\frac{J}{ed} + \frac{\langle n_e \rangle}{\tau_e} \right) dz = -\upsilon_g \langle n_{ph}(L,t) \rangle \tag{10}$$

Small deviations Δn_{ph}, and Δn_e, from their stationary values obey

$$\int_0^L \left(\frac{\partial \Delta n_e}{\partial t} + \frac{\Delta n_e}{\tau_e} \right) dz = -\upsilon_g \Delta n_{ph}(L,t) \tag{11}$$

The Fourier transform of (11) gives

$$\int_0^L \Delta N_e(z,f)dz = \frac{-\upsilon_g \Delta S_{out}(f)}{2\pi j f + 1/\tau_e} \tag{12}$$

where $\Delta S_{out}(f)$ and $\Delta N_e(z, f)$ are Fourier transforms of $\Delta n_{ph}(\mathrm{L},t)$ and $\Delta n_e(z, t)$, respectively.

The real refractive index n, is changed through the carrier density fluctuation

$$\Delta n_r(z,t) = K \Delta n_e(z,t) \tag{13}$$

$\Delta \Phi(f)$, the Fourier transform of the phase fluctuation of the amplified signal induced by the refractive-index variation, is then given from (12) and (13) as

$$\Delta \Phi = \frac{2\pi K \Gamma}{\lambda} \left(\frac{-\upsilon_g \Delta S_{out}}{2\pi j f + 1/\tau_e} \right) \tag{14}$$

where Γ denotes the optical confinement factor and λ is the wavelength. The fluctuation of the output power is related with the photon density variation as

$$\Delta P_{out}(t) = A \upsilon_g h \nu \Delta n_{ph}(L,t) \tag{15}$$

From (3), (5), (14), and (15), we have the expression for the phase-noise spectrum related to the process 2

$$S_{\phi 2} = \frac{\left(\frac{2\pi K \Gamma}{\lambda A} \right)^2 \cdot 4G(G-1) n_{sp} \tau_e{}^2 P_{in}}{\left[(2\pi f \tau_e)^2 + 1 \right] \cdot h \nu} \tag{16}$$

3.3.3 High-pass filtering Effect of SOA

In Agrawal's theory, the integrated gain h is defined as

$$h(\tau) = \int_0^L g(z,\tau)dz \qquad (17)$$

where g is gain, τ is reduced time, z is longitudinal position, and L is the length of the SOA. The differential equation based on the rate equation is given by

$$\frac{dh}{dt} = \left(\frac{g_0 L - h}{\tau_c}\right) - \frac{P_{in}(\tau)}{\tau_c P_s}\left(e^h - 1\right) \qquad (18)$$

where $P_{in}(\tau)$ is the power envelope of the input signal, g_o is the unsaturated small-signal gain, P_s is the saturation power, and τ is the carrier lifetime. Small-signal analysis of this equation gives the SOAs transfer function in the frequency domain (see the Appendix for details of the calculation) [3.8]

$$X(\omega) = G_{CW}\frac{1+\dfrac{\ln(G_0/G_{CW})}{G_{CW}-1} - i\omega\tau_c}{1+G_{CW}\dfrac{\ln(G_0/G_{CW})}{G_{CW}-1} - i\omega\tau_c} \qquad (19)$$

where ω is the modulation angular frequency, G_{CW} is CW amplifier gain, and Go is unsaturated amplifier gain. G_{CW} and G_o are given by

$$G_{CW} = G_0 \exp[-(G_{CW}-1)\frac{P_{in}}{P_s}] \qquad (20)$$

$$G_0 = \exp(g_0 L) \qquad (21)$$

where P_{in} is CW input power.

The frequency response of the SOA is defined by $|X(\omega)|^2$. This gives the frequency response of the SOAs output power to modulation of the input power. The schematic diagram of the frequency response is shown in Fig. 3.3.

