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Multi-objective Planning for Conjunctive Use of Surface and Subsurface Water Using Genetic Algorithm and Dynamics Programming

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Abstract The consideration of fixed cost and time-varying operating cost associated with the simultaneous conjunctive use of surface and subsurface water should be treated as a multi-objective problem due to the conflicting characteristics of these two objectives. In order to solve this multi-objective problem, a novel approach is developed herein by integrating the multi-objective genetic algorithm (MOGA), constrained differential dynamic programming (CDDP) and the groundwater simulation model ISOQUAD. A MOGA is used to generate the various fixed costs of reservoirs' scale, generate a pattern of pumping/recharge, and estimate the non-inferior solutions set. A groundwater simulation model ISOQUAD is directly embedded to handle the complex dynamic relationship between the groundwater level and the generated pumping/recharge pattern. The CDDP optimization model is then adopted to distribute the optimal releases among reservoirs provided that reservoir capacities are known. Finally, the effectiveness of our proposed integrated model is verified by solving a water resources planning problem for the conjunctive use of surface and subsurface water in southern Taiwan.

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1 Introduction

The conjunctive use of surface water and subsurface water (groundwater) can enhance the reliability of water supplies. Surface water can fulfill most of the demand during the wet season but it is sensitive to hydrological variation. On the other hand, groundwater is less sensitive to hydrological variation and can provide a stable water supply during the dry season. Therefore, how to plan the appropriate capacity of a reservoir and well network in a water resource system is urgent work for government. However, due to financial constraints, only a limited number of reservoirs and wells can be built in a river basin. Accordingly, an appropriate policy is necessary to consider the fixed costs and operating costs during the reservoir and well planning stage.

To assess state-of-the-art optimization of reservoir and well management and operation thereof, Dean Randall (1997) presented a simulation model applicable to water supply planning, with reference to the conjunctive use of a surface and subsurface system. Linear programming is embedded in a month-by-month simulation model and includes a priority-based objective function. The subsurface system limits the pumping and recharge of a groundwater basin to predefined capacities in the model of groundwater operations. All functions and constraints must be defined as linear because the programming is linear. However, the set of fixed costs is neglected. David and Daene (1998) described the application of two decomposition algorithms to a conjunctively managed surface and a groundwater system, with reference to cost functions that include both discrete and nonlinear terms. The hydraulics of the groundwater system are incorporated into the management model using the response matrix approach, according to which a groundwater simulation model is run repeatedly to calculate the response matrix which relates drawdown to pumping (recharge). Although its objective function has a discrete investment cost and continuous operating cost, these two terms are not considered in simultaneously minimizing a nonlinear programming formulation because the reservoir capacity is pre-specified, not being a decision variable. Philbrick and Peter (1998) proposed that intuitive rules based on experience may not be efficiently applicable to the management of water supply systems that involve both surface and subsurface storage. When the new gradient dynamic programming method is applied to the minimal operating cost problem, surface and subsurface storage are regarded as state variables for realizing the impact of conjunctive use. They did not consider fixed costs.

Hakan and Miguel (1999) developed a coupled simulation–optimization model of a hypothetical river basin to determine optimal operating policies for the joint use of surface and groundwater supplies. The response function approach is used here to incorporate the transient hydraulic interaction between stream and aquifer in the management model. The response function coefficients are derived from the results of the numerical simulation model. Furthermore, the associated objective function is linear and does not include the set of fixed costs. Barlow et al. (2003) presented a linear programming-based conjunctive management model to evaluate the tradeoffs between groundwater withdrawal and streamflow depletion for alluvialvalley stream aquifer systems representative of the northeastern United States. In their investigation, groundwater flow was simulated using the finite difference based program MODFLOW and groundwater stream interactions were simulated using a stream routing package along with MODFLOW. The objective function maximizes the sustained yield from the aquifer in a specified month for the given standard of stream depletion. All functions and constraints must be defined as linear because the programming is linear. Deepak Khare and Jat (2006) proposed a simple economic-engineering optimization model to explore the possibilities of conjunctive use of surface and groundwater using linear programming with various hydrological and management constraints, and to arrive at an optimal cropping pattern for optimal use of water resources for maximization of net benefits. Although their research was related to the fixed costs and operating costs of the conjunctive use of surface and subsurface systems, their problem did not address the issue of multi-objective planning.

To the authors' knowledge, no investigation has ever simultaneously considered the fixed costs and operating costs of the conjunctive use of surface and subsurface systems, and also treated fixed costs and operating costs as multi-objective. If the popular weighting factor approach is applied to solve the multi-objective problem, the non-inferior solutions set of multi-objective planning of surface water and subsurface water is not easy to obtain. Instead of combining these two objectives into just one objective using the weighting factor approach, this work develops a novel method by integrating a multi-objective genetic algorithm (MOGA), a constrained differential dynamic programming (CDDP) and a groundwater simulation model ISOQUAD. A MOGA is employed to generate the various combinations of reservoir and well capacity and estimate the non-inferior solutions set. A CDDP is herein adopted to distribute optimal releases among reservoirs to satisfy water demand. A groundwater simulation model ISOQUAD is directly embedded to handle the complex dynamic relationship between the groundwater level and the pumping/recharge. Finally, in recent years, municipal and industrial water demand in southern Taiwan have significantly increased due to the growing population, industry, and a general rise in living standards (Yen and Chen 2001). Therefore, the effectiveness of the proposed methodology is verified by solving a multi-objective planning problem of surface water and sub-surface water in southern Taiwan.

2 Methodology

The objective function of conjunctive use that considers fixed costs and operating costs can be described as follows and is denoted as Multi-objective Problem A in this study.

[Multi-objective Problem A: Original Form]

Objective:

$$\underset{\vec{\boldsymbol{a}},\,\tilde{\boldsymbol{u}},\,\tilde{\boldsymbol{q}},}{\text{Min}} \left\{ Z_1\left(\vec{\boldsymbol{a}}\right),\, Z_2\left[\tilde{\boldsymbol{u}}\left(\vec{\boldsymbol{a}}\right),\, \tilde{\boldsymbol{q}}\left(\vec{\boldsymbol{a}}\right)\right]\right\} \tag{1}$$

$$Z_1\left(\vec{\boldsymbol{a}}\right) = F\left(\vec{\boldsymbol{a}}\right) \tag{2}$$

$$Z_2\left[\tilde{\boldsymbol{u}}\left(\vec{\boldsymbol{a}}\right), \tilde{\boldsymbol{q}}\left(\vec{\boldsymbol{a}}\right)\right] = \sum_{t=1}^{n} \left\{g_t, (\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{q}})\vec{\boldsymbol{a}}\right\} \tag{3}$$

