國 立 交 通 大 學

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碩士論文

高效 **RSA** 密碼系統解密方法及實作

An Efficient Decryption Method for RSA Cryptosystem And Implementation

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摘　要

RSA密碼系統在電子商務與安全的網際網路存取等許多應用中是一種最有吸引力與歡迎的安全技巧。為了安全性的考量，RSA密碼系統必須在大的指數與模數下執行模指數運算，因此需要大量的計算成本。所以，在許多RSA的應用中，使用者會使用較小的公開金鑰來加快加密運算，相對的，在解密的運算還是需要大量的計算。本篇論文提出一有效率的解密實現方法，其架構在中國剩餘定理與RSA強質數的標準上。在TMS320C55x family of signal processors上實作，此新方法大約只須16%傳統解密方法的計算成本，與僅運用中國乘餘定理的解密法相比，大約只須55%的計算成本。換句話說，我們所提出的方法大約比運用中國乘餘定理的解密法快1.8倍。所以本方法非常適合加快RSA的解密運算。

An Efficient Decryption Method for RSA Cryptosystem And Implementation

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Abstract

In this thesis an efficient method to implement RSA decryption algorithm is proposed. In applications such as electronic commerce and internet access security, RSA cryptosystem is the most attractive and most popular security technique. For security reason, RSA cryptosystem has to execute modular exponentiation with large exponent and modulus. The RSA cryptosystem needs a very high computational cost. In many RSA applications, users use a small public key to speed up the encryption operation. However, the decryption operation has to take more computational cost to perform modular exponentiation by this case. In this thesis we propose an efficient decryption method not only based on Chinese Remainder Theorem (CRT) but also on the strong prime of RSA criterion. On the TMS320C55x family of signal processors, the proposed decryption method only needs 16% computational costs of the original decryption method. Compared with the computational cost of decryption method based on CRT, our proposed decryption method only needs 55% computational costs. Therefore, our proposed method is very useful in accelerating the speed of the RSA decryption operation.

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陳嘉耀

中 華 民 國　九 十 四 年 六 月

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Chapter 1 Introduction

With the rapid progress of modern information technology, security is an important technique of many applications. The RSA cryptosystem was proposed by R. Rivest, A. Shamir, L. Adleman in 1978 [2]. It is the most popular and well-defined security primary technique. RSA is a cryptosystem widely used to ensure data privacy in many fields such as communication and PKCS#1 standard lines out a way of encrypting data using the RSA cryptosystem [3]. Moreover, in digital signature and digital envelope, RSA provides non-repudiation and confidentiality of communication [3]. Actually, many good security protocols using RSA cryptosystem are applied in the modern information technology, for example, virtual private networks, electronic commerce, and secure Internet access.

RSA cryptosystem is easy to understand and implement. It is based on modular exponentiation. This modular exponentiation is performed by repeated modular multiplications. In general, the modular multiplication has to be performed a certain number of times to ensure security, but the consequence is that the RSA operation has to take much more computational cost for security consideration. In order to include RSA cryptosystem practically in many protocols, it is desired to devise faster encryption and decryption operations. Under this consideration, many hardware implementation methods have been proposed [4, 5, 6], in which high speed can be achieved but not flexibility.

In these applications, users usually select a small number such as 3, 17, or 65537 to be the public key to speed up the encryption operation [7].

However, by this way, the corresponding decryption operation costs more computational time because of the larger private key. Another choice is the Chinese Remainder Theorem (CRT). The decryption operation can be accelerated by applying the CRT [8, 9, 10] if the prime factors of modulus are known. It is reasonable that someone who holds the private key can get the prime factors of modulus. By means of the CRT, the speed for the RSA decryption operation could be 4 times faster [6]. In addition, Hayashi proposed a new modular exponentiation method [12] to improve the computational time of RSA. In his method, the modular exponentiation with the modulus n transforms into two substitute operations with factorable moduli n + 1 and n + 2. If moduli n + 1 and n + 2 can be factored, user can apply CRT to these modular operations modulo to n + 1 and n + 2 for each. The final result can be generated by Hayashi's formula. But this method is not very practical, especially when n is an odd number; it would be a difficult job to factor $n + 2$.

We propose an efficient method to implement RSA decryption operation in this thesis. This method is not only based on CRT but also on the strong prime of RSA criterion. The security of RSA is based on the difficulty of factoring problem. So, the prime factors of modulus of RSA algorithm must be strong primes. The large modular exponentiation result can be generated from small exponents and moduli. The proposed method enhances the performance of RSA algorithm.

The rest of this article is organized as follows: we will briefly review RSA algorithm in Chapter 2. In Chapter 3 we introduce our new decryption method. In Chapter 4 we analyze the computational complexity. In Chapter 5 we talk about the implementation of new decryption method. Finally, we make some conclusion in Chapter 6.

Chapter 2 RSA

RSA cryptosystem is a typical public-key cryptosystem. Although the cryptanalysis neither proved nor disproved RSA's security, it dose suggest confidence level in the algorithm. RSA gets its security from the difficulty of factoring large number. The public and private keys are functions of a pair of large prime numbers. Recovering the plaintext from the public key and the ciphertext is conjectured to be equal to factoring the product of the two primes.

2.1 Algorithm

RSA algorithm can be described briefly as follows:

Ⅰ **Key Generation:**

1. Choose two large strong primes, p and q.
2. Calculate $n = p \cdot q$.
3. Compute Euler value of n: $\Phi(n) = (p - 1)(q - 1)$
4. Find a random number e satisfying $1 < e < \Phi(n)$ and $\gcd(e, \Phi(n)) = 1$.
5. Compute a number d such that $d = e^{-1} \bmod \Phi(n)$

Public key = {e , n}

Private key = {d , n}

Ⅱ **Encryption:**

Plaintext:	m satisfying $m < n$,
Ciphertext:	$c = m^e \bmod n$

Ⅲ **Decryption:**

$$m = c^d \bmod n. \qquad \square$$

[Example 1]

Let p = 47 and q = 59, then n = pq = 2773 and (p-1)(q-1) = 2668. The value of e must be chosen somewhere between 1 and 2668. Assume e = 17. The value of d is 157.Assume further that the alphabet is represented by decimal values, i.e. a = 01, b = 02, c =03, etc. and a blank space is given the value 00. The plaintext is given as:

m = RSA CRYPTOSYSTEM

or in decimal representation by:

m = 1819 0100 0318 2516 2015 1925 1920 0513

The plaintext is enciphered by an individually encrypted message, which contains a block of four digits:

$m_1 = 1819$

$m_2 = 0100$

$m_3 = 0318$

$m_4 = 2516$

$m_5 = 2015$

$m_6 = 1925$

$m_7 = 1920$

$m_8 = 0513$

The first block is encrypted as:

$$1819^{17} \bmod 2773 = 0818$$

Performing the same operation on the subsequent blocks generates an encrypted message:

c = 0818 1952 0578 2666 0774 0246 2109 0772

Decrypting the message requires performing the same exponentiation using the decryption key of 157, so

$$0818^{157} \mathrm{mod} 2773 = 1819 = m_1$$

The rest of the plaintext can be recovered in this manner. □

2.2 The Security of RSA

There are three possible approaches to attack the RSA algorithm as follows: Brute force, Mathematical attacks, and time attacks.