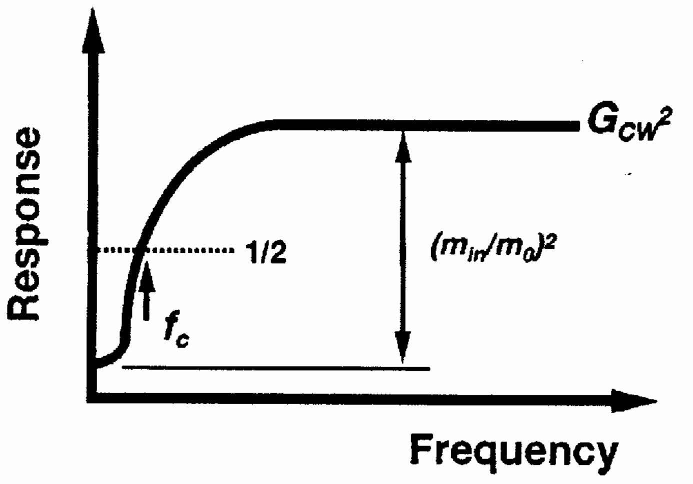

Fig. 3.3 Schematic diagram of the frequency response of an SOA (from IEEE J. Sel. Top. Quantum Electron., vol. 7, pp. 328-333, 2001).

At high frequencies, the response is flat and is proportional to the CW power gain. The modulation depth of the output signal is given by

$$m = m_{in}\left|X(\omega)\right| / G_{cw} \quad (22)$$

The modulation depth is, therefore, conserved at high frequencies. At low frequencies, the modulation depth is reduced. The modulated light changes the gain because it affects the carrier density. This effect is called as self-gain modulation. Self-gain modulation does not occur at high frequencies because it is limited by the carrier lifetime. The frequency at which the modulation depth becomes $1/\sqrt{2}m_{in}$ (-3 dB in electrical power) compared with the input signal is given by

$$f_c = \frac{1}{2\pi\tau_c}\sqrt{G_{CW}^2 b^2 + 2G_{CW} b - 2b^2 - 4b - 1} \quad (23)$$

$$b = \frac{\ln(G_0 / G_{CW})}{G_{CW} - 1} \quad (24)$$

The ratio of m/m_{in} gives the response reduction efficiency, and the bandwidth is limited by f_c. The modulation depth when the frequency is close to zero is given by

$$m_0 = m_{in}\frac{1+b}{1+G_{CW} b} \quad (25)$$

When $G_{CW}= Go$ (unsaturated case), $m_o= m_{in}$. As G_{CW} decreases, the modulation depth ratio m_o/m_{in} decreases.

This analysis indicates that an SOA acts as a high-pass filter. The limiting function of the SOA is only effective at low frequencies and is enhanced by increasing the input power. The transfer function can be used to reduce the mode partition noise because the noise mainly appears in low-frequency regions.

3.3.4 Jitter measurement

The SSB phase noise spectral power density of the mode-locked EDFL pulse-train are measured by a high-speed photodetector (New Focus Model 1014) and an RF spectrum analyzer (HP8565E). By subtracting the SSB phase noise spectrum at a higher harmonic frequency (for example, the 10^{th} harmonic component, n = 10) with that at the fundamental frequency (n = 1) of the EDFL pulse, the rms timing jitter σ in a bandwidth extending from f_L to f_H is given by [3.11]:

$$\sigma(f)=\frac{1}{2\pi f_0}\left\{2\int_{f_L}^{f_H}\left[\left(10^{L_n(f)/10}-10^{L_1(f)/10}\right)/\left(n^2-1\right)\right]df\right\}^{1/2} \tag{26}$$

where f_L and f_H are integration boundaries, $L_1(f)$ and $L_n(f)$ are phase noise power spectral densities of fundamental and nth harmonics signals, respectively. The n denotes the harmonic number and f_0 is the repetition frequency of the laser pulse.

