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貝氏門檻模型在財務議題之探討

Essays on the Bayesian Threshold Model in Finance

研 究 生：吳志強

指導教授：李昭勝 教授

鍾惠民 教授

中 華 民 國 九 十 六 年 十 月

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國 立 交 通 大 學

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本授權書所授權之學位論文，為本人於國立交通大學 財務金融 所， 96 學年度第 1 學期取得博士學位之論文。

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指導教授：李昭勝 教授；鍾惠民 教授

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中華民國　96　年　10　月　2　日

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授 權 人：吳志強

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指導教授：李昭勝 教授；鍾惠民 教授

茲同意將授權人擁有著作權之上列論文全文（含摘要），非專屬、無償授權國家圖書館，不限地域、時間與次數，以微縮、光碟或其他各種數位化方式將上列論文重製，並得將數位化之上列論文及論文電子檔以上載網路方式，提供讀者基於個人非營利性質之線上檢索、閱覽、下載或列印。

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民國 96 年 10 月 2 日

本授權書請以黑筆撰寫，並列印二份，其中一份影印裝訂於附錄三之二(博碩士紙本論文著作權授權書)之次頁；另一份於辦理離校時繳交給系所助理，由圖書館彙總寄交國家圖書館。

貝氏門檻模型在財務議題之探討

學生：吳志強　　　　　　　　　　　　　指導教授：李昭勝 博士

鍾惠民 博士

國立交通大學財務金融研究所博士班

中文摘要

這項研究包含兩篇貝氏門檻模型在金融市場議題探討之論文。

在第一篇論文中，我們提出四因子的貝氏門檻模型去比較基金經理人面對市場走空或走多時，對於系統風險調整是否存在非對稱性。我們證明不僅經理人有非對稱的擇時能力而且三區間的模型比二區間模型有更顯著的擇時能力。另外，我們使用縱橫資料模型檢查基金投資者的行為與基金績效和特性之間的關係。實證結果說明投資者的行為與過去基金選股績效和基金規模正相關，而與過去基金周轉率，銷售費，費用率負相關。另外，有較大即時現金流量的基金將有較好的預測上升趨勢市場的能力及較差的預測下降趨勢市場的能力。

第二篇論文提出穩健多變量門檻 VAR-GARCH-DCC 模型，這個模型可以描述在金融資產其條件平均值，波動，與相關存在的非對稱性。另外，我們把門檻變數假設成所有內生變數的線性組合。因為這樣不僅可以消除過分主觀的選取門檻變數，而且還可以作為決定哪一市場是價格領先者。我們用 MCMC 方法去估計模型中的參數。而且，介紹幾個有意義的準則去評估條件共變異矩陣的預測績效。最後，我們使用每日的 S&P500 期貨和現貨價格，和 S&P500 與 Nasdaq100 現貨價格作為實證研究。

關鍵字：門檻模型、共同基金、績效評估、波動預測、風險值、避險績效

Essays on the Bayesian Threshold Model in Finance

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ABSTRACT

This study contains two essays on the Bayesian threshold model in financial markets.

In essay 1, we propose a Bayesian three-regime threshold four-factor model to compare the asymmetric risk adjustment between the transitions from neutral to downside markets and those from neutral to upside markets and investigate the performance of mutual funds in changing market conditions. We show that not only fund managers have asymmetric timing ability but three-regime models are more powerful and exhibit significant timing ability more often than two-regime models. In addition, we use panel data model to examine fund investors' behavior and the relationships between fund performances and characteristics. Empirical results suggest that investor's behavior is positively associated with past selectivity performances and fund sizes, while it is negatively correlated to past turnover, load charges and expenses. In addition, funds with large contemporaneous net cash flows will results in better upside market timing ability but worse downside market timing ability.

Essay 2 proposes a robust multivariate threshold vector autoregressive (VAR) model with generalized autoregressive conditional heteroskedasticities (GARCH) and dynamic conditional correlations (DCC) to describe conditional mean, volatility and correlation asymmetries in financial markets. In addition, the threshold variable for regime switching is formulated as a weighted average of endogenous variables to eliminate excessively subjective belief in the threshold variable decision and to serve as the proxy in deciding which market to be the price leader. Estimation is performed using Markov chain Monte Carlo (MCMC) methods. Furthermore, several meaningful criteria are introduced to assess the forecasting performance in conditional covariance matrix. The proposed methodology is illustrated using two data sets including daily S&P500 futures and spot prices, and S&P500 and Nasdaq100 spot prices.

Keywords: Threshold Model; Mutual Fund; Performance Evaluation; Volatility Forecasting; Value at Risk; Hedge Performance

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韶光飛逝，隨著論文的完成，四年的博士班生涯即將劃下句點。回首這段時光，心中溢滿無限的感恩，感謝我周遭的每一個人，為我平凡的人生注入歡樂與喜悅。

從碩士班開始一路走來，李 教授 李昭勝博士不斷地給予指導與鼓勵，李教授不但是我的碩博士班的指導教授，亦是我挖掘知識的寶庫，是李教授細心與耐心的指導讓我對專業的技能有更深一層的領悟，待人處世上更是受益匪淺，繕寫至此筆觸間不免多了一點憂傷與惆悵，就在今年三月二日的傍晚，我一如往常待在研究室裡閱讀著我熟悉的書籍，突然電話響起，電話那頭傳來了一個晴天霹靂的消息，我敬愛的指導教授李昭勝博士被緊急的送到醫院，在還來不及思考狀態下，著一顆忐忑不安的心一手拉起我的同學狂奔到了現場，遺憾的是我依舊沒來得及趕上，當我推開病房的那扇門，頓時間現場的氣溫彷彿降到了冰點，空氣都凝結成了霜，令我感到呼吸困難，我拚命地想掩住心中的悲傷與不捨，但不爭氣的淚水如同潰堤般不聽使喚地崩下，這個事實我真不知該如何接受，我敬愛的李教授已與世長辭，這一切實在來的太突然，那封前一天已撰寫好的信件還躺在我的桌邊，但我知道再也來不及交到教授手上了，心中的痛楚實在無能言表，我就像迷了航的船，不知該向哪裡靠岸。

此時，財務金融所所長亦是我的指導教授 鍾 教授 鍾 惠民博士，不斷地給我鼓勵及安慰，讓我收拾起極度悲痛的情緒，記取李教授寄予我的期望，除了完成學業之外，更要延續李教授的一貫精神『嚴謹認真的治學原則和努力勤奮的工作態度』，在這段期間，財務金融所所長鍾 教授 鍾 惠民博士提供給我許多的資源與協助，並且不厭其煩的指導，使我能適時地修正缺失，在反覆地討論之中，我深刻地感受到老師廣闊的學術視野及對嚴謹治學的堅持，我由衷地感激 鍾 教授 鍾 惠民博士，在我的天空中添入了一道彩光，讓我的眼界得以更加寬廣遼闊，是鐘教授的鼓舞，使我重新振翅，鐘教授的教誨我會銘記在心。

論文口試期間，承蒙許 副校長 許 和鈞副校長、謝 教授 謝 文良教授、謝 教授 謝 國文教授與 王 教授 王 耀輝教授，在百忙之中撥冗指導，對於論文初稿的疏漏與不當之處，提供許多寶貴的意見，使本論文能夠更加完善。

當然還要一群同窗多年的好友，陳煒朋同學、劉炳麟同學、徐淑芳學姊、孫而音學姊，以及統計所的牛維方學長、張志浩學弟，感謝你們陪伴著我渡過這難忘的四年寒暑，讓我在課餘中體驗更精采多姿的生活；同時也要感謝文琪小姐、佳芸小姐及稚瑩小姐在行政上的協助與幫忙。

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CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1. Introduction

In the past decades several nonlinear time series models have been proposed and some of them have been used in practical applications. It is now fairly widely accepted stylized fact that financial time series, such as stock returns, exchange rates series and etc., exhibit strong signs of nonlinearity. The nonlinear phenomenon typically comes from the observation that many economic and financial time series are often characterized by regime specific behavior and asymmetric responses to shocks (see, Hansen (1996, 2000), Hsieh (1989, 1991, 1993), and Tsay (1989, 1998)). In addition, although the GARCH model can capture leptokurtosis and volatility clustering, structural breaks in the variance could lead to the spuriously high persistent in the GARCH model (see, Lamoureux and Lastrapes (1990) and Mikosch and Starica (2004)). Therefore, in the dissertation, we will use the threshold model to capture the nonlinear behavior in both returns and volatilities of financial assets.

In the threshold class of models, classical estimation of parameters is usually done by the maximum likelihood or the least squares methods with a joint grid search over the threshold values using information criteria such as AIC or BIC. However, Koop and Potter (2003) and So *et al.* (2005) showed that the inadequacy of these approaches is to fix the threshold parameters in advance before estimating the other parameters by least squares. Therefore, the uncertainty of the threshold parameters cannot be taken into account when implementing statistical inference for the other parameters. In addition, another problem is the large number of parameters in the threshold model must be estimated and the difficulty of estimation due to the positive definiteness restrictions of the covariance matrix. Thus, MLE will result in unstable estimates. Moreover, previous literatures (such as Koop and Potter (1999)) noted that likelihood functions of many nonlinear models are non-smooth and multimodal. So, it may be inappropriate to chooses one point in the parameter space and to use the large sample property to infer.

Therefore, to moderate the above problems, we adopt a Bayesian approach to estimate the threshold parameters as well as the other parameters simultaneously. And Bayesian method, by using information from the entire parameter space, captures this finite sample uncertainty about the true parameter values. In addition to the above inference advantage, a Bayesian method incorporates the investor's prior belief about the validity of the pricing model and managerial technique with the information in the data, and thus it will result in more appropriate conclusions (see, Pastor and Stambaugh (2000, 2002), and Jones and Shanken (2005)). The dissertation focuses on the application in several important issues in financial markets, including the mutual fund performance evaluation and the forecasting in conditional covariance matrix.

For the first issue in this dissertation, we propose three-regime Bayesian threshold models and use daily returns for each fund to examine the threshold effect and to test this effect resulting from the change of mutual fund managerial micro-forecasting ability and market exposures among different market conditions. Our results show that the effect of public information in the three-regime model is more significant than that in the two-regime model. The unconditional model will overestimate selectivity in the upside market, but underestimate selectivity in the downside market. It also underestimates fund market exposures whether in downside or upside markets. We also reveal clearly that much managerial timing ability decides by their skills to forecast the downside market. In addition, our results indicate that the conditional three-regime threshold model bring most powerful detection for significant timing activity. Our empirical results also indicate that selectivity and timing are negatively associated and the effect is especially significant in the downside market. This is very important for investors to allocate their portfolio.

In the second issue, we present a robust threshold VAR (or VECM)-DCC-GARCH model and use the Metropolis-Hastings (MH) algorithm and the Gibbs sampling algorithm to estimate the parameters simultaneously. Our model extends existing approaches by admitting thresholds in conditional means, conditional volatilities and correlations of multivariate time series. Such an extension, allows us to account for rich asymmetric effects and dependencies of conditional means, volatilities and correlations, as they are often encountered in practical financial applications. In addition, we use the concept of Chen and So (2006) to define the threshold variables as the linear combination of endogenous variables. This setting can eliminate excessively subjective belief in threshold variable decision. Besides, the weight coefficient can serve as the proxy in deciding which market is the price leader and which market is the price follower. Finally, threshold values in our model are not fixed ex ante, but they are estimated from the data, together with all other parameters in the model.

We investigate the empirical performance of our model in two data sets including daily S&P500 futures and spot prices, and S&P500 and Nasdaq100 spot prices. Our study attempts to use posterior odds ratio and Bayes factors as a formal tool for making comparison between competing models. We reduce our testing problem to a Bayesian model selection problem. We can then select the model with a higher posterior odds ratio. We also present the performance comparison results of the one-step-ahead forecast in the conditional covariance matrix. The forecast results are assessed by several criteria which include the views of statistical loss and risk managers.

Based on the estimation results, we find that the asymmetric dynamic structure is obvious in both the dynamic relationship between S&P500 futures and spot markets and between S&P500 and Nasdaq100 spot markets. We also detect that S&P500 futures market is the price leader between S&P500 futures and spot markets, and S&P500 spot market is the price leader between S&P500 and Nasdaq100 spot markets. Furthermore, based on several in-sample and out-of-sample performance measures in the conditional covariance matrix prediction, we find that the threshold model outperforms the linear model across most measure criteria.

Chapter 2.

Measuring Mutual Fund Asymmetric Performance in Changing Market Conditions

1. INTRODUCTION

The scale of U.S. mutual funds, especially equity funds, has grown quite rapidly in the past several decades. The trend illustrates that more and more investors prefer investing their capital in mutual funds rather than directly investing in the equity market. Due to the great number of funds in existence, it is significant to investigate whether managers of actively managed mutual funds rely on superior stock selection skills (micro-forecasting) and market timing capability (macro-forecasting) to outperform passive strategies. Furthermore, studies of relationships between mutual fund performances and characteristics are worthwhile for investors to reference when selecting funds. The investigation of mutual fund performance can serve as a guideline for fund investors and provide them detailed information and previous performance behavior about certain funds. In addition, the objective of identifying superior fund managers is also of interest to academia as a challenge to the efficient market hypothesis.

Therefore, over the past 30 years, numerous evaluation techniques have been proposed to examine the investment performance of mutual fund managers in the academic literature. For mutual funds' managerial stock selection skills, starting with Jensen (1968), numerous studies employed regular proxies for the market portfolio such as Capital Asset Pricing Model (CAPM) benchmark to evaluate the performance of mutual funds. Later, Grinblatt and Titman (1994), Carhart (1997), and Wermers (2000) argued that the use of CAPM as a benchmark would result in inconsistent outcomes and were centered on examining managerial stock picking talents with several comparable passive benchmarks. They also showed that performance tests are entirely sensitive to the chosen benchmark. In addition, Grinblatt and Titman (1989), Elton et al, (1993), Carhart (1997), and Wermers (2000) studied whether mutual fund managers possess superior talents for picking stocks with certain characteristics.

On the subject of managerial market timing ability, Treynor and Mazuy (1966), Merton and Henriksson (1981), and Ferson and Schadt (1996) used monthly fund returns to examine whether fund managers take advantage of superior information by increasing (decreasing) market exposure before the stock market turns bullish (bearish) and found that only a minority of fund managers have good market timing skills. Furthermore, Ferson and Schadt (1996) argued that traditional performance measures may be biased when fund managers use dynamic strategies resulting in time varying risk. Hence, Ferson and Schadt (1996) took public information into consideration and proposed conditional performance evaluation approach which is consistent with the semi-strong form of market efficiency hypothesis.

In addition, when the frequency of market timer (e.g. daily) is higher than that of measured fund returns (e.g. monthly), Goetzmann, Ingersoll, and Ivkovic (2000) found that widely used Henriksson-Merton (HM) parameteric test would result in weak and biased downward timing skill. Therefore, they adjusted HM type model to detect daily timing skill without requiring daily timer data. But simulations showed that the adjusted test was not as powerful as the classical HM test executed directly on daily timer returns. Moreover, Busse (1999), and Bollen and Busse (2001) documented that daily data take account of more efficient estimates of time variation in systematic risk and are more powerful in testing timing ability than monthly data.

Although a variety of evaluation methods have been proposed and implemented to study stock selection and market timing performance to date, they have only examined managerial stock selection ability without distinguishing between good and bad market conditions. Moreover, they have merely investigated the difference in funds' market exposure between upside and downside markets, but have not examined the asymmetric risk adjustment between the transitions from neutral (i.e. the market tends to be neither downside nor upside) to upside markets and those from neutral to downside markets. Additionally, in this essay, we take transaction costs into account when fund managers want to adjust their portfolios among different market conditions. Thus, managers may alter the overall risk composition of their portfolios in anticipation of excess market return being larger (smaller) than a certain positive (negative) level other than zero. We are also able to explore that managers' overall timing ability mainly comes from downside or upside market timing ability.

Furthermore, the majority of the earlier performance measure studies are based on least squares estimation or maximum likelihood estimation. However, a Bayesian approach of performance evaluation may be more adequate from an investor's viewpoint. It combines an investor's prior belief about the accuracy of the pricing model and managerial skills with the information in the data and obtains posterior distribution of the model's parameters. As a result, the conclusions based on the Bayesian method may be more informative for investors than those based on the traditional statistical approach.

To enable more powerful and efficient analyses of fund performance, we propose three-regime Bayesian threshold models and use daily returns for each fund to examine the threshold effect and to test this effect resulting from the change of mutual fund managerial micro-forecasting ability and market exposures among different market conditions. With regard to the chosen factor model, we select Carhart's (1997) four-factor model, which contains market factor (MKT), size factor (SMB), book-to-market factor (HML), and the momentum factor (UMD).

Our results show that the effect of public information in the three-regime model is more significant than that in the two-regime model. The unconditional model will overestimate selectivity in the upside market, but underestimate selectivity in the downside market. It also underestimates fund market exposures whether in downside or upside markets. We also reveal clearly that much managerial timing ability decides by their skills to forecast the downside market. In addition, our results indicate that the conditional three-regime threshold model bring most powerful detection for significant timing activity. Our empirical results also indicate that selectivity and timing are negatively associated and the effect is especially significant in the downside market. This is very important for investors to allocate their portfolio.

Subsequently, we adopt the annual estimation results based on the conditional three-regime threshold four-factor model and use a fixed effects panel data model to examine the relationships between investors' behavior and past fund performances and various fund characteristics. We find strong evidence that funds with better past performances, except for downside market timing ability, will attract more investors and bring larger net cash flows. In addition, net cash flows are persistent and positively associated with fund sizes. We also find the inverse relationships between net cash flows and lagged turnover, load charges, and expenses.

We also use a fixed effects panel data model to explore the relationships between fund performances and characteristics. Although numerous prior studies have examined their relationships, they mostly focused on selectivity performance and found the following results. For the relationship between performance and fund size, Grinblatt and Titman (1994) and Carhart (1997) found that managerial stock selection performance is not apparently related to fund size, while, Chen et al. (2004) found that fund size corrodes mutual fund performance. Moreover, prior evidence on the relationship between turnover and performance is mixed. Elton et al. (1993) and Carhart (1997) found that funds trading more actively have worse stock selection talents than those that trade less frequently, while Grinblatt and Titman (1994) found a positive relation between turnover and net mutual fund returns.

Again, for the relationship between total load and stock picking skills, Elton et al. (1993), Carhart (1997), and Dellva and Olson (1998) found that stock selection performance of the load fund is lower than that of the no-load fund. For the relation between expense and performance, Grinblatt and Titman (1994) and Carhart (1997) found that high expense funds have worse performances than low expense funds, while other extant literatures did not find any significant relations. In addition, Gruber (1996), Chevalier and Ellison (1997), Dellva and Olson (1998), and Sirri and Tufano (1998) found that funds with high cash positions and large net cash flows will produce superior overall stock selection performance.

Our empirical analyses using the fixed effects panel data model show the following findings. The selectivity and market timing performances are not long persistent. In addition, we find that fund investors have no good selection ability. Fund sizes will corrode selectivity performance in the upside market, while, they reduce fund performance loss in the downside market. Conversely, active managers will advance selectivity performance in the upside market, whereas increase the losses of abnormal returns in the downside market. Because managers of higher expense funds do not create more performances to recover their charged fees, investors prefer to select funds with lower expenses to maximize their net expense returns.

We also find that both managers with heavy trading as well as managers with low expenses have superior downside market timing ability, while they have worse upside market timing ability. In addition, net cash flows have negative impacts on timing the downside market but positive impacts on upside timing ability. This suggests that managers of large net cash flow funds might invest new cash in high beta stocks or call options to generate better timing ability in the upside market. Conversely, they might invest new cash in low beta stocks instead of put options and result in worse timing ability but better selectivity performances in the downside market. Finally, we also find that funds with high expenses will have alert insights into the predictions of downside and upside markets.

The rest of the essay is organized as follows. Section 2 presents the theoretical methodology and the estimation procedure. Data and the performance estimation results for an individual fund and all funds are described in Section 3. Section 4 investigates the behavior of funds investors. Section 5 documents the relationships between fund performance measures and fund characteristics. Section 6 concludes the essay.

2. UNCONDITIONAL AND CONDITIONAL BAYESIAN THRESHOLD MODEL

The model we proposed here is called the Bayesian threshold model, which allows the conditional variance to depend on its own previous realizations and to be drawn from different regimes. Since the purpose of this essay is to compare the stock selection and market timing ability among different market conditions, we assume the regression coefficients of other factor variables are identical in any market conditions. Therefore, the unconditional three-regime threshold model with conditional variances following GARCH(1,1) process based on Carhart's (1997) four factor variables, is specified as follows:

$$\begin{aligned} R_{p,t} = &\left(\alpha_p^{(1)} + \beta_{p,MKT}^{(1)} R_{MKT,t}\right) \cdot I_{\{-\infty = r_{p,0} \le R_{MKT,t} < r_{p,1}\}} + \left(\alpha_p^{(2)} + \beta_{p,MKT}^{(2)} R_{MKT,t}\right) \cdot I_{\{r_{p,1} \le R_{MKT,t} < r_{p,2}\}} \\ &+ \left(\alpha_p^{(3)} + \beta_{p,MKT}^{(3)} R_{MKT,t}\right) \cdot I_{\{r_{p,2} \le R_{MKT,t} < r_{p,3} = \infty\}} + \beta_{p,SMB} R_{SMB,t} + \beta_{p,HML} R_{HML,t} \\ &+ \beta_{p,UMD} R_{UMD,t} + \varepsilon_{p,t}, \end{aligned} \tag{2.1}$$

$$\varepsilon_{p,t} \left| \Im_{p,t-1} \sim N\left(0,\ h_{p,t}\right)\right., \tag{2.2}$$

$$\begin{aligned} h_{p,t} = &\left(a_{p,0}{}^{(1)} + a_{p,1}^{(1)} \varepsilon_{p,t-1}^2 + b_{p,1}^{(1)} h_{p,t-1}\right) \cdot I_{\{-\infty = r_{p,0} \le R_{MKT,t} < r_{p,1}\}} \\ &+ \left(a_{p,0}{}^{(2)} + a_{p,1}^{(2)} \varepsilon_{p,t-1}^2 + b_{p,1}^{(2)} h_{p,t-1}\right) \cdot I_{\{r_{p,1} \le R_{MKT,t} < r_{p,2}\}} \qquad , \\ &+ \left(a_{p,0}{}^{(3)} + a_{p,1}^{(3)} \varepsilon_{p,t-1}^2 + b_{p,1}^{(3)} h_{p,t-1}\right) \cdot I_{\{r_{p,2} \le R_{MKT,t} < r_{p,3} = \infty\}} \end{aligned} \tag{2.3}$$

where $R_{p,t}$ is the excess return of the fund *p* at time *t*, $R_{MKT,t}$ is the market excess return at time *t* and is also an observed variable determining the switching points, and the three extra factors ($R_{SMB,t}, R_{HML,t}, R_{UMD,t}$) are size, book-to-market, momentum factors at time *t*. The threshold parameters $r_{p,1}$ and $r_{p,2}$ satisfy $-\infty < r_{p,1} < 0 < r_{p,2} < \infty$, and the distribution of $\varepsilon_{p,t}$ conditional on information up to *t*-1, denoted by $\Im_{p,t-1}$, is $N\left(0,\ h_{p,t}\right)$. To allow heteroscedasticity in $R_{p,t}$ we have a GARCH formulation in the conditional variance equation for h_t. Standard restrictions on the GARCH parameters are $a_{p,0}^{(j)} > 0$, $a_{p,1}^{(j)} \geq 0$, $b_{p,1}^{(j)} \geq 0$, and $a_{p,1}^{(j)} + b_{p,1}^{(j)} < 1$ for $j = 1,2,3$.

The parameters $\left(\alpha_p^{(1)}, \alpha_p^{(2)}, \alpha_p^{(3)}\right)$ are the abnormal returns for the fund *p* in the downside market, the neutral market, and the upside market, respectively. Moreover, the parameters $\left(\beta_{p,MKT}^{(1)}, \beta_{p,MKT}^{(2)}, \beta_{p,MKT}^{(3)}\right)$ are the systematic risks for the fund *p* when the stock market conditions are downside, neutral, and upside, respectively. We motivate market timing from the fund managerial perspective, assuming that the manager attempts to time market exposure in the fund shareholder's best interests. The successful market timer should increase the portfolio weight of highly-risky equities prior to the market upswings. Therefore, the fund investors will earn more return when the market return is larger than a certain positive level. Conversely, the successful market timer should decrease the portfolio weight of highly-risky equities prior to the market downturns. Therefore, it will reduce the loss due to market factors when the market return is below a certain negative level. If a fund manager increases the market portfolio's exposure prior to the market rise and decreases the market portfolio's exposure prior to the market fall, then $\beta_{p,MKT}^{(2)} - \beta_{p,MKT}^{(1)}$, $\beta_{p,MKT}^{(3)} - \beta_{p,MKT}^{(2)}$, and $\beta_{p,MKT}^{(3)} - \beta_{p,MKT}^{(1)}$ will be significantly larger than zero. In addition, we can view $\beta_{p,MKT}^{(2)} - \beta_{p,MKT}^{(1)}$ and $\beta_{p,MKT}^{(3)} - \beta_{p,MKT}^{(2)}$ as indices of managers' timing ability to downside and upside markets, respectively. Furthermore, we can use $\bar{\alpha}_p$ to measure the fund managerial overall stock selection ability, where

$$\bar{\alpha}_p = \frac{1}{T}\left[\alpha_p^{(1)} \sum_{t=1}^{T} 1_{\{R_{MKT,t} < r_{p,1}\}} + \alpha_p^{(2)} \sum_{t=1}^{T} 1_{\{r_{p,1} \le R_{MKT,t} < r_{p,2}\}} + \alpha_p^{(3)} \sum_{t=1}^{T} 1_{\{R_{MKT,t} \ge r_{p,2}\}}\right] \tag{2.4}$$

is the weighted average of abnormal performance in the three different market conditions. If $\bar{\alpha}_p$ is significantly larger than zero, we say that the mutual fund manager, on average, has a superior selectivity performance.