Subject to:

$$\vec{s}_{t+1} = T\left(\vec{s}_t, \vec{u}_t\right) \quad t = 1 \ldots\ldots n \tag{4}$$

$$\vec{h}_{t+1} = T\left(\vec{h}_t, \vec{q}_t\right) \quad t = 1 \ldots\ldots n \tag{5}$$

$$f\left(\vec{s}_t, \vec{u}_t, \vec{\boldsymbol{a}}\right) \leq 0 \quad t = 1 \ldots\ldots n \tag{6}$$

$$f\left(\vec{h}_t, \vec{q}_t, \vec{\boldsymbol{a}}\right) \leq 0 \quad t = 1 \ldots\ldots n \tag{7}$$

$$0 \leq \vec{s}_t \leq \vec{\boldsymbol{a}} \quad t = 1 \ldots\ldots n \tag{8}$$

$$0 \leq \vec{u}_t \leq \vec{U}_t^{\max} \quad t = 1 \ldots\ldots n \tag{9}$$

$$\vec{\boldsymbol{a}}^{\min} \leq \vec{\boldsymbol{a}} \leq \vec{\boldsymbol{a}}^{\max} \tag{10}$$

$$\vec{h}^{\min} \leq \vec{h} \leq \vec{h}^{\max} \quad t = 1 \ldots\ldots n \tag{11}$$

$$0 \leq \vec{q}_t \leq \vec{Q}_t^{\max} \quad t = 1 \ldots\ldots n \tag{12}$$

Where,

Z_1	the objective of fixed cost
Z_2	the objective of operating cost
$F(\vec{\boldsymbol{a}})$	Fixed cost function based on capacity decisions
$\vec{\boldsymbol{a}}$	capacity decisions (for example, reservoir scale, capacity of well network ...)
$\tilde{\boldsymbol{u}}$	operating decisions concerning surface system (for example, release from reservoirs, release from weirs...)
$\tilde{\boldsymbol{q}}$	operating decisions concerning subsurface system (for example, groundwater pumping and recharging)

n	the number of periods
$g_t(\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{q}})$	operating cost function based on operating decisions for conjunctive use
$\vec{s}_t$	state variables in surface system (for example, reservoirs' storage)
$\vec{h}_t$	state variables in subsurface system (for example, groundwater level)
$T(\,)$	transition function
$f(\vec{s}_t, \vec{u}_t, \vec{\boldsymbol{a}})$	constraining function in the surface system
$f\left(\vec{h}_t, \vec{q}_t, \vec{\boldsymbol{a}}\right)$	constraining function in the subsurface system

The original multi-objective Problem A has two objectives, Z_1 and Z_2, called fixed cost and operating cost, respectively, which are to be minimized. The fixed cost Z_1 represents the design capacity cost, denoted as vector $\vec{\boldsymbol{a}}$ to represent the reservoirs and wells to be created and defined by a function $F(\vec{\boldsymbol{a}})$ (Eq. 2). The operating cost Z_2 is the cost of operating decisions identified by vectors $\tilde{\boldsymbol{u}}$ and $\tilde{\boldsymbol{q}}$ for each period, and defined by a function $g_t(\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{q}})$. The vector $\tilde{\boldsymbol{u}}$ represents the feasible decisions in surface water, such as reservoir outflow and spill, and the vector $\tilde{\boldsymbol{q}}$ also represents the feasible decisions in subsurface water, such as well pumping and recharge. These two vectors are required to satisfy the physical and policy constraints imposed on operational procedures (see Eqs. 6 to 12). In addition, the operating cost consists of the pumping, maintenance and other costs associated to the 'operation.' In Taiwan, the water deficit cost, which is also related to the system operation, is the major concern as compared with other operation costs for the Water Resources Agency. However, since the water deficit cost is difficult to estimate in general, the shortage index (SI) is used as a surrogate index for the water deficit cost in practice. Therefore, proposed by the U.S. Army Corps of Engineers (Hydrologic 1966, 1975), the shortage index (SI) is often adopted to reflect the water deficit in Taiwan (Hsu 1995) and is utilized as a surrogate index for the objective function of operating cost. Calculated by Eq. 13, this index specifies the sum of the indicated values for all periods.

$$\mathrm{SI} = \frac{100}{N} \sum_{i=1}^{N} \left(\frac{\mathrm{Sh}_i}{\mathrm{DT}_i} \right)^2 \tag{13}$$

Where N is the number of periods; Sh_i and DT_i are the shortage and the target demand at time period i.

Constraint (4) is the transition equation of a surface water system during time interval $[t, t+1]$, and the reservoir level at end of the stage S_{t+1} depends on the initial level state of reservoir S_t and the decision vector $\tilde{\boldsymbol{u}}$. Constraint (5) is a transition equation of a subsurface water during time interval $[t, t+1]$, and the groundwater level at end of the stage h_{t+1} depends on the initial level state of reservoir h_t and the decision vector $\tilde{\boldsymbol{q}}$. Constraint (6) to (7) represents the mass balance or other constraints that are functions of the decision and state variables of a surface water system or sub-surface water system. The system limitations of state variables and decision variables are articulated by Constraints (8) to (12).

An appropriate solution to a multi-objective problem is often difficult to obtain from the original form as expressed in Problem A. Therefore, multiple objective functions are usually merged into a scalar function by weighting factors, so that Problem A can be solved by single objective optimization methods, e.g. dynamic programming. Dynamic programming requires the objective function is separable in stage. A separable objective function can be represented as a summation of other functions in stage. The fixed cost is a function of the capacity decision variables $\vec{\boldsymbol{a}}$ that are independent in time. Thus, the fixed cost cannot be represented as a summation of other functions in time and it is not a separable function in stage t. Therefore, the fixed cost in the objective function prohibits the application of dynamic programming to solve the original problem (Hsiao and Chang 2002).

In this task, rather than combining these two competing objectives with a weighting factor, we propose a methodology employing MOGA embedded with CDDP and ISOQUAD through a two-stage formulation. To accomplish this, problem A is first modified into problem B, which comprises a main form and a minor form. The main form is formulated to estimate non-inferior solutions, while the minor form searches for optimal system operation Z_2^* for all time stages under specific capacity decision $\vec{\boldsymbol{a}}$ provided by the main form.

Problem A and Problem B essentially define the same problem. Problem A is the direct mathematical formulation of the multi-objectives problem and it is easier to realize the problem as a whole basing on that. However, basing only on the Problem A formulation, it is difficult to explain how the proposed hybrid algorithm is solving the problem. On the other hand, Problem B is the reformulation of Problem A to explore its internal problem structure. It is evident that the problem can be solved in two major steps, the system capacity and the groundwater recharge/pumping (the main problem) then the surface system operation (minor problem), basing on Problem B formulation. The proposed novel hybrid algorithm integrates the MOGA, CDDP and ISOQUAD, and mainly follows the structure of Problem B.

The decision variables in the main problem are entered into the minor problem as parameters. The optimal solution for each minor problem depends on the decision variable values from the main problem (Eq. 14). The minor problem is actually a parametric optimization sub-problem. When applying the MOGA to solve Problem B, the system capacity and the groundwater recharge/pumping are represented as chromosomes and the feasible domain for these decision variables are thoroughly searched for through the MOGA's evolution algorithm. Moreover, for each chromosome with specified system capacity and groundwater recharge/pumping, only the associated optimal surface water operation can be a candidate of the non-inferior solution. The optimal surface operation for a chromosome is then computed by the CDDP algorithm. This is because the reservoir operation problem is an optimal dynamic control problem and the numbers of decision variables increase over time. Therefore, using MOGA to solve the reservoir operation problem (the water allocation problem), the computational loading can significantly increase with the time steps. The CDDP is one of the optimal control algorithms that can explore the structure of optimal dynamic control problems and greatly reduce the computational loading. Therefore, the feasible domain for all the decision variables including surface water operation is fully searched and the computation will not lead to suboptimal solutions.