- Brute force: This involves trying all possible private keys.
- Mathematical attacks: There are several approaches, all equivalent in effect to factoring the product of two primes.
- Timing attacks: These depend on the running time of the decryption algorithm.

2.2.1 Brute Force:

To defense the Brute-force attack, the approach for RSA cryptosystem is the same as for other cryptosystems — use a large key space. That mean, the larger the number of bits in *e* and *d*, the better. However, because the calculations involved, both in key generation and in encryption/decryption, are complex, the larger the key length, the slower the system operates.

2.2.2 The Factoring Problem

Three approaches to attacking RSA mathematically can be identified as follows:

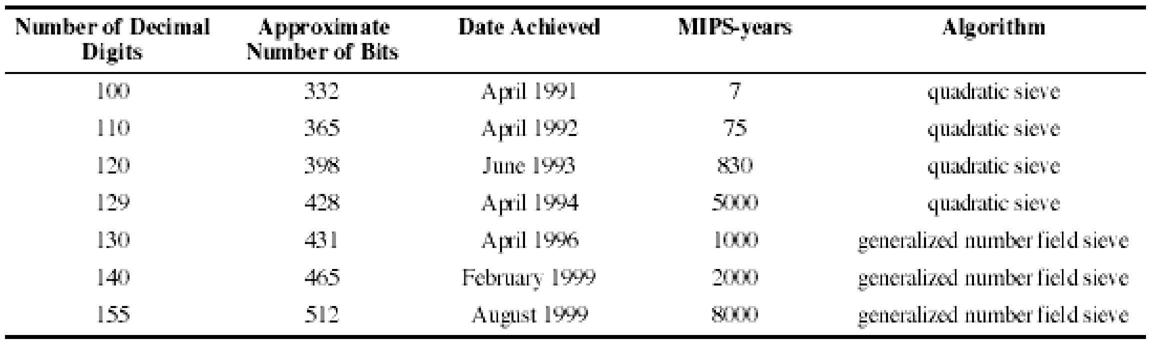
- Factor n into its two prime factors. This enables calculation of $\Phi(n) = (p - 1)(q - 1)$ and $d = e^{-1} \bmod \Phi(n)$.

- Direct Determine $\Phi(n)$. This enables determine of $d = e^{-1} \bmod \Phi(n)$.
- Direct Determine d.

Most discussions of the cryptanalysis of RSA have focused on the task of factoring n into two prime factors. Determine $\Phi(n)$ given n is equivalent to factoring n. With all known algorithms, determine d only given e and n, appears to be at least as time-consuming as the factoring problem.

To factor a large n with only two large prime factors is a hard problem, but not as hard as it used to be. With great computational capability, this problem can be solved in reasonable time. In table 1 shows the progress in factorization. The level of effort is measured in MIPS-years: a million-instructions-per-second processor running for

Table 1 Progress in Factorization [13]



Number of Decimal Digits	Approximate Number of Bits	Date Achieved	MIPS-years	Algorithm
100	332	April 1991	7	quadratic sieve
110	365	April 1992	75	quadratic sieve
120	398	June 1993	830	quadratic sieve
129	428	April 1994	5000	quadratic sieve
130	431	April 1996	1000	generalized number field sieve
140	465	February 1999	2000	generalized number field sieve
155	512	August 1999	8000	generalized number field sieve

one year, which is about 3×10^{13} instructions executed. A 200-MHz Pentium is about a 50-MIPS machine.

The threat to larger key sizes is twofold: the continuing increase in computing capability, and the continuing refinement of factoring algorithm. If a different algorithm is used, it can result in a tremendous speedup. It can expect further refinements in the generalized number field sieve, and the use of an even better algorithm is also a possibility. For example, a related algorithm, the special number field sieve, can factor number with a specialized from considerably faster than the generalized number field sieve. In Figure 1 we compar the performance of two algorithms. It is reasonable to