3.4 Results and Discussion

3.4.1 Phase noise, Timing jitter and Supermode noise suppression ratio with SOA

For a mode-locked EDFL without intra-cavity SOA, the pulsewidth and timing jitter are 36 ps and 0.6 ps, respectively. The SMN suppression ratio of such a general EDFL is only 32 dB, as illustrated in Fig. 3.4(a). The insertion of an SOA and the OBPF greatly enhances the SMN suppression ratio and reduces the intensity fluctuations, as shown in Figs. 3.4(c) and 3.4(d). When operating at nearly transparent condition, the SOA exhibits a small-signal gain of only 14 dB and a saturation output power of about 0.7 mW. Typically, the extremely long upper-level lifetime of excited erbium ions in EDFL (~10 ms) may lead to a large power fluctuation (see Fig. 3.4(b)) and a strong supermode beating effect of the output pulse. Since the frequency of the SMN in EDFL is primarily in the low frequency region, which can thus be suppressed by adding an SOA due to its relatively fast carrier lifetime (0.5-1 ns) and gain saturation effect [3.8]. Although the SOA based high-pass filter greatly enhances the SMN suppression ratio, the SSB phase noise and timing jitter of the EDFL are simultaneously degraded from -114 dBc/Hz to -96 dBc/Hz and from 0.6 ps to 1.4 ps, respectively. The degraded phase noise performance is mainly attributed to both the stimulated and spontaneous emissions generated from the SOA [3.12, 3.13]. In principle, the adding of SOA introduces SSB phase noise through the fluctuation in carrier density, which results from both the stimulated and the spontaneous emissions. The spontaneous emission induced SSB phase noise is written as [3.10]

$$S_{\phi 1}(f) = h\nu(G-1)n_{sp} / GP_{in}$$

where h is Planck's constant, ν is optical frequency, G is amplifier gain, n_{sp} is the spontaneous emission factor, and P_{in} is input optical power. In contrast, the SSB phase noise is also correlated with the intensity noise describe as [3.10]

$$S_{\phi 2}(f) = \frac{\left(\frac{2\pi K\Gamma}{\lambda A}\right)^2 4G(G-1)n_{sp}\tau_e^2 P_{in} / h\nu}{(2\pi f\tau_e)^2 + 1}$$

where K = $\Delta n_r (z, t)/\Delta n_e (z, t)$, Δn_r is real refractive index change, Δn_e is fluctuation of carrier density, Γ is optical confinement factor, λ is wavelength, A is amplifier cross section, and τ_e is carrier lifetime.

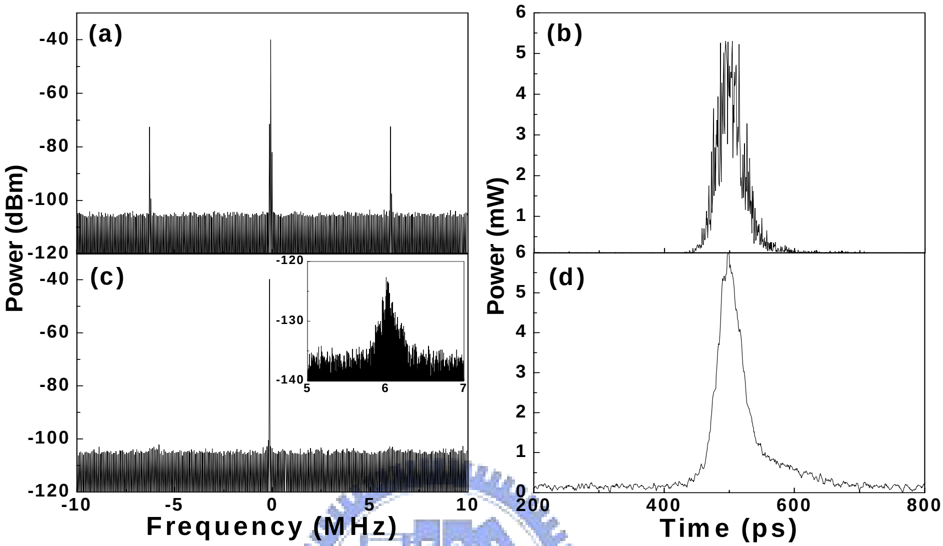

Fig. 3.4 Upper: (a) SMN spectrum (measured at VBW and RBW of 300 Hz) and (b) pulse shape of mode-locked EDFL without intra-cavity SOA filter. Lower: (a) SMN spectrum and (b) pulse shape of mode-locked EDFL with intra-cavity SOA and OBPF. Inset: the SMN spectrum measured at VBW and RBW of 1 Hz.