Because there may be public information that is correlated with future market returns, managers who use just public information to time market should get no credit for superior ability. Therefore, in order to eliminate this naïve market timing ability and allow time-varying returns and risk, we postulate that beta is a linear function of a vector Z_{t-1} of predetermined variables of information, as in Ferson and Schadt (1996). Therefore, the conditional three-regime threshold model based on four factor variables can be expressed as follows:

$$\begin{aligned} R_{p,t} = & \left(\alpha_p^{(1)} + \left(\beta_{p,MKT}^{(1)} + \Delta'_p z_{t-1}\right) R_{MKT,t}\right) \cdot I_{\{-\infty = r_{p,0} \le R_{MKT,t} < r_{p,1}\}} \\ & + \left(\alpha_p^{(2)} + \left(\beta_{p,MKT}^{(2)} + \Delta'_p z_{t-1}\right) R_{MKT,t}\right) \cdot I_{\{r_{p,1} \le R_{MKT,t} < r_{p,2}\}} \\ & + \left(\alpha_p^{(3)} + \left(\beta_{p,MKT}^{(3)} + \Delta'_p z_{t-1}\right) R_{MKT,t}\right) \cdot I_{\{r_{p,2} \le R_{MKT,t} < r_{p,3} = \infty\}} \\ & + \beta_{p,SMB} R_{SMB,t} + \beta_{p,HML} R_{HML,t} + \beta_{p,UMD} R_{UMD,t} + \varepsilon_{p,t}, \end{aligned} \tag{2.5}$$

where $z_{t-1} = Z_{t-1} - E(Z)$ represents the vector of deviations of Z_{t-1} from the average vector, Δ_p measures the response of the conditional beta to the information variables, and other notations are the same as equations (2.1) to (2.3). The criteria for measuring the fund's performance are also identical to those described in the unconditional model.

In the threshold class of models, classical estimation of parameters is usually done by the maximum likelihood or the least squares methods with a joint grid search over the threshold values $r_{p,1}$ and $r_{p,2}$ using information criteria such as AIC or BIC. (see, e.g., Tong (1990), Rabemananjara and Zakoian (1993), Li and Li (1996), Tsay (1998)) However, the inadequacy of these approaches is to fix the threshold parameters $(r_{p,1}, r_{p,2})$ in advance before estimating the other parameters by least squares. Therefore, the uncertainty of the threshold parameters cannot be taken into account when implementing statistical inference for the other parameters. To moderate the problems resulting from predetermining the threshold parameters, we adopt a Bayesian approach, which allows us to estimate the threshold parameters as well as the other parameters simultaneously. In addition to the above inference advantage, a Bayesian method incorporates the investor's prior belief about the validity of the pricing model and managerial technique with the information in the data, and thus it will result in more appropriate conclusions. Specifically, we can generate approximated samples from Markov chain Monte Carlo (MCMC) methods. In order to abridge the space, we only describe the Bayesian estimation procedures for conditional model, which is as follows.

Given the above assumptions, based on the conditional three-regime threshold four-factor model, it follows that the conditional likelihood function for the parameters $\left(\Theta_p^{(1)}, \Theta_p^{(2)}, \Theta_p^{(3)}, \Phi_p, \Upsilon_p\right)$ is

$$L\left(\boldsymbol{R}_p, \boldsymbol{R}_{MKT}, \boldsymbol{R}_{SMB}, \boldsymbol{R}_{HML}, \boldsymbol{R}_{UMD}, \boldsymbol{Z} \middle| \Theta_p^{(1)}, \Theta_p^{(2)}, \Theta_p^{(3)}, \Delta_p, \Phi_p, \Upsilon_p\right)$$

$$= \prod_{t=1}^{n} \left\{ \sum_{j=1}^{3} \left\{ \left[\frac{1}{\sqrt{2\pi h_{p,t}}} \exp\left(-\frac{\left(R_{p,t} - \left(\alpha_p^{(j)} + \left(\beta_{p,MKT}^{(j)} + \Delta_p' z_{t-1}\right) R_{MKT,t} + \Phi_p' f_t\right)\right)^2}{2h_{p,t}} \right) \right] \times I_{jt} \right\} \right\}, \quad (2.6)$$

where

$\Theta_p^{(j)} = \left(\alpha_p^{(j)}, \beta_{p,MKT}^{(j)}, a_{p,0}^{(j)}, a_{p,1}^{(j)}, b_{p,1}^{(j)}\right), \Phi = \left(\beta_{p,SMB}, \beta_{p,HML}, \beta_{p,UMD}\right)$,

$f_t = \left(R_{SMB,t}, R_{HML,t}, R_{UMD,t}\right)$, $\Upsilon_p = \left(r_{p,1}, r_{p,2}\right)$, $I_{jt} = 1$ indicates that $r_{p,j-1} \le R_{MKT,t} < r_{p,j}$

and $I_{jt}=0$ otherwise, and $h_{p,t}$ is the conditional variance of the model. Bayesian estimation requires us to specify the prior distributions for all unknown parameters. We adopt independent normal prior and uniform prior on the regression and GARCH parameters $\left(\Theta_p^{(1)},\Theta_p^{(2)},\Theta_p^{(3)},\Phi_p,\Delta_p\right)$. That is,

$$\Pi\left(\Theta_p^{(1)},\Theta_p^{(2)},\Theta_p^{(3)},\Phi_p,\Delta_p\right)=\left[\prod_{j=1}^{3}\Pi\left(\Theta_p^{(j)}\right)\right]\times\Pi\left(\Phi_p\right)\times\Pi\left(\Delta_p\right)$$

$$\begin{aligned}&=\prod_{j=1}^{3}\left[\Pi\left(\alpha_p^{(j)}\right)\Pi\left(\beta_{p,MKT}^{(j)}\right)\Pi\left(a_{p,0}^{(j)},a_{p,1}^{(j)},b_{p,1}^{(j)}\right)\right]\\&\quad\times\left[\Pi\left(\beta_{p,SMB}\right)\Pi\left(\beta_{p,HML}\right)\Pi\left(\beta_{p,UMD}\right)\right]\times\left[\prod_{k=1}^{K}\Pi\left(\delta_{p,k}\right)\right]\end{aligned}\tag{2.7}$$

$$\begin{aligned}&\propto\prod_{j=1}^{3}\left[N\left(\mu_{\alpha_p^{(j)}},\sigma^2_{\alpha_p^{(j)}}\right)N\left(\mu_{\beta_{p,MKT}^{(j)}},\sigma^2_{\beta_{p,MKT}^{(j)}}\right)\cdot I\left(a_{p,0}^{(j)}>0,\ a_{p,1}^{(j)}\ge 0,\ b_{p,1}^{(j)}\ge 0,\ a_{p,1}^{(j)}+b_{p,1}^{(j)}<1\right)\right]\\&\quad\times\left[N\left(\mu_{\beta_{p,SMB}},\sigma^2_{\beta_{p,SMB}}\right)N\left(\mu_{\beta_{p,HML}},\sigma^2_{\beta_{p,HML}}\right)N\left(\mu_{\beta_{p,UMD}},\sigma^2_{\beta_{p,UMD}}\right)\right]\times\prod_{k=1}^{K}N\left(\mu_{\delta_{p,k}},\sigma^2_{\delta_{p,k}}\right)\end{aligned}$$

where $N\left(\mu,\sigma^2\right)$ is a normal distribution with prior mean μ and prior variance σ^2, and $I(\cdot)$ is the indicator function with $I(S)=1$ if the event S is true, otherwise $I(S)=0$. Throughout the essay, we set prior mean $\mu_{\alpha_p^{(1)}}=\mu_{\alpha_p^{(2)}}=\mu_{\alpha_p^{(3)}}=\mu_{\alpha_{p,0}}$, where $\mu_{\alpha_{p,0}}$ equals to -1 multiplied by the fund's annual expense ratio divided by the number of trading date per year. Intuitively, this implies that we believe in advance that the fund manager does not possess superior skill of selecting stocks and that investing in mutual funds will underperform the benchmark assets by the charged management expense. In addition, the priors of factor parameters are set by $\mu_{\beta_{p,MKT}^{(1)}}=\mu_{\beta_{p,MKT}^{(2)}}=\mu_{\beta_{p,MKT}^{(3)}}=1$ and $\mu_{\beta_{p,SMB}}=\mu_{\beta_{p,HML}}=\mu_{\beta_{p,UMD}}=0$, which imply that systematic risks of fund and market are identical in any market conditions[1] and these three factors do not capture the behavior

[1] This can also be interpreted as that the fund manager does not have superior stock selection ability and expert market timing skill.

of expected mutual fund return, respectively. The prior mean of lagged public information are set by $\mu_{\delta_{p,k}} = 0$, for all *k*, which means that a portfolio manager does not use public information to predict the future market return. Leaving out overly subjective belief, we let prior variances of all regression coefficients equal to one. Furthermore, if the fund managers have ability to predict the market condition, they will adjust their portfolio positions on risky assets in different market conditions. That is to say, good fund managers should have prior beliefs about when to reallocate their positions on risky assets in the portfolio. In general, they have higher probability of adjusting their market exposure when the market condition attains certain level (i.e. $R_{MKT,t} < c_{p,1} < 0$ or $R_{MKT,t} > c_{p,2} > 0$). By contrast, they will have less motivation to change their highly risky assets ratio of the portfolio when the market condition will become a little better or a little worse ($R_{MKT,t} < c'_{p,1}$ or $R_{MKT,t} > c'_{p,2}$ where $c_{p,1} < c'_{p,1} < 0$ and $c_{p,2} > c'_{p,2} > 0$). In addition, they have a lower probability of adjusting the highly risky asset ratio when the market condition will be much better or much worse ($R_{MKT,t} < c''_{p,1}$ or $R_{MKT,t} > c''_{p,2}$ where $c''_{p,1} < c_{p,1} < c'_{p,1} < 0$ and $c''_{p,2} > c_{p,2} > c'_{p,2} > 0$). Therefore, we adopt a truncated normal prior on the threshold parameters $r_{p,1}$ and $r_{p,2}$, which can be written as

$$\begin{aligned}
&\Pi\left(\Upsilon_p\right) = \Pi\left(r_{p,1}\right) \times \Pi\left(r_{p,2}\right) \\
&\propto \left[N\left(\mu_{r_{p,1}}, \sigma^2_{r_{p,1}}\right) \cdot I\left(R_{MKT,[10]} < r_{p,1} < 0\right)\right] \times \left[N\left(\mu_{r_{p,2}}, \sigma^2_{r_{p,2}}\right) \cdot I\left(0 < r_{p,2} < R_{MKT,[90]}\right)\right],
\end{aligned} \tag{2.8}$$

where $\mu_{r_{p,1}}$ and $\mu_{r_{p,2}}$ are the prior means of $r_{p,1}$ and $r_{p,2}$, $\sigma^2_{r_{p,1}}$ and $\sigma^2_{r_{p,2}}$ are the prior variances of $r_{p,1}$ and $r_{p,2}$, and $R_{m,[k]}$ stands for the *k*th percentile of the market excess returns $\boldsymbol{R_{MKT}} = \left(R_{MKT,1}, R_{MKT,2}, \ldots, R_{MKT,T}\right)$. The amounts of $\sigma^2_{r_{p,1}}$ and $\sigma^2_{r_{p,2}}$ describe uncertainty of the managerial prior beliefs in selection of a switching point to adjust the market risk exposure. If $\sigma^2_{r_{p,1}}$ and $\sigma^2_{r_{p,2}}$ are large, it says that the manager has a more skeptical prior belief in selecting switching point to adjust fund's systematic risks. In contrast, the fund's manager has a more determined prior beliefs for small $\sigma^2_{r_{p,1}}$ and $\sigma^2_{r_{p,2}}$. To avoid excessively subjective prior belief, we use weakly informative prior and assume the prior means $\mu_{r_1} = R_{MKT,[25]}$ and $\mu_{r_2} = R_{MKT,[75]}$, and prior standard deviations $\sigma_{r_1} = \sigma_{r_2} = \tau/8$, where τ is the range of the market excess returns. The posterior density for the parameters is proportional to the product of the prior density and the conditional likelihood function. Given the conditional likelihood function in equation (2.6) and the prior densities in equations (2.7) and (2.8), the conditional joint posterior density can be written as

$$\begin{aligned} & p\left(\Theta_p^{(1)}, \Theta_p^{(2)}, \Theta_p^{(3)}, \Phi_p, \Delta_p, \Upsilon_p \middle| \boldsymbol{R}_p, \boldsymbol{R}_{MKT}, \boldsymbol{R}_{SMB}, \boldsymbol{R}_{HML}, \boldsymbol{R}_{UMD}, \boldsymbol{Z}\right) \\ & \propto L\left(\boldsymbol{R}_p, \boldsymbol{R}_{MKT}, \boldsymbol{R}_{SMB}, \boldsymbol{R}_{HML}, \boldsymbol{R}_{UMD}, \boldsymbol{Z} \middle| \Theta_p^{(1)}, \Theta_p^{(2)}, \Theta_p^{(3)}, \Phi_p, \Delta_p, \Upsilon_p\right) \\ & \quad \times \Pi\left(\Theta_p^{(1)}, \Theta_p^{(2)}, \Theta_p^{(3)}, \Phi_p, \Delta_p\right) \times \Pi\left(\Upsilon_p\right) \end{aligned} \tag{2.9}$$

Since this distribution does not have a standard form, we can compute moments of the joint posterior using the Metropolis-Hastings (MH) algorithm, which is a MCMC procedure introduced by Metropolis et al. (1953) and extended by Hastings (1970). The estimation procedures of the Bayesian analysis are concisely outlined as follows. Sample iteratively from $p\left(\boldsymbol{\Theta} \middle| \boldsymbol{R}_p, \boldsymbol{R}_{MKT}, \boldsymbol{R}_{SMB}, \boldsymbol{R}_{HML}, \boldsymbol{R}_{UMD}, \boldsymbol{Z}\right)$ to generate a posterior sample $\boldsymbol{\Theta}^{\mathbf{1}}, \boldsymbol{\Theta}^{\mathbf{2}}, \ldots, \boldsymbol{\Theta}^{N}$, where $\boldsymbol{N}$ is set to 10,000. Next, construct $\hat{\boldsymbol{\Theta}}$, the point estimate of $\boldsymbol{\Theta}$, as the sample mean of the posterior sample, which is written as $\hat{\boldsymbol{\Theta}} = \frac{1}{N-M} \sum_{\ell=M+1}^{N} \boldsymbol{\Theta}^{\ell}$, where $M = 5{,}000$ is the number of burn-in iterations to attain convergence. The detailed descriptions of above procedures are shown in Appendix. The unconditional or two-regime threshold models are also similar to above procedures.

3. DATA AND PERFORMANCE EVALUATION RESULTS

3.1. Data

Our primary mutual fund data is drawn from the Center for Research in Security Prices (CRSP) Survivor-Bias Free Mutual Fund Database. The database includes information collected from several sources and is designed to be a comprehensive sample of all funds from January 1962. Because the performance of disappearing funds is typically worse than that of surviving funds, inferring conclusions only on the selected sample of funds extant at the end of the time period will induce survivorship bias. Grinblatt and Titman (1989), Brown, Goetzman, Ibbotson, and Ross (1992), Brown and Goetzman (1995), and Malkiel (1995) concluded that the average return of surviving funds is 50 to 140 basis points per annum higher than returns on all funds. Following many prior studies and dropping out survivorship bias problems, we restrict our analysis to the subset of domestic diversified equity funds with a "common stock" investment policy and total net assets more than $500 million, as well as available daily net asset values (NAVs) and dividends (Ds). Therefore, by eliminating some redundant observations, our mutual fund sample consists of 622 open and domestic equity funds. These funds are classified as aggressive growth (141 funds), growth and income (202 funds), and long-term growth (279 funds) according to the fund objective of Investment Company Data, Inc. (ICDI). In our sample, there are thirteen funds which are not surviving on September 30, 2004. It is noted that the above objective code is based on the criterion specified by Standard & Poor's Fund Services in 1993. We take daily net asset values per share and dividends from CRSP Survivor-Bias Free US Mutual Fund Database. We then construct a daily return series for each fund from January 3, 2001 to September 30, 2004, by combining NAVs and Ds. We define return as

$$r_{p,t} = \frac{NAV_{p,t} + D_{p,t}}{NAV_{p,t-1}} - 1, \tag{2.10}$$

where $NAV_{p,t}$ and $D_{p,t}$ are the net asset value and the dividend of fund *p* at the end of date *t*, respectively. Because returns are calculated using net asset values, this measure of return is net of operating expenses.

We use the Standard and Poors 500 (S&P 500) market return proxies for the market portfolio. In addition, to compute daily excess returns on the funds and on the market return, we use the one-month T-Bill rate from Federal Reserve Bank of St. Louis' website to estimate the return on the riskless asset. We also capture the daily SMB, HML, and MOM factor data from Ken French's website.

To represent the public information for our empirical analyses, we choose the same variables used by Ferson and Warther (1996), which have been shown to be useful in predicting stock returns. The variables are (1) the lagged level of the one-month T-Bill yield, (2) the lagged dividend yield for the CRSP value-weighted NYSE and AMEX stock index, (3) the lagged slope of the U.S. Government term structure, measured as the difference between constant-maturity ten-year Treasury bond and three-month Treasury bill yields, (4) the lagged corporate bond quality spread, measure as the difference between Moody's BAA-rated and AAA-rated corporate bond yields, and (5) a dummy variable for month of January. The data of conditional information are from the data library of the Federal Reserve Bank of St. Louis.

3.2. Performance estimation results for an individual fund

We first compute the moments of the posterior distribution of the parameters for the model and the performance measure for a specific fund, which is randomly drawn from the investment objective category of aggressive growth funds. The main reason for this choice is that the fund whose investment objective is classified as aggressive growth can invest in more risky and flexible assets such as borrowing more than 10% of the value of its portfolio, short selling, investing in unregistered securities, and purchasing options. Therefore, the systematic risks in different market conditions for aggressive growth funds may have more significant differences. The total net assets of this fund at the end of 3rd quarter 2004 are $1,653.7 million. Annual turnover ratio, total load fees ratio, and expense ratio are 78 percent, 5.5 percent, and 1.3 percent, respectively.

As interpreted in the previous section, given the form of the conditional likelihood and the assumed prior independence among the regression parameters, the GARCH parameters, and the threshold parameters, the regression and GARCH parameters in each regime conditional on the threshold parameters depend only on the data and the prior distribution in its regime. For the specific aggressive growth fund, the Metropolis-Hastings algorithm is implemented to compute the posterior means and standard deviations of all parameters, as described in Appendix. We report the posterior means and standard deviations of the parameters based on the unconditional two-regime [2] and three-regime threshold models and some performance measure parameters in Panel A and B of Table 2.1, respectively.

[2] This model can be viewed as an extension of Merton-Henriksson (1981) model. Here, we also allow that funds' abnormal returns are different between up and down markets and assume abnormal return with a time-varying heteroskedasticity property.

TABLE 2.1 — Bayesian Estimation Results for a Specific Aggressive Growth Fund from Unconditional Two- and Three-Regime Threshold Four-Factor Models

Panel A. Unconditional two-regime threshold four-factor model

Parameter	Regime I $\left[R_{MKT,t}<0\right]$		Regime II $\left[R_{MKT,t}\geq 0\right]$	
$\alpha_p^{(j)}(\%)$	-0.0537	(0.0292)*	-0.0806	(0.0263)***
$\beta_{p,MKT}^{(j)}$	1.0216	(0.0283)***	1.1258	(0.0272)***
$a_{p,0}^{(j)}$	0.0088	(0.0047)*	0.0042	(0.0032)
$a_{p,1}^{(j)}$	0.1229	(0.0284)***	0.0572	(0.0278)**
$b_{p,1}^{(j)}$	0.8535	(0.0389)***	0.8969	(0.0374)***
Parameter	**Regime II- Regime I**			
$\alpha_p(\%)$	-0.0269	(0.0397)		
$\beta_{p,MKT}^{(j)}$	0.1042	(0.0366)***		
Parameter				
$\beta_{p,SMB}$	0.3768	(0.0244)***		
$\beta_{p,HML}$	-0.1629	(0.0346)***		
$\beta_{p,UMD}$	0.0348	(0.0193)*		
$\bar{\alpha}_p(\%)$	-0.0676	(0.0194)***		

Panel B. Unconditional three-regime threshold four-factor model

Parameter	Regime I $\left[R_{MKT,t}<r_1\right]$		Regime II $\left[r_1\leq R_{MKT,t}<r_2\right]$		Regime III $\left[R_{MKT,t}\geq r_2\right]$	
$\alpha_p^{(j)}(\%)$	-0.0381	(0.1282)	-0.0449	(0.0163)***	-0.1584	(0.0673)***
$\beta_{p,MKT}^{(j)}$	1.0435	(0.0708)***	1.0519	(0.0360)***	1.1688	(0.0439)***
$a_{p,0}^{(j)}$	0.0886	(0.0381)***	0.0062	(0.0053)	0.0288	(0.0157)**
$a_{p,1}^{(j)}$	0.2197	(0.0689)***	0.041	(0.0251)	0.1444	(0.0668)**
$b_{p,1}^{(j)}$	0.5794	(0.1694)***	0.8286	(0.0595)***	0.7872	(0.0833)***
Parameter	**Regime II- Regime I**		**Regime III- Regime II**		**Regime III- Regime I**	
$\alpha_p(\%)$	-0.0068	(0.1290)	-0.1135	(0.0691)	-0.1203	(0.1457)
$\beta_{p,MKT}^{(j)}$	0.0083	(0.0800)	0.117	(0.0571)**	0.1253	(0.0797)
Parameter						
$\beta_{p,SMB}$	0.3963	(0.0257)***				
$\beta_{p,HML}$	-0.1776	(0.0358)***				
$\beta_{p,UMD}$	0.0435	(0.0200)**				
$r_{p,1}(\%)$	-1.1203	(0.0748)***				
$r_{p,2}(\%)$	0.6634	(0.1884)***				
$\bar{\alpha}_p(\%)$	-0.0719	(0.0281)***				

Panels A and B in the table shows the coefficient estimates for unconditional two- and three-regime threshold four-factor models, respectively, which are based on the daily market excess returns and the daily excess returns of a specific growth fund for sample period from January 3, 2001 to September 30, 2004 (940 days). The unconditional two-regime model with daily conditional volatility estimates from GARCH (1, 1), which we adopt here, is as follows.

$$R_{p,t}=\left(\alpha_p^{(1)}+\beta_{p,MKT}^{(1)}R_{MKT,t}\right)\cdot 1_{\{R_{MKT,t}<0\}}+\left(\alpha_p^{(2)}+\beta_{p,MKT}^{(2)}R_{MKT,t}\right)\cdot 1_{\{0\leq R_{MKT,t}\}}+\beta_{p,SMB}R_{SMB,t}+\beta_{p,HML}R_{HML,t}+\beta_{p,UMD}R_{UMD,t}+\varepsilon_{p,t},$$

where $\varepsilon_{p,t}\left|\Im_{t-1}\sim N\left(0,\ h_{p,t}\right)\right.$ and $h_{p,t}=\left(a_{p,0}^{(1)}+a_{p,1}^{(1)}\varepsilon_{t-1}^2+b_{p,1}^{(1)}h_{t-1}\right)\cdot 1_{\{R_{MKT,t}<0\}}+\left(a_{p,0}^{(2)}+a_{p,1}^{(2)}\varepsilon_{t-1}^2+b_{p,1}^{(2)}h_{t-1}\right)\cdot 1_{\{0\leq R_{MKT,t}\}}.$

The unconditional three-regime threshold four-factor model is specified as follows.

$$R_{p,t}=\left(\alpha_p^{(1)}+\beta_{p,MKT}^{(1)}R_{MKT,t}\right)\cdot 1_{\{-\infty=r_0\leq R_{MKT,t}<r\}}+\left(\alpha_p^{(2)}+\beta_{p,MKT}^{(2)}R_{MKT,t}\right)\cdot 1_{\{r_1\leq R_{MKT,t}<r_2\}}+\left(\alpha_p^{(3)}+\beta_{p,MKT}^{(3)}R_{MKTt}\right)\cdot 1_{\{r_2\leq R_{MKT,t}<r_3=\infty\}}$$

$$+\beta_{p,SMB}R_{SMB,t}+\beta_{p,HML}R_{HML,t}+\beta_{p,UMD}R_{UMD,t}+\varepsilon_{p,t},$$ where $\varepsilon_{p,t}\left|\Im_{t-1}\sim N\left(0,\ h_{p,t}\right)\right.$ and

$$h_t=\left(a_{p,0}^{(1)}+a_{p,1}^{(1)}\varepsilon_{t-1}^2+b_{p,1}^{(1)}h_{t-1}\right)\cdot 1_{\{-\infty=r_{p,0}\leq R_{MKT,t}<r_{p,1}\}}+\left(a_{p,0}^{(2)}+a_{p,1}^{(2)}\varepsilon_{t-1}^2+b_{p,1}^{(2)}h_{t-1}\right)\cdot 1_{\{r_{p,1}\leq R_{MKT,t}<r_{p,2}\}}+\left(a_{p,0}^{(3)}+a_{p,1}^{(3)}\varepsilon_{t-1}^2+b_{p,1}^{(3)}h_{t-1}\right)\cdot 1_{\{r_{p,2}\leq R_{MKT,t}<r_{p,3}=\infty\}}$$

The numbers in parentheses are standard deviations. *,**, and *** indicate statistical significance at 10%, 5%, and 1% levels (two-tailed), respectively.

The results in Table 2.1 indicate that the systematic risk exposure increases when the stock market tends to rise regardless of two- or three-regime threshold models. However, the selectivity performance of fund manager decreases in the rising market, but the difference is not significant. We also find that the fund manager has a good market timing ability, which mainly results from the difference between neutral and upside markets ($0.117/0.1253 \approx 93.4\%$). In addition, two-regime threshold model may underestimate the quantity of changes in abnormal return and systematic risk exposure between downside and upside markets. The average abnormal returns are -0.0676 percent and -0.0719 percent a day (or about -16.9 percent and -17.9 percent a year) based on two- and three-regime threshold models, respectively. The intuition is that the manager has inferior ability to select undervalued securities and two-regime threshold model may overestimate fund manager's selectivity.

The GARCH parameters are almost significantly larger than zero in Panels A and B. This indicates that the abnormal return exhibits the volatility clustering phenomenon. That is to say, large changes in prices tend to cluster together and result in persistence of the amplitudes of price changes. In addition, the daily unconditional volatilities of abnormal returns in different regimes are 0.611 percent and 0.305 percent[3] (or about 9.66 percent and 4.82 percent a year) in the two-regime threshold model, and 0.664 percent, 0.218 percent, and 0.422 percent (or about 10.499 percent, 3.451 percent, and 6.672 percent a year) in the three-regime threshold model, respectively. This demonstrates that volatilities of stock selection performance in downside and upside markets are larger than that in the neutral market. Furthermore, the results show that the volatility of stock selection performance in the downside market is larger than that in the upside market. The reason for this phenomenon may be that investors usually have larger

[3] The daily unconditional volatility for the fund p in the jth regime is equal to $\sqrt{a_{p,0}^{(j)} \Big/ \left(1 - a_{p,1}^{(j)} - b_{p,1}^{(j)}\right)}$.

impulse responses when receiving bad news than good news.

The estimation results for threshold parameters reported in Panel B of Table 2.1 are -1.1203 percent and 0.6634 percent, and their standard deviations are 0.0748 percent and 0.1884 percent, respectively. This indicates, on average, that the manager will adjust his investment strategy when he predicts the market excess return is larger than 0.6634 percent and lower than -1.1203 percent. In addition, the asymmetric switching points may be explained as "languid" and "alert" anticipations in downside and upside markets, respectively. The manager's belief in choosing the switching point to the upside market is more diffuse than that to the downside market.

Furthermore, we take the public information into consideration to evaluate the specific fund's performance and show the results in Table 2.2. The differences between conditional two- and three-regime threshold models are similar to those between unconditional two- and three-regime threshold models. In Table 2.2, we find that only the variable dividend yield is statistically significant for this specific fund. The coefficient of this variable is negative, which would be expected since a lower level of dividend yield predicts higher market returns.

TABLE 2.2 — Bayesian Estimation Results for a Specific Aggressive Growth Fund from Conditional Two- and Three-Regime Threshold Four-Factor Models

Panel A. Conditional two-regime threshold four-factor model

Parameter	Regime I $[R_{MKT,t} < 0]$		Regime II $[R_{MKT,t} \geq 0]$	
$\alpha_p^{(j)}$ (%)	-0.0686	(0.0273)**	-0.0728	(0.0265)***
$\beta_{p,MKT}^{(j)}$	1.0293	(0.0271)***	1.1488	(0.0275)***
$a_{p,0}^{(j)}$	0.008	(0.0041)**	0.0042	(0.0030)
$a_{p,1}^{(j)}$	0.1382	(0.0312)***	0.0432	(0.0257)*
$b_{p,1}^{(j)}$	0.8392	(0.0394)***	0.911	(0.0334)***

Parameter	Regime II- Regime I	
$\alpha_p^{(j)}$ (%)	-0.0042	(0.0383)
$\beta_{p,MKT}^{(j)}$	0.1195	(0.0355)***

Parameter			Parameter		
$\beta_{p,SMB}$	0.3823	(0.0258)***	$\delta_{p,1}$	-0.0286	(0.0451)
$\beta_{p,HML}$	-0.1224	(0.0371)***	$\delta_{p,2}$	-0.3990	(0.1331)***
$\beta_{p,UMD}$	0.0671	(0.0212)***	$\delta_{p,3}$	0.0056	(0.0342)
$\bar{\alpha}_p$ (%)	-0.0708	(0.0188)***	$\delta_{p,4}$	-0.0996	(0.0720)
			$\delta_{p,5}$	0.0367	(0.0391)

Panel B. Conditional three-regime threshold four-factor model

Parameter	Regime I $[R_{MKT,t} < r_1]$		Regime II $[r_1 \leq R_{MKT,t} < r_2]$		Regime III $[R_{MKT,t} \geq r_2]$	
$\alpha_p^{(j)}$ (%)	-0.1473	(0.1296)	-0.0467	(0.0167)***	-0.1141	(0.0673)*
$\beta_{p,MKT}^{(j)}$	0.9940	(0.0723)***	1.0805	(0.0351)***	1.1672	(0.0447)***
$a_{p,0}^{(j)}$	0.0708	(0.0313)**	0.0056	(0.0041)	0.0200	(0.0136)
$a_{p,1}^{(j)}$	0.2050	(0.0684)***	0.0470	(0.0244)*	0.1175	(0.0645)*
$b_{p,1}^{(j)}$	0.6657	(0.1337)***	0.8417	(0.0550)***	0.8182	(0.0791)***

Parameter	Regime II- Regime I		Regime III- Regime II		Regime III- Regime I	
$\alpha_p^{(j)}$ (%)	0.1006	(0.1302)	-0.0674	(0.0698)	0.0332	(0.1540)
$\beta_{p,MKT}^{(j)}$	0.0866	(0.0808)	0.0867	(0.0574)	0.1732	(0.0783)**

Parameter			Parameter		
$\beta_{p,SMB}$	0.3923	(0.0263)***	$\delta_{p,1}$	-0.0095	(0.0502)
$\beta_{p,HML}$	-0.1372	(0.0382)***	$\delta_{p,2}$	-0.3095	(0.1442)**
$\beta_{p,UMD}$	0.0702	(0.0222)***	$\delta_{p,3}$	0.0098	(0.0384)
$r_{p,1}$ (%)	-1.1196	(0.0771)***	$\delta_{p,4}$	-0.0773	(0.0767)
$r_{p,2}$ (%)	0.6581	(0.2127)***	$\delta_{p,5}$	0.0464	(0.0434)
$\bar{\alpha}_p$ (%)	-0.0810	(0.0271)***			

The table shows the results of Bayesian estimation for conditional two- and three- regime threshold four-factor models in Panels A and B, respectively, which are based on the daily market excess returns and the daily excess returns of a specific growth fund for sample period from January 3, 2001 to September 30, 2004 (940 days). The conditional two-regime model with daily conditional volatility estimates from GARCH (1, 1), which we adopt here, is as follows.

$$R_{p,t} = \left(\alpha_p^{(1)} + \left(\beta_{p,MKT}^{(1)} + \sum_{i=1}^{5}\delta_{p,i}z_{i,t-1}\right)R_{MKT,t}\right)\cdot 1_{\{R_{MKT,t}<0\}} + \left(\alpha_p^{(2)} + \left(\beta_{p,MKT}^{(2)} + \sum_{i=1}^{5}\delta_{p,i}z_{i,t-1}\right)R_{MKT,t}\right)\cdot 1_{\{0\leq R_{MKT,t}\}} + \beta_{p,SMB}R_{SMB,t} + \beta_{p,HML}R_{HML,t} + \beta_{p,UMD}R_{UMD,t} + \varepsilon_{p,t}.$$

The conditional three-regime threshold four-factor model is specified as follows.

$$R_{p,t} = \left(\alpha_p^{(1)} + \left(\beta_{p,MKT}^{(1)} + \sum_{i=1}^{5}\delta_{p,i}z_{i,t-1}\right)R_{MKT,t}\right)\cdot 1_{\{-\infty=r_0\leq R_{MKT,t}<r\}} + \left(\alpha_p^{(2)} + \left(\beta_{p,MKT}^{(2)} + \sum_{i=1}^{5}\delta_{p,i}z_{i,t-1}\right)R_{MKT,t}\right)\cdot 1_{\{r_1\leq R_{MKT,t}<r_2\}}$$
$$+ \left(\alpha_p^{(3)} + \left(\beta_{p,MKT}^{(3)} + \sum_{i=1}^{5}\delta_{p,i}z_{i,t-1}\right)R_{MKTt}\right)\cdot 1_{\{r_2\leq R_{MKT,t}<r_3=\infty\}} + \beta_{p,SMB}R_{SMB,t} + \beta_{p,HML}R_{HML,t} + \beta_{p,UMD}R_{UMD,t} + \varepsilon_{p,t}.$$

The numbers in parentheses are standard deviations. *,**, and *** indicate statistical significance at 10%, 5%, and 1% levels (two-tailed), respectively.

We also compare the dissimilarities between unconditional and conditional models. In two-regime threshold models, the momentum effect in the conditional model becomes more significant, and the unconditional model will overestimate fund's abnormal return. We find more significant inconsistency on evaluating selectivity and market timing skills in three-regime threshold models. First, the abnormal return in the unconditional model will be overestimated in the downside market and underestimated in the upside market. Second, the market exposure will be overestimated and underestimated in downside and neutral markets, respectively, without taking predetermined information into account. In the conditional model, the market timing performance will averagely involve the skills to forecast downside and upside markets, instead of the pure aptitude for forecasting the upside market. Third, we find that the quantity of market timing measurement will be underestimated and the average abnormal return will be overevaluated compared to the conditional model.

3.3. Performance estimation results for all funds

Table 2.3 presents the average coefficient estimates for unconditional and conditional two- and three-regime threshold four-factor models of the 622 mutual funds with a sample period of January 3, 2001 to September 30, 2004. In Panel A, we find that the market exposure decrease, moving down the table from the AG funds to the GI funds, no matter in up or down markets based on the unconditional two-regime model. However, when we divide market conditions into three regimes, Panel B of Table 2.3 shows that the market exposure increase from the AG funds to GI funds in the downside market and demonstrates that the two-regime threshold model will overestimate or underestimate the market exposure in the downside market. In addition, for the AG and LG funds, we find that their market exposure increases from downside to neutral markets, while it decreases from neutral to upside markets. The result is opposite to the GI funds. Moreover, the GI funds have lowest down-threshold and highest up-threshold. This indicates that the managers of GI funds have more laggard sensibility to predict downside and upside markets.

TABLE 2.3 — Bayesian Estimation Results for All Funds from Unconditional and Conditional Two- and Three-Regime Threshold Four-Factor Models

Panel A. Unconditional two-regime threshold four-factor model

Fund Types	$\alpha_p^{(1)}$(%)	$\beta_{p,MKT}^{(1)}$	$\alpha_p^{(2)}$(%)	$\beta_{p,MKT}^{(2)}$	$\beta_{p,SMB}$	$\beta_{p,HML}$	$\beta_{p,UMD}$
AG	-0.05489	1.00695	-0.01597	1.03696	0.49184	-0.02554	-0.01910
LG	-0.04413	0.99968	-0.01014	1.02284	0.04403	-0.07130	-0.00974
GI	-0.00305	0.97831	-0.01031	0.98447	-0.12064	0.08191	0.04707
Overall	-0.03323	0.99439	-0.01152	1.01358	0.09207	-0.01117	0.00659

Panel B. Unconditional three-regime threshold four-factor model

Fund Types	$\alpha_p^{(1)}$(%)	$\beta_{p,MKT}^{(1)}$	$\alpha_p^{(2)}$(%)	$\beta_{p,MKT}^{(2)}$	$\alpha_p^{(3)}$(%)	$\beta_{p,MKT}^{(3)}$	$\beta_{p,SMB}$	$\beta_{p,HML}$	$\beta_{p,UMD}$	r_1(%)	r_2(%)
AG	-0.16234	0.95403	-0.02518	1.04212	0.01335	1.02741	0.49702	-0.01188	-0.02418	-0.88907	0.85977
LG	-0.08505	0.97696	-0.01968	1.03212	0.01166	1.01289	0.04208	-0.07100	-0.01133	-0.82026	0.86728
GI	0.01311	0.98499	-0.00887	0.96885	0.01258	0.97094	-0.12027	0.07269	0.04623	-0.91900	0.87863
Overall	-0.07069	0.97437	-0.01741	1.01384	0.01234	1.00256	0.09248	-0.01093	0.00445	-0.86792	0.86927

Panel C. Conditional two-regime threshold four-factor model

Fund Types	$\alpha_p^{(1)}$(%)	$\beta_{p,MKT}^{(1)}$	$\alpha_p^{(2)}$(%)	$\beta_{p,MKT}^{(2)}$	$\beta_{p,SMB}$	$\beta_{p,HML}$	$\beta_{p,UMD}$
AG	-0.05082	1.01934	-0.02763	1.06075	0.48465	-0.00302	-0.01905
LG	-0.03238	1.01508	-0.02028	1.03808	0.03358	-0.08140	-0.01763
GI	0.00327	0.97240	-0.00398	0.96336	-0.11983	0.04020	0.04029
Overall	-0.02498	1.00219	-0.01665	1.01896	0.08601	-0.02414	0.00086

Panel D. Conditional three-regime threshold four-factor model

Fund Types	$\alpha_p^{(1)}$(%)	$\beta_{p,MKT}^{(1)}$	$\alpha_p^{(2)}$(%)	$\beta_{p,MKT}^{(2)}$	$\alpha_p^{(3)}$(%)	$\beta_{p,MKT}^{(3)}$	$\beta_{p,SMB}$	$\beta_{p,HML}$	$\beta_{p,UMD}$	r_1(%)	r_2(%)
AG	-0.16189	0.96118	-0.02483	1.03888	-0.01147	1.05371	0.49312	0.00471	-0.02105	-0.87826	0.80234
LG	-0.04867	1.00494	-0.01860	1.02780	-0.02103	1.03865	0.03549	-0.07918	-0.01719	-0.76992	0.75371
GI	0.04233	0.99274	-0.00819	0.95922	0.01085	0.95625	-0.12003	0.03986	0.03994	-0.82127	0.75383
Overall	-0.04478	0.99106	-0.01663	1.00804	-0.00851	1.01530	0.08873	-0.02150	0.00049	-0.81116	0.76477

Listed are average coefficient estimates for unconditional and conditional two- and three-regime threshold four-factor models, which are based on the daily market excess returns and the daily excess returns of the 622 mutual funds with sample period from January 3, 2001 to September 30, 2004. The models, which we apply in Panels A and C are the same as those that we mention in the Table 2.1. Moreover, the models which we adopt in Panels C and D are specified in the Table 2.2.

Comparing unconditional with conditional two-regime threshold models, we find controversial results. In Panel A, we find the GI funds with slight market timing ability. But separating the performance from public information, there is no expert market timing skill any more for the GI funds, which is shown in Panel C of Table 2.3. In Panel D, the market exposures become monotonically increasing or decreasing followed by market conditions compared with Panel B. The amounts of change in market exposures are more significant than those in the unconditional model. Comparing Panel A with Panel C, we find that the unconditional model results in the descending and ascending biases in the abnormal return by 2.061 percent and 1.283 percent a year in down and up markets. However, the phenomenon is more obvious for the three-regime threshold model. The unconditional model will bias the abnormal return downward by 6.478 percent and upward by 5.213 percent a year in downside and upside markets, respectively. Furthermore, on average, down-threshold and up-threshold get higher and lower, respectively. Therefore, the unconditional model may underestimate or overestimate fund manager's anticipations in choosing switching points to downside and upside markets.

Table 2.4 lists the count of funds that have positive and negative changes in abnormal return and risk exposure coefficients and the number of funds that have significantly positive and negative shifts in abnormal return and risk exposure coefficients. We compare the discrepancy between unconditional and conditional models. In Panels A and C, even if the fraction of managers who have positive or negative changes in selectivity are similar, the change in selectivity between down and up markets gets less significant in the conditional model. The unconditional model biases the timing coefficients upward especially for the GI funds. For the three-regime threshold model, we find that the differences between unconditional and conditional models are similar to those for two-regime threshold model but the differences are more apparent. For example, the unconditional model overestimates seriously the timing ability to forecast downside market and underestimates less the timing ability to predict upside market. In addition, although the fractions of managers who have significantly positive or negative timing coefficients, $\beta_{p,MKT}^{(2)} - \beta_{p,MKT}^{(1)}$ and $\beta_{p,MKT}^{(3)} - \beta_{p,MKT}^{(2)}$, are low, the portion of funds whose timing coefficient between downside and neutral markets is significantly high. The conclusion is contrastive to the unconditional three-regime models whose timing coefficients, between downside and neutral markets and between neutral and upside markets, are more significant, but timing coefficient between downside and upside markets is less significant.

TABLE 2.4 — Unconditional and Conditional Measures of Selectivity and Timing Based on Two- and Three-Regime Threshold Four-Factor Models

Panel A. Unconditional two-regime threshold four-factor model

Fund Types	Sizes	$\alpha_p^{(2)}-\alpha_p^{(1)}$		$\beta_{p,MKT}^{(2)}-\beta_{p,MKT}^{(1)}$	
		—	+	—	+
AG	141	48(17)	93(43)	35(3)	106(38)
LG	279	92(26)	187(99)	71(6)	208(66)
GI	202	91(33)	111(36)	56(12)	146(30)
Overall	622	231(76)	391(178)	162(21)	460(134)

Panel B. Unconditional three-regime threshold four-factor model

Fund Types	Sizes	$\alpha_p^{(2)}-\alpha_p^{(1)}$		$\alpha_p^{(3)}-\alpha_p^{(2)}$		$\alpha_p^{(3)}-\alpha_p^{(1)}$	
		—	+	—	+	—	+
AG	141	34(13)	107(63)	55(11)	86(19)	35(19)	106(58)
LG	279	96(28)	183(88)	106(23)	173(55)	90(34)	189(89)
GI	202	101(41)	101(21)	56(11)	146(36)	83(32)	119(40)
Overall	622	231(82)	391(172)	217(45)	405(110)	208(85)	414(187)

Fund Types	Sizes	$\beta_{p,MKT}^{(2)}-\beta_{p,MKT}^{(1)}$		$\beta_{p,MKT}^{(3)}-\beta_{p,MKT}^{(2)}$		$\beta_{p,MKT}^{(3)}-\beta_{p,MKT}^{(1)}$	
		—	+	—	+	—	+
AG	141	45(13)	96(55)	71(20)	70(25)	28(4)	113(43)
LG	279	94(24)	185(96)	162(60)	117(29)	101(14)	178(55)
GI	202	92(33)	110(30)	121(49)	81(23)	112(27)	90(9)
Overall	622	231(70)	391(181)	354(129)	268(77)	241(45)	381(107)

Panel C. Conditional two-regime threshold four-factor model

Fund Types	Sizes	$\alpha_p^{(2)}-\alpha_p^{(1)}$		$\beta_{p,MKT}^{(2)}-\beta_{p,MKT}^{(1)}$	
		—	+	—	+
AG	141	49(4)	92(14)	28(4)	113(44)
LG	279	114(24)	165(28)	91(19)	188(58)
GI	202	100(17)	102(7)	142(30)	60(6)
Overall	622	263(45)	359(49)	261(53)	361(108)

Panel D. Conditional three-regime threshold four-factor model

Fund Types	Sizes	$\alpha_p^{(2)}-\alpha_p^{(1)}$		$\alpha_p^{(3)}-\alpha_p^{(2)}$		$\alpha_p^{(3)}-\alpha_p^{(1)}$	
		—	+	—	+	—	+
AG	141	36(11)	105(62)	64(8)	77(6)	31(18)	110(51)
LG	279	122(44)	157(66)	142(32)	137(26)	125(46)	154(54)
GI	202	170(69)	32(5)	56(8)	146(31)	139(29)	63(11)
Overall	622	328(124)	294(133)	262(48)	360(63)	295(93)	327(116)

Fund Types	Sizes	$\beta_{p,MKT}^{(2)}-\beta_{p,MKT}^{(1)}$		$\beta_{p,MKT}^{(3)}-\beta_{p,MKT}^{(2)}$		$\beta_{p,MKT}^{(3)}-\beta_{p,MKT}^{(1)}$	
		—	+	—	+	—	+
AG	141	41(11)	100(43)	57(4)	84(12)	32(5)	109(52)
LG	279	124(32)	155(38)	131(11)	148(32)	106(28)	173(60)
GI	202	157(35)	45(6)	135(14)	67(13)	162(85)	40(4)
Overall	622	322(78)	300(87)	323(29)	299(57)	300(118)	322(116)

The table shows the numbers of funds which their changes in selectivity and systematic risk are negative and positive between different market conditions based on unconditional and conditional two- and three-regime threshold four-factor models. The number in parentheses represents the number of statistically significant parameter estimates at 5% level (two-tailed). The sample consists of 622 mutual funds and sample period is January 2, 2001, to September 30, 2004, a total of 940 trading days.

Furthermore, we compare the conditional two- with three-regime threshold models. We find that the fraction of funds with significant changes in selectivity between different market conditions is higher, when the three-regime threshold is used instead of two-regime. In addition, the three-regime threshold model adds the fraction of funds with significant timing coefficient between downside and upside markets especially for the GI funds. In Table 2.5, we try to know that the manager's timing ability is mainly generated from the skill to predict downside or upside markets. We find that most fund managers with significant timing coefficients will adjust slightly their market exposure to a certain direction. Therefore, there are less fraction of funds with significant timing coefficients, $\beta_{p,MKT}^{(2)} - \beta_{p,MKT}^{(1)}$ and $\beta_{p,MKT}^{(3)} - \beta_{p,MKT}^{(2)}$. But, as long as we only examine the timing coefficient between downside and upside markets, this will result in a significant timing coefficient. In addition, we find that the significant timing coefficients most result from the changes in the market exposure between downside and neutral markets.

TABLE 2.5 — Frequency Counts of Significantly Positive and Negative Market Timing Coefficients Based on the Conditional Three-Regime Threshold Model

		Significant at 5 percent level		
$\left(\beta_{MKT}^{(2)} - \beta_{MKT}^{(1)}, \beta_{MKT}^{(3)} - \beta_{MKT}^{(2)}, \beta_{MKT}^{(3)} - \beta_{MKT}^{(1)}\right)$	N	$\beta_{MKT}^{(2)} - \beta_{MKT}^{(1)}$	$\beta_{MKT}^{(3)} - \beta_{MKT}^{(2)}$	$\beta_{MKT}^{(3)} - \beta_{MKT}^{(1)}$
$(+,+,+)$	117	30	7	75
$(+,-,+)$	129	53	7	31
$(-,+,+)$	76	9	31	10
$(-,-,-)$	140	17	8	85
$(-,+,-)$	106	52	19	29
$(+,-,-)$	54	4	14	4

Divide total sample into six possible regions based on the manager's market timing skills. The frequency counts of significantly positive and negative market timing coefficients based on the conditional three-regime threshold model are reported. The sample consists of 622 mutual funds with sample period from January 3, 2001 to September 30, 2004.

To examine the relationship between selectivity and timing skills, we compute pair-wise correlation coefficients of seven parameter estimates which are used to evaluate funds' performance. The correlation coefficients for the full sample are presented in Table 2.6. For the selectivity correlations between any two different market conditions, we find that the selectivity in the downside market is positively correlated with that in the neutral market, but is slightly negatively correlated with that in the upside market. In addition, we find that the average selectivity mainly depend on the managers' selectivity ability in the downside market, and latter are those in neutral and upside markets. The similar results happen in the market timing. Fund managers' market timing coefficients between downside and upside markets are highly positively correlated with those between downside and neutral markets. The managers with superior timing ability to forecast the downside market will posses inferior timing ability to predict the upside market simultaneously.