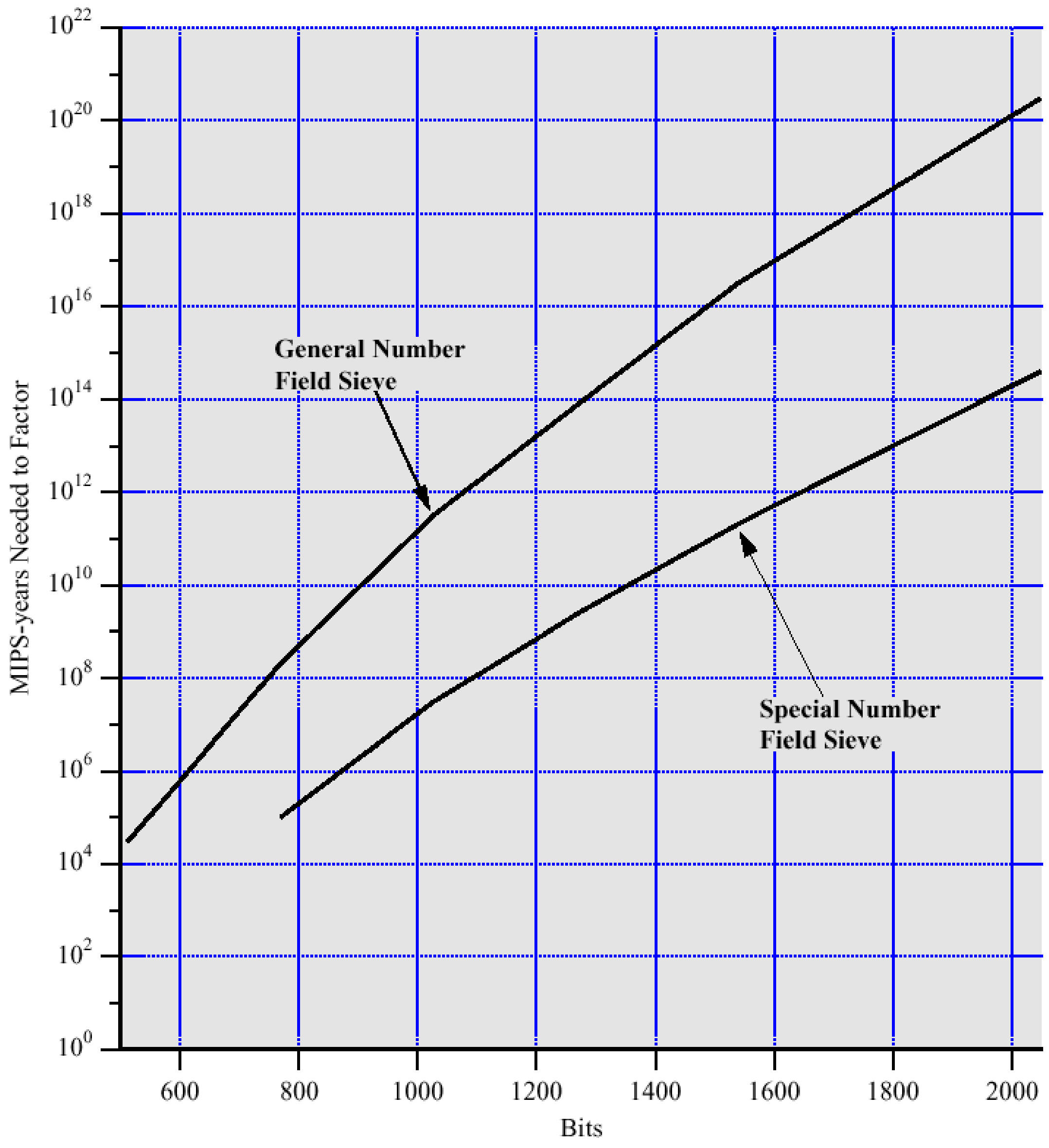


Figure 1 MIPS-years Needed to Factor [13]

expect a breakthrough that would enable a general factoring performance in about the same time as the special number field sieve, or even better. Thus, we need to be careful in choosing a key size for

RSA. In the near future, a key size in the range of 1024 to 2048 bits seems reasonable.

In some special case, there are some factoring algorithms, which can easy to factor *n* such like Pollard's *p - 1* algorithm.

- Pollard's *p - 1* algorithm

Pollard's *p - 1* algorithm [17] is efficient only if *n* has a prime factor *p* such that *p - 1* is smooth. The algorithm can be described as follows:

1. Select $a \in \mathbb{Z}/N\mathbb{Z}$ at random. Select a positive integer *k* that is divisible by many prime powers, for example, $k = \mathrm{lcm}(1,2,\cdots,B)$ for a suitable bound *B* (the larger *B* is the more likely the method will be to succeed in producing a factor, but the longer the method will take to work).
2. Compute $a_k = a^k \bmod N$.
3. Compute $d = \gcd(a_k - 1, N)$.
4. If $1 < d < N$, then d is a nontrivial factor of N, output d and go to step 6.
5. If *d* is not a nontrivial factor of N and still want to try more experiment, then go to step 2 to start all over again with a new *a* and/or a new *k*, else go to step 6.
6. Terminate the algorithm. □

The Pollard's *p - 1* algorithm is usually successful in the fortunate case where N has a prime divisor p for which $p - 1$ has no large prime factors. Suppose that $(p - 1) | k$ and that $p \nmid a$.

Since $|(\mathbb{Z}/p\mathbb{Z})^{*}| = p - 1$ and $a^{k} \equiv 1 \pmod{p}$, thus $p |$ gcd $(a_k - 1, N)$.

In many cases, $p =$ gcd $(a_k - 1, N)$.

[Example 2]

Input N = 540143, and B = 8

let $k = 840 = 2^3 * 3 * 5 * 7$, and a = 2

gcd $(2^{840} - 1 \bmod 540143, 540143)$

= gcd (53046 , 540143) = 421

421 is a factor of 540143. In fact, 540143 = 421*1283. □

The drawback of this algorithm is that it requires N to have a prime factor p such that $p - 1$ has only "small" prime factors. In RSA algorithm, it would be very easy to Factor n into its two prime factors, if $p - 1$ or $q - 1$ has only "small" prime factors. Note that the $p + 1$ algorithm, proposed by H. C. Williams in 1982, is an algorithm very similar to Pollard's $p - 1$ algorithm. It is efficient only if n has a prime factor p such that $p + 1$ is smooth.

2.2.3 Timing Attack

The main idea of timing attack is that a snooper can determine a private key by keeping track of how long a computer takes to decipher message [20]. Timing attacks are applicable not only to RSA, but also to other public-key cryptography system. This attack is alarming for two reasons: It comes from a completely unexpected direction and it is a ciphertext-only attack.

The attack assumes that the attacker knows the design of the target system, although practically this could probably be inferred from timing information. The attack can be tailored to work with virtually any implementation that does not run in fixed time, but is first outlined using the simple modular exponentiation algorithm below which computes $R = c^d \bmod n$, where d is *w* bits long:

```
Let $s_0 = 1$.
For $k = 0$ to $w - 1$:
    If (bit k of d) is 1 then
        Let $R_k = (s_k \bullet c) \bmod n$.
    Else
        Let $R_k = s_k$.
    Let $s_k+1 = R_k^2 \bmod n$.
    End.
Return ($R_{w-1}$).
```
□

The attack allows someone who knows exponent bits 0…(b-1) to find bit b. To obtain the total exponent, begin with b equal to 0 and repeat the attack until the total exponent is known.

Because the first b exponent bits are known, the attacker can compute the first b iterations of the "**For**" loop to find the value of s_b. The next iteration requires the first unknown exponent bit. If this bit is "**1**", $R_b = (s_b \bullet c) \bmod n$ will be computed. If it is "**0**", the operation will be skipped. The attack will be described first in an extreme hypothetical case. Suppose the target system uses a modular multiplication function that is normally extremely fast but occasionally takes much more time than an entire normal modular exponentiation. For a few s_b and c values the calculation of $R_b = (s_b \bullet$

c) mod n will be extremely slow, and by using knowledge about the target system's design the attacker can determine which these are. If the total modular exponentiation time is ever fast when $R_b = (s_b \bullet c)$ mod n is slow, exponent bit b must be zero. Conversely, if slow $R_b = (s_b \bullet c)$ mod n operations always result in slow total modular exponentiation times, the exponent bit is probably set. Once exponent bit b is known, the attacker can verify that the overall operation time is slow whenever $s_{b+1} = R_b^2$mod n is expected to be slow. The same set of timing measurements can then be reused to find the following exponent bits.

Although the timing attack is a serious threat, there are simple countermeasures that can be used, including the following:

- **Constant exponentiation time:** Ensure that all exponentiation takes the same amount of time before returning result. This is a simple fix but does degrade performance.
- **Random delay:** Better performance could be achieved by adding a random delay to the exponentiation algorithm to confuse the timing attack. If defenders do not add enough noise, attackers could still succeed by collecting additional measurements to compensate for the random delay.
- **Blinding:** Multiply the ciphertext by a random number before performing exponentiation. This processed prevents the attacker from knowing what ciphertext bits are being processed inside the computer and therefore prevents the bit-by-bit analysis essential to the timing attack.