In summary, reformulating the Problem A into Problem B helps to justify the proposed novel algorithm without adding other limitations or assumptions.

[Main form of multi-objective Problem B]

Objective:

$$\underset{\vec{\boldsymbol{a}},\tilde{\boldsymbol{q}}}{\text{Min},}\left\{Z_1\left(\vec{\boldsymbol{a}}\right),\,Z_2^*\left(\vec{\boldsymbol{a}},\tilde{\boldsymbol{q}}\right)\right\}$$

$$Z_1\left(\vec{\boldsymbol{a}}\right)=F\left(\vec{\boldsymbol{a}}\right) \tag{14}$$

Subject to:

$$\vec{h}_{t+1}=T\left(\vec{h}_t,\vec{q}_t\right)\quad t=1\ldots\ldots n \tag{5}$$

$$f\left(\vec{h}_t,\vec{q}_t,\vec{\boldsymbol{a}}\right)\le 0\quad t=1\ldots\ldots n \tag{7}$$

$$\vec{\boldsymbol{a}}^{\,\min}\le\vec{\boldsymbol{a}}\le\vec{\boldsymbol{a}}^{\,\max} \tag{10}$$

$$\vec{h}^{\min}\le\vec{h}\le\vec{h}^{\max}\quad t=1\ldots\ldots n \tag{11}$$

$$0\le\vec{q}_t\le\vec{Q}_t^{\max}\quad t=1\ldots\ldots n \tag{12}$$

[Minor form of multi-objective Problem B]

Objective:

$$Z_2^*\left(\vec{\boldsymbol{a}},\tilde{\boldsymbol{q}}\right)=\underset{\tilde{u}}{\text{Min}}\sum_{t=1}^{n}\left\{g_t,\left(\tilde{\boldsymbol{u}},\tilde{\boldsymbol{q}}\right)\vec{\boldsymbol{a}},\tilde{\boldsymbol{q}}\right\} \tag{15}$$

Subject to:

$$\vec{s}_{t+1}=T\left(\vec{s}_t,\vec{u}_t\right)\quad t=1\ldots\ldots n \tag{4}$$

$$f\left(\vec{s}_t,\vec{u}_t,\vec{\boldsymbol{a}}\right)\le 0\quad t=1\ldots\ldots n \tag{6}$$

$$0\le\vec{u}_t\le\vec{U}_t^{\max}\quad t=1\ldots\ldots n \tag{9}$$

$$0\le\vec{s}_t\le\vec{\boldsymbol{a}}\quad t=1\ldots\ldots n \tag{8}$$

Under the given vectors $\vec{\boldsymbol{a}}$ and $\tilde{\boldsymbol{q}}$ from MOGA, the minor form of Problem B for a single objective $Z_2\left(\vec{\boldsymbol{a}},\tilde{\boldsymbol{q}}\right)$ is expected to define the best surface system operation $\tilde{\boldsymbol{u}}^*$

and its optimal value Z_2^*. Since $\vec{\boldsymbol{a}}$ is a constant rather than a decision variable in the minor form of problem B, the difficulty of a non-separable problem for DP vanishes.

The above-mentioned methods of CDDP, ISOQUAD and MOGA are described below.

2.1 CDDP

Murray and Yakowitz (1981) presented a constrained differential dynamic programming (CDDP) algorithm and applied it to a multi-reservoir control problem. They formulated the problem as a discrete optimal control problem with a linear transition function and linear constraints on the state and control variables. Their algorithm adapted the quadratic programming method into the DDP framework. They also point out the state and control vectors of the problem need not be discrete, implying that CDDP overcomes the "curse of dimensionality," a serious limitation of conventional DP. CDDP can reduce a significant "working" dimensionality of the algorithm over that of mathematical programming algorithms. Based on those advantages, we adopt the CDDP instead of DP. The CDDP used herein is a modified procedure suggested by Murray and Yakowitz (1981). Within each iteration, Quadratic Programming is applied at each stage of the backward and forward sweep. The iterations are repeated until the solution converges. More detailed discussion of the CDDP algorithm and application is provided in Murray and Yakowitz (1981) and Chang et al. (1992).

2.2 ISOQUAD

The groundwater simulator used here is ISOQUAD (Pinder 1978). The ISOQUAD was obtained by applying Galerkin's finite element method and an implicit finite difference scheme. Originally, ISOQUAD is a groundwater flow and contamination simulation program for a confined aquifer. Hsiao and Chang (2002) proposed a modified ISOQUAD for unconfined aquifer and is adopted in our investigation. The modified ISOQUAD program is embedded into the water resource management model and serves as a transition function of groundwater flow. A detailed description of the modified ISOQUAD algorithm and application is provided by Hsiao and Chang (2002).

2.3 MOGA

In real life, most of the water resources optimization problems involve conflicting objectives, for which there is no efficient method for finding multiple trade-off optimal solutions (Reddy and Kumar 2006). If compared with traditional multi-objective programming approaches, the genetic algorithm-based solution method has two advantages: first, it can generate both convex and concave points on the trade-off curve; second, it can generate large portions of the trade-off curve in a single run. Moreover, MOGA differs significantly from traditional optimization techniques in that it operates on a coding of the parameters themselves. A search is performed over a population of solutions rather than over sequences of individual solutions as in traditional optimal search methods. MOGA directly utilizes information from the fitness functions (objective functions), and applies probabilistic transition rules during the solution process. In this study, the operation procedures of MOGA are modified from the study of the Pareto-optimal ranking method (Goldberg 1989) and elitist conservation (Yeh and Labadie 1997).

2.4 Integration of the MOGA, ISOQUAD and CDDP

CDDP and ISOQUAD are embedded in the structure of MOGA. ISOQUAD is applied to calculate the groundwater level and CDDP is employed to distribute the release among reservoirs. The flow chart of this integrated model is shown in Fig. 1, and the detail operational procedures are described step by step as follows.

1. Select the potential scale of the reservoir and well network.

MOGA requires encoding schemes that transform the decision variable vectors into a structure (chromosome) that enables genetic operations: reproduction, crossover and mutation. These genetic operations generate new sets of chromosomes with, on average, enhanced performance. This step mainly focuses on encoding the decision variable as a chromosome, randomly generating an initial population of given size, 100 in this case. Each decision variable can denote the possible capacity of a reservoir, the possible capacity of a well network, and the volume of pumping/recharge in every time step. Each chromosome denotes a combination of these three decision variables.

2. Calculate water demand for surface water.

For each chromosome, since the groundwater pumping is defined in step 1, the water demand to be fulfilled by the surface water system then can be calculated by subtracting the groundwater pumping from the original water demand.

