

整數階與分數階統一渾沌系統之渾沌與渾沌同步

學生：楊坤偉

指導教授：戈正銘

國立交通大學機械工程研究所碩士班

摘 要

本論文探討整數階與分數階統一渾沌系統的渾沌與渾沌同步。首先利用兩種耦合方法：狀態耦合與速度耦合，使得兩個完全相同的整數階統一渾沌系統達到同步。接著對統一渾沌系統做更詳細的研究，藉由相圖、分歧圖與 Lyapunov 指數來驗證其結果，不僅對參數 $\alpha \in [0,1]$ 內的非渾沌範圍也對 $\alpha \in [0,1]$ 外的渾沌範圍給予標定；我們另提出了一個新參數 $\beta = f(\alpha)$ 使得延伸的統一渾沌系統仍只在 $\beta \in [0,1]$ 存在渾沌。然後我們使統一渾沌系統參數 α 作週期性的變換，其中變化模式包括一次方、二次方、四次方與六次方之正弦波、三角波與鋸齒波，並且研究各種波所激發出的動態行為。最後我們對分數階統一渾沌系統研究其渾沌現象及控制，更對相同階與不同階的系統研究其同步。

Chaos and Chaos Synchronization of Integral and Fractional Order Unified Chaotic System

student : Kun-Wei Yang

Advisor : Zheng-Ming Ge

Department of Mechanical Engineering
National Chiao Tung University

ABSTRACT

The chaos and chaos synchronization of integral and fractional order unified chaotic system are studied in this thesis. First, two methods, state coupling and speed coupling, are used to synchronize two identical unified chaotic systems. Next, unified chaotic system is studied in detail and its dynamics are studied by phase portrait, bifurcation diagram and Lyapunov exponent. Not only non-chaotic ranges within $\alpha \in [0,1]$ but also chaotic ranges besides $\alpha \in [0,1]$ for unified chaotic system are found. We also introduce a new parameter $\beta = f(\alpha)$ so that the extended unified chaotic system is chaotic only when $\beta \in [0,1]$. Then a unified chaotic system with various periodic switches are put forward. Four functions $\sin^2 \omega t$, $\sin \omega t$, triangular wave and sawtooth wave are used to replace α which is the original constant parameter of unified chaotic system. Finally, chaos, chaos control and synchronization of the fractional order unified chaotic systems are studied.

誌 謝

本論文得以完成，首先必須感謝我的指導老師 戈正銘教授，戈老師有教無類與因材施教的教育方式使我在課業上與做研究方面獲得極大的幫助。在這段期間，老師不僅在專業的學術領域給予指導，其達觀的待人處世態度與文學、史學、哲學涵養上，也讓學生受益匪淺。

在這段研究生活裏，非常感謝陳炎生、鄭普建、楊振雄學長在渾沌知識的指導與經驗傳承；其次要感謝的是同學莊為任與林國樺，與他們相處不論在課業上與生活上都獲得極大的幫助；也謝謝學弟歐展義、易昌賢、張安瑞與徐茂原在平常生活上的幫助。

最後我要感謝我的阿公、父母親與家人，在多年來的求學生涯，不論我決定的方向如何，他們總是默默地支持與鼓勵。也感謝從小到大教導過我的老師與一直陪在我身邊的同學與好友。最後還是要再次感謝我的恩師 戈正銘教授。