2.3 Strong Prime

The security of RSA depends critically on the problem of factoring n into its prime factors p and q. Therefore it is important for the user to select primes p and q in such a way that the problem of factoring $n = p \times q$ is computationally infeasible for an adversary. The recommended way of maximizing the difficulty of factoring n is to choose p and q as strong primes. Ogiwara [21, 22, 23] defined that a prime p is said to be strong if p satisfies the following constraints:

1. $p - 1$ should contain a large prime factor p_1 so that $p - 1 = p_1 \times \alpha$, where $\gcd(p_1, \alpha) = 1$.
2. $p + 1$ should contain a large prime factor p_2 so that $p + 1 = p_2 \times \beta$, where $\gcd(p_2, \beta) = 1$.
3. $p_1 - 1$ has a large prime factor r_1 so that $p_1 - 1 = r_1 \times \chi$, where $\gcd(r_1, \chi) = 1$.
4. $p_1 + 1$ has a large prime factor s_1 so that $p_1 + 1 = s_1 \times \delta$, where $\gcd(s_1, \delta) = 1$.
5. $p_2 - 1$ has a large prime factor r_2 so that $p_2 - 1 = r_2 \times \varepsilon$, where $\gcd(r_2, \varepsilon) = 1$.
6. $p_2 + 1$ has a large prime factor s_2 so that $p_2 + 1 = s_2 \times \gamma$, where $\gcd(s_2, \gamma) = 1$.

These 'level-3 prime' numbers r_1, s_1, r_2 and s_2 can be easily found by a probabilistic primarily test. There are many methods have been proposed to generate strong primes for RSA [21, 24, 25, 26]. These methods first use level-3 primes to find 'level-2 primes'

p_1 or p_2. Similarly we use the level-2 primes to find the 'level-1 prime' p. In these methods, the bit length of the level-(i + 1) prime is about half of the bit length of the level-i prime. Figure 2 shows the strong prime structure. Another prime q also can be generated by the same method. The private key holder knows these secret values essentially, because he should select these secret values to generate his private key by himself.

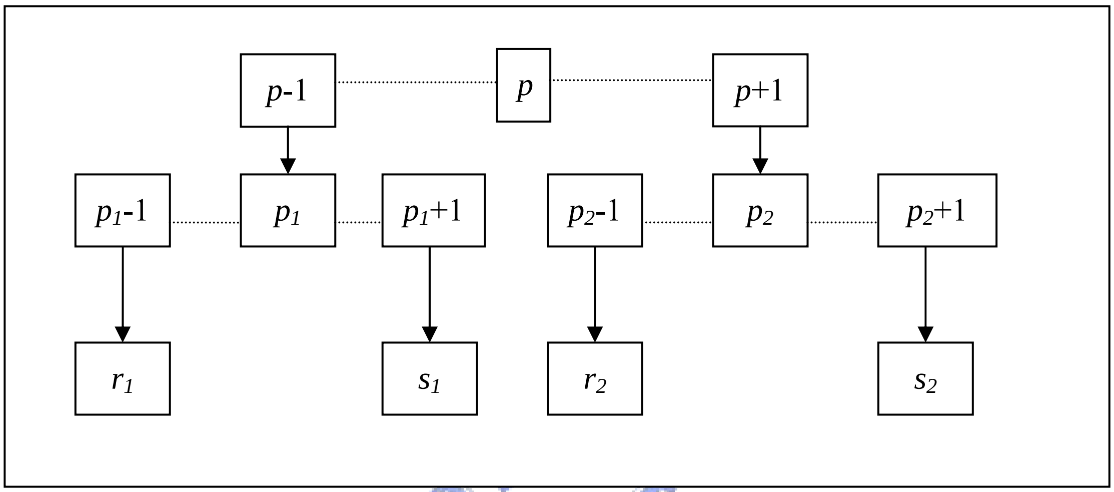


Figure 2 The Structure of Strong Prime

2.4 Chinese Remainder Theorem

The Chinese remainder Theorem (CRT) is a really method of solving certain system of congruence. Suppose $m_1, m_2, \ldots, m_r$ are pairwise relatively prime positive integers, and suppose $a_1, a_2, \ldots, a_r$ are integers. Then the system of r congruences

$$\begin{aligned} x &\equiv a_1 \bmod m_1 \\ x &\equiv a_2 \bmod m_2 \\ &\vdots \\ x &\equiv a_r \bmod m_r \end{aligned}$$

has a unique solution modulo $M = m_1 \times m_2 \times \ldots \times m_r$, which is given by

$$x \equiv \sum_{i=1}^{r} a_i M_i Y_i \bmod M$$

where $M_i = M/m_i$ and $y_i = M_i^{-1} \bmod m_i$, for $1 \le i \le r$.

[Example 3]

$$\begin{aligned} \text{Let } x &\equiv 2 \bmod 3 \\ x &\equiv 3 \bmod 5 \\ x &\equiv 2 \bmod 7 \end{aligned}$$

$$M = m_1 m_2 m_3 = 105$$

$$M_1 = M / m_1 = 35$$

$$y_1 = M_1^{-1} \bmod m_1 = 35^{-1} \bmod 3 = 2$$

$$M_2 = 21 \text{ and } y_2 = 1$$

$$M_3 = 15 \text{ and } y_3 = 1$$

$$\begin{aligned} x &= (a_1M_1y_1 + a_2M_2y_2 + a_3M_3y_3) \bmod M \\ &= (2*35*2 + 3*21*1 + 2*15*1) \bmod 105 \\ &= 23 \end{aligned}$$ □

The CRT is very useful in Cryptography and its applications very broad. When the factors of the modulus N (i.e., p and q) are assumed to be known, the RSA decryption operation can be speeded up by using the CRT [8, 9]. By using the CRT, the computation of $M = C^d \bmod N$ can be partitioned into two parts:

$$M_p = C_p^{dp} \bmod p,$$

$$M_q = C_q^{dq} \bmod q,$$

where

$$C_p = C \bmod p,\ d_p = d \bmod (p - 1),$$

$$C_q = C \bmod q,\ d_q = d \bmod (q - 1).$$

Finally, we use CRT to compute M as follows:

$$M = (M_p (q^{-1} \bmod p)) q + M_q (p^{-1} \bmod q)) p) \bmod N. \quad □$$

This reduces computation time since d_p, $d_q < d$ and C_p, $C_q < C$. In fact, their sizes are bout half the original sizes. In the ideal case we can have a speedup of about 4 times. In this case, d_p, d_q, $q^{-1} \bmod p$ and $p^{-1} \bmod q$ can be predict. The extra operations are $C_p = C \bmod p$, $C_q = C \bmod q$ and $M = M_p (q^{-1} \bmod p) q + M_q (p^{-1} \bmod q) p) \bmod N$. Compared with the calculation of M_p and M_q, the time spent on the extra operations is negligible. Therefore, in most cases, the speedup is close to 4 times.