3. Define the system network and prepare hydrologic data.
4. Distribute the optimal releases among reservoirs in the surface system using CDDP.

After the reservoir capacity of chromosomes in the initial population has been determined as in step 1, the release of the surface system in every period is calculated by the CDDP corresponding to each chromosome. This procedure is repeated for all chromosomes in each generation. The CDDP is embedded in the MOGA to calculate the optimal release considering the water demand for surface water. Finally, the release of the surface system for each chromosome is returned to the MOGA to measure the operating cost.

5. Evaluate the fixed cost using the fixed cost-coefficient, reservoir capacity and well network scale; and the operating cost using the shortage index. In this study, however, the Shortage Index (SI) surrogates the operating cost.
6. Calculate the groundwater level of the designed pumping/recharging via ISOQUAD.

A groundwater model ISOQUAD is adopted herein as a transition function of groundwater flow to calculate the groundwater level under various combinations of pumping/recharge. If the groundwater level of a chromosome combination in

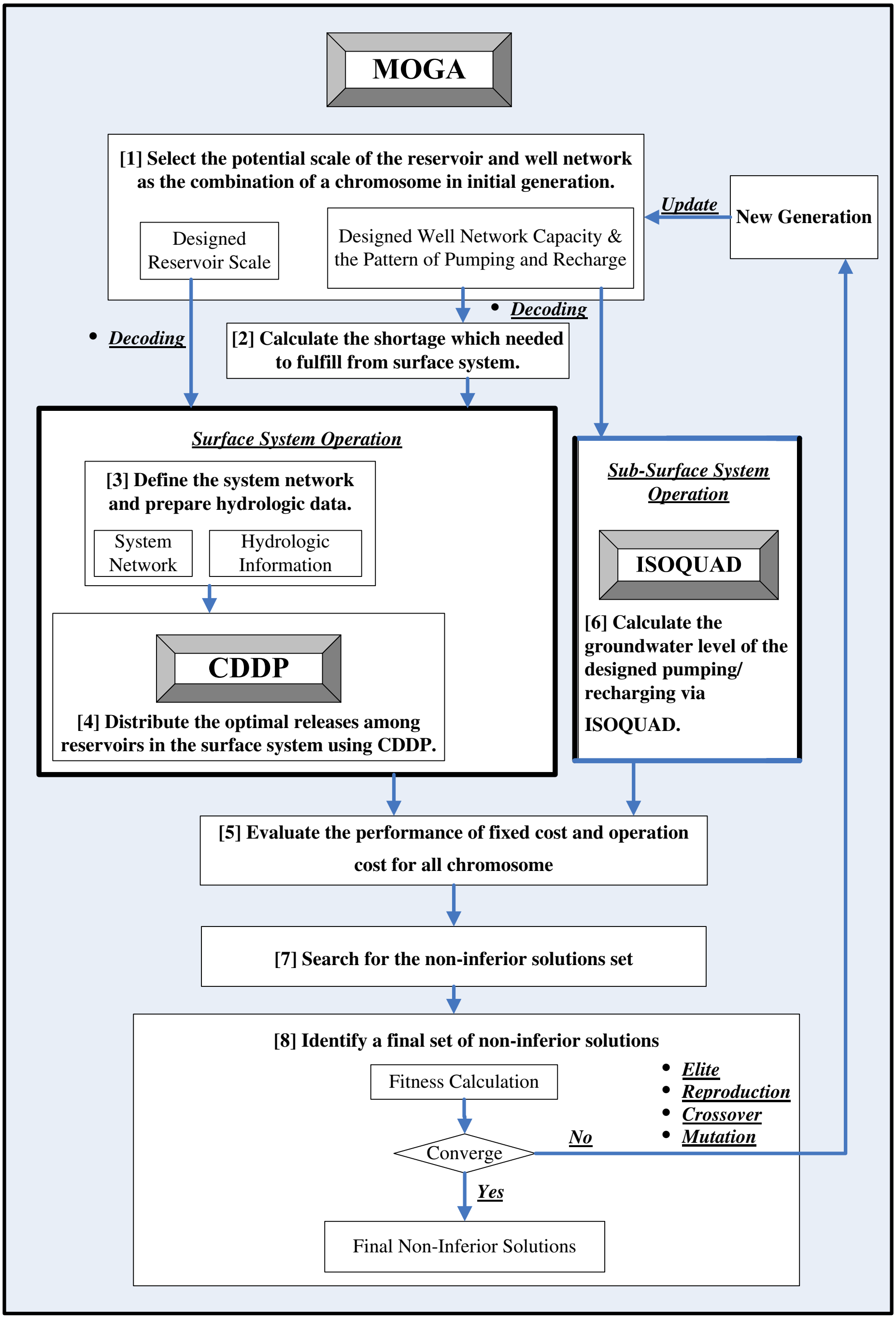


Fig. 1 Flow chart of our proposed model, integration of the MOGA, ISOQUAD and CDDP

any time period is below our proposed safety limit, the penalty term is then added in its objective functions (making both values for fixed costs and operating costs enormous). Therefore, a chromosome with such large values can be automatically eliminated when choosing the non-inferior solutions set (our costs are to be minimized).

7. Search for the non-inferior solutions set.

The values of the fixed cost (Z_1) and operating cost (Z_2) can be identified for every chromosome of one generation through the above steps, and MOGA finds the non-inferior solution set based on these values at Step 7. Because our multi-objective problem belongs to the minimal problem, a chromosome $a2$ is defined as inferior to chromosome $a1$ ($a1$ dominate $a2$) if the following condition holds.

$$Z_1(a1) \leq Z_1(a2) \text{ and } Z_2(a1) \leq Z_2(a2)$$

The non-inferior solutions set at each generation is composed of chromosomes that are non-dominated by any other chromosome in the population. Additionally, the non-inferior solutions for each generation are added into a non-inferior solutions pull. The non-inferior solution pull accumulates the non-inferior solutions from generation to generation. When a new non-inferior solutions set is adding into the pull, it will compare and update the non-inferior solutions set in the pull. The computation will stop when the non-inferior solutions set in the pull does not change for a specified number of generations. The non-inferior solutions set in the pull also serve as stopping criteria for the MOGA.