In experiment, the SSB phase noise is changed from -96 to -100 dBc/Hz (measured at 100 kHz offset frequency from carrier) and the SMN suppression ratio is enhanced from 62.4 to 76 dB by increasing the driving current of the SOA from 45 to 76 mA. The minimum SSB phase noise of -104.2 dBc/Hz is observed by driving the nearly transparent current (~66 mA), as shown in Fig. 3.5. It is evident that the SMN suppression ratio slightly improves as the SOA gain increases, especially when the SOA switches from absorption to gain regimes. However, the SSB phase noise increases rapidly at higher driving current of the SOA, whereas the SMN suppression ratio remains unchanged. These results are interpreted that the optimized driving current of the SOA based high-pass filter for the EDFL is nearly at its transparent current condition, while the SMN suppression ratio and SSB phase noise (-104.2 dBc/Hz) can be improved by 13.6 dB and 8.2 dB, respectively. Nonetheless, the nearly transparent SOA still causes the EDFL pulsewidth broadening from 36 ps (without the SOA) to 61 ps (with the SOA).

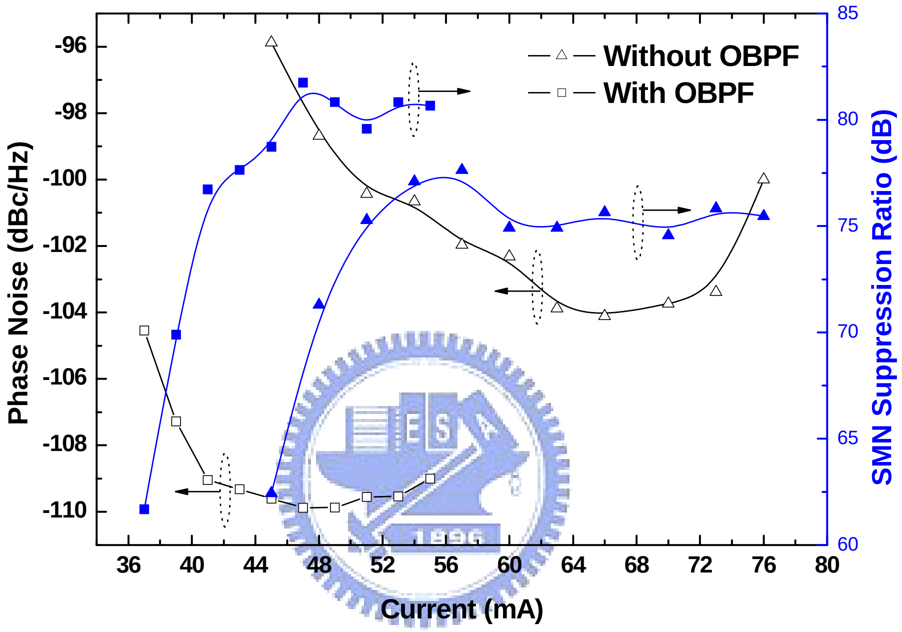

Fig. 3.5 The SMN suppression ratios and the SSB phase noises of the mode-locked EDFL with SOA filter (hollow and solid triangles) or with SOA and OBPF filters (hollow and solid squares) at different SOA currents.