Another RSA decryption algorithm based on CRT can be described as follows:

Compute:
$$a = c^d \bmod p.$$
$$b = c^d \bmod q.$$
$$u*q = 1 \bmod p$$

If $a \geq b \bmod p$, then

$$c^d \bmod n = (((a - (b \bmod p))*u) \bmod p)*q + b$$

If $a < b \bmod p$, then

$$c^d \bmod n = (((a + p - (b \bmod p))*u) \bmod p)*q + b \quad □$$

If a and b can be easily computed, this algorithm will be more efficient.

[Example 4]

From example 1, $m_1 = 1819$, $c_1 = 0818$, $m_3 = 0318$, $c_3 = 0578$, e = 17, d = 157, p = 47 and q = 59,

$$a_1 = c^d \bmod p = 818^{157} \bmod 47 = 33$$

$$b_1 = c^d \bmod q = 818^{157} \bmod 59 = 49$$

$$u = 59^{-1} \bmod 47 = 4$$

$$a_1 < b_1$$

$$m_1 = (((33 + 47 - (49 \bmod 47))*4) \bmod 47)*59 + 49 = 1819$$

$$a_2 = c^d \bmod p = 578^{157} \bmod 47 = 36$$

$$b_2 = c^d \bmod q = 578^{157} \bmod 59 = 23$$

$$a_2 \geq b_2$$

$$m_3 = (((36 - (23 \bmod 47))*4) \bmod 47)*59 + 23 = 0318$$

The result is the same with the result in example 1. □

Chapter 3 New Decryption Method

In this Chapter, we propose an efficient RSA decryption method based on strong prime criterion. If the highest concern is to ensure data security, the prime factors p and q of modulus n in RSA cryptosystem must be strong primes. It is reasonable that the private key holder knows the prime factors of $p-1$, $p+1$, $q-1$ and $q+1$. The proposed decryption method is based on the strong primes of RSA criterion and Chinese Remainder Theorem (CRT). The private key holder performs the decryption procedure: c^d mod n by our method as the following steps:

Step 1: Factor $p-1$, $p+1$, $q-1$, and $q+1$ to get their prime factors. We assume that moduli $p-1$, $p+1$, $q-1$, and $q+1$ be expressed as follows.

$$p-1 = 2 \cdot r_1 \cdot \ldots \cdot r_i,$$

$$p+1 = 2 \cdot s_1 \cdot \ldots \cdot s_j,$$

$$q-1 = 2 \cdot t_1 \cdot \ldots \cdot t_k,$$

$$q+1 = 2 \cdot u_1 \cdot \ldots \cdot u_l.$$

Step 2: Compute the modular exponentiation with prime factors of $p-1$ as the modulus. Then, apply the CRT to generate individually

$$y_1 = c^d \bmod (p-1),$$

$$y_2 = c^d \bmod (p+1),$$

$$y_3 = c^d \bmod (q-1),$$

$$y_4 = c^d \bmod (q+1),$$

The detail steps are as follows:

2.1 Compute $c_{(p-1),i} = c \bmod r_i^{\alpha_i}$, $i = 1, \ldots, h$.

2.2 Compute $d_{(p-1),i} = d \bmod \Phi(r_i^{\alpha_i})$, $i = 1, \ldots, h$.

2.3 Compute the modular exponentiation

$$m_{(p\text{-}1),i} = c_{(p\text{-}1),i}^{\,d_{(p\text{-}1),i}} \bmod r_i^{\alpha_i},\ i = 1, \ldots, h.$$

2.4 Apply the CRT to generate $y_1 = c^d \bmod (p - 1)$ based on $m_{(p\text{-}1),i}$, $i = 1, \ldots, h$.

Step 3: Compute

$$X_1 = 2^{-1}(y_1 + y_2 - z) \bmod p$$

where $z = 0$ if $y_1 \geq y_2$; otherwise $z = 1$,and

$$X_2 = 2^{-1}(y_3 + y_4 - z) \bmod q$$

where $z = 0$ if $y_3 \geq y_4$; otherwise $z = 1$.

Step 4: Apply the CRT to calculate the final result $m = c^d \bmod n$ based on X_1 and X_2.

Figure 3 shows the diagram of our decryption method.

The numbers $p - 1$, p + 1, q - 1 and q + 1 can be individually decomposed into three prime factors at least in step 1. The bit lengths of $r_i^{\alpha_i}$ ($i = 1, \ldots, h$) are shorter than p -1. The bit lengths of $d_{(p\text{-}1),\,i}$ ($i = 1, \ldots, h$) are shorter than d. In general, the complexity of modular exponentiation depends on the bit length of exponent and modulus. The total computation time of Step 2 to get $y_1 = c^d \bmod (p - 1)$ is shorter than compute y_1 by computing $c^d \bmod (p - 1)$ directly. The efficient results are similar to $y_2 = c^d \bmod (p + 1)$, $y_3 = c^d \bmod (q - 1)$ and $y_4 = c^d \bmod (q + 1)$. The proposed decryption method is more efficient than the original method. In chapter 4 we will prove these results in detail.

In Step 3, we use (y_1, y_2) to compute X_1, and (y_3, y_4) to compute X_2. Theorem 1 demonstrates that X_1 and X_2 are generated as Step 3 correctly.