8. Identify a final set of non-inferior solutions through the procedures (procedure a–f) of fitness calculation, elite, reproduction, crossover and mutation of MOGA until reaching converge conditions.

   a. Evaluate the fitness for each chromosome
   The solutions of the non-inferior solutions set determined through step 7 are assigned as rank 1. The fitness of all feasible solutions is estimated by Eq. 16.

   $$f_i = d_{\max} - d_{i_{\min}} \tag{16}$$

   Where $d_{\max}$ denotes the maximum distance between all feasible solutions and all non-inferior solutions in rank 1, i.e. $d_{\max} = \max\left\{d_{ij(i=1\sim\text{pop},\, j=1\sim N)}\right\}$; $d_{i_{\min}}$ represents the minimal distance between the feasible solution i and the non-inferior solution j in rank 1, $d_{i_{\min}} = \min\left\{d_{ij(j=1\sim N)}\right\}$; d_{ij} is the distance between the feasible solution i and any non-inferior solution j in rank 1; f_i is the fitness of feasible solution i; pop represents the total number of feasible solutions, and N is the total number of non-inferior solutions in rank 1. Figure 2 clearly demonstrates that the distance between any feasible solution of chromosomes and the members of rank 1 is closer with respect to bigger fitness values compared to the other chromosomes.

   b. Store the elite solutions
   This study proposes a new way of storing the elite solutions to ensure that the set of non-inferior solutions can proceed with the crossover step so as to avoid any component among this set from disappearing in the reproduction process. This procedure also performs the function of a diversity maintaining mechanism – one of the key ingredients of a MOGA. If the number of

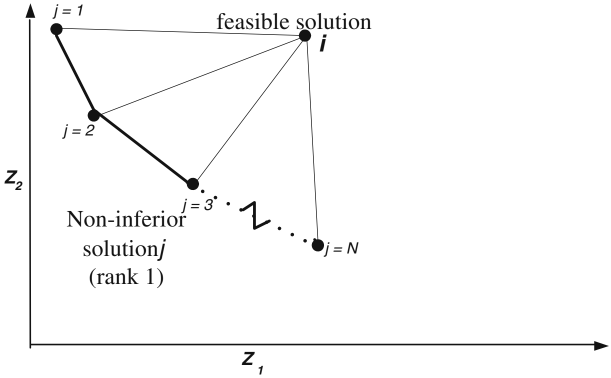


Fig. 2 Definition of fitness (Z_1, Z_2 are objectives in minimal problem)

solutions with rank 1 is lower than that of the elite set, all of these solutions are included in this set. The remaining elite set comprises feasible solutions with better fitness. Otherwise, a portion of solutions with rank 1 is chosen to compose the elite set. The number of the elite set in this study is 30, and thus the number of reproductions is 70.

c. Reproduce the best strings
This investigation undertakes reproduction by tournament selection. The selection mechanism plays an important role in driving the search for superior individuals and maintaining high genotypic diversity in the population. MOGA selects parents from a population of strings based on fitness. In each tournament selection, a group of five individuals are randomly selected from the population, and the fittest individual(s) is selected for reproduction. The procedure is repeated until the number of chromosomes required for crossover is met.

d. Crossover
Crossover involves randomly coupling the newly reproduced strings and exchanging information within a pair of strings. Crossover occurs with a constant probability of p_{cross} for each pair of strings. In this instance, p_{cross} was set to 0.7 with a uniform crossover operator.

e. Mutation
Mutation restores lost or unexplored genetic material to the population to prevent the GA from converging prematurely to a local optimum. A mutation probability p_{mutat}, $p_{\text{mutat}} = 0.03$ is specified in this study, with mutation applied randomly to individual genes. If a random number generated from a uniform distribution function is smaller than the mutation probability, then mutation is conducted by changing the binary value of the gene in the offspring strings produced by the crossover operation.

f. Termination mechanism
A new population for the next generation is created after the mutation operation, and the non-inferior solutions set is extracted as from in Steps 2 to 8. The stopping criterion in this study is based on the variation rate, which is defined as the change ratio in the non-inferior solutions sets. The procedure finishes if the user-defined stopping criterion is met or the maximum allowed number of generations is reached; otherwise, step 8 proceeds for another cycle (another generation).

3 Application

3.1 Description of Study Area

The study area is located in southern Taiwan and includes two major watersheds, Tsengwen and Kaopin River, and two metropolitan areas, Tainan and Kaohsiung. The Tainan area is supplied by Wushantou and Nanhwa Reservoir that have 81.45×10^6 and 149.46×10^6 m^3 effective storage capacity, respectively. The Kaohsiung area is supplied by the Nanhwa Reservoir and Kaopin River Weir. A Tongkou Weir with 581.23×10^6 m^3 effective storage capacity is located downstream from the Tsengwen Reservoir and transports the released water from the Tsengwen Reservoir into the Wushantou reservoir. A Chiahsien Weir located at Chiahsien Creek transports the water into the Nanhwa Reservoir. The future demand to be met by this surface system is set to 2011. In the following simulation study, the capacities of the three reservoirs (Nanhwa, Wushantou, Tsengwen) are assumed to be varied between half and double of their original capacities.

Furthermore, a groundwater supply system is assumed and conjunctively operates with the surface water system. The conjunctive operating system is depicted in Fig. 3. The groundwater system supplies the water demand of Kaohsiung and the recharge water of the groundwater system comes from the Kaopin weir. The set up of the hypothetical groundwater system and well network is as follows.

The aquifer of groundwater is unconfined and is assumed to be homogeneous and isotropic. The site, $1{,}575 \times 1{,}125$ m, was laid with 165 finite element nodes, 20 wells and 3 observed wells. Each well possesses pumping and recharge functions.

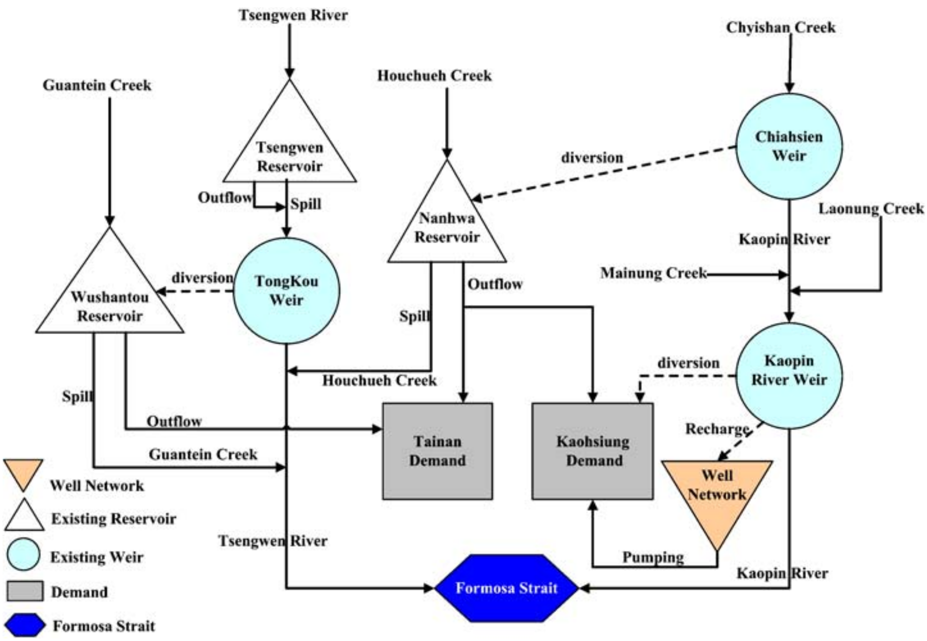


Fig. 3 Water system diagram of conjunctive use for southern Taiwan

Constant-head and no-flow boundaries circumvent the flow domain. The Constant-head boundary at the east and west sides were $h_a = 75$ m, $h_b = 61$ m, and initial heads of domain are demonstrated in Fig. 4. The distance (L) between the ground surface and the aquifer bottom was 100 m.