TABLE 2.6 — Correlation Coefficients for Selectivity and Timing Estimated Parameters Based on the Conditional Three-Regime Threshold Model

	$\bar{\alpha}$	$\alpha^{(1)}$	$\alpha^{(2)}$	$\alpha^{(3)}$	$\beta_{MKT}^{(2)}-\beta_{MKT}^{(1)}$	$\beta_{MKT}^{(3)}-\beta_{MKT}^{(2)}$	$\beta_{MKT}^{(3)}-\beta_{MKT}^{(1)}$
$\bar{\alpha}$	1.0000	0.8266***	0.6407***	0.3554***	-0.7660***	-0.2289***	-0.9315***
$\alpha^{(1)}$		1.0000	0.4196***	-0.1499***	-0.8880***	0.1308***	-0.8088***
$\alpha^{(2)}$			1.0000	0.1895***	-0.4663***	0.0543	-0.4346***
$\alpha^{(3)}$				1.0000	0.0895**	-0.7395***	-0.4155***
$\beta_{MKT}^{(2)}-\beta_{MKT}^{(1)}$					1.0000	-0.3551***	0.7686***
$\beta_{MKT}^{(3)}-\beta_{MKT}^{(2)}$						1.0000	0.3251***
$\beta_{MKT}^{(3)}-\beta_{MKT}^{(1)}$							1.0000

Pairwise correlation coefficients of selectivity, market timing, and threshold estimated parameters for the full sample are reported. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels (two-tailed test), respectively

For the relationship between managers' selectivity and timing ability, the correlation between the average return and the timing coefficient in our sample is -0.9315. This means that managers with good selection ability tend to be poor market timers, which is consistent with the findings in Henriksson (1984), Chang and Lewellen (1984), and Glosten and Jagannathan (1994). And, this appearance is more obvious in the downside market than in the upside market. The reason may be that the manager purchase put options to hedge the market exposures in the downside market and this will result in a naïvely superior timing ability. Clearly, this type of timing is artificial and is irrelevant to manager ability. Meanwhile, the hedge cost is reflected in lower manager selectivity. However, the evidence in the upside market is not as powerful as that in the downside market. This indicates that although more managers use options to hedge market exposures in the downside market, fewer managers earn more market premiums by options in the upside market.

4. BEHAVIOR OF MUTUAL FUND CLIENTS

In this section, we conduct an empirical analysis of the determinants of mutual fund flows, focusing on the past performance and characteristics variables. We use annual total net asset values from Standard & Poor's Micropal to calculate annual normalized net cash flow for each fund. We define normalized net cash flow for the fund p during year t as

$$NCF_{p,t} = \frac{TNA_{p,t} - TNA_{p,t-1}(1 + R_{pt})}{TNA_{p,t-1}}, \tag{2.11}$$

where $TNA_{p,t}$ is the fund's total net asset value at the end of year t, and $R_{p,t}$ is the fund's return during year t.

We use conditional three-regime threshold four-factor model to estimate managers' selectivity, market timing ability, and thresholds as measurement of funds' previous one year performance. The fund characteristics which we adopt in this essay are total net assets (TNA) under management in millions of dollars, turnover (TURN) defined as the minimum of aggregate purchases and sales of securities divided by the average TNA over the calendar year, total loads ratio (LOAD) which include all maximum front, deferred, and redemption charges as a percentage of new investment, expense (EXPEN) which is the total annual management fees and expenses divided by year-end TNA, percentage invested in cash equivalents (CASH), and the dummy variable for the fund's status on September 30, 2004 ($I(p\in$LIVE$)$), where ($I(p\in$LIVE$)$) equals to one if the fund p is still surviving on September 30, 2004, and equals to zero otherwise.

To control the impacts of both investment objective and year of observation, we estimate the following fixed effects panel regression.

$$\begin{aligned} NCF_{p,t} &= \theta_0 + \sum_{i=1}^{3}\theta_i YEAR_t + \sum_{i=4}^{5}\theta_i INVOB_p + \theta_6\alpha_{p,t}^{(1)} + \theta_7\alpha_{p,t}^{(2)} + \theta_8\alpha_{p,t}^{(3)} + \theta_9\left(\beta_{p,t}^{(2)} - \beta_{p,t}^{(1)}\right) \\ &+ \theta_{10}\left(\beta_{p,t}^{(3)} - \beta_{p,t}^{(2)}\right) + \theta_{11}\ln\left(TNA_{p,t-1}\right) + \theta_{12}TURN_{p,t-1} + \theta_{13}LOAD_{p,t-1} \\ &+ \theta_{14}EXP_{p,t-1} + \theta_{15}CASH_{p,t-1} + \theta_{16}NCF_{p,t-1} + \theta_{17}I\left(p \in LIVE\right) + \varepsilon_{p,t} \end{aligned} \tag{2.12}$$

where INVOB_p and YEAR_t are dummy variables for investment objective and year of observation, respectively. The first five variables in Equation (2.12) comprise the fixed effect model and account for period-specific fund net cash flows attributable to the same period economy-wide and investment-style effects. In addition, to control for autocorrelation and heteroscedasticity in the panel regression, we allow depend variable to follow a common AR(1) process and for each fund it has its own variance. Then again, we allow for disturbances to be contemporaneously correlated across funds.

The results presented in Table 2.7 indicates that the net cash flow are significantly and positively associated with all past selectivity and timing performance measures at 1 percent level except for the market timing ability between downside and neutral markets. The finding provides supporting evidence that the naïve downside market timing ability resulted from the operation of options is not valuable to fund investors. The behavior of Investors mainly relies on past managers' selectivity and upside market timing ability. In addition, there is a strong positive relationship between mutual fund flows and last-year threshold points of downside and upside markets. The results demonstrate that investors prefer managers with alert insight to forecast the downside market but with languid perception to forecast the upside market.

TABLE 2.7 — Regression of Net Cash Flow on Fund's Past Performance and Characteristics Based on the Conditional Three-Regime Threshold Four-Factor Model

Dependent variable	Explanation variables													
	Performance variables for lagged one year							Characteristics variables for lagged one year						Status
	$\alpha^{(1)}(\%)$	$\alpha^{(2)}(\%)$	$\alpha^{(3)}(\%)$	$\beta_{MKT}^{(2)}-\beta_{MKT}^{(1)}$	$\beta_{MKT}^{(3)}-\beta_{MKT}^{(2)}$	$r_1(\%)$	$r_2(\%)$	Ln(TNA)	TURN(%)	LOAD(%)	EXPEN(%)	CASH(%)	NCF	LIVE
NCF	0.10911	0.55963	0.06686	0.05302	0.09394	0.06984	0.01832	0.00410	-0.01102	-0.00082	-4.71237	0.00056	0.44941	0.02558
	(0.02206)***	(0.04276)***	(0.01891)***	(0.03545)	(0.02549)***	(0.00634)***	(0.00828)**	(0.00113)***	(0.00315)***	(0.00027)***	(0.87444)***	(0.00030)*	(0.07071)***	(0.00807)***

This table reports the coefficients and standard errors from the fixed-effect panel data model of net cash flow on seven lagged performance measure indexes, six lagged fund's characteristic variables, and one status variables. The seven lagged performance measure indexes are ($\alpha^{(1)}$, $\alpha^{(2)}$, $\alpha^{(3)}$), which are fund's selectivity in three different market conditions last year , ($\beta^{(2)}$- $\beta^{(1)}$, $\beta^{(3)}$- $\beta^{(2)}$), which mean manager's skill to predict downside and upside markets last year, and threshold points, (r_1, r_2), in previous one year. The six lagged characteristics variables are total net assets (TNA), turnover (TURN), total load (LOAD), expense (EXPEN), percentage invested in cash equivalents (CASH), net cash flow of the fund (NCF) in previous one year. The status variables are the dummy variable for the fund's status on September 30, 2004. The standard errors for the coefficients are below the coefficients in parentheses. The regression factors are [$\alpha^{(1)}$, $\alpha^{(2)}$, $\alpha^{(3)}$,$\beta^{(2)}$- $\beta^{(1)}$, $\beta^{(3)}$- $\beta^{(2)}$, r_1, r_2,TNA, TURN, LOAD, EXPEN, CASH, NCF, , $I(p\in\text{LIVE})$], where $I(p\in\text{LIVE})$ equal to one if fund *p* is surviving on September 30, 2004, and equal to zero otherwise. The estimation results for dummy variables for the investment objectives and year of observation are ignored here. The sample consists of 622 mutual funds. The mutual fund sample period is from January 3, 2001 to September 30, 2004. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels (two-tailed test), respectively.

For the impacts of characteristics on mutual fund flows, we find that a positive relationship between fund flow and fund size. One possible explanation for this result is that large funds spend more on advertising and are likely to attract greater investor cash flows. This finding is consistent with Jain and Wu (1998) who show that flows are significantly larger for those equity funds that are advertised in the financial magazines. Turnover, load fee, and management expense are significantly and negatively associated with money flows. The reason might be that investors think that fine managers should not operate their portfolio too frequently. In addition, Carhart (1997) found that load funds and funds with higher expense ratios have worse performance than funds with lower fees. Again, because investors would like to maximize net-of-fee earnings, higher load fee and expense ratio will reduce investor's desires. Finally, we find that the net cash flows are persistent and positively related to live funds, suggesting that clients as a whole invest in funds expected to survive.

5. RELATIONSHIPS BETWEEN PERFORMANCES AND CHARACTERISTICS

In this section, we investigate the relationships between fund managerial performance and several fund characteristics. In addition, we examine if fund performance is persistent and if mutual fund investors have good selection ability. The fund characteristics which we adopt in this section are the same as those in previous section. To investigate the relationships between performance and characteristics, we estimate the following fixed effects panel regression.

$$\begin{aligned} PMI_{p,t} = {} & \theta_0 + \sum_{i=1}^{3} \theta_i YEAR_t + \sum_{i=4}^{5} \theta_i INVOB_p + \theta_6 \ln\left(TNA_{p,t}\right) + \theta_7 TURN_{p,t} \\ & + \theta_8 LOAD_{p,t} + \theta_9 EXP_{p,t} + \theta_{10} CASH_{p,t} + \theta_{11} NCF_{p,t} + \theta_{12} PMI_{p,t-1} \\ & + \theta_{13} NCF_{p,t-1} + \theta_{14} I\left(p \in LIVE\right) + \varepsilon_{p,t}, \end{aligned} \tag{2.13}$$

where $PMI_{p,t}$ is performance measure indices (ex, selectivity or market timing indices) during year t based on the conditional three-regime threshold model and the definitions of other variables are the same to those in Equation (2.12). As before, the regression is estimated allowing autocorrelation with lag one period (AR(1)) of dependent variable to checking whether there is persistent fund performance. The lagged variable, $NCF_{p,t-1}$, is used to examine if fund investors have good selection ability. We also allow for disturbances to be contemporaneously correlated and heteroscedasticity across funds.

Panel A of Table 2.8 reports the coefficient estimates with selectivity as dependent variables. The negative and significant coefficient estimate of total net asset in the upside market supports the hypothesis that fund size erodes performance because of liquidity mentioned by Chen et al. (2004). However, owing to small funds with more illiquid stocks, we find strong evidence that small funds earn significantly lower abnormal returns than larger funds in the downside markets. Size impacts on selectivity are as a whole not significant, which is consistent with the findings of Carhart (1997) and Grinblatt and Titman (1994). In addition, the coefficients on the turnover are significantly negative in downside and neutral markets but significantly positive in the upside markets. This indicates that turnover has positive impacts on performance provided that managers are acting on good information in the upside market. However, the abnormal returns generated by more frequent operations do not cover the greater transaction costs, such as brokerage fees and bid-ask spreads, in downside and neutral markets and it results in worse selectivity performance.

We also find that selectivity of load funds is significant and superior to that of no-load funds in downside and upside markets. Panel A further indicates that the expense ratio is apparently and negatively associated with managers' selectivity in neutral and upside markets. The significantly negative estimate for expense ratio coefficient suggests that management expenditures are not effectively used to support research on searching for undervalued stocks and managerial expertise. Hence, they do not help investors creating satisfactory selectivity performance, which at least can recover the charged expenses. Moreover, cash position is positively related to stock selection talents in any market conditions. One reason may be that if funds do not have enough cash on hand, the manager is forced to sell stocks to meet redemptions. This will produce more transaction costs and generate worse selectivity performance.

In addition, we find that net cash flows are significantly and positively related to selectivity performance, consistent with the results in Gruber (1996), Chevalier and Ellison (1997), Della and Olson (1998), and Sirri and Tufano (1998). This suggests that managers can invest in undervalued stocks by new cash flows but need not to sell existing stocks, and leave out some transaction costs or capital losses. This effect is more apparent in downside and neutral markets. Again, the results indicate that fund selectivity performances are not persistent, and investors can not only predict future selectivity performance but have an adverse prediction, especially in downside and neutral markets. This suggests that mutual fund investors have bad selection ability. The coefficient for the status variable is positive in the neutral market but negative in the upside market, implying that dead-fund managers will adopt investment strategies with greater risk. Hence, they will earn more abnormal return in the upside market.

TABLE 2.8 — Relationships between Fund Performances and Characteristics Based on the Conditional Three-Regime Threshold Four-Factor

Dependent variable		Explanation variables								
	PMI	Fund's immediate characteristic variables						Lagged variables		Status
		Ln(TNA)	TURN(%)	LOAD(%)	EXPEN(%)	CASH(%)	NCF	PMI(-1)	NCF(-1)	LIVE
Panel A. Selectivity	$\bar{\alpha}(\%)$	0.00017	-0.00330	0.00047	-0.45584	0.00119	0.02927	0.19622	-0.02092	-0.00818
		(0.00070)	(0.00247)	(0.00026)*	(0.13718)***	(0.00030)***	(0.00467)***	(0.16821)	(0.00709)***	(0.00533)
	$\alpha^{(1)}(\%)$	0.00257	-0.01498	0.00225	0.18837	0.00276	0.08382	-0.08731	-0.05308	-0.03429
		(0.00087)***	(0.00735)**	(0.00052)***	(0.46461)	(0.00104)***	(0.01078)***	(0.16527)	(0.01812)***	(0.01826)*
	$\alpha^{(2)}(\%)$	0.00006	-0.00398	-0.00002	-0.49543	0.00051	0.01944	0.05186	-0.00653	0.00846
		(0.00021)	(0.00104)***	(0.00013)	(0.15653)***	(0.00011)***	(0.00233)***	(0.10699)	(0.00194)***	(0.00148)***
	$\alpha^{(3)}(\%)$	-0.00209	0.00698	0.00087	-1.18825	0.00092	0.00797	0.05751	0.00207	-0.02748
		(0.00104)**	(0.00193)***	(0.00023)***	(0.40176)***	(0.00013)***	(0.00707)	(0.10008)	(0.00672)	(0.00969)***
Panel B. Timing	$\beta^{(2)}-\beta^{(1)}$	-0.00157	0.01422	-0.00375	-0.95618	-0.00311	-0.09219	-0.25614	0.01477	0.00038
		(0.00142)	(0.00313)***	(0.00067)***	(0.42543)**	(0.00087)***	(0.02033)***	(0.16008)	(0.02471)	(0.01364)
	$\beta^{(3)}-\beta^{(2)}$	0.00407	-0.02327	-0.00093	2.18563	-0.00099	0.03830	-0.06973	-0.02844	0.05811
		(0.00129)***	(0.00416)***	(0.00077)	(0.50196)***	(0.00045)**	(0.00282)***	(0.11088)	(0.00529)***	(0.00798)***
	$\beta^{(3)}-\beta^{(1)}$	0.00237	-0.00203	-0.00264	1.42494	-0.00362	-0.04472	0.06385	0.01367	0.07232
		(0.00256)	(0.00563)	(0.00103)**	(0.45853)***	(0.00150)**	(0.00916)***	(0.19456)	(0.01644)	(0.02493)***
Panel C. Threshold	$r_1(\%)$	0.01018	-0.02156	0.00084	4.42396	0.00088	0.28121	0.20681	-0.33437	0.13253
		(0.00687)	(0.01102)*	(0.00558)	(2.18383)**	(0.00138)	(0.09836)***	(0.25251)	(0.09448)***	(0.03335)***
	$r_2(\%)$	-0.00991	0.01087	0.00289	-3.18407	0.00024	0.00455	-0.11925	0.07660	-0.02009
		(0.00475)**	(0.01061)	(0.00229)	(1.49532)**	(0.00139)	(0.05333)	(0.22155)	(0.07174)	(0.03357)

This table presents the coefficients and standard errors form the fix-effect panel data model of annual performance measures (PMI), which are based on conditional four-factor model, on total net assets (TNA), turnover (TURN), total load (LOAD), expense (EXPEN), percentage invested in cash equivalents (CASH), net cash flow of the fund (NCF), lagged performance (PMI(-1)), lagged net cash flow (NCF(-1)), and the dummy variable for the fund's status on September 30, 2004. The standard errors for the coefficients are below the coefficients in parentheses. The regression factors are [TNA, TURN, LOAD, EXPEN, CASH, NCF, , PMI(-1), NCF(-1), I(p∈LIVE)], where I(p∈LIVE) equal to one if fund p is surviving on September 30, 2004, and equal to zero otherwise. The sample consists of 622 mutual funds. The mutual fund sample period is from January 3, 2001 to September 30, 2004. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels (two-tailed test), respectively.

The relationships between timing performance and characteristics are exhibited in Panel B of Table 2.8. The results indicate that managers of larger size funds tend to have better upside market timing ability. Combining with the finding, significantly negative relationship between selectivity and fund size in the downside market, in Panel A, this suggests that larger funds are more likely to generate naïve upside market timing ability by operating options. We also find that market exposures of high-turnover funds decrease more than that of low-turnover funds when market transits from neutral to downside. In contrast, the market exposure of low-turnover funds increases more significantly than that of high-turnover funds when market transits from neutral to upside. Moreover, managers of load funds have worse market timing ability and are specifically significant for the downside market. The market exposures of high-expense funds decrease less and increase more than those of low-expense funds when market transits from neutral to upside and from downside to upside. Again, funds with higher cash position have worse downside and upside market timing ability.

We also find that net cash flows have significant and negative impacts on overall market timing ability, which supports the cash-flow hypothesis described in Warther (1995), Ferson and Warther (1996), and Edelen (1999). The hypothesis implies that investors increase subscriptions to mutual funds in the downside market and this leads to a temporarily large cash position and lower fund beta. However, dividing timing ability into two parts, we find that funds with larger net cash flows have worse downside market timing performance and better upside market timing performance, which is opposite to the cash-flow hypothesis. The reason might be that managers of fund with larger net cash flows are relatively likely to invest their capital in newly discovered and undervalued stocks instead of buying put options to reduce their market exposures in the downside market. Conversely, they might invest the new capital in high beta stocks or options to earn more risk premium in the upside market. As before, we find that managers' market timing ability is not persistent whether for downside or upside markets and fund investors have bad selection ability. Furthermore, the surviving fund has a larger increase in market exposure than the non-surviving fund when market transits from neutral to upside conditions and from downside to upside conditions.

We also explore the relationships between threshold points and fund characteristics, and the results are reported in Panel C of Table 2.8. Fund sizes are significantly and negatively correlated to the up threshold, suggesting larger size funds are more sensitive to predict the upside market. In addition, funds with high expenses have positive impacts on the down threshold, and negative impacts on the up threshold. This implies managers of higher expense funds are more alert to forecasting both downside and upside markets. Contemporaneous net cash flows can improve managers' sensitivity of the downside market prediction. But, higher lagged net cash flows cause managers with the languid discovery of the downside market. Moreover, managers of surviving funds have more alert insights to forecast the downside market.

6. CONCLUSIONS

In this essay, we use Bayesian unconditional and conditional threshold four-factor models to study how fund managers react to change in market conditions. We divide the market condition into three regimes (downside, neutral, and upside markets) and investigate managerial stock selection and market timing performance in different market conditions. The advantages of using our approach to analyze stock selection and market timing performance of mutual funds are not only incorporating the investor's prior belief and managerial information but also transparently detecting the performance behavior across three different market conditions rather than only two conditions.

The empirical analyses show that there are more apparent differences between unconditional and conditional three-regime threshold models instead of two-regime threshold models. In addition, we find that most managers' market timing ability comes from the skills to forecast the downside market. The three-regime threshold models have more power to detect significant timing activity when lagged public information is taken into account.

We also estimate annual fund performances by the conditional three-regime threshold four-factor model and predict the behavior of fund investors by fund performances and various characteristics in previous one year. We find that investors prefer to select funds with better past selectivity performance and upside market timing ability instead of downside market timing skill. Moreover, fund clients favor large size funds and funds with lower turnover, total load charges, and expenses. Again, the behavior of investors is persistent and positively related to surviving funds.

Finally, we investigate the relationship between fund performances and characteristics. We find that fund sizes erode and add fund selectivity performances in upside and downside markets, respectively. High turnover funds tend to have worse (better) selectivity performances in the downside (upside) market. We also find that contemporaneous net cash flows are negatively associated with downside market timing ability, but are positively correlated to upside market timing skills. In addition, funds with higher expenses have alert sensitivities to discover downside and upside markets.

7. APPENDIX—BAYESIAN ESTIMATION OF THE CONDITIONAL THREE-REGIME THRESHOLD FOUR-FACTOR MODEL

From the conditional joint posterior density in equation (2.9), we have the following conditional posterior of parameters $\Theta_p^{(j)}$, for $j = 1,2,3$:

$$\begin{aligned} & p\left(\Theta_p^{(j)} \middle| -\Theta_p^{(j)}, \Phi_p, \Delta_p, \Upsilon_p, \boldsymbol{R}_p, \boldsymbol{R}_{MKT}, \boldsymbol{R}_{SMB}, \boldsymbol{R}_{HML}, \boldsymbol{R}_{UMD}, \boldsymbol{Z}\right) \\ & \propto L\left(\boldsymbol{R}_p, \boldsymbol{R}_{MKT}, \boldsymbol{R}_{SMB}, \boldsymbol{R}_{HML}, \boldsymbol{R}_{UMD}, \boldsymbol{Z} \middle| \Theta_p^{(1)}, \Theta_p^{(2)}, \Theta_p^{(3)}, \Phi_p, \Delta_p, \Upsilon_p\right) \Pi\left(\Theta_p^{(j)}\right) \end{aligned} \tag{A2.1}$$

where $-\Theta_p^{(j)}$ represents the vector $\left(\Theta_p^{(1)}, \Theta_p^{(2)}, \Theta_p^{(3)}\right)$ without $\Theta_p^{(j)}$.

Expressing the target density in (A2.1) by f and $\Theta_p^{(j)}$ by $\boldsymbol{\omega}_j$, we apply the Metropolis-Hasting (MH) algorithm to draw the regression parameters $\boldsymbol{\omega}_j$. The algorithm is described as follows.

At the *i*th iteration, for $i \le \boldsymbol{M}$ and $j = 1,2,3$, we generate a point $\boldsymbol{\omega}_j^*$ from the random kernel, $\boldsymbol{\omega}_j^* = \boldsymbol{\omega}_j^{[i-1]} + \varepsilon$, where $\varepsilon \sim N_2(0, I_2)$ and $\boldsymbol{\omega}_j^{[i-1]}$ is the (*i*-1)th iteration of $\boldsymbol{\omega}_j$. Thereupon, we accept as $\boldsymbol{\omega}_j^{[i]}$ with probability $p = \min\left\{1, f\left(\boldsymbol{\omega}_j^*\right) / f\left(\boldsymbol{\omega}_j^{[i-1]}\right)\right\}$ or else we set $\boldsymbol{\omega}_j^{[i]} = \boldsymbol{\omega}_j^{[i-1]}$. For *i* between $\boldsymbol{M}$ and $\boldsymbol{N}$, we generate a point $\boldsymbol{\omega}_j^*$ from the independent kernel, $\boldsymbol{\omega}_j^* = \mu_{\omega_j} + \varepsilon$, $\varepsilon \sim N_2\left(0, \Omega_{\omega_j}\right)$, where μ_{ω_j} and Ω_{ω_j} are the sample mean and sample covariance calculated by the first $\boldsymbol{M}$ iterates of $\boldsymbol{\omega}_j$. Then, we accept as $\boldsymbol{\omega}_j^{[i]}$ with probability $p = \min\left\{1, \dfrac{f\left(\boldsymbol{\omega}_j^*\right) g\left(\boldsymbol{\omega}_j^{[i-1]}\right)}{f\left(\boldsymbol{\omega}_j^{[i-1]}\right) g\left(\boldsymbol{\omega}_j^*\right)}\right\}$, where we set the Gaussian proposal density $g(\boldsymbol{\omega}) \propto \exp\left\{-\frac{1}{2}\left(\boldsymbol{\omega} - \mu_{\omega_j}\right)^T \Omega_{\omega_j}^{-1} \left(\boldsymbol{\omega} - \mu_{\omega_j}\right)\right\}$. Otherwise, we set $\boldsymbol{\omega}_j^{[i]} = \boldsymbol{\omega}_j^{[i-1]}$.

For the extra three factors' parameters, Φ_p, we have the following target density,

$$p\left(\Phi_p \middle| \Theta_p^{(1)}, \Theta_p^{(2)}, \Theta_p^{(3)}, \Delta_p, \Upsilon_p, \boldsymbol{R}_p, \boldsymbol{R}_{MKT}, \boldsymbol{R}_{SMB}, \boldsymbol{R}_{HML}, \boldsymbol{R}_{UMD}, \boldsymbol{Z}\right)$$
$$\propto L\left(\boldsymbol{R}_p, \boldsymbol{R}_{MKT}, \boldsymbol{R}_{SMB}, \boldsymbol{R}_{HML}, \boldsymbol{R}_{UMD}, \boldsymbol{Z} \middle| \Theta_p^{(1)}, \Theta_p^{(2)}, \Theta_p^{(3)}, \Phi_p, \Delta_p, \Upsilon_p\right) \Pi\left(\Phi_p\right) \quad \text{(A2.2)}$$

The conditional posterior density for the public information parameters, Δ_p, is

$$p\left(\Delta_p \middle| \Theta_p^{(1)}, \Theta_p^{(2)}, \Theta_p^{(3)}, \Phi_p, \Upsilon_p, \boldsymbol{R}_p, \boldsymbol{R}_{MKT}, \boldsymbol{R}_{SMB}, \boldsymbol{R}_{HML}, \boldsymbol{R}_{UMD}, \boldsymbol{Z}\right)$$
$$\propto L\left(\boldsymbol{R}_p, \boldsymbol{R}_{MKT}, \boldsymbol{R}_{SMB}, \boldsymbol{R}_{HML}, \boldsymbol{R}_{UMD}, \boldsymbol{Z} \middle| \Theta_p^{(1)}, \Theta_p^{(2)}, \Theta_p^{(3)}, \Phi_p, \Delta_p, \Upsilon_p\right) \Pi\left(\Delta_p\right) \quad \text{(A2.3)}$$

The conditional posterior density for the threshold parameters, Υ_p, is

$$p\left(\Upsilon_p \middle| \Theta_p^{(1)}, \Theta_p^{(2)}, \Theta_p^{(3)}, \Phi_p, \boldsymbol{R}_p, \boldsymbol{R}_{MKT}, \boldsymbol{R}_{SMB}, \boldsymbol{R}_{HML}, \boldsymbol{R}_{UMD}, \boldsymbol{Z}\right)$$
$$\propto L\left(\boldsymbol{R}_p, \boldsymbol{R}_{MKT}, \boldsymbol{R}_{SMB}, \boldsymbol{R}_{HML}, \boldsymbol{R}_{UMD}, \boldsymbol{Z} \middle| \Theta_p^{(1)}, \Theta_p^{(2)}, \Theta_p^{(3)}, \Phi_p, \Delta_p, \Upsilon_p\right) \Pi\left(\Upsilon_p\right) \quad \text{(A2.4)}$$

We also use the MH algorithm described above to implement the Bayesian estimates of all parameters. For full details about how to complete the MCMC methods, see Chib and Greenberg (1995), and Gilk, Richardson, and Spiegelhalter (1996).

Chapter 3.

On a Robust Bayesian Threshold VAR-DCC-GARCH Model and the Forecasting Performance Comparison in the Conditional Covariance Matrix

1. INTRODUCTION

The Vector autoregressive (VAR) model, popularized by Sims (1980), has been used widely and extensively by economists to study the dynamic behavior of economic variables. The appeal of this model is likely owing to several attractive features relative to other econometric modeling methods. For example, although it includes a lot of parameters, meanwhile it has a very elastic and simple structure. In addition, it only requires the order of the autoregressive process to be considered. Its specification and estimation are simple, since it is neither necessary to specify the variables entering each equation nor to incorporate restrictions derived from economic theory. Thus, estimation and forecasting using a VAR model are purely mechanical processes.

Most covariance matrices of financial asset returns are serial correlated, so multivariate generalized autoregressive conditional heteroskedastic (GARCH) models are more and more popular in financial econometrics in the past decade. A number of different multivariate GARCH models have been proposed, including the simplified diagonal VECH model of Bollerslev *et al.* (1988), the BEKK model of Engle and Kroner (1995), the constant conditional correlation (CCC) model of Bollerslev (1990), the factor ARCH model of Engle *et al.* (1990) and the dynamic conditional correlation (DCC) model of Engle (2002). Especially, the DCC-GARCH model is simpler and has successfully solved many practical problems. For example, hedges require estimates of the correlation between the returns of the assets. It is well known that financial market volatility changes over time. If the correlations and volatility are changing, then the hedge ratio should be adjusted to account for the most recent information. Also, construction of an optimal portfolio with a set of constraints requires a forecast of the covariance matrix of returns. Similarly, the calculation of the standard deviation of today's portfolio requires the covariance matrix of all the assets in the portfolio. These functions use estimation and forecasting of large covariance matrices that potentially have thousands of assets.

Furthermore, multivariate threshold models are widely applicable, including co-integrated systems. A growing body of research in the recent time series literature has concentrated on incorporating nonlinear behavior in conventional linear reduced form specifications such as autoregressive and moving average models. The motivation for moving away from the traditional linear model with constant parameters has typically come from the observation that many economic and financial time series are often characterized by regime specific behavior and asymmetric responses to shocks. For such series the linearity and parameter constancy restrictions are typically inappropriate and may lead to misleading inferences about their dynamics.

We next consider the situation in which each linear regime follows an autoregressive process. For instance, we have the well known threshold autoregressive class of models, the statistical properties of which have been investigated in the early work of Tong (1983, 1990) and Tsay (1989). Multivariate threshold VARs are piecewise linear models with different autoregressive matrices in each regime, which is determined by a transition variable (one of the endogenous variables), a delay and a threshold (Tsay, 1998). They were more recently reconsidered and extended in Hansen (1996, 2000), Caner and

Hansen (2001) among others. Hansen and Seo (2002) examined a two-regime vector error-correction model with a single co-integrated vector and a threshold.

Bayesian methods are also increasingly becoming popular to researchers in financial econometrics recently. The VAR model usually has a large number of parameters, which are often estimated through maximum likelihood or least squares. In the threshold VAR, however, finite-sample frequentist analysis of the nonlinear functions is difficult. For instance, for some distributions of data, the maximum likelihood estimation (MLE) does not have an analytical form or simply does not exist, or in some applications of VAR models, nonlinear functions of VAR parameters are the focus of research. The difficulties faced in the frequentist approach of VAR inference can be circumvented by the Bayesian approach, which combines information from observations with researcher's priors. When the objective of the model is to forecast, the Bayesian approach is more satisfactory. This approach consists of imposing prior restrictions over the VAR model parameters. The estimates of the model parameters are obtained by combining the prior belief and the likelihood, so more accurate forecasts can be achieved. This kind of models is known in the literature as Bayesian VAR or BVAR models (see, Zellner (1971), Bauwens *et al.* (1999)). Later, Ni and Sun (2003, 2005) proposed Bayesian estimates for VAR models under different priors. Vrontos *et al.* (2003) and Osiewalski and Pipień (2004) used Bayesian methodology to assess relative predictive and explanatory powers of various multivariate GARCH models. So *et al.* (2005) developed a Bayesian testing scheme for threshold nonlinearity in financial time series. Chen and So (2006) performed Bayesian diagnostic checking for the threshold heteroscedastic model.

In this essay, we present a robust threshold VAR(or VECM)-DCC-GARCH model and use the Metropolis-Hastings (MH) algorithm and the Gibbs sampling algorithm to estimate the parameters simultaneously. Our model extends existing approaches by admitting thresholds in conditional means, conditional volatilities and correlations of multivariate time series. Such an extension, allows us to account for rich asymmetric effects and dependencies of conditional means, volatilities and correlations, as they are often encountered in practical financial applications. In addition, we use the concept of Chen and So (2006) to define the threshold variables as the linear combination of endogenous variables. This setting can eliminate excessively subjective belief in threshold variable decision. Besides, the weight coefficient can serve as the proxy in deciding which market is the price leader and which market is the price follower. Finally, threshold values in our model are not fixed ex ante, but they are estimated from the data, together with all other parameters in the model.

We investigate the empirical performance of our model in two data sets including daily S&P500 futures and spot prices, and S&P500 and Nasdaq100 spot prices. Our study attempts to use posterior odds ratio and Bayes factors as a formal tool for making comparison between competing models. We reduce our testing problem to a Bayesian model selection problem. We can then select the model with a higher posterior odds ratio. We also present the performance comparison results of the one-step-ahead forecast in the conditional covariance matrix. The forecast results are assessed by several criteria which include the views of statistical loss and risk managers.

Based on the estimation results, we find that the asymmetric dynamic structure is obvious in both the dynamic relationship between S&P500 futures and spot markets and between S&P500 and Nasdaq100 spot markets. We also detect that S&P500 futures market is the price leader between S&P500 futures and spot markets, and S&P500 spot market is the price leader between S&P500 and Nasdaq100 spot markets. Furthermore, based on several in-sample and out-of-sample performance measures in the conditional covariance matrix prediction, we find that the threshold model outperforms the linear model across most measure criteria.

The rest of the essay is organized as follows. In the next section, a robust multivariate threshold vector autoregressive is introduced. We assume the model with the error having multivariate DCC-GARCH. Then, Bayesian approach is specified including the priors setting, and then conditional posterior distributions for relevant parameters are derived. Also, the Markov chain Monte Carlo (MCMC) simulation methods and implementation algorithm are taken up. We then illustrate the empirical applications and model performance comparisons. Finally brief conclusions are given.

2. THRESHOLD VAR-DCC-GARCH MODEL

First, we consider the multivariate threshold vector autoregressive model with a dynamic correlated and heteroscedastic error. Let $\boldsymbol{Y}_t$ be a K-dimensional time series with threshold variable z_{t-d}, so the mean equation and the conditional distribution of the innovation process are given by the following equations:

$$\boldsymbol{Y}_t = \sum_{g=1}^{G}\left[\left(\boldsymbol{\Phi}_0^{(g)} + \sum_{l=1}^{L_g}\boldsymbol{\Phi}_l^{(g)}\boldsymbol{Y}_{t-l}\right)\cdot I\left(r_{g\text{-}1} \le z_{t-d} < r_g\right)\right] + \boldsymbol{\varepsilon}_t \ , \tag{3.1}$$

$$\boldsymbol{\varepsilon}_t \mid \Im_{t\text{-}1} \sim N(0, \boldsymbol{H}_t)\,, \tag{3.2}$$

where $\boldsymbol{Y}_t = \begin{pmatrix} y_{1t} \\ \vdots \\ y_{Kt} \end{pmatrix}$, $\boldsymbol{\Phi}_0^{(g)} = \begin{pmatrix} \phi_{10}^{(g)} \\ \vdots \\ \phi_{K0}^{(g)} \end{pmatrix}$, $\boldsymbol{\Phi}_l^{(g)} = \begin{pmatrix} \phi_{11l}^{(g)} & \cdots & \phi_{1Kl}^{(g)} \\ \vdots & \ddots & \vdots \\ \phi_{K1l}^{(g)} & \cdots & \phi_{KKl}^{(g)} \end{pmatrix}$, $z_t = \sum_{k=1}^{K} w_k y_{kt}$, $0 \le w_k \le 1$, $\sum_{k=1}^{K} w_k = 1$, for $t = 1,\ldots,T$, $g = 1,\ldots,G$, $l = 1,\ldots,L_g$. $\boldsymbol{\Phi}_0^{(g)}$ is K-dimensional intercept vector, $\boldsymbol{\Phi}_l^{(g)}$ for $l = 1,\ldots,L_g$ are the $K \times K$ regression coefficient matrices in the gth regime, $L_g \in \mathbb{N}$ is the lag of VAR in the gth regime, $\boldsymbol{\varepsilon}_t$ is a $K \times 1$ innovation term, and $d \in \mathbb{N}$ is the threshold lag of the model with maximum delay d_0. $\Im_{t\text{-}1}$ is the information set up to t-1, $\boldsymbol{H}_t$ is a $K \times K$ covariance matrix with elements $h_{ii,t}$ and

$h_{ij,t}, i=1,\ldots,K$, $j=i+1,\ldots,K$, where $h_{ii,t}$ is the variance of the *i*th variable at time *t* and $h_{ij,t}$ is the covariance between the *i*th and *j*th variables at time *t*, and the number of regime is *G*. In addition, the threshold values r_g must satisfy $-\infty = r_0 < r_1 < \cdots < r_G = \infty$, and thus the intervals $[r_{g-1}, r_g), j=1,\ldots,G$, form a partition of the space of z_{t-d}.

The threshold variable z_{t-d} is defined by a weighted average of y_{it-d} and this can be viewed as a extension of Tsay (1998) and Brooks (2001), who set threshold variable to be a specific endogenous variables, y_{it-d}. We think that this setting of the threshold variable may have the following advantages and economic meanings. First, when y_{it-d} are returns of different markets, we can regard z_{t-d} as the return of a portfolio without short sales, and the dynamic structure or leverage effect may be influenced by the portfolio return. That is, the structural change of markets may rely on the global economic condition instead of a specific market condition. Second, it can eliminate excessively subjective belief in threshold variable decision and let the data choose a more appropriate z_{t-d} by estimating the weights, w_k. Third, the weights w_k can reflect the relative significance of each endogenous variable y_{it-d}. This can not only govern the time series behavior of Y_t but also find which market is the lead price leader and which markets are price followers.

In addition, the accurate structure of the conditional covariance matrix is of primary importance in the practice of portfolio analysis, asset pricing, and risk management. Recently, multivariate GARCH is used more and more extensively in modeling the correlation among financial time series and multivariate volatility. Bollerslev (1990) proposed the simple constant correlation coefficient (CCC) multivariate GARCH model, which is allowed to have time-varying conditional variances and covariance matrices, but constant conditional correlations. He decomposed the conditional covariance matrix $\boldsymbol{H}_t$ as

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{R} \boldsymbol{D}_t \tag{3.3}$$

where $\boldsymbol{R}$ is the $K \times K$ time-invariant correlation matrix with element ρ_{ij}, $i = 1, \ldots, K$, $j = i+1, \ldots, K$, $i \neq j$ and $\boldsymbol{D}_t$ is the $K \times K$ time-varying diagonal matrix with element $\sqrt{h_{ii,t}}$, $i = 1, \ldots, K$. The individual variances $h_{ii,t}$ follow standard univariate GARCH models.

To relax the restriction of the CCC model, Engle (2002) proposed flexible and parsimoniously parameterized generalizations of the CCC model. The main difference between CCC and DCC models is that the DCC model allows the correlation matrix $\boldsymbol{R}$ to be time-varying. Consequently, the time varying conditional covariance matrix $\boldsymbol{H}_t$ is defined as follows:

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t, \tag{3.4}$$

where $\boldsymbol{R}_t$ is time-varying correlation matrix of innovation $\boldsymbol{\varepsilon}_t$.

Because, in this essay, the structure of the dynamic covariance matrix mainly focuses on the DCC model with the threshold structure, it can be viewed as an extension of DCC-GARCH of Engle (2002). Therefore, by means of the specification for $\boldsymbol{Q}_t$ considered in Engle (2002), the threshold DCC-GARCH structure of covariance matrix is given by

$$\boldsymbol{D}_t^2 = \sum_{g=1}^{G} \left(\mathrm{diag}\{\omega_i^{(g)}\} + \sum_{p=1}^{m_g} \mathrm{diag}\{\alpha_{ip}^{(g)}\} \circ \varepsilon_{t-p} \varepsilon'_{t-p} + \sum_{q=1}^{n_g} \mathrm{diag}\{\beta_{iq}^{(g)}\} \circ \boldsymbol{D}_{t-q}^2 \right) \cdot I\left(r_{g\text{-}1} \leq z_{t-d} < r_g\right), \tag{3.5}$$

$$\boldsymbol{R}_t = \mathrm{diag}\left\{\boldsymbol{Q}_t\right\}^{-1/2} \boldsymbol{Q}_t \mathrm{diag}\left\{\boldsymbol{Q}_t\right\}^{-1/2}, \tag{3.6}$$

$$\boldsymbol{Q}_t = \sum_{g=1}^{G} \left((\boldsymbol{\iota}\boldsymbol{\iota}' - \boldsymbol{A}^{(g)} - \boldsymbol{B}^{(g)}) \circ \bar{\boldsymbol{Q}}^{(g)} + \boldsymbol{A}^{(g)} \circ \boldsymbol{\eta}_{t-1} \boldsymbol{\eta}'_{t-1} + \boldsymbol{B}^{(g)} \circ \boldsymbol{Q}_{t-1} \right) \cdot I\left(r_{g\text{-}1} \leq z_{t-d} < r_g\right), \tag{3.7}$$

where $\omega_i^{(g)} > 0$, $\alpha_{ip}^{(g)} \geq 0$, $\beta_{iq}^{(g)} \geq 0$, $\sum_{p=1}^{m_g} \alpha_{ip}^{(g)} + \sum_{q=1}^{n_g} \beta_{iq}^{(g)} < 1$, $\circ$ is the Hadamard matrix product operator, $\boldsymbol{\eta}_t = \boldsymbol{D}_t^{-1} \boldsymbol{\varepsilon}_t$ is the vector of standardized errors, $\bar{\boldsymbol{Q}}^{(g)}$ is the unconditional correlation matrix of ε_t in the *g*th regime, and the parameters $\boldsymbol{A}^{(g)}$ and $\boldsymbol{B}^{(g)}$ are symmetric and positive semidefinite matrices. To ensure $\boldsymbol{Q}$ to be positive semidefinite, we also restrict $(\boldsymbol{\iota\iota}' - \boldsymbol{A}^{(g)} - \boldsymbol{B}^{(g)})$ to be positive semidefinite.

Under the assumption of conditional normality for error process in Equation (3.2), the likelihood function for the parameters can be expressed as

$$\begin{aligned} L(\boldsymbol{y}|\boldsymbol{\theta}) &= \prod_{t=P+1}^{T} (2\pi)^{-\frac{K}{2}} |\boldsymbol{H}_t|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\boldsymbol{\varepsilon}_t' \boldsymbol{H}_t^{-1} \boldsymbol{\varepsilon}_t\right\} \\ &= \prod_{t=P+1}^{T} (2\pi)^{-\frac{K}{2}} |\boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\boldsymbol{\varepsilon}_t' (\boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t)^{-1} \boldsymbol{\varepsilon}_t\right\} \end{aligned} \tag{3.8}$$

where θ is the set of all parameters, $P = Max(L_1, \ldots, L_G, m_1, \ldots, m_g, n_1, \ldots, n_g, d_0)$, $\boldsymbol{\varepsilon}_t$, $\boldsymbol{D}_t$, and $\boldsymbol{R}_t$ obey Equations (3.1), (3.5), and (3.6), respectively

However, when the variables $\boldsymbol{Y}_t$ in the model are integrated and of order one or more, estimating by Equation (3.1) is subject to the hazard of regressions involving nonstationary variables. In addition, Engle and Yoo (1987) also argued that, in the presence of co-integration, a VAR model with an error correction mechanism should outperform a VAR over longer forecasting horizons. Therefore, taking into account the explicitly long-run equilibrium relationship, the mean Equation (3.1) is modified to the threshold vector error correction model, which can be written as

$$\boldsymbol{\Delta Y}_t = \sum_{g=1}^{G} \left[\left(\boldsymbol{a}^{(g)} \boldsymbol{b}' \boldsymbol{Y}_{t-1} + \Phi_0^{(g)} + \sum_{l=1}^{P_g} \Phi_l^{(g)} \boldsymbol{\Delta Y}_{t-l} \right) \cdot I\left(r_{g\text{-}1} \le z_{t-d} < r_g\right) \right] + \boldsymbol{\varepsilon}_t \ , \tag{3.9}$$

where $\boldsymbol{\Delta}$ denotes the difference operator, $z_t = \sum_{k=1}^{K} w_k \cdot \Delta y_{kt}$, $\boldsymbol{b}$ is $K \times \gamma$ full rank matrix of co-integrating vectors, $\boldsymbol{a}^{(g)}$ is $K \times \gamma$ full rank matrix of coefficients associated with the error correction terms, and the value γ determines the number of co-integrating relationships ($\gamma < K$). Avoiding the identification problem, we impose the so-called linear normalization where $\boldsymbol{b} = \left(\boldsymbol{I}_\gamma \ \ \boldsymbol{b}_o\right)'$. So, the likelihood function for the parameters is similar to Equation (3.8) except $\boldsymbol{\varepsilon}_t$ must follow Equation (3.9).

3. BAYESIAN INFERENCE AND MCMC IMPLEMENTATION

In this section, first, we explain why the Bayesian method is used in this essay to analyze the threshold VAR(or VECM)-DCC-GARCH models. The reason is that using maximum likelihood method is difficult to estimate the parameters in the threshold VAR-DCC-GARCH models. The main problems are the large number of parameters to be estimated and the difficulty of estimation due to the positive definiteness restrictions of the covariance matrix. Thus, this will result in unstable estimates. In addition, due to the unknown threshold variable z_t in this essay, this hinders us to implement a two step estimation procedure similar to that considered by Tsay (1998) and Brooks (2001). Even if the threshold variable is known, Tsay (1998) showed that the asymptotic properties for the threshold lag, d, and threshold parameter, r_g, are hard to infer. Thus, to deal with the above infeasible procedure by maximum likelihood method, we extend the Bayesian method using MCMC techniques introduced by Chen and So (2006).

Implementation of the Bayesian analysis depends on a willingness to assign probability distributions not only to data variable $\boldsymbol{y}$, but also to all unknown parameters. Consider a situation in which absolutely weakly previous subjective information is known about the phenomenon of interest, so as to mitigate frequentist criticisms of intentional subjectivity. Thus, in this essay, we choose noninformative or weakly informative priors for most parameters to interject the least amount of prior knowledge and the specification of priors is listed below.

First, we assume the discrete uniform prior for the threshold lag parameter d with maximum delay d_0, and it can be written as $\pi(d) = 1/d_0$, $d = 1,\ldots,d_0$, which does not favor any one of the candidate d values over any other. Since the weighted vector $\boldsymbol{w} = (w_1,\ldots,w_K)$ relies on d, the conditional prior of $\boldsymbol{w}$ given $d, \pi(\boldsymbol{w}|d)$, is taken as symmetric Dirichlet distribution, $\boldsymbol{w}|d \sim D(\delta,\ldots,\delta)$, where the hyper-parameter $\delta > 0$. Identically, z_{t-d} depends on d and $\boldsymbol{w}$, so we assume the conditional prior of every threshold parameter, r_g, for $g = 1,\ldots,G-1$, to be a continuously bounded uniform distribution, $\pi(r_g|d,\boldsymbol{w}) = 1/\left(r_{\text{up}}^{(g)} - r_{\text{low}}^{(g)}\right), r_{\text{low}}^{(g)} \le r_g \le r_{\text{up}}^{(g)}$. The lower and upper bounds for threshold parameters are employed to constrain that there are at least τ percent of observations in each regime. As well, τ depends on the number of observations. When the sample size is small, higher τ is recommended. The purpose of this setting is to make parameter estimates more efficient and more reliable.

Subsequently, we divide the parameters in each regime into four independent blocks, which are the parameters in the VAR model (3.1, 3.9), the error correction term (3.9), the volatility process (3.5), and the dynamic correlation procedure (3.7), and we assume the priors are independent among any two regimes. For the priors of VAR parameters, Litterman (1980) and Kinal and Ratner (1986) have indicated that VAR sometime suffers from overparameterization. The requirement to estimate a large number of coefficients in VAR often leads to large standard errors for inferences and forecasts. The Bayesian VAR imposing some prior restrictions on parameters will usually provide more accurate forecasts. In this essay, we adopt Litterman's (1980) Minnesota prior for VAR parameters and make some proper modifications. For convenience, we define $\boldsymbol{\Phi}^{(g)} = \left[\boldsymbol{\Phi}_0^{(g)}, \boldsymbol{\Phi}_1^{(g)}, \cdots, \boldsymbol{\Phi}_{L_g}^{(g)}\right]'$, which is a $K \times (1 + K \cdot L_g)$ matrix, and consider vectorized $\boldsymbol{\Phi}^{(g)}$, $\text{vec}\left(\boldsymbol{\Phi}^{(g)}\right)$,which we denote by $\boldsymbol{\phi}^{(g)}$. We assume that the vector $\boldsymbol{\phi}^{(g)}$ follows a multivariate normal distribution with zero mean and diagonal covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\phi}}$. That is, we believe in advance the unconditional mean and short run dynamics center around zero, and investors are unable to earn excess returns by this short run dynamic relationship. In addition, the priors are made independently across elements of $\boldsymbol{\phi}^{(g)}$ and the standard deviation of the coefficient $\phi_{ijl}^{(g)}$, which is an element of $\boldsymbol{\phi}^{(g)}$ and describes how variable *i* is affected by variable *j* of lag *l* in the *g*th regime, is given by

$$\mathbf{S}\left(\phi_{ijl}^{(g)}\right)=\begin{cases}\dfrac{\lambda}{l} & \text{if } i=j\\ \dfrac{\eta\lambda}{l}\cdot\dfrac{\tau_i}{\tau_j} & \text{if } i\neq j\end{cases}, \tag{3.10}$$

where the hyper-parameter λ controls the tightness of beliefs on $\boldsymbol{\phi}^{(g)}$, τ_i/τ_j is a correction for the scale of series *i* compared with series *j*, and the restriction $0<\eta<1$ implies that the series are more likely to be influenced by their own lags than by the lags of other series.

The above model requires us to choose specific values for the hyper-parameters λ, τ_i, τ_j ,and η. The correction term τ_i/τ_j is engaged in modifying the inconsistency in variation of each series variable. While in principle these should be chosen on the basis of a priori reasoning or knowledge, we will in practice follow Litterman (1986) in choosing these as the sample standard deviations of residuals from univariate autoregressive models fit to the individual series in the sample. For the remaining hyper-parameters, λ is commonly set from 0.1 to 0.9, and η ranges from 0.2 to 0.5 (see, Litterman (1986), Kinal and Ratner (1986), Doan (1990)). In addition, Villani (2001) showed that the selection of these two parameters is not sensitive to the forecasting results. So, we use $\lambda=0.5$ and $\eta=0.4$ in our empirical analysis. We also set the standard deviation of the intercept coefficient as 1 to employ a more diffuse prior.

For the prior on the long term structure (ie, error correction term) in each regime, we follow Geweke (1996) to choose uniform prior for both the matrix of co-integrating vectors $\boldsymbol{b}^{(g)}$ and the associated weighting matrix $\boldsymbol{a}^{(g)}$, and the prior can be written as $\pi\left(\boldsymbol{a}^{(g)},\boldsymbol{b}^{(g)}\right)\propto 1$. Furthermore, a uniform prior with some restrictions is assumed for the parameters of the GARCH and is written as

$$\pi\left(\boldsymbol{\omega}^{(g)},\boldsymbol{\alpha}^{(g)},\boldsymbol{\beta}^{(g)}\right)\propto\prod_{i=1}^{K} I\left(\omega_i^{(g)}>0;\alpha_{ip}^{(g)}\geq 0;\beta_{iq}^{(g)}\geq 0;\sum_{p=1}^{m_g}\alpha_{ip}^{(g)}+\sum_{q=1}^{n_g}\beta_{iq}^{(g)}<1\right), \tag{3.11}$$

where $I(\cdot)$ is the indicator function which takes unity if the constraint holds and otherwise zero, and $\boldsymbol{\omega}^{(g)}=\left(\omega_1^{(g)},\ldots,\omega_K^{(g)}\right)$, $\boldsymbol{\alpha}^{(g)}=\left(\alpha_{i1}^{(g)},\ldots,\alpha_{im_g}^{(g)},\ldots,\alpha_{K1}^{(g)},\ldots,\alpha_{Km_g}^{(g)}\right)$, and $\boldsymbol{\beta}^{(g)}=\left(\beta_{i1}^{(g)},\ldots,\beta_{im_g}^{(g)},\ldots,\beta_{K1}^{(g)},\ldots,\beta_{Km_g}^{(g)}\right)$. Finally, we also choose a uniform prior for the dynamic correlation structure,

$$\pi\left(\boldsymbol{A}^{(g)},\boldsymbol{B}^{(g)},\bar{\boldsymbol{Q}}^{(g)}\right)\propto I(\Upsilon), \tag{3.12}$$

where Υ is the set of $\left(\boldsymbol{A}^{(g)},\boldsymbol{B}^{(g)},\bar{\boldsymbol{Q}}^{(g)}\right)$, which must satisfy that $\boldsymbol{A}^{(g)}$, $\boldsymbol{B}^{(g)}$, and $(\boldsymbol{\iota\iota}'-\boldsymbol{A}^{(g)}-\boldsymbol{B}^{(g)})$ are symmetric and positive semidefinite matrices, and $\bar{\boldsymbol{Q}}^{(g)}$ is a form of correlation coefficient matrix. Therefore, the prior of all unknown parameters in the threshold VAR-DCC-GARCH model can be expressed as:

$$\begin{aligned}\pi(\boldsymbol{\theta})\propto\ &\pi(d)\pi(\mathbf{w}|d)\left(\prod_{g=1}^{G-1}\pi\left(r_g|d,\mathbf{w}\right)\right)\\ &\cdot\left(\prod_{g=1}^{G}\pi\left(\boldsymbol{\phi}^{(g)}\right)\pi\left(\boldsymbol{\omega}^{(g)},\boldsymbol{\alpha}^{(g)},\boldsymbol{\beta}^{(g)}\right)\pi\left(\boldsymbol{A}^{(g)},\boldsymbol{B}^{(g)},\bar{\boldsymbol{Q}}^{(g)}\right)\right).\end{aligned} \tag{3.13}$$

Bayesian inference about the parameter vector $\boldsymbol{\theta}$ conditional on data matrix $\boldsymbol{y}$ are constructed through the posterior density $p(\boldsymbol{\theta}|\boldsymbol{y})$. Using Bayes theorem, the posterior density is formed by the prior density $\pi(\boldsymbol{\theta})$ and the likelihood $L(\boldsymbol{y}|\boldsymbol{\theta})$, and it can be written:

$$p(\boldsymbol{\theta}|\boldsymbol{y})=\frac{L(\boldsymbol{y}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})}{\int L(\boldsymbol{y}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})d\boldsymbol{\theta}}\propto L(\boldsymbol{y}|\boldsymbol{\theta})\pi(\boldsymbol{\theta}). \tag{3.14}$$

Therefore, the optimal Bayes estimator of $\boldsymbol{\theta}$ under quadratic loss is simply the posterior mean, which is

$$\hat{\boldsymbol{\theta}}=\mathrm{E}(\boldsymbol{\theta}|\mathbf{Y})=\int\boldsymbol{\theta}p(\boldsymbol{\theta}|\boldsymbol{y})d\boldsymbol{\theta}. \tag{3.15}$$

However, for many realistic problems, the posterior distribution $p(\boldsymbol{\theta}|\boldsymbol{y})$ may not have an analytically tractable form, particularly in high dimensions, so calculating the posterior mean is a difficult task. In fact, to settle our major problems, we can use numerical or asymptotic methods to compute the approximate posterior mean for the full Bayesian model. Because, the posterior density is with very high dimension and only known up to a constant, in this essay we adopt the MCMC sample algorithm as our tool for this purpose. The idea is based on the Hammersley-Clifford theorem (1968), which says that a joint distribution can be characterized by its complete conditional distributions. For example, the posterior distribution, $p(\theta_1,\theta_2|\boldsymbol{y})$, can be characterized by the complete conditional distribution, $p(\theta_1|\boldsymbol{y},\theta_2)$ and $p(\theta_2|\boldsymbol{y},\theta_1)$.

Given the initial values, $\theta_1^{(0)}$ and $\theta_2^{(0)}$, we draw $\theta_1^{(1)}$ from $p\left(\theta_1^{(1)}\middle|\boldsymbol{y},\theta_2^{(0)}\right)$ and then $\theta_2^{(1)}$ from $p\left(\theta_2^{(1)}\middle|\boldsymbol{y},\theta_1^{(1)}\right)$. Iterating the steps, we produce a series of sample, $\left\{\theta_1^{(m)},\theta_2^{(m)}\right\}_{m=1}^{M}$, which will converge to the joint posterior distribution under some mild conditions. Thus, we can obtain the Bayesian estimator $\hat{\theta}_i$ by computing the mean from the sample of the stationary distribution of the simulated $\theta_i^{(m)}$, which is after some burn-in iterations . When the complete conditional distribution is known, such as Normal distribution or Beta distribution, we only use Gibbs sampler to draw the random variables. On the contrary, if it is unknown, we will use a hybrid method consisting of both Gibbs steps and Metropolis-Hastings (MH) steps. Assuming the complete conditional distribution $p(\theta_1|\boldsymbol{y},\theta_2)$ is unknown, in the MH algorithm, we generate a value θ_1^* from its proposal distribution $g(\cdot)$ and accept the proposal value, i.e. $\theta_1^{(g+1)}=\theta_1^*$, with probability $\lambda\left(\theta_1^{(g)},\theta_1^*\right)$, which is

$$\lambda\left(\theta_1^{(g)},\theta_1^*\right)=\min\left\{\frac{p\left(\theta_1^*|\boldsymbol{y},\theta_2\right)}{p\left(\theta_1^{(g)}|\boldsymbol{y},\theta_2\right)}\Big/\frac{g\left(\theta_1^*\right)}{g\left(\theta_1^{(g)}\right)},1\right\}. \tag{3.16}$$

Theoretically, we can use almost any distribution for the proposal distribution. However, in practice, we need to choose proposal distribution very carefully to ensure fast convergence of MCMC samples. See Chib and Greenberg (1995), and Gilk et al. (1996) among others for full detailed explanations and discussions about the MH algorithm as well as the MCMC methods.

Furthermore, in order to reduce the possible bias due to the selection of starting values, generated sample within an initial transient or burn-in period are usually discarded. Since rate of convergence on different target distributions vary considerably, it is also difficult to determine the required length of burn-in. In this essay, we choose the length of burn-in period by the convergence diagnostic (CD) in Geweke (1992). The idea and calculation procedure of this method are described as follows. Let θ denote a parameter of interest and $\theta^{(j)}$ denote the parameter at the *j*th iteration. Suppose observations of the chain for $m+n$ iterations and form averages

$$\bar{\theta}_b = \frac{1}{n_b}\sum_{j=m+1}^{m+n_b}\theta^{(j)} \text{ and } \bar{\theta}_a = \frac{1}{n_a}\sum_{j=m+n-n_a+1}^{m+n}\theta^{(j)}$$

where $n_a + n_b < n$. If *m* is the length of burn-in period, then $\bar{\theta}_a$ and $\bar{\theta}_b$ are the ergodic averages at the end and beginning of the convergence period and should behavior similarly. Thus, the statistic CD is given as

$$\mathrm{CD} = \frac{\bar{\theta}_a - \bar{\theta}_b}{\sqrt{\hat{V}ar\left(\bar{\theta}_a\right) + \hat{V}ar\left(\bar{\theta}_b\right)}}, \tag{3.17}$$

and it asymptotically follows standard normal distribution. So, if convergence has been achieved, the statistic CD should not be large. Following the suggestion of Geweke (1992), we calculate the statistic by setting $n_a = 0.5n$ and $n_b = 0.1n$, and using spectral density estimators for the variances to take into account any autocorrelation.

Subsequently, we will use Bayes factors to select the appropriate order of the VAR process and to choose between linear ($G=1$) and non-linear ($G \geq 2$) versions of our model. When comparing any two competing parametric Bayesian models (M_i, M_j) for the same data matrix $\boldsymbol{y}$, the Bayes factor (BF) can be calculated by marginal likelihood concept. In general terms letting $\boldsymbol{\theta}_j$ be the appropriate set of parameters under model M_j, the marginal likelihood can be written:

$$p\left(\boldsymbol{y}\middle|M_j\right)=\int_{\Theta_j} p\left(\boldsymbol{y}\middle|\boldsymbol{\theta},M_j\right)p\left(\boldsymbol{\theta}\middle|M_j\right)d\boldsymbol{\theta}_j\,, \tag{3.18}$$

where $p\left(\boldsymbol{y}\middle|\boldsymbol{\theta},M_j\right)$ and $p\left(\boldsymbol{\theta}\middle|M_j\right)$ are the sampling density function and the prior density function, respectively.

For the Bayesian model selection, we can determine the posterior odds ratio (POR) of M_i against M_j by the Bayes factors $B_{ij}=p\left(\boldsymbol{y}\middle|M_i\right)\big/p\left(\boldsymbol{y}\middle|M_j\right)$ and the prior odds ratio $p(M_i)/p(M_j)$, and it can be expressed as

$$\mathrm{POR}_{ij}=\frac{p(M_i|\boldsymbol{y})}{p(M_j|\boldsymbol{y})}=\frac{p(M_i)}{p(M_j)}B_{ij}\,. \tag{19}$$

If there is absence of a prior preference for either model, i.e. $p(M_i)=p(M_j)=1/2$, the Bayes factor can be interpreted as a measure of the extent to which the data support M_i over M_j. When $B_{ij}>1$, the data prefer M_i over M_j, and when $B_{ij}<1$, the data favor M_j over M_i. Thus, in this essay we compare any two threshold VAR-GARCH time series models of different orders and regimes by computing Bayes factors as the ratio of marginal likelihood evaluated along the route of Chib (1995).

4. EMPIRICAL APPLICATIONS

4.1. Data description

Our data consist of daily closing prices for S&P500 index, Nasdaq 100 index, and S&P500 index futures for the period between January 3, 1995 and December 31, 2004.

They are collected from *TICK DATA* and the sample size for each stock market is 2519. In addition, intraday 5-minute prices are used to construct the series of realized covariance as done in the past related literature.

The descriptive statistics of daily returns from January 3, 1995, to December 31, 2004, are summarized in Table 3.1. The statistics reported are sample mean, standard deviation, maximum, minimum, Jarque-Bera (JB) statistics, and the Ljung-Box (LB) statistics for the return and the square return series. For the S&P500 stock market, the standard deviation of futures returns is larger than that of the spot returns, indicating that the futures market is more volatile than the spot market. In addition, the Nasdaq100 spot returns are more volatile than S&P500 spot and futures returns. The spot and futures returns of S&P500 index are negatively skewed, whereas the returns of Nasdaq100 index are positively skewed. All spot and futures returns present concerns for excess kurtosis. The Jarque-Bera test statistics provide clear evidence to reject the null hypothesis of normality for each returns series and this is mainly due to the presence of skewness and excess kurtosis. The augmented Dickey-Fuller unit root test results suggest that all return series are stationary. The Ljung-Box Q statistics indicate possible serial correlations for all returns series. Moreover, the Q^2 statistics suggest that there are autoregressive conditional heteroskedasticity effects on each returns series. The Johansen co-integration test indicates a long-run equilibrium relationship between S&P500 spot and futures indices.

TABLE 3.1 – Descriptive Statistics for the Returns of Daily S&P500 Index, S&P500 Index Futures, and Nasdaq 100 Index, from 1995 to 2004

	$R_{\text{S\&P500,F}}$	$R_{\text{S\&P500,S}}$	$R_{\text{Nasdaq100,S}}$
Mean(%)	0.0383	0.0385	0.0551
SD(%)	1.1935	1.1446	2.2737
Skewness	-0.1544	-0.1184	0.1185
Kurtosis	6.3738	6.0544	6.1907
Max(%)	5.8141	5.3080	17.2030
Min(%)	-7.7058	-7.1127	-10.3777
JB	1204.6951	985.0505	1074.4440
ADF	-51.8382	-50.6566	-38.8717
$Q(12)$	21.2310	24.9340	35.2010
$Q^2(12)$	605.1300	636.5300	1028.5000
Johansen Test	58.1941		

Note: $R_{\text{S\&P500,F}}$, $R_{\text{S\&P500,S}}$, and $R_{\text{Nasdaq100,S}}$ refer to S&P500 spot, S&P500 futures, and Nasdaq 100 spot returns, respectively. Returns for stock indices are calculated by $100\times(\ln(P_t)-\ln(P_{t-1}))$. The values in rows JB, ADF, Johansen are statistics of Jarque-Bera normality test, the augmented Dickey-Fuller unit root test, and the Johansen cointegration test. Q(12) and $Q^2(12)$ report the Ljung-Box (LB) portmanteau test statistics including 12 lags for the return and square return series. The Critical values at 5% of JB, ADF, Johansen, and LBP are 5.991, 2.862, 3.433, and 21.026, respectively.

4.2. Goodness-of-fit results

In this section, we use bivariate threshold time series model, which was introduced earlier, to two data sets. One data set covers the returns series of S&P500 spot and futures markets, and thus the bivariate threshold VECM-GARCH model is appropriate. The other data set consists of the returns series of S&P500 and Nasdaq100 spot markets, so we adopt the bivariate threshold VAR-GARCH model. We also use two different time lengths (five years and ten years) to perform model comparison. In addition, we simplify our model by assuming $L_1 = \cdots = L_G = L$, and GARCH(1,1) model is considered as a parsimonious model which is found to be appropriate in most applications. The matrix parameters $A^{(g)}$ and $B^{(g)}$ in Equation (3.7) are also reduced to scale parameters. So, to ensure Q to be positive semidefinite, we restrict $A^{(g)} + B^{(g)} < 1$, for $g = 1, \ldots, G$. Therefore, we only need to choose proper L and G by the Bayes factor criterion.

To implement our MCMC sampling scheme, we carry out 30,000 iterations which are performed with the first 10,000 burn-in iterations discarded, to reach the convergence of every parameter. The logarithm marginal likelihood, the logarithm Bayes factors, which are on the basis of simple linear VECM(1)-DCC-GARCH(1,1) or VAR(1)-DCC-GARCH(1,1) model, and the overall ranking of models for different L and G are shown in Tables 3.2 and 3.3. For the period between 2000 and 2004, Table 3.2 shows that the best model to describe the dynamic relationship between S&P500 futures and spot markets is the two-regime threshold VECM(3)-DCC-GARCH(1,1) model with a logarithm of marginal likelihood value of -1866.171. If we compare the best model with linear VECM(1)-DCC-GARCH(1,1) model, i.e. a model where there is no asymmetric effect on mean, variance, and correlation equations, then the difference in logarithm marginal likelihood is $Ln\left(B_{6,1}\right)=59.833$, which yields a Bayes factor of $B_{6,1}=9.664\times10^{25}$. This means the two-regime threshold VECM(3)-DCC-GARCH(1,1) model is 9.664×10^{25} times more likely than the linear VECM(1)-DCC-GARCH(1,1) model. Table 3.2 also indicates that the two-regime threshold VECM(3)-DCC-GARCH(1,1) model is more satisfactory than the other competitors considered here regardless of five- or ten-year data. Generally speaking, the results in Table 3.2 show that models which consider asymmetric effects and longer lags have larger logarithms of marginal likelihood. This suggests that there are asymmetric effects on mean, covariance, or error correction processes for the dynamic relationship between S&P500 futures and spot markets. In addition, considering longer lags will benefit model explanatory power.

In Table 3.3, for the period between 2000 and 2004, we find that the best model to characterize the dynamic relationship between S&P500 and Nasdaq100 spot returns is the two-regime threshold VAR(1)-DCC-GARCH(1,1) model which is 8.405×10^{14} times more likely than the linear VAR(1)-DCC-GARCH(1,1) model. However, for ten-year time length, we find that the three-regime threshold VAR(1)-DCC-GARCH(1,1) model is most appropriate among our competitive models. On the whole, our findings suggest that the threshold model is more suitable than the linear model in the dynamic relationship between S&P500 and Nasdaq100 spot returns. In addition, the logarithm marginal likelihood of the model with shorter lags is larger than that with longer lags except for the linear models with five years data.

TABLE 3.2 – Logarithms of Marginal Likelihood and Bayes Factors-The Dynamic Relationship between S&P500 Futures and Spot Markets

Model (M_i)	Number of parameters	2000-2004 (5 years)			1995-2004 (10 years)		
		$Ln\left[p\left(\boldsymbol{y}\middle\|M_i\right)\right]$	Ln($B_{i,1}$)	Rank	$Ln\left[p\left(\boldsymbol{y}\middle\|M_i\right)\right]$	Ln($B_{i,1}$)	Rank
M_1, VECM(1)-DCC-GARCH(1,1)	18	-1926.004	0.000	9	-3654.369	0.000	9
M_2, VECM(2)-DCC-GARCH(1,1)	22	-1909.399	16.605	8	-3604.425	49.944	8
M_3, VECM(3)-DCC-GARCH(1,1)	26	-1896.580	29.424	5	-3584.793	69.576	7
M_4, 2R-VECM(1)-DCC-GARCH(1,1)	38	-1898.022	27.982	7	-3583.782	70.587	6
M_5, 2R-VECM(2)-DCC-GARCH(1,1)	46	-1888.237	37.767	2	-3532.965	121.404	3
M_6, 2R-VECM(3)-DCC-GARCH(1,1)	54	-1866.171	59.833	1	-3513.248	141.120	1
M_7, 3R-VECM(1)-DCC-GARCH(1,1)	56	-1897.335	28.669	6	-3567.204	87.165	5
M_8, 3R-VECM(2)-DCC-GARCH(1,1)	68	-1888.504	37.500	3	-3540.125	114.244	4
M_9, 3R-VECM(3)-DCC-GARCH(1,1)	80	-1896.414	29.590	4	-3516.374	137.995	2

*Note:*The table shows the number of parameter, logarithms of marginal likelihood, logarithms of Bayes factor, and the ranks of marginal likelihood for several models and two different time lengths. The data are based on the daily S&P500 futures and spot market prices for the sample period from January 3, 1995 to December 31, 2004.

TABLE 3.3 – Logarithms of Marginal Likelihood and Bayes Factors- The Dynamic Relationship between S&P500 and Nasdaq100 Spot Returns

Model (M_i)	Number of parameters	2000-2004 (5 years)			1995-2004 (10 years)		
		$Ln\left[p\left(\boldsymbol{y}\middle\|M_i\right)\right]$	$Ln(B_{i,1})$	Rank	$Ln\left[p\left(\boldsymbol{y}\middle\|M_i\right)\right]$	$Ln(B_{i,1})$	Rank
M_1, VAR(1)-GARCH(1,1)	15	-3863.507	0.000	9	-7388.013	0.000	7
M_2, VAR(2)-GARCH(1,1)	19	-3857.392	6.115	6	-7394.210	-6.197	8
M_3, VAR(3)-GARCH(1,1)	23	-3862.844	0.663	8	-7398.697	-10.684	9
M_4, 2R-VAR(1)-GARCH(1,1)	33	-3829.141	34.365	1	-7365.001	23.012	2
M_5, 2R-VAR(2)-GARCH(1,1)	41	-3831.830	31.677	2	-7371.956	16.056	4
M_6, 2R-VAR(3)-GARCH(1,1)	49	-3841.109	22.398	5	-7383.827	4.186	6
M_7, 3R-VAR(1)-GARCH(1,1)	49	-3833.646	29.860	3	-7349.133	38.880	1
M_8, 3R-VAR(2)-GARCH(1,1)	61	-3840.603	22.904	4	-7362.203	25.810	3
M_9, 3R-VAR(3)-GARCH(1,1)	73	-3857.999	5.508	7	-7380.973	7.040	5

*Note:*The table reports the number of parameter, logarithms of marginal likelihood, logarithms of Bayes factor, and the ranks of marginal likelihood for several different models and two different time lengths. The data are based on the daily S&P500 and Nasdaq100 market returns for the sample period from January 3, 1995 to December 31, 2004.

4.3. Bayesian estimation results

In this section, we will illustrate the Bayesian estimation results of two data sets, which are used in the above section, for full sample periods. Table 3.4 presents the linear and threshold VECM(3)-DCC-GARCH(1,1) models estimation results for the mean equation (3.9) and variance-covariance matrix equations (3.5, 3.6, 3.7) for the dynamic relationship between S&P500 futures and spot markets. The feedback effects between each pair of S&P500 futures and spot markets are observed except for the upside market. That is, lagged spot (futures) returns help to predict current futures (spot) returns. More specifically, the lagged spot (futures) returns have positive effects on current futures (spot) returns. In addition, the lagged spot (futures) returns have negative effects on current spot (futures) returns. That is to say, each market exhibits mean reversion behavior with a stronger degree occurring in the spot market except for the upside market with a stronger degree occurring in the futures market.

TABLE 3.4 – Bayesian Estimation Results for S&P500 Futures and Spot Markets

		One Regime		Two Regime				Three Regime					
				$z_t \le r_1$		$z_t > r_1$		$z_t \le r_1$		$r_1 < z_t \le r_2$		$r_2 < z_t$	
		$\Delta \ln F_t$	$\Delta \ln S_t$	$\Delta \ln F_t$	$\Delta \ln S_t$	$\Delta \ln F_t$	$\Delta \ln S_t$	$\Delta \ln F_t$	$\Delta \ln S_t$	$\Delta \ln F_t$	$\Delta \ln S_t$	$\Delta \ln F_t$	$\Delta \ln S_t$
Mean Equations	ϕ_0	0.047	0.094	-0.203	-0.168	0.054	0.103	-0.145	-0.161	0.023	0.035	0.176	0.213
		(0.042)	(0.053)	(0.120)	(0.106)	(0.036)	(0.058)	(0.122)	(0.107)	(0.033)	(0.055)	(0.227)	(0.212)
	$\phi_{\cdot F1}$	-0.162	0.392	-0.330	0.266	-0.209	0.318	-0.338	0.292	-0.213	0.331	-0.315	0.149
		(0.067)**	(0.060)**	(0.079)**	(0.071)**	(0.080)**	(0.072)**	(0.083)**	(0.072)**	(0.073)**	(0.069)**	(0.116)**	(0.135)
	$\phi_{\cdot F2}$	0.012	0.306	-0.009	0.268	-0.084	0.242	-0.066	0.242	0.021	0.323	-0.313	0.094
		(0.083)	(0.076)**	(0.115)	(0.098)**	(0.110)	(0.098)*	(0.122)	(0.112)*	(0.081)	(0.070)**	(0.120)**	(0.123)
	$\phi_{\cdot F3}$	-0.101	0.053	0.066	0.212	-0.110	0.061	-0.008	0.154	-0.020	0.140	-0.058	0.149
		(0.065)	(0.060)	(0.142)	(0.128)	(0.078)	(0.072)	(0.123)	(0.115)	(0.074)	(0.070)*	(0.199)	(0.188)
	$\phi_{\cdot S1}$	0.179	-0.372	0.207	-0.389	0.256	-0.287	0.230	-0.401	0.257	-0.291	0.284	-0.208
		(0.070)**	(0.064)**	(0.076)**	(0.068)**	(0.080)**	(0.072)**	(0.080)**	(0.066)**	(0.072)**	(0.068)**	(0.097)**	(0.113)*
	$\phi_{\cdot S2}$	-0.021	-0.309	-0.055	-0.314	0.078	-0.246	-0.034	-0.324	-0.005	-0.308	0.153	-0.252
		(0.086)	(0.078)**	(0.121)	(0.103)**	(0.112)	(0.100)*	(0.123)	(0.112)**	(0.082)	(0.071)**	(0.131)	(0.131)*
	$\phi_{\cdot S3}$	0.080	-0.070	-0.123	-0.255	0.116	-0.059	-0.084	-0.231	0.042	-0.126	0.024	-0.168
		(0.066)	(0.062)	(0.144)	(0.129)*	(0.080)	(0.075)	(0.126)	(0.118)*	(0.076)	(0.072)	(0.197)	(0.185)
	a	-0.020	0.076	-0.130	0.000	0.015	0.103	-0.106	0.011	0.016	0.107	0.091	0.156
		(0.046)	(0.044)	(0.110)	(0.106)	(0.056)	(0.053)*	(0.109)	(0.104)	(0.050)	(0.048)*	(0.236)	(0.232)
	b	-1.001		-1.001				-1.000					
		(0.001)**		(0.001)**				(0.001)**					
Threshold Parameters	d	1#		1#				1#					
	w_1			0.790				0.904					
				(0.118)**				(0.068)**					
	r_1			-0.599				-0.629					
				(0.024)**				(0.028)**					
	r_2							1.762					
								(0.059)**					
Variance Covariance Equations	ω	0.025	0.022	0.304	0.284	0.033	0.023	0.303	0.275	0.038	0.025	0.724	0.584
		(0.006)**	(0.005)**	(0.046)**	(0.043)**	(0.007)**	(0.006)**	(0.046)**	(0.042)**	(0.007)**	(0.006)**	(0.179)**	(0.171)**
	α	0.063	0.070	0.069	0.079	0.056	0.065	0.080	0.091	0.015	0.046	0.035	0.046
		(0.009)**	(0.010)**	(0.011)**	(0.013)**	(0.010)**	(0.012)**	(0.012)**	(0.015)**	(0.010)	(0.012)**	(0.017)*	(0.018)**
	β	0.916	0.910	0.864	0.844	0.858	0.858	0.840	0.822	0.860	0.858	0.683	0.700
		(0.012)**	(0.013)**	(0.019)**	(0.022)**	(0.016)**	(0.017)**	(0.025)**	(0.030)**	(0.015)**	(0.016)**	(0.086)**	(0.090)**
	A	0.136		0.147		0.223		0.160		0.245		0.123	
		(0.016)**		(0.039)**		(0.024)**		(0.043)**		(0.029)**		(0.066)8	
	B	0.128		0.172		0.173		0.221		0.147		0.385	
		(0.052)**		(0.094)		(0.060)**		(0.128)		(0.059)*		(0.228)	
	$\bar{\rho}$	0.975		0.971		0.977		0.969		0.977		0.967	
		(0.001)**		(0.004)**		(0.001)**		(0.009)**		(0.001)**		(0.067)**	

*Note:*This table shows the parameter estimates for one, two and three regime threshold VECM(3)-DCC-GARCH(1,1) models, respectively, which are based on the daily S&P500 futures and spot market returns for sample period from January 3, 1995 to December 31, 2004. The numbers in parentheses are standard deviations. *and** indicate statistical significance at 5%, and 1%, respectively. # indicates the posterior mode of the threshold lag parameter.

In the two-regime threshold model, the spot returns tend to increase when the spread is large in order to restore the long-run equilibrium relationship only in the upside market. The evidence suggests that the S&P500 spot price tends to converge to the futures price and the effect is more apparent in the upside market. Furthermore, the futures price seems to converge to the spot price in the downside market, but the effect is not significant. In the three-regime threshold model, the coefficients of error correction terms are only significant in spot returns in the neutral market. Although only a few coefficients of error correction terms are significantly different from zero, these coefficients increase from downside to upside markets in both futures and spot returns. This exhibits that the structures of error correction terms are asymmetric in different market conditions. In both two- and three-regime models, the weighted coefficient in threshold variable, w_1, is significantly larger than 0.5, especially in the three-regime threshold model. This indicates that S&P500 futures market is the price leader whereas the spot market is the price follower. Additionally, we find that the threshold values, r_1 and r_2, are not symmetric and point out the asymmetric dynamic structures between downside and upside markets. For the coefficients of variance covariance equations, we find that the volatility is most persistent in the downside market and followed by neutral and upside markets in both futures and spot markets. However, the persistence of the correlation between futures and spot returns has an opposite outcome. In addition, the unconditional correlation coefficients, $\bar{\rho}$, are all above 0.95 and the coefficients in DCC structure, B, are all below 0.4. This shows that futures and spot returns are highly correlated and the lagged correlation does not heavily influence the current correlation.

The linear and threshold VAR(1)-DCC-GARCH(1,1) models estimation results for the dynamic relationship between S&P500 and Nasdaq100 spot markets are reported in

Table 3.5 and we have the following findings. First, the S&P500 market is the price leader whereas the Nasdaq100 is the price follower during the sample period. Second, the persistence in volatility and correlation is heaviest in the neutral market. Finally, we find that the dynamic volatility and correlation structures between S&P500 and Nasdaq100 returns is very significant and has obvious variations among different market conditions.

TABLE 3.5 – Bayesian Estimation Results for S&P500 and Nasdaq100 Spot Markets

		One Regime		Two Regime				Three Regime					
				$z_t \le r_1$		$z_t > r_1$		$z_t \le r_1$		$r_1 < z_t \le r_2$		$r_2 < z_t$	
		$R_{SP,t}$	$R_{NAS,t}$	$R_{SP,t}$	$R_{NAS,t}$	$R_{SP,t}$	$R_{NAS,t}$	$R_{SP,t}$	$R_{NAS,t}$	$R_{SP,t}$	$R_{NAS,t}$	$R_{SP,t}$	$R_{NAS,t}$
Mean Equations	ϕ_0	0.080	0.137	-0.717	-0.775	0.070	0.108	-0.672	-0.673	0.067	0.100	0.299	1.000
		(0.017)**	(0.031)**	(0.251)**	(0.394)	(0.018)**	(0.032)**	(0.294)**	(0.452)	(0.018)**	(0.032)**	(0.336)	(0.635)
	$\phi_{\cdot SP1}$	0.053	0.059	-0.330	-0.324	0.082	0.121	-0.336	-0.324	0.098	0.155	0.019	-0.249
		(0.033)	(0.059)	(0.094)**	(0.129)**	(0.034)**	(0.062)*	(0.093)**	(0.118)**	(0.037)**	(0.067)**	(0.138)	(0.208)
	$\phi_{\cdot NAS1}$	-0.017	-0.056	-0.029	-0.167	-0.024	-0.058	-0.016	-0.148	-0.021	-0.068	-0.058	-0.039
		(0.015)	(0.031)	(0.054)	(0.111)	(0.015)	(0.032)	(0.054)	(0.113)	(0.016)	(0.033)**	(0.050)	(0.111)
Threshold Parameters	d	$1^{\#}$		$1^{\#}$				$1^{\#}$					
	w_1			0.894				0.911					
				(0.059)**				(0.044)**					
	r_1			-1.713				-1.698					
				(0.073)**				(0.075)**					
	r_2							1.797					
								(0.100)**					
Variance Covariance Equations	ω	0.013	0.036	0.025	0.032	0.011	0.036	0.010	0.028	0.008	0.032	0.274	0.433
		(0.003)**	(0.007)**	(0.031)	(0.029)	(0.003)**	(0.009)**	(0.006)**	(0.025)	(0.002)**	(0.008)**	(0.107)**	(0.243)
	α	0.070	0.068	0.123	0.131	0.049	0.052	0.141	0.155	0.046	0.051	0.037	0.053
		(0.007)**	(0.007)**	(0.018)**	(0.019)**	(0.009)**	(0.008)**	(0.018)**	(0.022)**	(0.008)**	(0.008)**	(0.017)*	(0.017)**
	β	0.916	0.921	0.782	0.823	0.934	0.932	0.810	0.805	0.941	0.935	0.682	0.780
		(0.008)**	(0.007)**	(0.046)**	(0.031)**	(0.009)**	(0.008)**	(0.032)**	(0.030)**	(0.008)**	(0.008)**	(0.062)**	(0.064)**
	A	0.044		0.068		0.043		0.109		0.045		0.060	
		(0.007)**		(0.023)**		(0.006)**		(0.029)**		(0.007)**		(0.025)*	
	B	0.942		0.739		0.953		0.753		0.952		0.873	
		(0.010)**		(0.070)**		(0.008)**		(0.069)**		(0.008)**		(0.058)**	
	$\bar{\rho}$	0.802		0.812		0.740		0.662		0.726		0.723	
		(0.036)**		(0.066)**		(0.153)**		(0.131)**		(0.150)**		(0.166)**	

Note: This table presents theparameter estimates for one, two and three regime threshold VAR(1)-DCC-GARCH(1,1) models, respectively, which are based on the daily S&P500 and Nasdaq100 market returns for sample period from January 3, 1995 to December 31, 2004. The numbers in parentheses are standard deviations. *and** indicate statistical significance at 5%, and 1%, respectively. # indicates the posterior mode of the threshold lag parameter.

5. FORECASTING PERFORMANCE COMPARISIONS IN CONDITIONAL COVARIANCE MATRIX

In this section, the data from January 1, 2000 to December 31, 2004, consisting of 1256 trading days, are used for in-sample estimation and forecasting performance evaluation in the one-step-ahead conditional covariance matrix. The 61 observations from January 1, 2005 to March 31, 2005 are used for the out-of-sample performance evaluation purposes. In addition, we present two categories of criteria to measure forecasting performance of different competitive models. One category is based on the views of the statistical loss function, which is a non-negative function that generally increases as the distance between the actual value and the forecasted value increases, and three different types of criteria are adopted here. The other category of performance measure is based on the views of risk managers and two types of criteria are introduced.

5.1. Statistical Loss Performance

Thereinafter, we will introduce three types of loss functions. First, a multivariate version of the classical mean absolute error (MAE) statistic is addressed, which is

$$\text{MAE} = \frac{1}{K^2}\sum_{i,j=1}^{K}\frac{1}{n}\sum_{t=1}^{n}\left|\hat{h}_{ij,t} - h_{ij,t}\right|, \tag{3.20}$$

where $\hat{h}_{ij,t}$ and $h_{ij,t}$ denote the forecast and the actual of the covariance between assets *i* and *j* at time *t*, respectively.

The second type of loss function is a multivariate version of the mean square error (MSE) statistic, which is

$$\text{MSE} = \frac{1}{K^2}\sum_{i,j=1}^{K}\frac{1}{n}\sum_{t=1}^{n}\left(\hat{h}_{ij,t} - h_{ij,t}\right)^2. \tag{3.21}$$

The above two loss functions are symmetric, and we will also use an asymmetric loss function, which is linear-exponential (LINEX) loss with the form:

$$\text{LINEX} = \frac{1}{K^2}\sum_{i,j=1}^{K}\frac{1}{n}\sum_{t=1}^{n}\left\{\exp\left[\zeta(\hat{h}_{ij,t}-h_{ij,t})\right]-\zeta\left(\hat{h}_{ij,t}-h_{ij,t}\right)-1\right\}, \tag{3.22}$$

where ζ is a given parameter. When ζ is close to 0, the LINEX loss function is nearly symmetric and is not much different from the MSE statistic. In the LINEX loss function, positive errors are weighed differently from the negative errors when $\zeta \neq 0$. If $\zeta > 0$ ($\zeta < 0$), the LINEX loss function is approximately linear (exponential) for $\hat{h}_{ij,t} - h_{ij,t} < 0$ and exponential (linear) for $\hat{h}_{ij,t} - h_{ij,t} > 0$. This implies that an overestimate (underestimate) needs to be taken into consideration more seriously. More specifically, in all above cases, a lower loss measure indicates a higher forecasting power.

As a result of the unobservable property of the covariance matrices, here we use intraday 5-minute data to construct the proxies for the daily-realized covariance observations. The concept of the realized volatility has been proposed by French, Schwert, and Stambaugh(1987) and Andersen *et al.*(2001). The realized volatility is nothing more than the sum of squared high-frequency returns over a given sampling period. Similarly, we can directly express the realized covariance (RCOV) as:

$$\text{RCOV}\left(t,\Delta\right) = \sum_{j=1}^{1/\Delta} R\left(t-1+j\cdot\Delta,\Delta\right)R\left(t-1+j\cdot\Delta,\Delta\right)', \tag{3.23}$$

where $R\left(t,\Delta\right)$ denotes the $K\times 1$ vector of logarithm returns over the $\left[t-\Delta,t\right]$ time interval.

The compared results of the forecast performance measures in the conditional covariance matrix between S&P500 futures and spot returns and between S&P500 and Nasdaq100 spot returns for the different models are presented in Figures 3.1, 3.2 and Tables 3.6, 3.7, respectively. When looking at the in-sample prediction in Tables 3.6 and 3.7, we observe that the threshold model yields better performance relative to the linear model no matter what symmetric (MAE and MSE) or asymmetric (LINEX) loss functions are adopted. In Table 3.6, we also find that the LINEX(1) indictor for the linear model is about twice as large as that for the threshold model, but the LINEX(-1) indictor is close for all competitive models. This indicates that the linear model may overestimate the covariance matrix. For the out-of-sample forecast, the threshold model performs more appropriately for the covariance between S&P500 and Nasdaq100 spot returns. However, the linear model in predicting covariance between S&P500 futures and spot returns seems better than the threshold model.

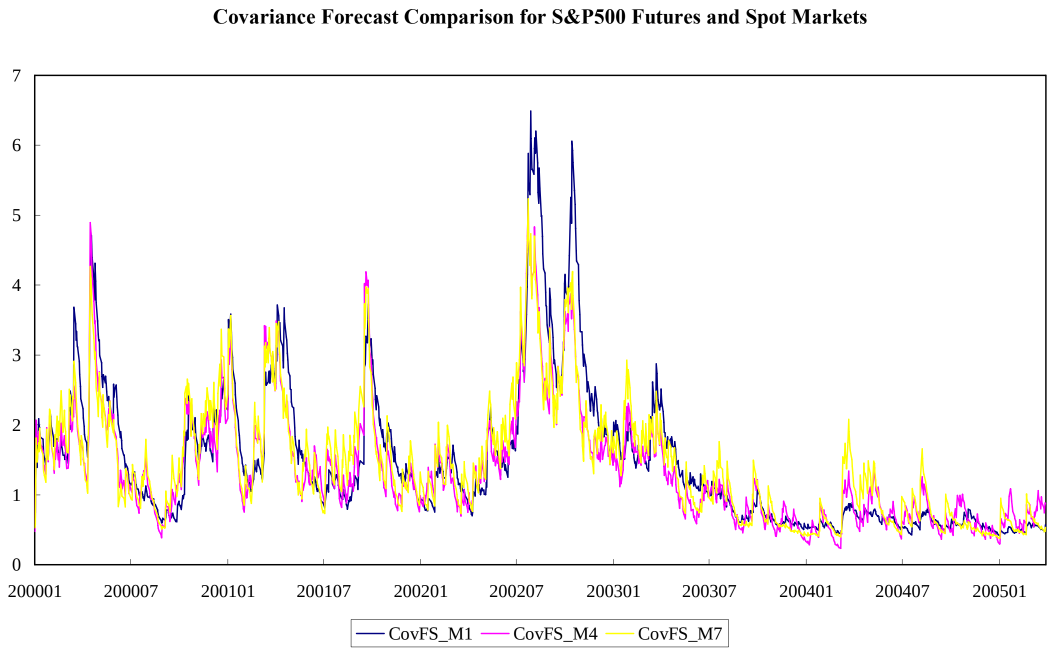

FIGURE 3.1－One Period Ahead Covariance Forecast Comparison for S&P500 Futures and Spot Markets, 2000/1-2005/3.

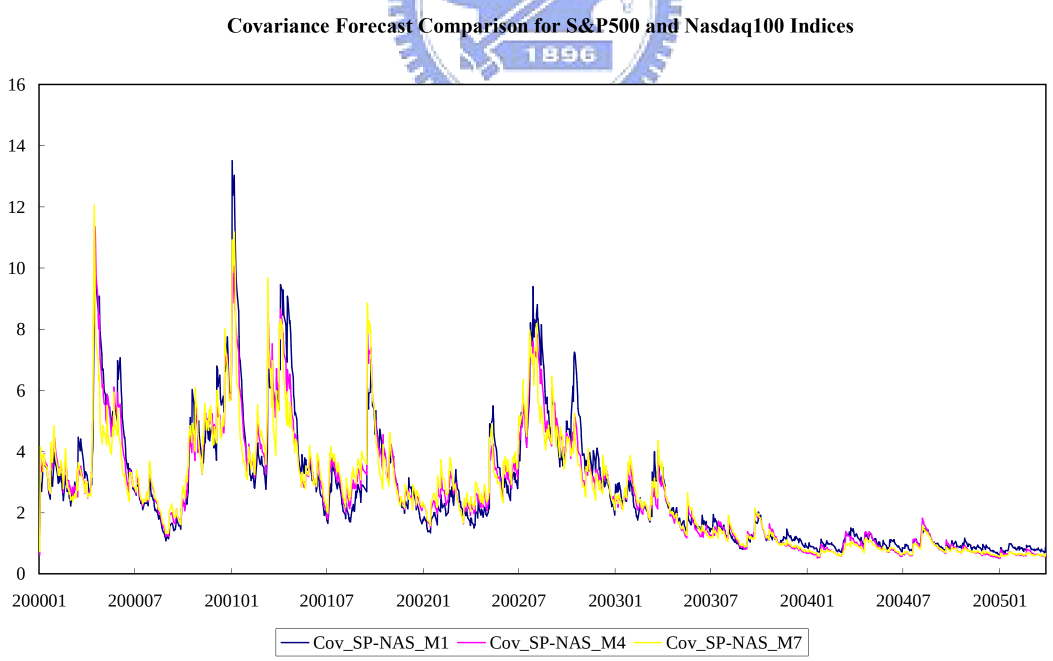

FIGURE 3.2－One Period Ahead Covariance Forecast Comparison for S&P500 and Nasdaq100 Indices, 2000/1-2005/3.

TABLE 3.6 – In- and Out-of-sample Covariance Matrix Forecast Comparison of Alternative Models for S&P500 Futures and Spot Markets

	In Sample				Out-of-Sample			
Model (M_i)	MAE	MSE	LINEX(1)	LINEX(-1)	MAE	MSE	LINEX(1)	LINEX(-1)
M_1, VECM(1)-DCC-GARCH(1,1)	0.788	2.054	0.829	9.329	**0.212**	**0.058**	**0.032**	**0.026**
M_2, VECM(2)-DCC-GARCH(1,1)	0.808	2.101	0.965	9.215	0.215	0.062	0.035	0.028
M_3, VECM(3)-DCC-GARCH(1,1)	0.819	2.138	0.824	9.885	0.233	0.071	0.040	0.032
M_4, 2R-VECM(1)-DCC-GARCH(1,1)	**0.668**	1.762	**0.441**	9.430	0.248	0.075	0.043	0.035
M_5, 2R-VECM(2)-DCC-GARCH(1,1)	0.715	1.788	0.467	9.498	0.254	0.082	0.047	0.037
M_6, 2R-VECM(3)-DCC-GARCH(1,1)	0.682	**1.720**	0.512	**8.763**	0.259	0.091	0.052	0.040
M_7, 3R-VECM(1)-DCC-GARCH(1,1)	0.713	1.785	0.472	9.301	0.287	0.106	0.061	0.046
M_8, 3R-VECM(2)-DCC-GARCH(1,1)	0.731	1.768	0.522	8.798	0.276	0.107	0.063	0.046
M_9, 3R-VECM(3)-DCC-GARCH(1,1)	0.752	1.829	0.540	9.193	0.285	0.113	0.067	0.049

Note: This table computes the mean-absolute-errors (MAE), mean-square-errors (MSE), and asymmetric LINEX loss (LINEX) with $\zeta = 1$ and $\zeta = -1$. The measured covariance matrices are daily realized covariance matrices which are calculated by intra-day 5-minute returns. The data used are daily and intra-day 5-minute S&P500 index futures and spot price. The in-sample data period is from January 1, 2000 to December 31, 2004 and out-of-sample data period is from January 1, 2005 to March 31, 2005.

TABLE 3.7 – In- and Out-of-sample Covariance Matrix Forecast Comparison of Alternative Models for S&P500 and Nasdaq100 Indices

	In Sample				Out-of-Sample			
Model (M_i)	MAE	MSE	LINEX(1)	LINEX(-1)	MAE	MSE	LINEX(1)	LINEX(-1)
M_1, VAR(1)-GARCH(1,1)	1.773	13.745	30.487	29.830	0.393	0.213	0.135	0.088
M_2, VAR(2)-GARCH(1,1)	1.743	13.441	28.818	30.076	0.378	0.197	0.123	0.082
M_3, VAR(3)-GARCH(1,1)	1.751	13.489	29.346	29.801	0.388	0.205	0.129	0.086
M_4, 2R-VAR(1)-GARCH(1,1)	1.673	12.078	26.935	28.182	0.253	0.095	0.051	0.047
M_5, 2R-VAR(2)-GARCH(1,1)	1.677	11.778	29.244	27.743	**0.240**	**0.089**	**0.048**	**0.044**
M_6, 2R-VAR(3)-GARCH(1,1)	1.690	12.002	29.107	27.427	0.253	0.097	0.052	0.047
M_7, 3R-VAR(1)-GARCH(1,1)	1.626	11.270	23.487	27.773	0.242	0.092	0.050	0.045
M_8, 3R-VAR(2)-GARCH(1,1)	1.657	11.638	25.190	**26.604**	0.260	0.102	0.056	0.049
M_9, 3R-VAR(3)-GARCH(1,1)	**1.598**	**11.034**	**22.299**	27.971	0.252	0.095	0.051	0.047

*Note:*This table computes the mean-absolute-errors (MAE), mean-square-errors (MSE), and asymmetric LINEX loss (LINEX) with $\zeta = 1$ and $\zeta = -1$. The measured covariance matrices are daily realized covariance matrices which are calculated by intra-day 5-minute returns. The data used are daily and intra-day 5-minute S&P500 and Nasdaq 100 indices prices. The in-sample data period is from January 1, 2000 to December 31, 2004 and out-of-sample data period is from January 1, 2005 to March 31, 2005.

5.2. Risk Management Performance

Predictability in covariance between two assets' returns, as measured by traditional criteria that focus on the size of the forecast error, does not necessarily imply that an investor can make profits or reduce risk from a trading strategy based on such forecasts. Therefore, we also use the other category of performance measure which is based on the views of risk managers, and two types of criteria are used. One is to calculate the value at risk (VaR) as an evaluation of the estimator. For a two-asset portfolio with δ invested in the first asset and $(1-\delta)$ in the second asset, the one-step-ahead VaR at time *t* and at $\alpha\%$, assuming normality, is

$$\mathrm{VaR}_t(\alpha) = -\left[\left(\delta\hat{r}_{1,t} + (1-\delta)\hat{r}_{2,t}\right) - z_\alpha \cdot \sqrt{\delta^2\hat{h}_{11,t} + (1-\delta)^2\hat{h}_{22,t} + 2\delta(1-\delta)\hat{h}_{12,t}}\right], \tag{3.24}$$

where z_a is the right quantile at $\alpha\%$. To compare and evaluate model performances on several different model specifications, we choose the following criteria. By definition, the failure rate (FR) is the proportion of returns (in absolute value) exceed the forecasted VaR, i.e. $\mathrm{FR} = (1/T)\sum_{t=1}^{T} I\left(\delta r_{1,t} + (1-\delta) r_{2,t} < -\mathrm{VaR}_t(\alpha)\right)$, where $I(\cdot)$ is the indicator function. Hence, if the VaR model is correctly specified, the failure rate should be equal to the prespecified VaR level. We also use Kupiec LR test (1995) to examine if the model is correctly specified. To test $\mathrm{H}_0{:}f = \alpha$ against $H_1 : f \neq \alpha$, the LR statistic is $LR = -2\ln\left(\alpha^{T-N}(1-\alpha)^N\right) + 2\ln\left((N/T)^{T-N}\left(1-(N/T)\right)^N\right)$, where N is the number of VaR violations, T is the total number of observations and f is the theoretical failure rate. Under the null hypothesis, the LR test statistic is asymptotically distributed as $\chi^2(1)$.

Figure 3.3 plots one period ahead 5% VaR forecast comparison for a hedged portfolio with weights (1,-1) and three weighted portfolios with weights (0.5, 0.5), (0.3, 0.7), and (0.7, 0.3). The fractions of VaR violations and p-values of the Kupiec (1995) failure rate test are reported in Table 3.8. For in-sample data, the p-values for the null hypothesis of the hedged portfolio are all smaller than 0.05 when the linear model is considered. The threshold model performs very well as there are no p-values smaller than 0.05. Thus the switch from the linear model to threshold model yields a significant improvement in the VaR performance in the hedged portfolio. For the weighted portfolios consisting of S&P and Nasdaq100 spot markets, the failure rates are very close to the prespecified VaR level and p-values are larger than 0.05 for in-sample predictions in all linear and threshold models. For the out-of-sample VaR performance comparison, we find that there are no p-values smaller than 0.05 no matter what kinds of portfolios or models which we consider here. But, the fractions of VaR violation based on the threshold model are closer to the prespecified VaR level than those based on the linear model except the weighted portfolio with weights (0.7, 0.3). In view of in-sample and out-of-sample empirical results, we find that the linear model may sometimes overestimate the VaR, so using the threshold model to calculate the portfolio's VaR may be more appropriate.

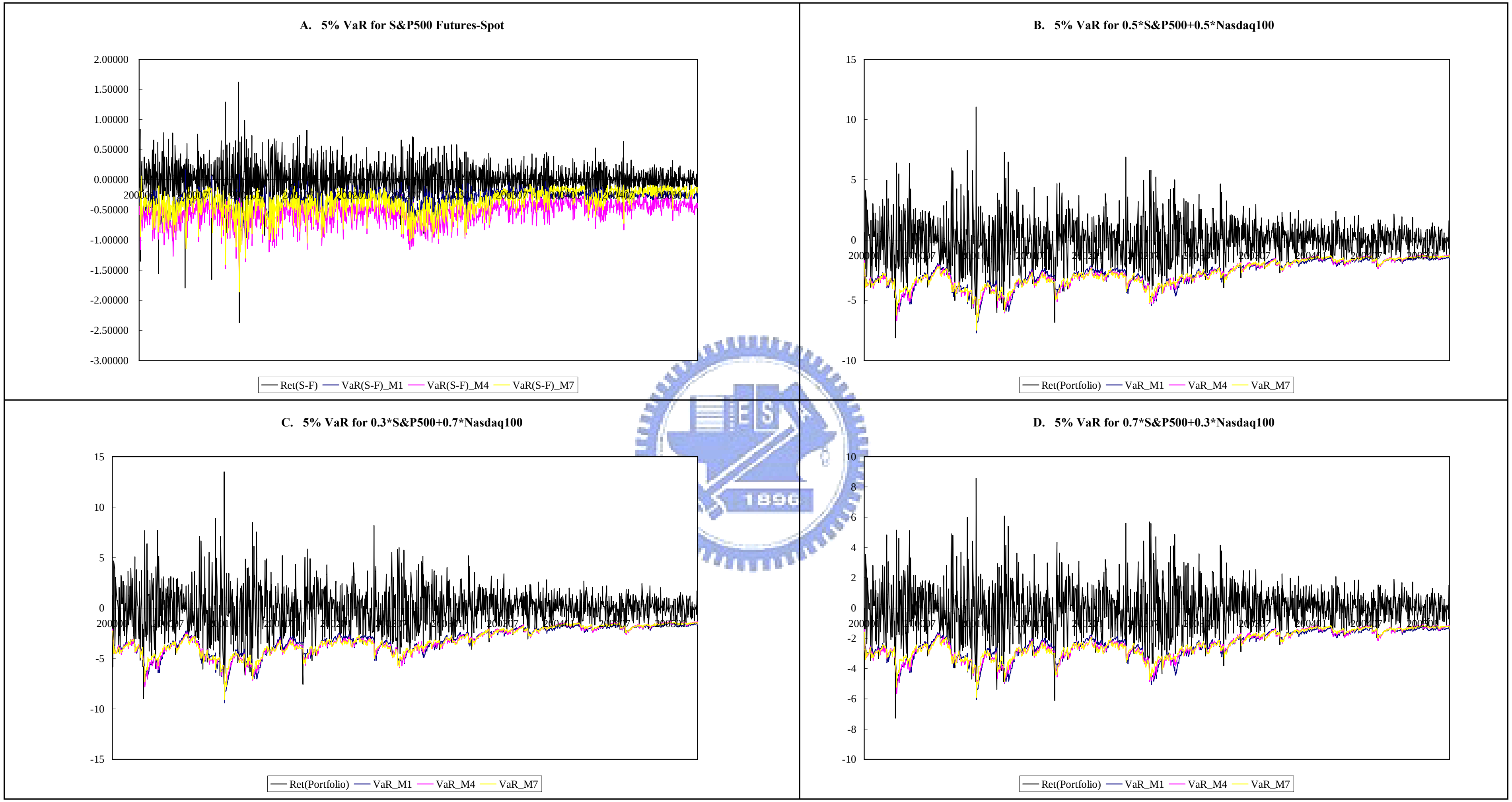

FIGURE 3.3－One Period Ahead 5% VaR Forecast Comparison for Various Portfolios, 2000/1-2005/3.

TABLE 3.8 – In- and Out-of-sample 5% VaR Failure Rate Results for the S&P500 Futures-Spot and S&P500-Nasdaq100 Spot Markets

A. Fraction of VaR Violation

	In sample				Out of sample			
	S&P500 Fut-Spot	S&P500-Nasdaq100			S&P500 Fut-Spot	S&P500-Nasdaq100		
Model (M_i)	VaR (1,-1)	VaR (0.5,0.5)	VaR (0.3,0.7)	VaR (0.7,0.3)	VaR (1,-1)	VaR (0.5,0.5)	VaR (0.3,0.7)	VaR (0.7,0.3)
M_1	1.911	5.096	5.016	4.697	1.639	1.639	1.639	3.279
M_2	1.433	5.255	5.096	4.936	1.639	1.639	1.639	3.279
M_3	2.389	5.255	5.096	4.857	1.639	1.639	1.639	3.279
M_4	4.459	5.334	5.096	5.016	3.279	3.279	4.918	3.279
M_5	4.379	5.096	5.255	5.414	3.279	6.557	4.918	3.279
M_6	6.131	5.255	5.096	5.175	3.279	6.557	4.918	3.279
M_7	5.732	5.175	5.096	5.096	6.557	4.918	4.918	3.279
M_8	5.096	5.334	5.255	4.857	4.918	3.279	4.918	3.279
M_9	5.334	5.334	5.255	5.175	4.918	3.279	4.918	3.279

B. P-Value of Kupiec LR Test

	In sample				Out of sample			
	S&P500 Fut-Spot	S&P500-Nasdaq100			S&P500 Fut-Spot	S&P500-Nasdaq100		
Model (M_i)	VaR (1,-1)	VaR (0.5,0.5)	VaR (0.3,0.7)	VaR (0.7,0.3)	VaR (1,-1)	VaR (0.5,0.5)	VaR (0.3,0.7)	VaR (0.7,0.3)
M_1	0.000	0.877	0.979	0.619	0.164	0.164	0.164	0.512
M_2	0.000	0.681	0.877	0.917	0.164	0.164	0.164	0.512
M_3	0.000	0.681	0.877	0.815	0.164	0.164	0.164	0.512
M_4	0.370	0.590	0.877	0.979	0.512	0.512	0.977	0.512
M_5	0.303	0.877	0.681	0.506	0.512	0.594	0.977	0.512
M_6	0.075	0.681	0.877	0.777	0.512	0.594	0.977	0.512
M_7	0.244	0.777	0.877	0.877	0.594	0.977	0.977	0.512
M_8	0.877	0.590	0.681	0.815	0.977	0.512	0.977	0.512
M_9	0.590	0.590	0.681	0.777	0.977	0.512	0.977	0.512

*Note:*This table shows the 5% VaR forecast results of four different portfolios for alternative models. Panel A is the fraction of VaR violations, and the results of Kupiec LM test (1995) are showed in Panel B. The data used are daily S&P500 index futures, S&P500 and Nasdaq 100 index prices. The in-sample data period is from January 1, 2000 to December 31, 2004 and out-of-sample data period is from January 1, 2005 to March 31, 2005. $M_1, M_2, \ldots, M_9$ denote the same models in Tables 3.2.

While futures contracts are popular among investors as a class of speculative assets, they are important in the financial markets due to their use as a hedging instrument. Furthermore, hedging with futures contracts may be the simplest method to manage market risk resulting from adverse movements in the price of various assets. In this section, we assume the hedger attempts to minimize the conditional variance of the spot-futures portfolio. It is well known that the optimal hedge ratio (OHR) is the ratio of the conditional covariance between spot and futures returns over the conditional variance of the futures return. So, the one-step-ahead forecasts of optimal hedge ratios can then be calculated as

$$HR_t^* = Cov_{t+1|t}\left(r_s, r_f\right) \big/ Var_{t+1|t}\left(r_f\right), \tag{3.25}$$

where r_s and r_f are spot and futures returns, respectively. The variance of the estimated optimal hedged portfolio can be characterized as

$$Var\left(r_{s,t} - HR_t^* \cdot r_{f,t}\right).$$

To evaluate hedging performance, the typical criterion is based on the percentage variance reduction (PVR) of the hedged portfolio relative to the unhedged position. It can be calculated as

$$\mathrm{PVR}(\%) = \left[1 - \left(\frac{\mathrm{Var}\left(\text{Hedged Portfolio}\right)}{\mathrm{Var}\left(\text{Unhedged Portfolio}\right)}\right)\right] \times 100\%. \tag{3.26}$$

When the futures contract completely eliminates risk, PVR=100 is obtained, otherwise PVR=0 is obtained when hedging with the futures contract does not reduce risk. Hence, a larger PVR indicates better hedging performance.

The in- and out-of-sample hedged portfolio variances and hedging effectiveness of alternative models for the S&P500 futures contract are presented in Table 3.9. The variances of hedged portfolio returns are calculated under the following eleven alternative models: three linear VECM-DCC-GARCH models with different lag parameter L (M_1,M_2,M_3), six threshold VECM-DCC-GARCH models with different lag parameter L and the number of regime G (M_4,---,M_9), hedging with a constant OHRs estimate using regression methods of returns and the naïve hedge with hedge ratio of 1 at all times. The results show that the three-regime threshold VECM(3)-DCC-GARCH model has the lowest in-sample hedged portfolio variance, with a 93.037% in-sample variance reduction compared to the variance of the unhedged position. In addition, the in-sample hedging performance of the linear model is even worse than that of OLS or naïve strategy.

Table 3.9. In- and Out-of-sample Hedging Effectiveness of Alternative Models for S&P500 Spot and Futures Markets

Model (M_i)	In Sample			Out of Sample		
	Variance	*Variance Reduction*	*Rank*	*Variance*	*Variance Reduction*	*Rank*
M_1	0.09457	92.55636	10	0.02249	96.48434	9
M_2	0.09854	92.24371	11	0.03114	95.13240	10
M_3	0.09342	92.64672	9	0.02356	96.31607	11
M_4	0.09053	92.87463	6	0.01891	97.04412	2
M_5	0.08872	93.01648	2	0.01909	97.01488	4
M_6	0.08912	92.98510	4	**0.01877**	**97.06492**	**1**
M_7	0.08911	92.98586	3	0.01896	97.03593	3
M_8	0.08943	92.96117	5	0.01916	97.00495	6
M_9	**0.08846**	**93.03718**	**1**	0.01910	97.01378	5
OLS	0.09091	92.84419	7	0.01922	96.99463	7
Naïve	0.09182	92.77321	8	0.01929	96.98378	8

Note: The table reports the variance of the hedged portfolio, percentage variance reductions, and the ranks of hedging effectiveness for several different models. The data used are daily index futures and spot price. The in-sample data period is from January 1, 2000 to December 31, 2004 and out-of-sample data period is from January 1, 2005 to March 31, 2005. M_1,M_2,…,M_9 denote the same models in Tables 3.2.

However, active hedgers are likely to be more concerned about future hedging performance. Therefore, the comparison of out-of-sample performance is a better way to evaluate our hedging strategy. We find that the ranking of out-of-sample hedging effectiveness is the same as that of in-sample hedging effectiveness. The two-regime threshold VECM(3)-DCC-GARCH model has a 97.065% out-of-sample variance reduction and outperforms all linear dynamic and static hedging models we considered. Overall, the dynamic hedge with threshold model has a better hedging performance than that with linear VECM-DCC-GARCH model. Both naïve and OLS strategies tend to outperform the linear VECM-DCC-GARCH model. This fact may indicate that the threshold model has superior ability to forecast the optimal hedge ratios.

6. CONCLUSIONS

We proposed a robust multivariate VAR-DCC-GARCH model that extends existing approaches by admitting multivariate thresholds in conditional means, conditional volatilities and conditional correlations. In addition, such threshold variables are defined by a weighted average of endogenous variables and the weights are estimated from the data. This threshold setting can not only enhance the robustness of the model but also has some economic meanings or values. Moreover, the Markov chain Monte Carlo method is implemented for the Bayesian inference. We studied the performance of our model in an application to two data sets consisting of daily S&P500 futures and spot prices and S&P500 and Nasdaq100 spot prices.

We develop a Bayesian testing scheme for model selection among several competing models and select the model with a higher posterior probability. We also adopt several criteria, which are based on the views of statistical loss and risk managers, to evaluate the prediction performance of the conditional covariance matrix.

In our real data application we find that estimated conditional volatilities are strongly characterized by both GARCH and multivariate threshold effects. Dynamic correlations are still apparent between S&P500 and Nasdaq100 spot returns, while slighter between S&P500 futures and spot markets. In addition, the estimation results suggest that S&P500 futures market is price leader between S&P500 futures and spot markets and S&P500 spot market is price leader between S&P500 and Nasdaq100 spot markets. For the comparison in covariance matrix forecasting performance, the threshold model has a better in-sample and out-of-sample forecasting performance relative to the linear model across most measure criteria.

Chapter 4. Summary and Conclusions

There are more and more academics shows that financial time series, such as stock returns, exchange rates series and etc., exhibit strong signs of nonlinearity. For this reason, some of traditional financial models should be modified appropriately. In this dissertation, we focus on the threshold model and use the Bayesian approach to settle the difficulty in the maximum likelihood estimation and inference. In addition, the applications in several important issues in financial markets are discussed, including the mutual fund performance evaluation and the forecasting in conditional covariance matrix.

The first essay in this dissertation uses three-regime Bayesian unconditional and conditional threshold four-factor models to study how fund managers react to change in market conditions. Our empirical analyses show that there are more apparent differences between unconditional and conditional three-regime threshold models instead of two-regime threshold models. In addition, we find that most managers' market timing ability comes from the skills to forecast the downside market. The three-regime threshold models have more power to detect significant timing activity when lagged public information is taken into account.

In addition, for the relationship between fund performances and various characteristics, we find that investors prefer to select funds with better past selectivity performance and upside market timing ability instead of downside market timing skill. Moreover, fund clients favor large size funds and funds with lower turnover, total load charges, and expenses. High turnover funds tend to have worse (better) selectivity performances in the downside (upside) market. We also find that contemporaneous net cash flows are negatively associated with downside market timing ability, but are positively correlated to upside market timing skills. In addition, funds with higher expenses have alert sensitivities to discover downside and upside markets.

In the second essay, we proposed a robust multivariate VAR-DCC-GARCH model that extends existing approaches by admitting multivariate thresholds in conditional means, conditional volatilities and conditional correlations. In addition, such threshold variables are defined by a weighted average of endogenous variables and the weights are estimated from the data. We studied the performance of our model in an application to two data sets consisting of daily S&P500 futures and spot prices and S&P500 and Nasdaq100 spot prices.

Our empirical analyses find that estimated conditional volatilities are strongly characterized by both GARCH and multivariate threshold effects. Dynamic correlations are still apparent between S&P500 and Nasdaq100 spot returns, while slighter between S&P500 futures and spot markets. In addition, the estimation results suggest that S&P500 futures market is price leader between S&P500 futures and spot markets and S&P500 spot market is price leader between S&P500 and Nasdaq100 spot markets. For the comparison in covariance matrix forecasting performance, the threshold model has a better in-sample and out-of-sample forecasting performance relative to the linear model across most measure criteria.

References

Andersen T., Bollerslev T., Diebold F., Ebens H., 2001. The distribution of realized stock return volatility. Journal of Financial Economics 61: 43-76.

Bauwen L., Lubrano M., Richard J.F., 1999. Bayesian Inference in Dynamic Econometric Models. Oxford University Press: New York.

Bollen N.P.B., Busse, J.A., 2001. On the timing ability of mutual fund managers. Journal of Finance 56: 1075-1094.

Bollerslev T., 1990. Modeling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model. Review of Economics and Statistics 72: 498-505.

Bollerslev T., Engle R., Wooldridge J.M., 1988. A capital asset pricing model with time varying covariances. Journal of Political Economy 96: 116-131.

Brooks C., 2001. A double-threshold GARCH model for the French Franc/Deutschmark exchange rate. Journal of Forecasting 20: 135-143.

Brown S.J., Goetzmann W.N., Ibbotson R.G., Ross S.A., 1992. Survivorship bias in performance studies. Review of Financial Studies 5: 553–580.

Brown S.J., Goetzman W.N., 1995. Performance persistence. Journal of Finance 50: 679-698.

Busse J.A., 1999. Volatility timing in mutual funds: Evidence from daily returns. Review of Financial Studies 12: 1009-1041.

Caner M., Hansen B.E., 2001. Threshold autoregression with a unit root. Econometrica 69: 1555-1596.

Carhart M.M., 1997. On persistence in mutual fund performance. Journal of Finance 52: 57-82.

Chang E.C., Lewellen W.G., 1984. Market timing and mutual fund investment performance.

Journal of Business 57: 57-72.

Chen C.W.S., So M.K.P., 2006. On a threshold heteroscedastic model. International Journal of Forecasting 22: 73-89.

Chen J., Huang H., Hong M., Kubik J.D., 2004. Does fund size erode mutual fund performance? The role of liquidity and organization. American Economic Review 94: 1276-1302.

Chevalier J., Ellison G., 1997. Risk taking by mutual funds as a response to incentives. Journal of Political Economy 105: 1167-1200.

Chib S., Greenberg E., 1995. Understanding the Metropolis-Hastings algorithm. American Statistician 49, 327-335.

Dellva W.L., Gerard T.O., 1998. The relationship between mutual fund fees and expenses and their effects on performance. The Financial Review 33: 85-104.

Doan T.A., 1990. Regression Analysis of Time Series, Evanston, IL: VAR Econometrics.

Edelen R.M., 1999. Investor flows and the assessed performance of open-end mutual funds. Journal of Financial Economics 53: 439-466.

Elton E.J., Gruber M.J., Das S., Hlavka M., 1993. Efficiency with costly information: a reinterpretation of evidence from managed portfolios. Review of Financial Studies 6:1-22.

Fama E.F, French K.R., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics 33: 3-56.

Engle R.F., Yoo B., 1987. Forecasting and testing in cointegrated systems. Journal of Econometrics 35: 143-159.

Engle R.F., Ng V., Rothschild M., 1990. Asset pricing with a factor-ARCH covariance structure: empirical estimates for Treasury bills. Journal of Econometrics 45: 213-238.

Engle R.F., Kroner K., 1995. Multivariate simultaneous GARCH. Econometric Theory 11: 122-150.

Engle R.F., 2002. Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. Journal of Business and Economic Statistics 20: 339-350.

Ferson W.E., Schadt R., 1996. Measuring fund strategy and performance in changing economic conditions. Journal of Finance 51: 425-462.

French K.R., Schwert G.W., Stambaugh R.F., 1987. Expected stock returns and volatility. Journal of Financial Economics 19: 3-29.

Geweke J., 1992. Evaluating the accuracy of sampling-based approaches to the calculation of posterior moments. Bayesian Statistics 4: 169-193.

Geweke J., 1996. Bayesian reduced rank regression in econometrics. Journal of Econometrics 75: 121-146.

Gilks W.R., Richardson S., Spiegelhalter D.J., 1996. Markov chain Monte Carlo in practice. Chapman & Hall.

Glosten L.R., Jagannathan R., 1994. A contingent claim approach to performance evaluation. Journal of Empirical Finance 1: 133-160.

Goetzmann W.N., Ingersoll J., Ivkovic Z., 2000. Monthly measurement of daily timers. Journal of Financial and Quantitative Analysis 35: 257-290.

Grinblatt M., Titman S., 1989. Mutual fund performance: An analysis of quarterly portfolio holdings. Journal of Business 62: 393-416.

Grinblatt M., Titman S., 1994. A study of monthly mutual fund returns and performance evaluation techniques. Journal of Financial and Quantitative Analysis 29: 419-444.

Gruber M., 1996. Another puzzle: the growth in activity managed and mutual funds. Journal of Finance 51: 783-810.

Hammersley J.M., Clifford P., 1968. Markov fields of finite graphs and lattices. Preprint, Univ. of Calif.-Berkeley.

Hansen B.E., 1996. Inference when a nuisance parameter is not identified under the null hypothesis. Econometrica 64: 413-430.

Hansen B.E., 2000. Sample splitting and threshold estimation. Econometrica 68: 575-603.

Hansen B.E, Seo B., 2002. Testing for two-regime threshold cointegration in vector error-correction models. Journal of Econometrics 110: 293-318.

Hastings W.K., 1970. Monte Carlo sampling methods using Markov chains and their applications. Biometrika 57: 97-109.

Henriksson R.D., 1984. Market timing and mutual fund performance: An empirical investigation. Journal of Business 57: 73-96.

Hsieh D., 1989. Testing for Nonlinearity in Daily Foreign Exchange Rate Changes. Journal of Business 62: 339-368.

Hsieh D., 1991. Chaos and Nonlinear Dynamics: Application to Financial markets. Journal of Finance 46: 1839-1877.

Hsieh D., 1993. Implication of nonlinear dynamics for financial risk management. Journal of Financial and Quantitative Analysis 28: 41-64.

Ippolito R., 1989. Efficiency with costly information：A study of mutual fund performance 1965-1984. The Quarterly Journal of Economic 104: 1-23.

Jain P.C., Wu J.S., 2000. Truth in mutual fund advertising: Evidence on future performance and fund flows. Journal of Finance 55: 937-958.

Jensen M., 1968. The performance of mutual funds in the period 1945-1964. Journal of Finance 23: 389-416.

Jones Christopher S., Shanken Jay, 2005. Mutual fund performance with learning across funds. Journal of Financial Economics 78: 507-552.

Kinal T, Ratner J., 1986. A VAR forecasting model of a regional economy: Its construction and comparative accuracy. International Regional Science Review 10: 113-26.

Koop, G., Potter, S.M., 1999. Bayes factors and nonlinearity: Evidence from economic time series. Journal of Econometrics 88: 251–281.

Koop, G., Potter, S.M., 2003. Bayesinan analysis of endogenous delay threshold models. Journal of Business & Economic Statistics 21: 93–103.

Kupiec P., 1995. Techniques for verifying the accuracy of risk management models. Journal of Derivatives 3: 73-84.

Lamoureux C.G., Lastrapes W.D., 1990. Persistence in variance, structural change, and the GARCH model. Journal of Busines & Economic Statistics 8: 225-234.

Li C.W., Li W.K., 1996. On a double-threshold autoregressive heteroscedastic time series model. Journal of Applied Econometrics 11: 253-274.

Litterman R.B., 1980. Techniques for forecasting with vector autoregressions. Working paper, Massachusetts Institute of Technology.

Litterman R.B., 1986. Forecasting with Bayesian vector autoregressive-five years of experience. Journal of Business & Economic Statistics 4: 25-38.

Malkiel B., 1995. Returns from investing in equity mutual funds 1971 to 1991. Journal of Finance 50: 549-572.

Merton R.C., Henriksson R.D., 1981. On the market timing and investment performance of managed portfolios II – statistical procedures for evaluating forecasting skills. Journal of Business 54: 513-533.

Metropolis N., Rosenbluth A.W., Rosenbluth M.N., Teller A.H., Teller E., 1953. Equation of state calculations by fast computing machines. Journal of Chemical Physics 21,:1087-1092.

Mikosch T., Starica C., 2004. Nonstationarities in Financial Time Series, the Long Range Dependence and the IGARCH Effects. Review of Economics and Statistics 86: 378-390.

Ni S, Sun D., 2003. Noninformative priors and frequentist risks of Bayesian estimators of vector-autoregressive models. Journal of Econometrics 115: 159-197

Ni S, Sun D., 2005. Bayesian estimates for vector autoregressive models. Journal of Business & Economic Statistics 23: 105-117.

Osiewalski J., Pipień M., 2004. Bayesian comparison of bivariate ARCH-type models for the main exchange rates in Poland. Journal of Econometrics 123: 371-391.

Pastor L., Stambaugh Robert F., 2000. Comparing Asset Pricing Models: An Investment Perspective. Journal of Financial Economics 56: 335–381

Pastor L., Stambaugh Robert F., 2002. Investing in Equity Mutual Funds. Journal of Financial Economics 63: 351–380.

Rabemananjara R., Zakoian J.M, 1993. Threshold ARCH models and asymmetries in volatility. Journal of Applied Econometrics 8: 31-49.

Sharpe W.F., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. Journal of Finance 19: 425-442.

Sirri E., Tufano P., 1998. Costly search and mutual fund flows. Journal of Finance 53: 1589-1622.

Sims C.A., 1980. Macroeconomics and reality. Econometrica 48: 1-48.

So M.K.P., Chen C.W.S., Chen M.T., 2005. A Bayesian threshold nonlinearity test for financial time series. Journal of Forecasting 24: 61-75.

Tong H., 1983. Threshold Models in Non-linear Time Series Analysis: Lecture Notes in Statistics 21. Berlin: Springer-Verlag.

Tong, H., 1990. Non-linear time series: A dynamical system approach. Oxford University Press: Oxford.

Treynor J.L., Mazuy, K., 1966. Can mutual funds outguess the market? Harvard Business Review 44: 131-136.

Tsay R.S., 1989. Testing and modeling threshold autoregressive process. Journal of the American Statistical Association 84: 231- 240.

Tsay R.S., 1998. Testing and modeling multivariate threshold models. Journal of the American Statistical Association 93: 1188-1202.

Villani M., 2001. Bayesian prediction with cointegrated vector autoregressions. International Journal of Forecasting 17: 585-605.

Vrontos D, Dellaportas P, Politis D.N., 2003. Inference for some multivariate ARCH and GARCH models. Journal of Forecasting 22: 427-446.

Warther V.A., 1995. Aggregate mutual fund flows and security returns. Journal of Financial Economics 39: 209-235.

Wermers R., 2000. Mutual fund performance: An empirical decomposition into stock-picking talent, style, transactions costs, and expenses. Journal of Finance 4: 1655-1703.

Zellner A., 1971. An Introduction to Bayesian Inference in Econometrics. New York: Wiley.