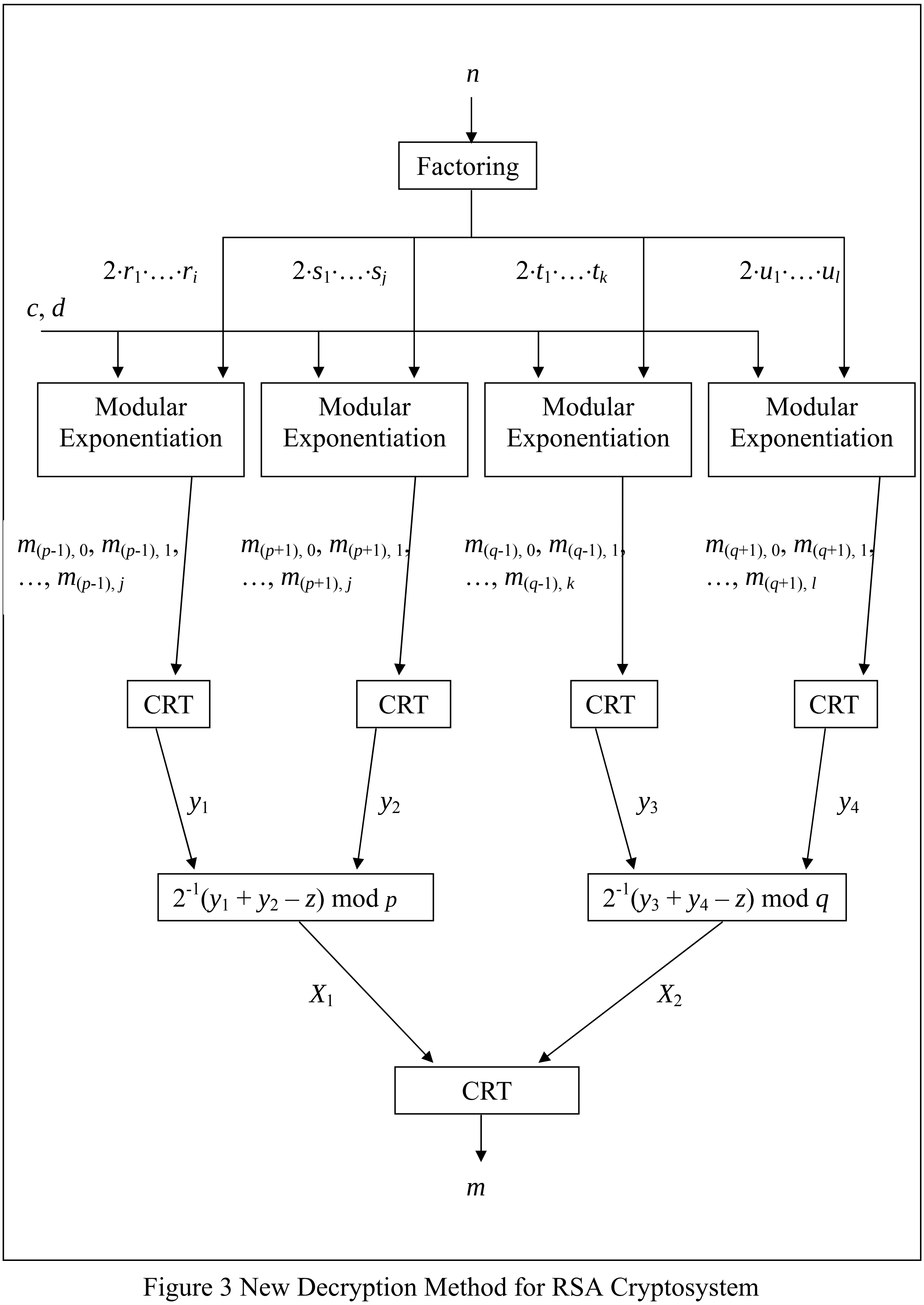

n
Factoring
$2 \cdot r_1 \cdot \ldots \cdot r_i$
$2 \cdot s_1 \cdot \ldots \cdot s_j$
$2 \cdot t_1 \cdot \ldots \cdot t_k$
$2 \cdot u_1 \cdot \ldots \cdot u_l$
c, d
Modular Exponentiation
Modular Exponentiation
Modular Exponentiation
Modular Exponentiation
$m_{(p-1),0}, m_{(p-1),1}, \ldots, m_{(p-1),j}$
$m_{(p+1),0}, m_{(p+1),1}, \ldots, m_{(p+1),j}$
$m_{(q-1),0}, m_{(q-1),1}, \ldots, m_{(q-1),k}$
$m_{(q+1),0}, m_{(q+1),1}, \ldots, m_{(q+1),l}$
CRT
CRT
CRT
CRT
y_1
y_2
y_3
y_4
$2^{-1}(y_1 + y_2 - z) \bmod p$
$2^{-1}(y_3 + y_4 - z) \bmod q$
X_1
X_2
CRT
m

Figure 3 New Decryption Method for RSA Cryptosystem

[Lemma 1]

Given

$$y_1 = \mathrm{X} \bmod (p-1),$$

$$y_2 = \mathrm{X} \bmod (p+1)$$

where $0 \le \mathrm{X} < (p^2 - 1)/2$, and p is a prime, then

$$\mathrm{X} = (p+1)y_1/2 - (p-1)y_2/2 + (p-1)(p+1)z/2$$

where $z = 1$ or 0.

Proof:

From $y_1 = X \bmod (p-1)$ and $y_2 = X \bmod (p+1)$, we have

$$y_1 + (p-1)z_1 = \mathrm{X} \qquad (1)$$

$$y_2 + (p+1)z_2 = \mathrm{X} \qquad (2)$$

z_1 and z_2 are two positive integers

Equation (1) $\times$ $(p+1)$:

$$(p+1)y_1 + (p+1)(p-1)z_1 = (p+1)\mathrm{X}. \qquad (3)$$

Equation (2) $\times$ $(p-1)$:

$$(p-1)y_2 + (p+1)(p-1)z_2 = (p-1)\mathrm{X}. \qquad (4)$$

Equation (3) – Equation (4):

$$2\mathrm{X} = (p+1)y_1 - (p-1)y_2 + (p-1)(p+1)(z_1 - z_2)$$

Let $z = z_1 - z_2$

$$\mathrm{X} = (p+1)y_1/2 - (p-1)y_2/2 + (p-1)(p+1)z/2 \qquad (5)$$

We will proof z is 0 or 1 in Equation (5) by contradiction as follows:

Case 1:

Assume that $z < 0$.Since $y_1 \le p - 2$ and $y_2 \ge 0$, it follows that

$$\mathrm{X} = (p+1)y_1/2 - (p-1)y_2/2 + (p-1)(p+1)z/2$$

$$\mathrm{X} \le (p+1)(p-2)/2 - (p-1)(p+1)/2$$

$$= -(p+1)/2$$

$$\mathrm{X} < 0.$$

This result contradicts the given condition $\mathrm{X} \ge 0$. Hence, z is not smaller than 0.

Case 2:

Assume that $z > 1$.Since $y_1 \geq 0$ and $y_2 \leq p$, it follows that

$$X = (p + 1)y_1/2 - (p - 1)y_2/2 + (p - 1)(p + 1)z/2$$

$$X \geq -(p - 1)p/2 + (p - 1)(p + 1)$$

$$= (p - 1)(p + 2)/2$$

$$X > (p - 1)(p + 1)/2.$$

This result contradicts the given condition $X < (p^2 - 1)/2$. Hence, z is not larger than 1.

By Cases 1, 2 and z_1, z_2 are integers, z must be either 0 or 1. Q.E.D.

[Theorem 1]

Given

$$y_1 = X \bmod (p - 1),$$

$$y_2 = X \bmod (p + 1)$$

where $0 \leq X < (p^2 - 1)/2$, and p is a prime, then

$$X = 2^{-1}(y_1 + y_2 - z) \bmod p$$

The value of z is

$$\text{if } y_1 \geq y_2 \text{ ; } z = 0,$$

$$\text{otherwise ; } z = 1.$$

Proof:

By Lemma 1,

$$X = (p + 1)y_1/2 - (p - 1)y_2/2 + (p - 1)(p+1)z/2,$$

where z is either 0 or 1.