Table 1 lists the aquifer properties and simulation parameters. In the groundwater management model, the total planning horizon is divided into 120 stages over a 10-year period. The sum of pumping for each stage must satisfy the water shortage as soon as possible, with maximum and minimum well capacities of 0.1 and 0 m^3/s. The minimum requirement of hydraulic head was set to 47 m to prevent damage such as land subsidence caused by over-pumping. Similarly, the maximum on the hydraulic

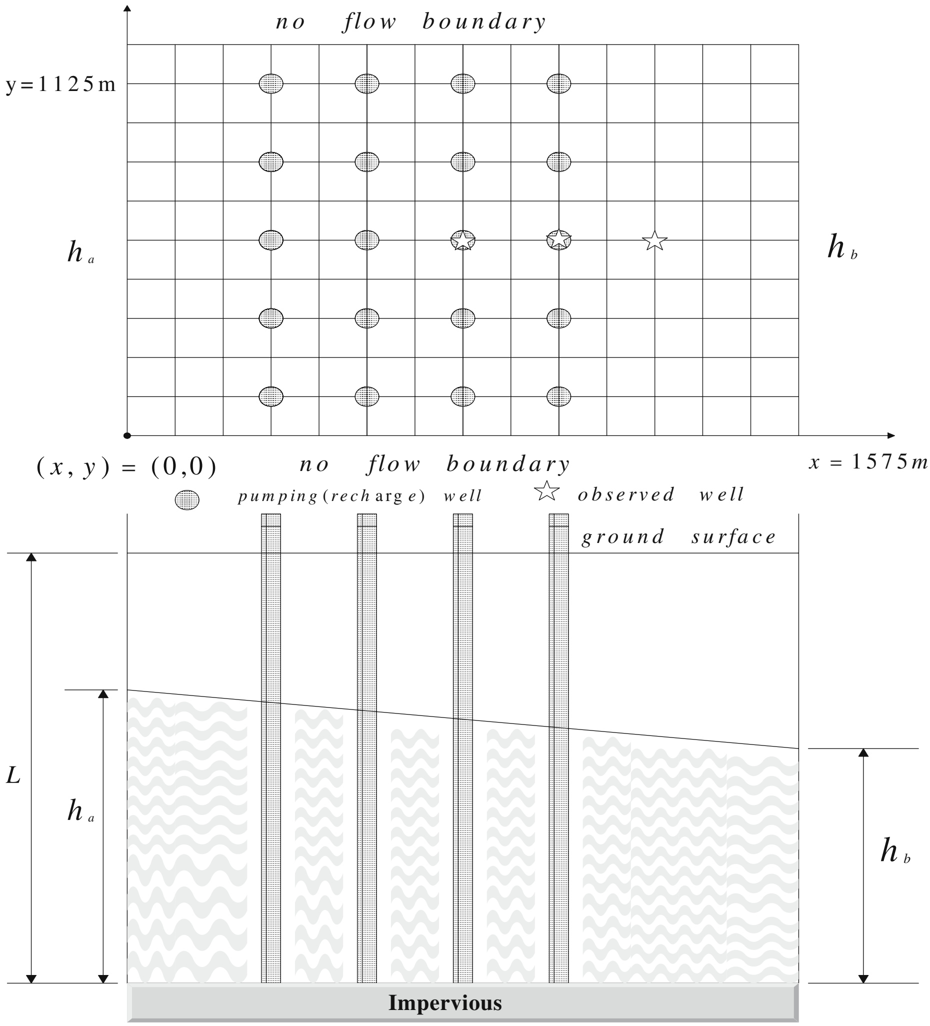


Fig. 4 The set up of groundwater with visual well network

Table 1 Aquifer properties and simulation parameters of example application

Parameter	Value
Hydraulic conductivity	5.0×10^{-4} m/s
Specific yield	0.1
Unit cost of groundwater pumping (or recharge)	NT 0.48/m^3
Stimulation period	10 years
Time step	1 month
The highest limit on the hydraulic head	100 m
The lowest limit on the hydraulic head	47 m
Minimum well capacity	0.0 cm s
Maximum well capacity	0.1 cm s

head was set to 100 m so as to avoid over-recharge that would cause hydraulic head over the ground surface.

3.2 Problem Definition

The objective function and system dynamics in the main form for the southern Taiwan problem are formulated as below.

[Application Problem; main form]

Objective:

$$\underset{\vec{\boldsymbol{a}},\tilde{\boldsymbol{q}}}{\text{Min,}} \left\{ Z_1\left(\vec{\boldsymbol{a}}\right), Z_2^*\left(\vec{\boldsymbol{a}}, \tilde{\boldsymbol{q}}\right)\right\} \tag{14}$$

$$Z_1\left(\vec{\boldsymbol{a}}\right) = \sum_{i=1}^{m} C_i \times a_i \tag{17}$$

Subject to:

$$\vec{\boldsymbol{a}}^{\min} \le \vec{\boldsymbol{a}} \le \vec{\boldsymbol{a}}^{\max} \tag{10}$$

$$\vec{h}_{t+1} = \text{ISOQUAD}\left(\vec{h}_t, \vec{q}_t\right) \quad t = 1\ldots\ldots n \tag{18}$$

$$\vec{h}^{\min} \le \vec{h} \le \vec{h}^{\max} \quad t = 1\ldots\ldots n \tag{11}$$

$$0 \le \left|\vec{q}_t\right| \le \vec{Q}_t^{\max} \quad t = 1\ldots\ldots n \tag{19}$$

Where m is the total number of reservoir and well networks. In this case, three reservoirs and a well network are selected for optimization so that $m = 4$. a_1, a_2 and a_3 denote the installation capacities of Wushantou Reservoir, Tsengwen Reservoir and Nanhwa Reservoir, respectively. a_4 is the installation capacity of the well network. The assumption of a linear function and the coefficients for the unit construction cost of reservoirs (C_1–C_3 = 2.62 N.T. dollars/ton) and the well network (if the designed life of wells is 10 years, C_4 = 0.48 N.T. dollars/ton; if the designed life of the wells is 30 years, C_4 = 0.11 N.T. dollars/ton; Wu (1997)). The constraint 19 is the pumping or recharge limitation of the well network. If $q \geq 0$, it represents pumping; otherwise the network is recharging. In addition, a groundwater model ISOQUAD is adopted herein as a transition function of groundwater flow, Eq. 18, to calculate the groundwater level under various combinations of pumping/recharge. If the groundwater level of a chromosome in any time period is below our proposed safety groundwater level, constraint 11, the penalty term is then added in its objective functions. Chromosomes with huge value objectives can be automatically eliminated when choosing the non-inferior solutions set.

With the shortage index (Hsu 1995), the minor form of the problem discussed in this application is formulated as below.