3.4.2 Phase noise, Timing jitter and Supermode noise suppression ratio with SOA and OBPF

The pulsewidth broadening and phase-noise degradation problems are solved by concurrently adding an intra-cavity OBPF and adjusting the SOA driving current. The SOA exhibits a minimum phase-noise when operating at unitary gain regime, and the SSB phase noise can further be decreased by suppressing the ASE in the

SOA-EDFL link with an OBPF. In more detail, the best SMN suppression ratio is improved from 76 to 81 dB (see inset of Fig. 3.4(c)) and the SSB phase noise is reduced from -104 to -110 dBc/Hz by increasing the driving current of the SOA from 37 mA to 47 mA, as shown in Fig. 3.5. This also leads to the shortened pulsewidth and reduced timing jitter of 42 ps and 0.9 ps, respectively. In comparison, the SMN suppression ratio of the mode-locked EDFL with intra-cavity SOA and OBPF is already 10 dB better than those systems using only OBPF [3.3-3.5]. The minimum SSB phase noise is found at the transparent current of the SOA (~47 mA), which is also comparable with previous reports [3.2]. The mode-locked pulse is distorted and fluctuated at SOA beyond transparent current. Note that the transparent current of the SOA is slightly varied with the intra-cavity power of the EDFL.

Obviously, the SSB phase noise of the EDFL can be greatly suppressed by driving the SOA at its transparent current without sacrificing the SMN suppression ratio. To clarify, the theoretically simulated SSB phase noise of the SOA at different gain conditions is illustrated in Fig. 3.6, which is obtained by fitting Eqs. (7) and (16).

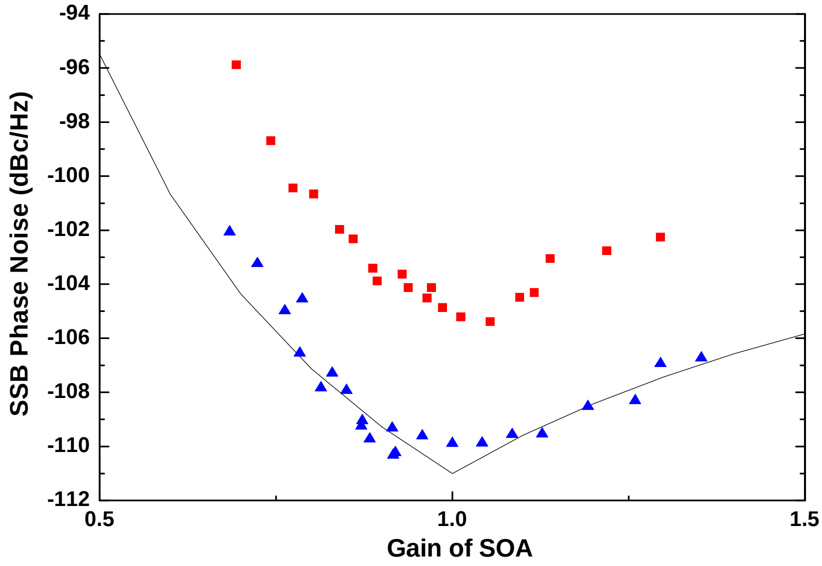

Fig. 3.6 The simulated (solid line) and measured SSB phase noises as a function of the SOA gain for the SOA-filtered EDFL without (solid square) and with OBPF (up triangle).

The spontaneous emission dominates the phase noise of the EDFL-SOA link as the SOA operates at below transparent gain condition, whereas the SSB phase noise is mainly contributed by the stimulated emission of the SOA operated at beyond transparent region. Note that the signal-spontaneous beating induced noise is inevitable, however, the spontaneous-spontaneous beating noise can be reduced by loading a narrow-band OBPF matched to the signal frequency since it arises from the beating of the ASE components themselves over a wide gain spectrum. The reduction in spontaneous beating induced SSB phase noise by using OBPF has been confirmed in our experiments. When the OBPF is not inserted in the EDFL, the measured SSB phase noise is far beyond the theoretical curve and is significantly degraded at higher SOA gain conditions. The inserted OBPF reduces the SSB phase noise to the theoretical limit. After passing through a fiber link with 11-m DCF (with an normal group velocity dispersion of -92 ps/km·nm) and 800-m standard SMF (with a anomalous group velocity dispersion of 18 ps/km·nm), the amplified EDFL pulse can be further compressed to 3.1 ps with a spectral linewidth of 1.6 nm, as shown in Figs. 3.7(a) and (b). The estimated time-bandwidth product of 0.63 is nearly transform-limit.

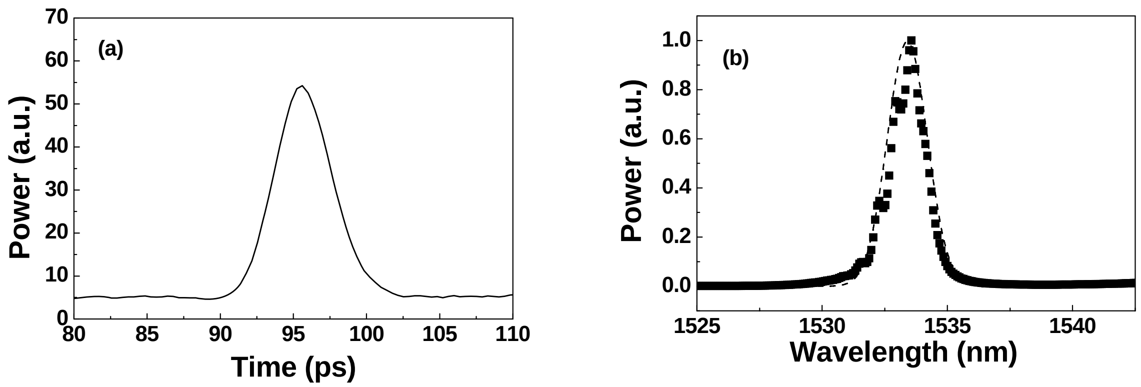

Fig. 3.7 (a) Autocorrelation trace and (b) lasing spectrum of a compressed EDFL pulse after passing through a fiber link with 11-m DCF and 800-m standard SMF.

3.5 Conclusion

In conclusion, we use an intra-cavity SOA and OBPF to improve the supermode noise suppression ratio without sacrificing the SSB phase noise of an actively harmonic mode-locked EDFL. The inserted SOA greatly enhances the SMN suppression ratio from 32 dB to 76 dB at a cost of larger SSB phase noise (degrading from -114 dBc/Hz to -96 dBc/Hz) and timing jitter (degrading from 0.6 ps to 1.4 ps). The SMN suppression ratio saturates at higher driving current of SOA, whereas the pulsewidth of the EDFL is significantly broadened from 36 ps to 61 ps. By driving the SOA at transparent condition and adding an OBPF, the SMN suppression ratio is up to 81 dB and the SSB phase noise further reduces to -110 dBc/Hz even. The EDFL pulsewidth and jitter can be reduced to 42 ps and 0.98 ps, respectively. The amplified pulse can be shortened to 3.1 ps with a time-bandwidth product of 0.63 after pulse compression with a DCF and SMF link. Theoretical and experimental results conclude that the optimized driving current of the SOA based high-pass filter for the EDFL is its transparent current, while the SSB phase noise can be greatly suppressed without sacrificing the SMN suppression ratio of the EDFL.

3.6 References

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3.7 Appendix

The modulated input power to the SOA is given by [3.8]

$$P_{in}(\tau) = P_{in} + \delta P_{in}\exp(-i\omega\tau) \tag{A1}$$

The effect of modulation of the input power is to introduce a deviation δh of the integrated gain, which is derived from (18)

$$\delta h = \frac{[1 - G_{CW}]\delta P_{in} / P_s}{1 + G_{CW}\delta P_{in} / P_s - i\omega\tau_c} \tag{A2}$$

The output power from the SOA is given by [3.14]

$$P_{out}(\tau) = P_{in}(\tau)\exp[h(\tau)] \tag{A3}$$

The deviation of the output power δP_{out} can be approximated by

$$\delta P_{out} = G_{CW}(\delta P_{in} + P_{in}\delta h) \tag{A4}$$

The transfer function of the SOA is defined as

$$X(\omega) \equiv \delta P_{out} / \delta P_{in} \tag{A5}$$

From (A2), (A4), and (20), we obtain (19).