We get

$$X = 2^{-1}(y_1 + y_2 - z) \bmod p, \qquad (6)$$

where z is either 0 or 1.

In addition, we will demonstrate the conditions of Equation (6) that z

= 0 or $z = 1$.

By Equation (2) – Equation (1)

$$y_2 - y_1 = (p-1)z_1 - (p+1)z_2 = (p-1)(z_1 - z_2) - 2z_2$$

where $y_1 \geq 0$, $y_2 \geq 0$, $p > 0$, both z_1 and z_2 are two positive integers.

Let $(z_1 - z_2) = z$.

$$y_2 - y_1 = (p-1)z - 2z_2 \tag{7}$$

By Equation (2), and $0 \leq X < (p^2 - 1)/2$, $0 \leq y_2 \leq p$, we get

$$0 \leq y_2 + (p+1)z_2 < (p^2 - 1)/2.$$

Hence, we get

$$0 \leq z_2 < (p-1)/2.$$

By Equation (7),

$$y_2 - y_1 > (p-1)z - (p-1) = (p-1)(z-1). \tag{8}$$

Case 1: $y_1 \geq y_2$

Since $y_1 \geq y_2$, from Equation(8)

$$0 \geq y_2 - y_1 > (p-1)(z-1)$$

$$0 > (p-1)(z-1)$$

and, $(p-1) > 0$,

$$0 > z - 1$$

$$1 > z$$

By Lemma 1, $z = 0$.

Case 2: $y_1 < y_2$

The value of z must be a positive integer. Thus $z = 1$.

To sum up, $z = 0$ if $y_1 \geq y_2$; otherwise $z = 1$. Q.E.D.

[Example 4]($y_1 < y_2$)

Let p = 19, X = 169

p + 1= 20 = 4*5

p – 1 = 18 =3*6

$$y_1 = 169 \bmod 18 = 7$$

$$y_2 = 169 \bmod 20 = 9$$

$$y_1 < y_2$$

$$y \equiv 2^{-1}(7 + 9 - 1) \bmod 19$$

$$\equiv 17 \bmod 19$$ □

[Example 5]($y_1 \geq y_2$)

$$\text{Let } p = 2773,\ X = 920^2$$

$$p+1 = 2774 = 38*73$$

$$p-1 = 2772 = 36*77$$

$$y_1 = 920^2 \bmod 2772 = 940$$

$$y_2 = 920^2 \bmod 2774 = 330$$

$$y_1 \geq y_2$$

$$y \equiv 2^{-1}(330 + 940) \bmod 2773$$

$$\equiv 635 \bmod 2773$$ □

Although we need to compute the multiplicative inverse of 2 modular *p* in Step 3, Theorem 2 provides an efficient method to compute the inverse value.

[Theorem 2]

Let *p* be a prime, the multiplicative inverse of 2 modulo *p* can be computed by $2^{-1} \bmod p = (p + 1)/2$.

Proof:

$$2 \times (p + 1)/2 = p + 1 \equiv 1 \bmod p.$$

The multiplicative inverse of 2 modulo *p* is equal to $(p + 1)/2$.   Q.E.D

Chapter 4 Computational complexity

In this chapter we will demonstrate our proposed decryption method, which is more efficient than the original decryption method and decryption method based on CRT. First, we will define some denotations as follows:

- $\text{MOD}_E(y, z)$ denotes an operation of modular exponentiation (x^y mod z).
- M(w), A(w) and Mod(w) denote operations of multiplication, addition and modulus with the bit length of operand is w.
- l(w) denotes lengths of w.
- S denotes an operation of shift.

By the additional chain method [27] the modulo operation c^d mod n can be expressed as:

$$\text{MOD}_E(d, n) = 1.5 \times \text{l}(d)[\text{M}(\text{l}(n) + 2\,\text{Mod}(\text{l}(n)) + 1). \quad (9)$$

The multiplication and addition operations can be expressed as follows [28]:

$$\text{M}(w) = 3M(w/2) + 5A(w) + 2S, \quad (10)$$

$$\text{A}(w) = w/32. \quad (11)$$

Also, the modular operation could be expressed as the following equations based on the divide and conquer concept [29]:

$$\text{Mod}(w) = \text{Mod}(w/2) + 4\text{M}(w/2) + 1.5\text{A}(w) + 3\text{S}. \quad (12)$$

Without loss of generality, we assume all of Mod(32), M(32), A(32) and S take one clock cycle. By the equations (10) and (11), we get

$$\begin{aligned} \text{M}(1024) &= 3\text{M}(512) + 5\text{A}(1024) + 2\text{S} \\ &= 3\text{M}(512) + 162 \\ &= 3[3\text{M}(256) + 5\text{A}(512) + 2\text{S}] + 162 \end{aligned}$$

$$\begin{aligned}
&= 9M(256) + 408\\
&= 9[3M(128) + 5A(256) + 2S] + 408\\
&= 27M(128) + 786\\
&= 27[3M(64) + 5A(128) + 2S] + 786\\
&= 81M(64) + 1380\\
&= 81[3M(32) + 5A(64) + 2S] + 1380\\
&= 243M(32) + 2352\\
&= 2595.
\end{aligned}$$

By the equations (10), (11) and (12), we get

$$\begin{aligned}
\mathrm{Mod}\,(1024) &= \mathrm{Mod}(512) + 4M(512) + 1.5A(1024) + 3S\\
&= [\mathrm{Mod}(256) + 4M(256) + 1.5A(512) + 3S] + 3295\\
&= [\mathrm{Mod}(128) + 4M(128) + 1.5A(256) + 3S] + 4294\\
&= [\mathrm{Mod}(64) + 4M(64) + 1.5A(128) + 3S] + 4577\\
&= [\mathrm{Mod}(32) + 4M(32) + 1.5A(64) + 3S] + 4646\\
&= 4657
\end{aligned}$$

We use the recursion down to the 32-bit level, then

$M(128) = 9M(32) + 50A(32) + 8S,$

$M(256) = 27M(32) + 190A(32) + 26S,$

$M(512) = 81M(32) + 650\,A(32) + 80\,S,$

$M(1024) = 243M(32) + 2110A(32) + 242S,$

$\mathrm{Mod}(128) = \mathrm{Mod}(32) + 16M(32) + 49A(32) +14S,$

$\mathrm{Mod}(256) = \mathrm{Mod}(32) + 52M(32) + 261A(32) +49S,$

$\mathrm{Mod}(512) = \mathrm{Mod}(32) +160M(32) +1045A(32) + 156S,$

$\mathrm{Mod}\,(1024) = \mathrm{Mod}\,(32) + 484M(32) + 3693A(32) + 479S.$

In order to ensure data security, the bit length of modulus should be 1024 at least. By Equation (9), the original decryption method can be repressed as

$$\begin{aligned}\mathrm{MOD}_E(d, n) &= 1.5 \times 1024[\mathrm{M}(1024) + 2\,\mathrm{Mod}(1024)+1].\\ &= 3072\mathrm{Mod}(32) + 1860096\mathrm{M}(32) + 14585856\mathrm{A}(32)\\ &\quad + 1843200\mathrm{S} + 1536.\\ &= 18293760\end{aligned}$$

That is to say, the original decryption method should take 18293760 clock cycles.