Contents

Chinese Abstract	i
Abstract	ii
Acknowledgment	iii
Contents	iv
List of Figures	vii
Chapter 1 Introduction	1
Chapter 2 Differential Equations of Motion and Description of the System	3
2.1 Unified Chaotic System	3
2.2 Lyapunov Exponent	3
2.3 Phase Portrait	4
Chapter 3 Synchronization of Unified Chaotic System	5
3.1 State Feedback Synchronization of Unified Chaotic System	5
3.1.1 Case1: add state coupling as $k_1(x_1 - x_2)$ to first equation of the system	5
3.1.2 Case2: add state coupling as $k_2(x_1 - x_2)$ to second equation of the system	8
3.1.3 Case3 : add state coupling as $k_3(x_1 - x_2)$ to third equation of the system	10
3.2 Speed Feedback Synchronization of Unified Chaotic System	12
3.2.1 Case1: add state coupling as $G(\dot{x}_1 - \dot{x}_2)$ to second equation of the system	12
3.2.2 Case2: add state coupling as $G(\dot{x}_1 - \dot{x}_2)$ to third equation of the system	14
3.3 $\alpha \sim G$ Diagram of Speed Coupling Adding to Second Equation of the System	16

Chapter 4 Detailed Study of the Chaos of Unified Chaotic System	17
4.1 Non-chaotic Ranges within $\alpha \in [0,1]$ for Unified Chaotic System	17
4.2 Chaotic Ranges besides $\alpha \in [0,1]$	18
4.3 An Extended Unified Chaotic System with Parameter $\beta \in [0,1]$	18
4.4 An Extended Unified Chaotic System with Two Parameter Ranges Excited by Continuous Periodic Switches	18
Chapter 5 A Unified Chaotic System with Four Different Periodic Switches	20
5.1 A Unified Chaotic System with $\sin^2 \omega t$ Continuous Periodic Switch	20
5.2 A Unified Chaotic System with $\sin \omega t$ Continuous Periodic Switch	21
5.3 A Unified Chaotic System with 0~1 Triangular Wave Switch	21
5.4 A Unified Chaotic System with -1~1 Triangular Wave Switch	22
5.5 A Unified Chaotic System with 0~1 Sawtooth Wave Switch	23
Chapter 6 Chaos Control and Synchronization of the Fractional Order Unified Chaotic System	25
6.1 Introduction of Fractional Order System	25
6.2 Fractional Derivative and Its Approximation	25
6.3 The Fractional Order Unified Chaotic System	26
6.4 Chaos Control of Fractional Order Unified Chaotic System	27
6.5 Chaos Synchronization of the Same Fractional Order Unified Chaotic Systems	28

6.6	Chaos Synchronization of Different Fractional Order Unified Chaotic Systems	29
Chapter 7	Conclusions	32
References		34



List of Figures

Fig.2.2.1	The Lyapunov exponents for α between -2 and 2.	38
Fig.2.3.1(a)	3D phase portrait for unified chaotic system with $\alpha = 0$.	38
Fig.2.3.1(b)	2D(xy) phase portrait for unified chaotic system with $\alpha = 0$.	39
Fig.2.3.1(c)	2D(yz) phase portrait for unified chaotic system with $\alpha = 0$.	39
Fig.2.3.1(d)	2D(xz) phase portrait for unified chaotic system with $\alpha = 0$.	40
Fig.2.3.2(a)	3D phase portrait for unified chaotic system with $\alpha = 1$.	40
Fig.2.3.2(b)	2D(xy) phase portrait for unified chaotic system with $\alpha = 1$.	41
Fig.2.3.2(c)	2D(yz) phase portrait for unified chaotic system with $\alpha = 1$.	41
Fig.2.3.2(d)	2D(xz) phase portrait for unified chaotic system with $\alpha = 1$.	42
Fig.2.3.3(a)	3D phase portrait for unified chaotic system with $\alpha = -1$ and initial condition $x=2, y=5, z=7$.	42
Fig.2.3.3(b)	2D(xy) phase portrait for unified chaotic system with $\alpha = -1$ and initial condition $x=2, y=5, z=7$.	43
Fig.2.3.3(c)	2D(yz) phase portrait for unified chaotic system with $\alpha = -1$ and initial condition $x=2, y=5, z=7$.	43
Fig.2.3.3(d)	2D(xz) phase portrait for unified chaotic system with $\alpha = -1$ and initial condition $x=2, y=5, z=7$.	44
Fig.2.3.4(a)	3D phase portrait for unified chaotic system with $\alpha = 1.7$.	44
Fig.2.3.4(b)	2D(xy) phase portrait for unified chaotic system with $\alpha = 1.7$.	45
Fig.2.3.4(c)	2D(yz) phase portrait for unified chaotic system with $\alpha = 1.7$.	45
Fig.2.3.4(d)	2D(xz) phase portrait for unified chaotic system with $\alpha = 1.7$.	46
Fig.3.1.1(a)	Time history of error with $\alpha = 0$ $k_1 = 270$.	46
Fig.3.1.1(b)	Time history of errors with $\alpha = 0$ $k_1 = 20$.	47
Fig.3.1.2(a)	Time history of error with $\alpha = 1$ $k_1 = 1000$ $t=20$.	47
Fig.3.1.2(b)	Time history of error with $\alpha = 1$ $k_1 = 1000$ $t=1000$.	48
Fig.3.1.3(a)	Time history of error with $\alpha = 0$ $k_2 = 30$.	48
Fig.3.1.3(b)	Time history of errors with $\alpha = 0$ $k_2 = 10$.	49
Fig.3.1.4(a)	Time history of error with $\alpha = 1$ $k_2 = 25$.	49
Fig.3.1.4(b)	Time history of errors with $\alpha = 1$ $k_2 = 18$.	50

Fig.3.1.5(a)	Time history of error with $\alpha = 0$ $k_3 = 50$.	50
Fig.3.1.5(b)	Time history of error with $\alpha = 0$ $k_3 = 500$.	51
Fig.3.1.5(c)	Time history of error with $\alpha = 1$ $k_3 = 50$.	51
Fig.3.1.5(d)	Time history of error with $\alpha = 1$ $k_3 = 500$.	52
Fig.3.2.1	Time history of error with $\alpha = 0$ $G = 10$.	52
Fig.3.2.2	Time history of error with $\alpha = 1$ $G = 10$.	53
Fig.3.2.3	Time history of error with $\alpha = 0$ $G = 10$.	53
Fig.3.2.4	Time history of error with $\alpha = 1$ $G = 10$.	54
Table3.3.1	αG value for $\alpha \in [0,1]$, where G is the TLV for synchronization.	54
Fig.3.3.1	Curve fitting diagram for $\alpha \in [0,1]$.	55
Fig.4.1.1	Power spectrum of unified chaotic system with $\alpha = 1$ (Chen system).	56
Fig.4.1.2(a)	Lyapunov exponents of Unified Chaotic System for α between -1 and 1.	56
Fig.4.1.2(b)	Lyapunov exponents of Unified Chaotic System for α between -1 and 1.	57
Fig.4.1.3(a)	Phase portrait of unified chaotic system with $\alpha = 0.583$.	57
Fig.4.1.3(b)	Power spectrum of unified chaotic system with $\alpha = 0.583$.	58
Fig.4.1.4	Some non-chaotic points of unified chaotic system for $\alpha \in [0,1]$.	58
Fig.4.2.1	The magnified part with $\alpha > 1$ of unified chaotic system.	59
Fig.4.2.2	The magnified part with $\alpha < 0$ of unified chaotic system.	59
Fig.4.3.1	Lyapunov exponent of extended unified chaotic system.	60
Fig.4.4.1(a)	Lyapunov exponent of extended unified chaotic system($\sin^2 \omega t$).	60
Fig.4.4.1(b)	Lyapunov exponent of extended unified chaotic system($\sin^2 \omega t$).	61
Fig.4.4.2(a)	Lyapunov exponent of extended unified chaotic system($\sin^4 \omega t$).	61
Fig.4.4.2(b)	Lyapunov exponent of extended unified chaotic system($\sin^4 \omega t$).	62
Fig.4.4.3(a)	Lyapunov exponent of extended unified chaotic system($\sin^6 \omega t$).	62
Fig.4.4.3(b)	Lyapunov exponent of extended unified chaotic system($\sin^6 \omega t$).	63
Fig.5.1.1	Lyapunov exponent of unified chaotic system with $\alpha = \sin^2 \omega t$.	63
Fig.5.1.2	Lyapunov exponent of unified chaotic system with $\alpha = 2 \sin^2 \omega t$.	64
Fig.5.1.3	Lyapunov exponent of unified chaotic system with $\alpha = 4 \sin^2 \omega t$.	64
Fig.5.2.1	Lyapunov exponent of unified chaotic system with $\alpha = \sin \omega t$.	65

Fig.5.2.2	Lyapunov exponent of unified chaotic system with $\alpha = 2 \sin \omega t$.	65
Fig.5.2.3	Lyapunov exponent of unified chaotic system with $\alpha = 4 \sin \omega t$.	66
Fig.5.2.4	Lyapunov exponent of unified chaotic system with $\alpha = 6 \sin \omega t$.	66
Fig.5.2.5	Lyapunov exponent of unified chaotic system with $\alpha = 8 \sin \omega t$.	67
Fig.5.2.6	Lyapunov exponent of unified chaotic system with $\alpha = 10 \sin \omega t$.	67
Fig.5.3.1	Lyapunov exponent of unified chaotic system with $\alpha = f(t), L = \pi$.	68
Fig.5.3.2	Lyapunov exponent of unified chaotic system with $\alpha = f(t), L = 2\pi$.	68
Fig.5.3.3	Lyapunov exponent of unified chaotic system with $\alpha = f(t), L = 3\pi$.	69
Fig.5.3.4	Lyapunov exponent of unified chaotic system with $\alpha = f(t), k = 1$.	69
Fig.5.3.5	Lyapunov exponent of unified chaotic system with $\alpha = f(t), k = 2$.	70
Fig.5.3.6	Lyapunov exponent of unified chaotic system with $\alpha = f(t), k = 3$.	70
Fig.5.3.7	Lyapunov exponent of unified chaotic system with $\alpha = f(t), k = 10$.	71
Fig.5.3.8	Bifurcation diagram of the Lyapunov exponent of unified chaotic system with $\alpha = f(t), k = 1, \omega = 2\pi / L$.	71
Fig.5.4.1	Lyapunov exponent of unified chaotic system with $\alpha = g(t) k = 1$.	72
Fig.5.4.2	Lyapunov exponent of unified chaotic system with $\alpha = g(t) k = 2$.	72
Fig.5.4.3	Lyapunov exponent of unified chaotic system with $\alpha = g(t) k = 4$.	73
Fig.5.4.4	Lyapunov exponent of unified chaotic system with $\alpha = g(t) k = 6$.	73
Fig.5.4.5	Lyapunov exponent of unified chaotic system with $\alpha = g(t) k = 8$.	74
Fig.5.4.6	Lyapunov exponent of unified chaotic system with $\alpha = g(t) k = 10$.	74
Fig.5.5.1	Lyapunov exponent of unified chaotic system with $\alpha = q(t) k = 1$.	75
Fig.5.5.2	Lyapunov exponent of unified chaotic system with $\alpha = q(t) k = 2$.	75
Fig.5.5.3	Lyapunov exponent of unified chaotic system with $\alpha = q(t) k = 4$.	76
Fig.5.5.4	Lyapunov exponent of unified chaotic system with $\alpha = q(t) k = 6$.	76
Fig.5.5.5	Lyapunov exponent of unified chaotic system with $\alpha = q(t) k = 8$.	77
Fig.5.5.6	Lyapunov exponent of unified chaotic system with $\alpha = q(t) k = 10$.	77
Fig.6.3.1	Bifurcation diagram of 2.7(3*0.9) order unified chaotic system.	78
Fig.6.3.2	Phase portrait of 2.7(3*0.9) order unified chaotic system with $\alpha = 0.8$ (Lü).	78
Fig.6.3.3	Phase portrait of 2.7(3*0.9) order unified chaotic system with $\alpha = 2.0$.	79

Fig.6.3.4	Phase portrait of 2.7(3*0.9) order unified chaotic system with $\alpha = 2.2$.	79
Fig.6.3.5	Phase portrait of 2.7(3*0.9) order unified chaotic system with $\alpha = 2.25$.	80
Fig.6.3.6	Bifurcation diagram of 2.4(3*0.8) order unified chaotic system.	80
Fig.6.3.7	Phase portrait of 2.4(3*0.8) order unified chaotic system with $\alpha = 1.7$.	81
Fig.6.3.8	Phase portrait of 2.4(3*0.8) order unified chaotic system with $\alpha = 1.8$.	81
Fig.6.3.9	Phase portrait of 2.4(3*0.8) order unified chaotic system with $\alpha = 1.9$.	82
Fig.6.3.10	Phase portrait of 2.4(3*0.8) order unified chaotic system with $\alpha = 2.05$.	82
Fig.6.3.11	Phase portrait of 2.1(3*0.7) order unified chaotic system with $\alpha = 3.7$.	83
Fig.6.3.12	Phase portrait of 2.1(3*0.7) order unified chaotic system with $\alpha = 3.8$.	83
Fig.6.3.13	Phase portrait of 2.1(3*0.7) order unified chaotic system with $\alpha = 4.0$.	84
Fig.6.3.14	Phase portrait of 1.8(3*0.6) order unified chaotic system with $\alpha = 2.5$.	84
Fig.6.3.15	Phase portrait of 1.8(3*0.6) order unified chaotic system with $\alpha = 3.0$.	85
Fig.6.3.16	Phase portrait of 1.5(3*0.5) order unified chaotic system with $\alpha = 2.0$.	85
Fig.6.3.17	Phase portrait of 1.5(3*0.5) order unified chaotic system with $\alpha = 2.5$.	86
Fig.6.4.1(a)	Time history of the controlled fractional order unified chaotic system with order $a = 0.9$ and system parameter $\alpha = 0.8$.	86
Fig.6.4.1(b)	Time history of the controlled fractional order unified chaotic system with order $a = 0.9$ and system parameter $\alpha = 1.8$.	87
Fig.6.4.2(a)	Time history of the controlled fractional order unified chaotic system with order $a = 0.8$ and system parameter $\alpha = 1.4$.	87

Fig.6.4.2(b)	Time history of the controlled fractional order unified chaotic system with order $a = 0.8$ and system parameter $\alpha = 1.9$.	88
Fig.6.4.3	Time history of the controlled fractional order unified chaotic system with order $a = 0.7$ and system parameter $\alpha = 3.7$.	88
Fig.6.5.1(a)	Time history of errors with $\alpha = 0.8$ order=0.9 $k_1 = k_2 = k_3 = 1$.	89
Fig.6.5.1(b)	Time history of errors with order=0.9 $\alpha = 2.2$ $k_1 = k_2 = k_3 = 1$.	89
Fig.6.5.2(a)	Time history of errors with order=0.8 $\alpha = 0.8$ $k_1 = k_2 = k_3 = 2$.	90
Fig.6.5.2(b)	Time history of errors with order=0.8 $\alpha = 1.9$ $k_1 = k_2 = k_3 = 2$.	90
Fig.6.5.3	Time history of errors with order=0.7 $\alpha = 3.7$ $k_1 = k_2 = k_3 = 4$.	91
Fig.6.6.1	Time history of errors with $\alpha = 1.2$.	91
Fig.6.6.2	Time history of errors with master $\alpha = 1.2$ and slave $\alpha = 3.7$.	92
Fig.6.6.3	Time history of errors with master $\alpha = 3.7$ and slave $\alpha = 2.2$.	92