[Application Problem; minor form]

Objective:

$$Z_2^*(\vec{\boldsymbol{a}}, \widetilde{\boldsymbol{q}}) = \underset{\mathrm{UO,WD}}{\mathrm{Min}} \frac{100}{n} \sum_{t=1}^{n} \left\{ \sum_{j=1}^{s} \left[\frac{\mathrm{UO}_{j,t} + \mathrm{WD}_{j,t} + q_t - D_{j,t}}{D_{j,t}} \right]^2 \right\}, \text{ for known } \vec{\boldsymbol{a}}, \widetilde{\boldsymbol{q}}. \tag{20}$$

Subject to:

Transition equation of reservoir:

$$S_{i,t+1} = S_{i,t} + \mathrm{RI}_{i,t} + \mathrm{WD}_{i,t} - \mathrm{UO}_{i,t} - \mathrm{US}_{i,t}, \quad i = 1 \ldots m-1, \; t = 1 \ldots n \tag{21}$$

Mass balance of weir:

$$\mathrm{WI}_{k,t} + \mathrm{RS}_{k,t} + \mathrm{RO}_{k,t} = \mathrm{WQ}_{k,t} + \mathrm{WD}_{k,t}, \quad k = 1 \ldots g, \; t = \ldots 1 \ldots n \tag{22}$$

Water level of reservoir:

$$0 \leq \vec{s}_t \leq \vec{\boldsymbol{a}} \quad t = 1 \ldots\ldots n \tag{8}$$

Capacity Constraints:

(1) The upper limits of capacities for reservoirs and pipelines. (23)

(2) Supply capacity: $\mathrm{UO}_{j,t} + \mathrm{WD}_{j,t} \leq D_{j,t} - q_t$ (24)

Non-negativity: all variables are larger than or equal to zero.

Where, $D_{j,t}$ is the demand in the supply area j at time t; $S_t + 1$ and S_t denote the storage of the reservoir at time $t + 1$ and t, respectively; UO_t, US_t, and RI_t represent the amounts of outflow, spill, and inflow of reservoir at time t; WO_t, WD_t, and WI_t are the amounts of outflow, diversion, and inflow of weir at time t; g is the total number of weirs ($g = 3$), and s is the total number of demands ($s = 2$).

Some key values or parameters in this paper's optimization problem refer to several papers about Taiwan's reservoir system operations (Water Resources Planning Commission 1986; Chang and Yang 2002; Hsu 1995; Wu 1997).

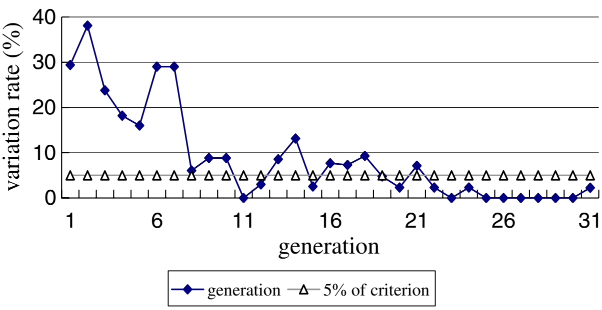
4 Results

The integrated model estimates the non-inferior solutions consisting of fixed costs and operating costs for the area of interest. The decision variables must be encoded as a chromosome before the MOGA is applied. Due to the number of our well networks is one and its predefined capacity, the scale setting of the decision variable of the well network is neglected. Moreover, a total planning horizon is divided into 120 stages over a 10-year period, making the total number of decision variables of pumping in the well network 120. In this case, each decision variable is represented by six binary bits so that 123 decision variables (three reservoir capacities and 120 pumping/recharge) are involved, and a chromosome consists of a total of 738 loci. In the problem considered here, there are 100 chromosomes in each population, and the initial population is randomly generated. As indicated in Fig. 1, the CDDP and ISOQUAD are used repeatedly within each generation to simulate the conjunctive operation of the surface and sub-surface system, according to reservoir scale and the pumping/recharge via the chromosomes. The stopping criterion for MOGA is the variation rate of non-inferior solutions over ten consecutive generations should be under 5%. Figure 5 shows that the variation rate decreases from the initial value to convergence, and that the final non-inferior solution appears in generation number 31.

Figure 6 indicates the results (the designed life of wells is 10 years, $C_4 = 0.48$ N.T. dollars/ton) with the initial and final non-inferior solutions set, plotting the fixed cost against the shortage index. The trend of the results for the final non-inferior solutions set are significant compared with the initial set. Solutions of the final non-inferior solutions set with fixed costs range from 4,578,260,000 N.T. dollars to 3,366,480,000 N.T. dollars with the shortage index from 18.29 to 28.02.

Figure 7 is the outcome of the final non-inferior solutions set with four scenarios and also examines the impact of the well unit cost on the non-inferior solutions sets. Scenario 1 is conjunctive use without the consideration of fixed cost in the subsurface system. Scenario 2 is conjunctive use with a well design-life of 10 years, $C_4 = 0.48$ N.T. dollars/ton. Scenario 3 is conjunctive use with a well design-life of 30 years, $C_4 = 0.11$ N.T. dollars/ton. Finally, scenario 4 is not conjunctive use, but only a surface system for the water supply. As shown in Fig. 7, the conjunctive use is a better strategy than using only surface water (scenario 4) since all the non-inferior solutions of conjunctive use (scenario 1 to 3) are below that of using only surface water. The non-inferior solutions of scenario 1 have the lowest value as expected since scenario 1 represents using the groundwater system without fixed cost. The non-inferior solutions of conjunctive use with other well unit cost should be within that of scenario 1 and 4. The differences of the non-inferior solutions between scenario 1 and 4 are the largest contribution one can get for applying the groundwater system.

Fig. 5 Variation rates by generation



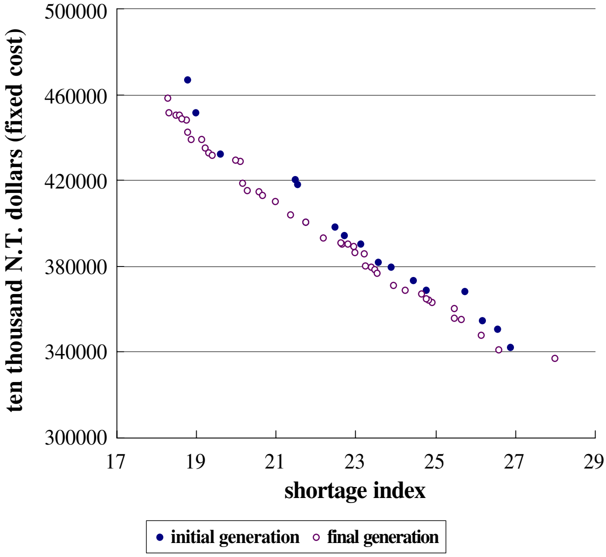


Fig. 6 Initial and final non-inferior solutions set (the design-life of wells is 10 years, C_4 = 0.48 N.T. dollars/ton)

Figures show the volumes of all non-inferior solutions for the three concerned reservoirs in scenario 1. The y-axis indicates reservoir volume, while the x-axis indicates the non-inferior solutions. In Fig. 8, most non-inferior solutions for Wushantou Reservoir are located in and close proximity to the volumes 66 $\times$ 10^6 and 125 $\times$ 10^6 m^3. In Fig. 9, most non-inferior solutions for the Tsengwen Reservoir are located in and close proximity to the volumes 1,000 $\times$ 10^6 and 1,200 $\times$ 10^6 m^3. In Fig. 10, most non-inferior solutions for the Nanhwa Reservoir are located in and close proximity to the volumes 190 $\times$ 10^6 and 220 $\times$ 10^6 m^3.