In the decryption method based on CRT, we assume that l(p) and l(q) are equal. Thus the length of the exponent is about n/2. The total number of operations of the decryption method based on original CRT is equal to:

$$2\mathrm{MOD}_E(d/2, n/2) + \mathrm{A}(3d/2) + 4\mathrm{M}(n/2) + 2\mathrm{Mod}(n/2) + \mathrm{Mod}(n),$$

that is:

$$\begin{aligned}\mathrm{MOD}_E(d, n) &= 2\{1.5\times 512[\mathrm{M}(512) + 2\,\mathrm{Mod}(512) + 1]\}\\ &\quad + \mathrm{A}(1536) + 4\mathrm{M}(512) + 2\mathrm{Mod}(512) + \mathrm{Mod}(1024)\\ &= \mathrm{Mod}(1024) + 1540\mathrm{M}(512) + 3074\mathrm{Mod}(512)\\ &\quad + \mathrm{A}(1536) + 1536\\ &= 2[243\mathrm{M}(32) + 2110\mathrm{A}(32) + 242\mathrm{S}]\\ &\quad + \mathrm{Mod}\,(32) + 484\mathrm{M}(32) + 3693\mathrm{A}(32) + 479\mathrm{S}\\ &\quad + 1540[81\mathrm{M}(32) + 650\,\mathrm{A}(32) + 80\mathrm{S}]\\ &\quad + 3074[\mathrm{Mod}(32) +160\mathrm{M}(32) +1045\mathrm{A}(32) + 156\mathrm{S}]\\ &\quad + 48\mathrm{A}(32) + 1536\\ &= 3075\mathrm{Mod}(32) + 617550\mathrm{M}(32) + 4221291\mathrm{A}(32)\\ &\quad + 603707\mathrm{S} + 1536\\ &= 5447159\end{aligned}$$

By this case, the decryption method takes 5447159 clock cycles.

In our proposed method, p - 1, p + 1, q - 1 and q + 1 can be factored into at least three numbers. Without loss of generality, we assume that bit length of the largest prime factor is about $l(n)/4$ and others are about $l(n)/8$ [21, 22, 24, 25]. The total number of operations of our proposed method is

$$4[2\mathrm{MOD}_E(d/4, n/4) + A(3d/4) + 4M(n/4) + 2\mathrm{Mod}(n/4) + \mathrm{Mod}(n/2)] + 2[2A(n/2) + M(n/2) + \mathrm{Mod}(n/2)] + 4A(n/2) + 2M(n/2) + \mathrm{Mod}(n/2)$$

$$\begin{aligned}\mathrm{MOD}_E(d, n) = {} & 4\{2\{1.5\times 256[M(256) + 2\,\mathrm{Mod}(256) + 1]\} + A(768) \\ & + 4M(256) + 2\mathrm{Mod}(256) + \mathrm{Mod}(512)\} \\ & + 2[2A(512) + M(512) + \mathrm{Mod}(512)] \\ & + 4A(512) + 2M(512) + \mathrm{Mod}(512) \\ & = 8A(512) +4M(512) + 7\mathrm{Mod}(512) \\ & + 3088M(256) + 6152\mathrm{Mod}(256) \\ & + 4A(192) + 3072 \\ & =6159\mathrm{Mod}(32) + 404724M(32) + 2202459A(32) \\ & + 383148S + 3072 \\ & =2995062\end{aligned}$$

It takes 2995062 clock cycles.

The original decryption method take 18293760 clock cycles. The decryption method based on CRT takes 5447159 clock cycles; however, our proposed method only takes 2995062 clock cycles. If we suppose the computational costs of the original decryption method is 100%, Compared with the original decryption method, the decryption method based on CRT takes 30% computational costs, but our proposed method takes only approximately 16%. The result will show in the table 2. The speed of our proposed method is almost 1.8 times faster than the decryption method based on CRT only.

Table 2 Comparison Among Three Types of Decryption Methods

	Clock Cycles	Computational Cost
Original Decryption Method	18293760	100%
Decryption Method Based on CRT	5447159	29.78%
New Decryption Method	2995062	16.37%

Chapter 5 Implementation

In order to evaluate the performance of our method, we examine these methods mentioned above based on Texas Instruments TMS320C55x family of signal processors. In table 3[34], we show the features of TMS320C55x. In addition, we assume the calculated CPU clock cycles of the realization of the RSA decryption with the following parameters as Table 4. The CPU clock cycles needed for processing the original RSA decryption method, decryption method based on the CRT, and our proposed method are given in Table 5, Table 6, and Table 7. The critical factor in influencing the speed of RAS decryption is the computation of modular multiplication. We apply the square and multiply algorithm to compute the modular exponentiation [27]. The square and multiply algorithm needs $3n/2$ modular multiplications for an n-bit exponent. The original RSA decryption method needs modular multiplication only one time. The decryption method based on the CRT needs modular-multiplication two times. Because there is a difference in the length of d and n in modular multiplication, the clock cycles needed by the decryption method based on the CRT are less and the speed of this method is faster than the speed of the original decryption method. However, as our proposed method needs modular multiplication four times and the length of d and n is much shorter, the clock cycles needed are much less and the speed is much faster.

As demonstrated in Table 6, due to the characteristics of chip TMS320C55x, the clock cycles needed by original decryption method is more than the clock cycles assessed in chapter four. Nevertheless, the

Table 3 The Features of TMS320C55x [32]

High-Performance, Low-Power, Fixed-Point TMS320C55x Digital Signal Processor (DSP)	– 6.25-/5-ns Instruction Cycle Time – 160-/200-MHz Clock Rate – One/Two Instructions Executed per Cycle – Dual Multipliers (Up to 400 Million Multiply-Accumulates Per Second (MMACS)) – Two Arithmetic/Logic Units – One Internal Program Bus – Three Internal Data/Operand Read Buses – Two Internal Data/Operand Write Buses
Instruction Cache	24K Bytes
160K x 16-Bit On-Chip RAM	– Eight Blocks of 4K × 16-Bit Dual-Access RAM (DARAM) (64K Bytes) – 32 Blocks of 4K × 16-Bit Single-Access RAM (SARAM)(256K Bytes)
16K × 16-Bit On-Chip ROM	32K Bytes
8M × 16-Bit Maximum Addressable External Memory Space	
32-Bit External Memory Interface (EMIF)	– Asynchronous Static RAM (SRAM) – Asynchronous EPROM – