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Hyperchaos of four state autonomous system with three positive Lyapunov exponents

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ABSTRACT

This Letter gives the results of numerical simulations of Quantum Cellular Neural Network (Quantum-CNN) autonomous system with four state variables. Three positive Lyapunov exponents confirm hyperchaotic nature of its dynamics.

Nonlinear systems are capable of exhibiting a variety of behaviors, ranging from fixed points via limit cycles and tori to the more complex chaotic and hyperchaotic attractors. It is well known that continuous time systems of integer order must be at least third order in order for chaos to appear. Such systems are characterized by one positive Lyapunov exponent (PLE) in the Lyapunov spectrum [1–11]. A chaotic signal is characterized by having a single positive Lyapunov exponent which indicates that the dynamics of the underlying chaotic attractor expands only in one direction. Whenever a chaotic attractor is characterized by more than one positive Lyapunov exponent, it is termed hyperchaos. In this case, the dynamics of the chaotic attractor expands in more than one direction giving rise to a "thick" chaotic attractor [12–15].

A hyperchaotic system is characterized by the presence of two or more PLEs in its Lyapunov spectrum, indicating that it is unstable in more than one direction. Besides the theoretical interest in the dynamics of such nonlinear systems, there has been a practical interest in chaos and hyperchaos as means for secure communication. Hyperchaos was first reported from computer simulations of hypothetical ordinary differential equations in [17–21]. The first observation of hyperchaos from a real physical system, a fourth-order electrical circuit, was later reported in [16]. Very few hyperchaos generators have been reported since then [22–26]. As the numerical example, recently developed Quantum Cellular Neural Network (Quantum-CNN) autonomous oscillator with four state variables is used. Quantum-CNN oscillator equations are derived from a Schrödinger equation taking into account quantum dots cellular automata structures to which in the last decade a wide interest has been devoted with particular attention towards quantum computing [4]. For this system four Lyapunov exponents are not zero. Although by traditional theory [27], for four-dimensional continuous-time systems, there must be a zero Lyapunov exponent, however, on the history of science, as said by T.S. Kuhn [28] in his book "The Structure of Scientific Revolution", the unexpected discovery or anomality (counterinstance) is not simply factual in its import and the scientist's world is qualitatively transformed as well as quantitatively enriched by fundamental novelties of either fact or theory. "Conversion as a feature of revolutions in science" is the conclusion of the book "Revolution in Science" written by Cohen [29]. One of the patterns of the evolution of science is: current paradigm → normal science → anomality (counterinstance) → crisis → emergence of scientific theories → new paradigm.

For a two-cell Quantum-CNN, the following differential equations are obtained [4]:

$$\begin{cases} \dot{x}_1 = -2a_1\sqrt{1-x_1^2}\sin x_2, \\ \dot{x}_2 = -\omega_1(x_1-x_3) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2, \\ \dot{x}_3 = -2a_2\sqrt{1-x_3^2}\sin x_4, \\ \dot{x}_4 = -\omega_2(x_3-x_1) + 2a_2\frac{x_3}{\sqrt{1-x_3^2}}\cos x_4, \end{cases} \tag{1}$$

where x_1 and x_3 are polarizations, x_2 and x_4 are quantum phase displacements, a_1 and a_2 are proportional to the inter-dot energy

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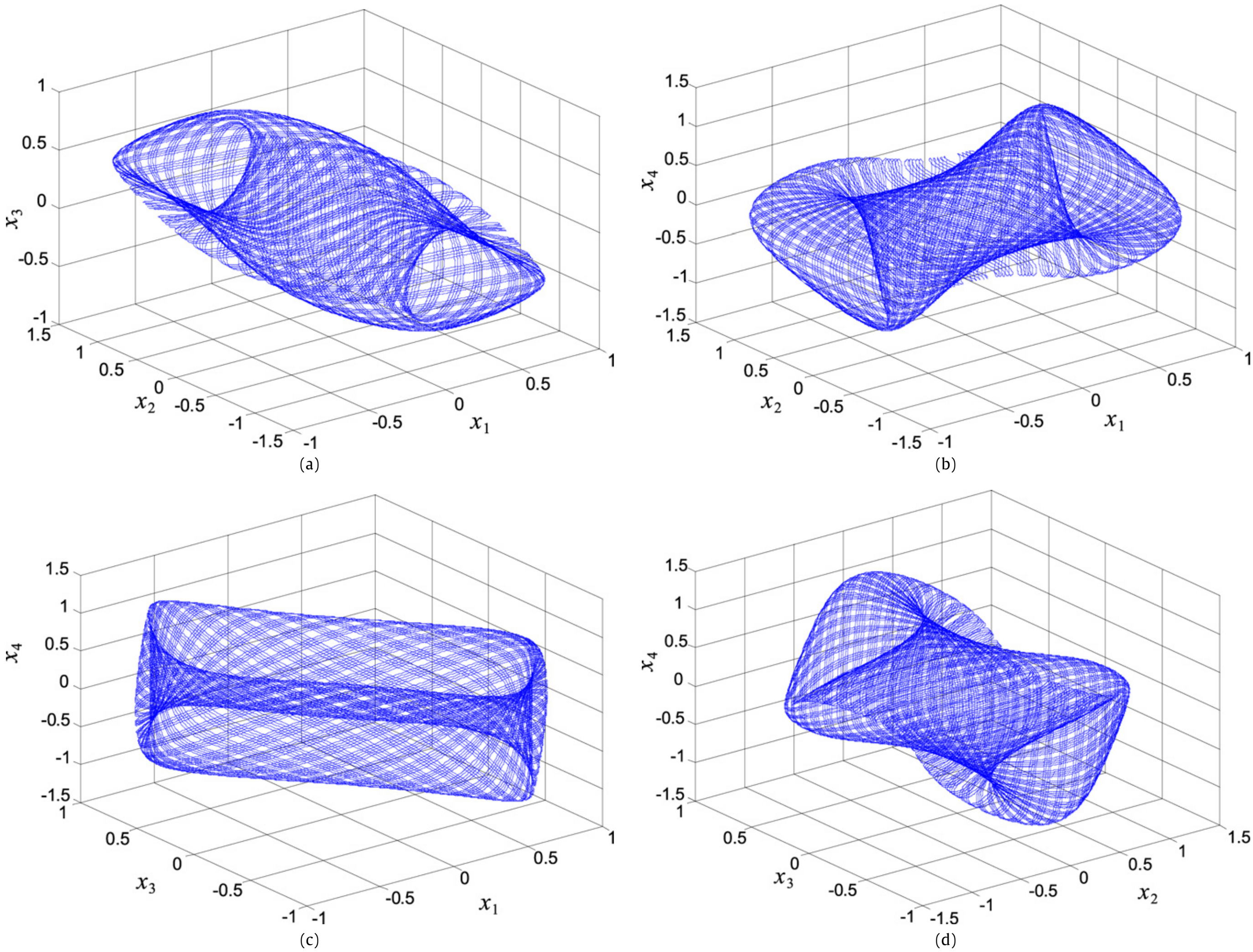

Fig. 1. The three-dimensional projections of four-dimensional phase portrait of Quantum-CNN, with $a_1 = 0.00009$, $a_2 = 0.00009$, $\omega_1 = 0.0001$, and $\omega_2 = 0.0001$.

inside each cell and ω_1 and ω_2 are parameters that weigh effects on the cell of the difference of the polarization of neighboring cells, like the cloning templates in traditional CNNs.

The evolution of a set of trajectories emanating from various initial conditions is presented in the phase space. When the solution becomes stable, the asymptotic behaviors of the phase trajectories are particularly interested and the transient behaviors in the system are neglected. The three-dimensional projections of fourdimensional phase portrait of the Quantum-CNN system, Eq. (1), are plotted in Fig. 1.

If the states are not periodic, their spectrum must be in terms of oscillations with a continuum of frequencies. Such a representation of the spectrum is called Fourier integral. The power spectrum analysis of the nonlinear dynamical system, Eq. (1), is shown in Fig. 2. The noise-like spectrum is the characteristics of chaotic dynamical system.

The Lyapunov exponents of the solutions of the nonlinear dynamical system, Eq. (1), is plotted in Figs. 3–8.

Furthermore, the parameter values, a_1, a_2, ω_1, and ω_2, are varied to observe the regions of chaotization of the system. By varying any two of a_1, a_2, ω_1, and ω_2, and fixing the other two, the parametric diagrams are described and shown as Figs. 9–14. The enriched information of chaotic behaviors of the system can be obtained from the diagrams. Three positive Lyapunov exponents occur in many regions.

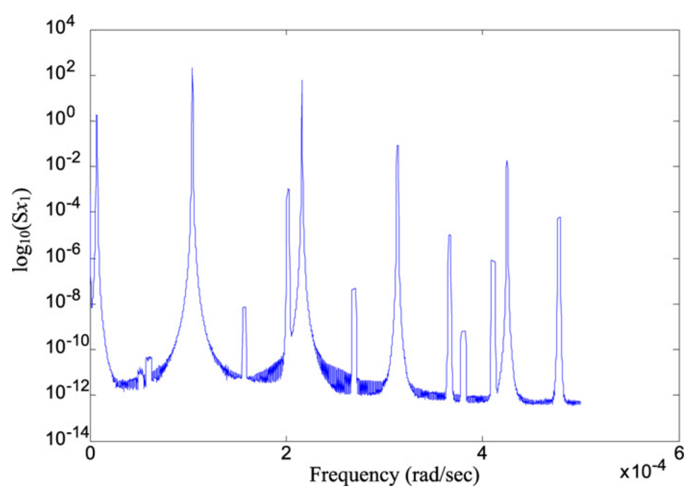

Fig. 2. Power spectrum of x_1 for Quantum-CNN, with $a_1 = 0.00009$, $a_2 = 0.00009$, $\omega_1 = 0.0001$, and $\omega_2 = 0.0001$.

In this Letter, we have shown that the autonomous continuous-time Quantum Cellular Neural Network (Quantum-CNN) system with four state variables as described by (1) can exhibit hyperchaos with three positive Lyapunov exponents.

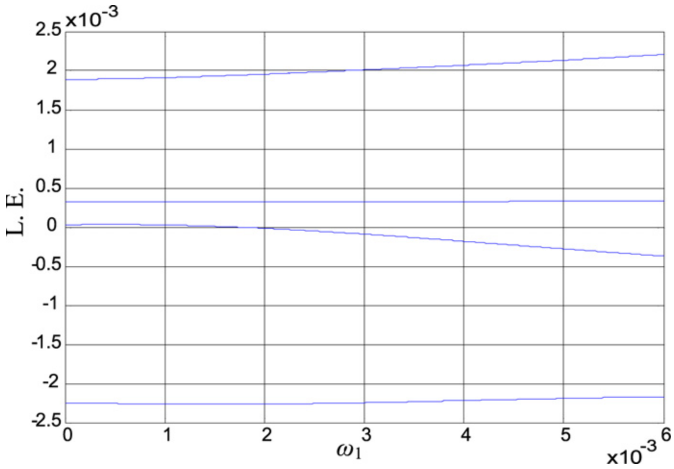

Fig. 3. Lyapunov exponents of Quantum-CNN, with $a_1 = 0.00003$, $a_2 = 0.00007$, and $\omega_2 = 0.0054$.

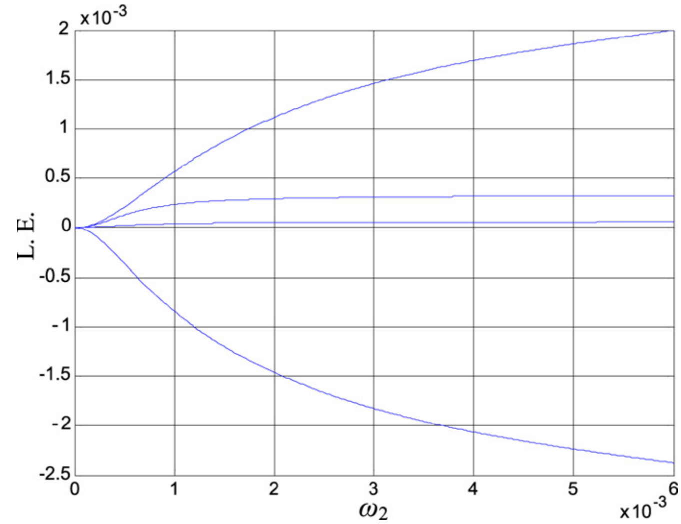

Fig. 6. Lyapunov exponents of Quantum-CNN, with $a_1 = 0.000049$, $a_2 = 0.00005$, and $\omega_1 = 0.00031$.

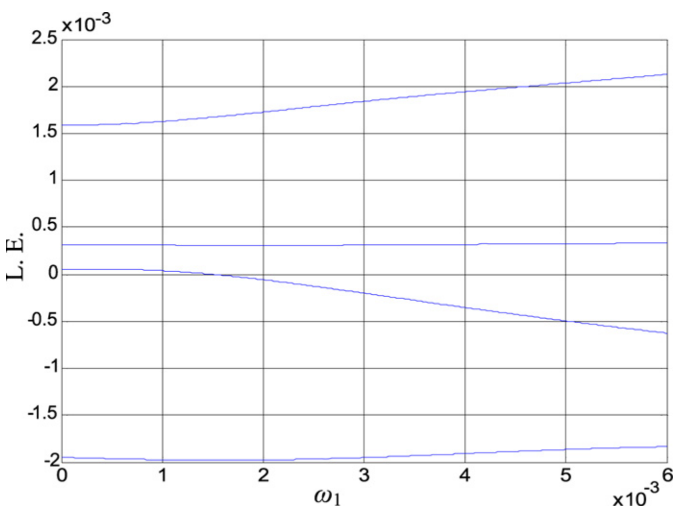

Fig. 4. Lyapunov exponents of Quantum-CNN, with $a_1 = 0.00005$, $a_2 = 0.00004$, and $\omega_2 = 0.0035$.

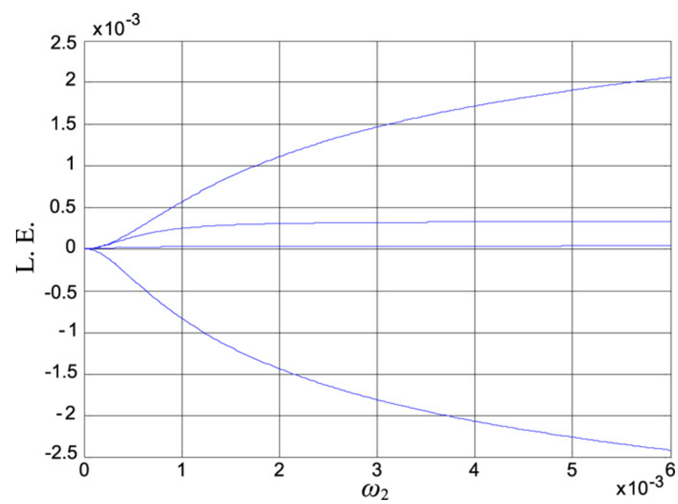

Fig. 7. Lyapunov exponents of Quantum-CNN, with $a_1 = 0.00003$, $a_2 = 0.000023$, and $\omega_1 = 0.00021$.

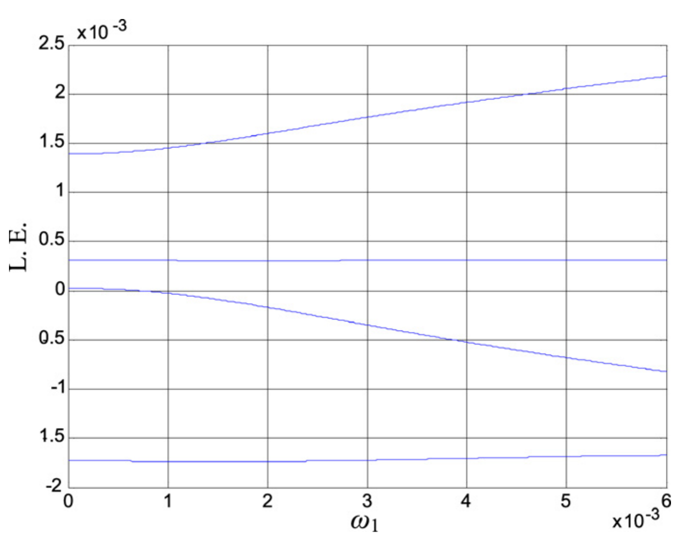

Fig. 5. Lyapunov exponents of Quantum-CNN, with $a_1 = 0.000023$, $a_2 = 0.00007$, and $\omega_2 = 0.0027$.

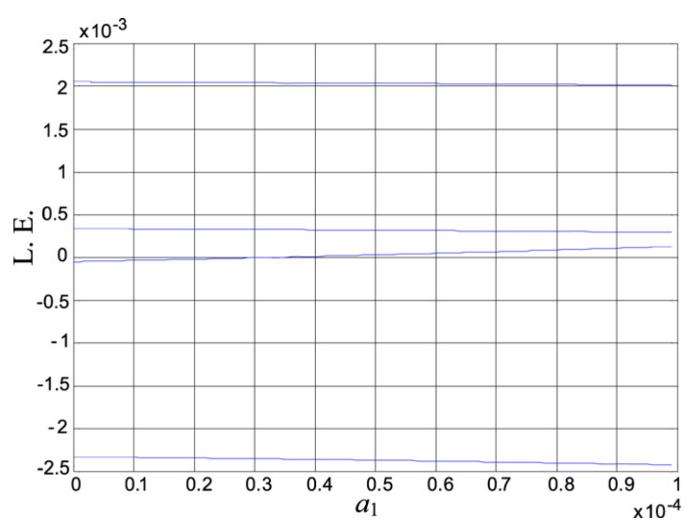

Fig. 8. Lyapunov exponents of Quantum-CNN, with $a_2 = 0.00006$, $\omega_1 = 0.0021$, and $\omega_2 = 0.006$.

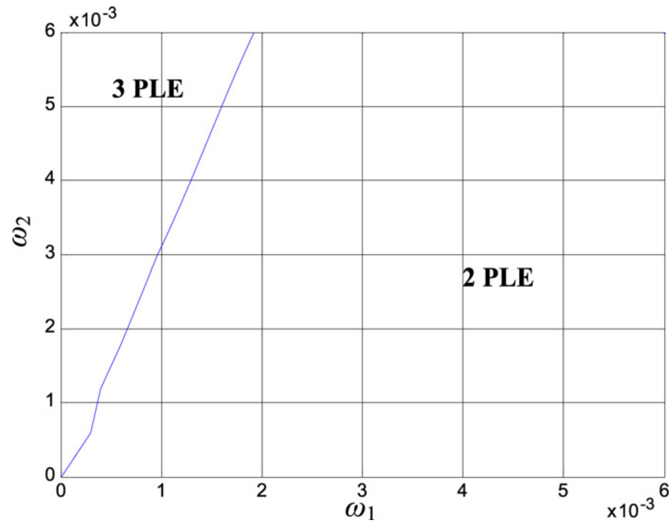

Fig. 9. Parameter diagrams of Quantum-CNN, with $a_1 = 0.00003$ and $a_2 = 0.00007$.

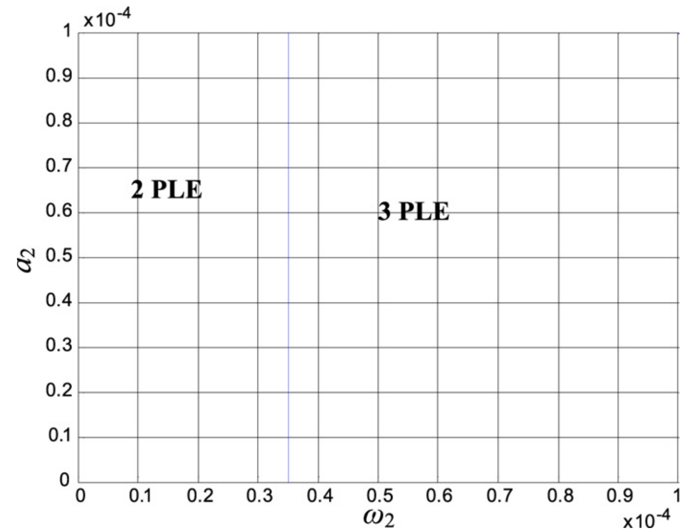

Fig. 12. Parameter diagrams of Quantum-CNN, with $a_1 = 0.000049$ and $\omega_1 = 0.00031$.

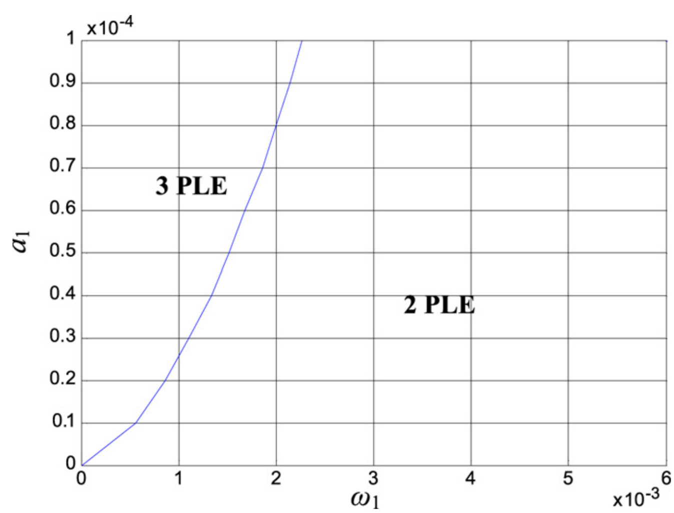

Fig. 10. Parameter diagrams of Quantum-CNN, with $a_2 = 0.00004$ and $\omega_2 = 0.0035$.

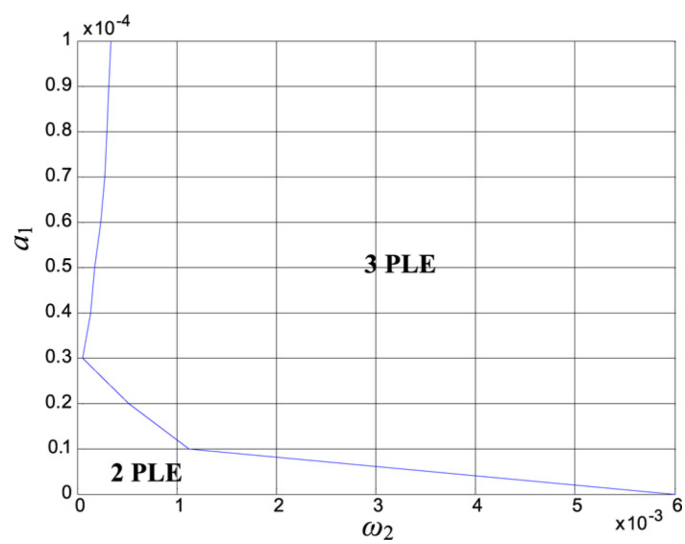

Fig. 13. Parameter diagrams of Quantum-CNN, with $a_2 = 0.000023$ and $\omega_1 = 0.00021$.

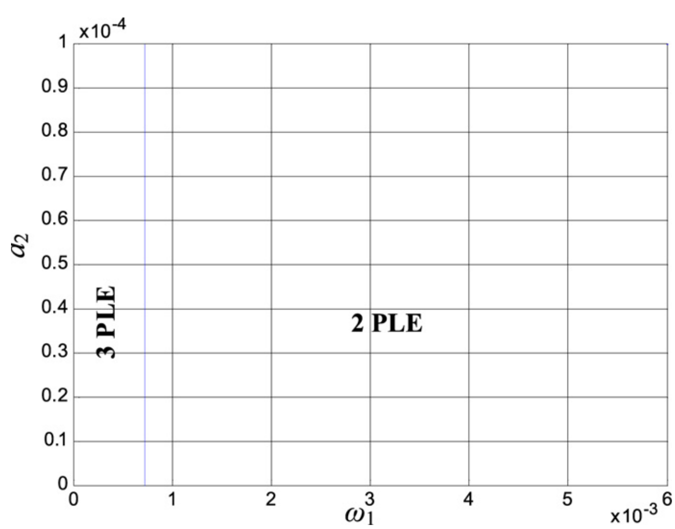

Fig. 11. Parameter diagrams of Quantum-CNN, with $a_1 = 0.000023$ and $\omega_2 = 0.00027$.

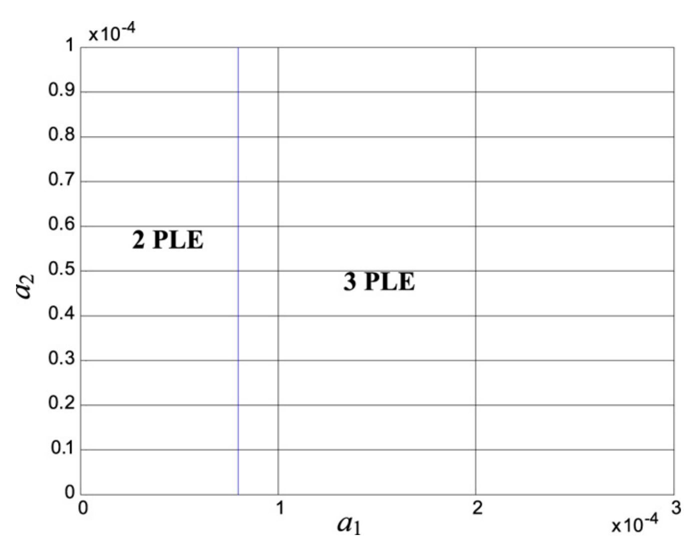

Fig. 14. Parameter diagrams of Quantum-CNN, with $\omega_1 = 0.0021$ and $\omega_2 = 0.006$.

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