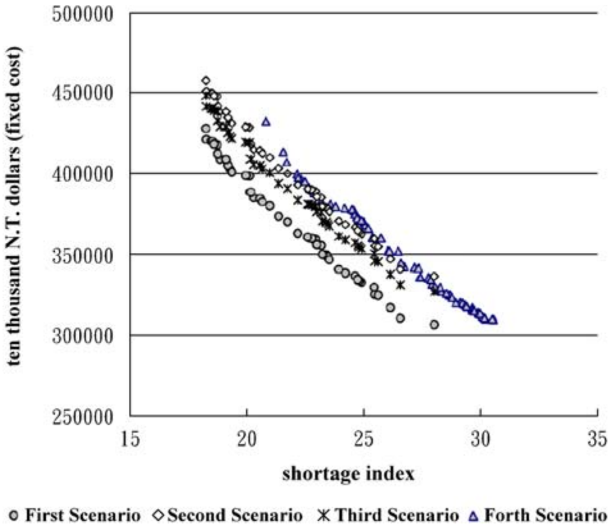
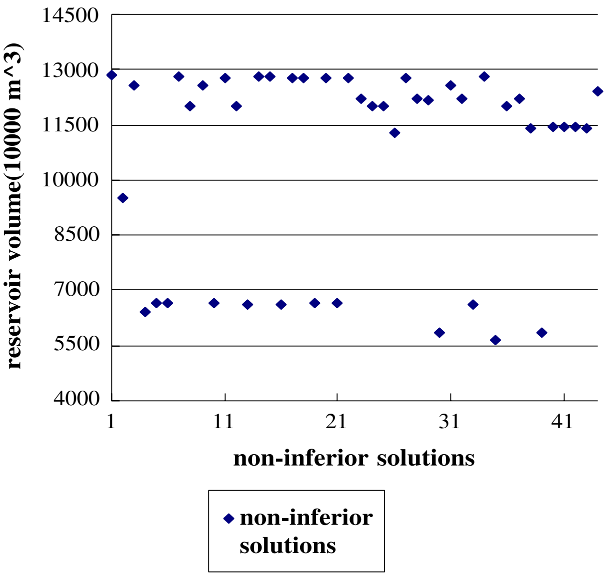


Fig. 7 Final non-inferior solutions set with four scenarios

Fig. 8 The capacity of all non-inferior solutions for Wushantou reservoir in scenario 1



From the previous description, this study recommended two strategies for governmental authorities depending on the following budget conditions:

1. The appropriate scales for Wushantou, Tsengwen, and Nanhwa are 66×10^6, $1{,}000 \times 10^6$, and 190×10^6 m^3, respectively, in the event of a constrained budget.
2. The appropriate scales for Wushantou, Tsengwen, and Nanhwa are 125×10^6, $1{,}200 \times 10^6$, and 220×10^6 m^3, respectively, if there are no serious budget constraints.

Fig. 9 The capacity of all non-inferior solutions for Tsengwen reservoir in scenario 1

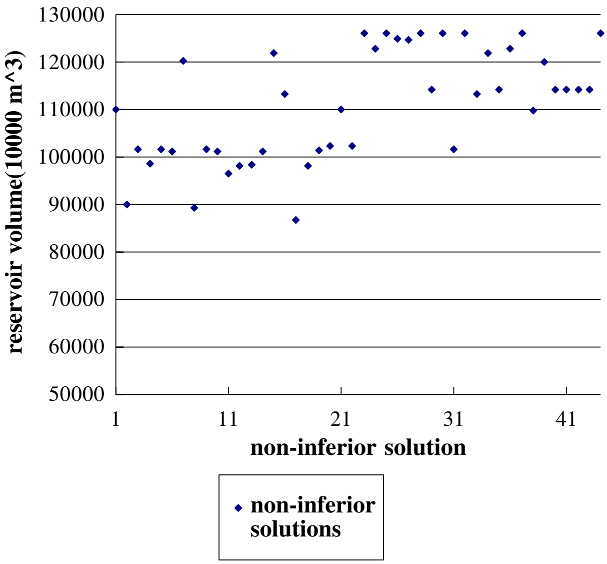
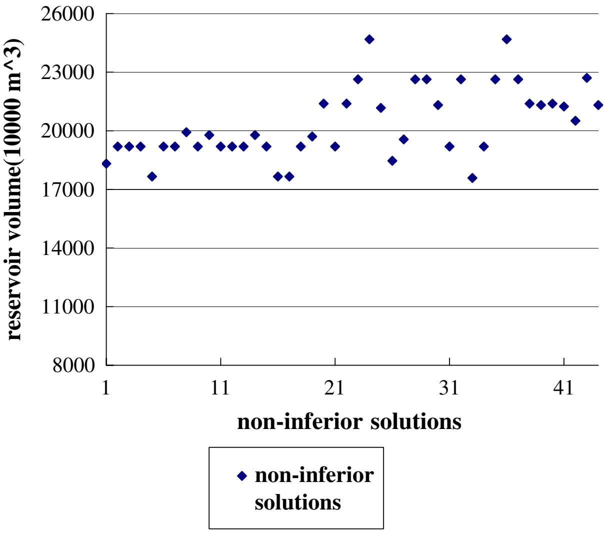


Fig. 10 The capacity of all non-inferior solutions for Nanhwa reservoir in scenario 1



5 Conclusions

The consideration of fixed cost and time-varying operating cost associated with the simultaneous conjunctive use of surface and subsurface water should be treated as a multi-objective problem with non-commensurable objectives. In order to solve this multi-objective problem effectively, a novel approach is developed herein by integrating the multi-objective genetic algorithm (MOGA), constrained differential dynamic programming (CDDP) and a groundwater simulation model ISOQUAD. Unlike in single objective optimization, the solution to this kind of problem is not a single point, but a family of points known as the non-inferior solutions set so that a number of solutions can be found to provide the decision-maker with insight into the characteristics of the problem before a final solution is determined. Therefore, the main focus of this research is to demonstrate how a complicated multi-objective problem can be solved using state of the art optimization schemes. Beside the methodology, the other objective is to illustrate the trade-off between the cost (increasing the capital cost) and benefit (reducing the water deficit) caused by a system capacity expansion. In light of above reason, to reduce the complexity of model development, several details of the existing water supply system in the study area are simplified. In addition, there is no existing groundwater supply system in the area although it is on the government listing of future developing alternatives. Despite this, the case study is meant to demonstrate the feasibility of the proposed novel algorithm, and the existing system is only a framework for the case study. We still believe that the study can offer a good reference to a general water resources engineer who would apply our proposed methodology to their work.

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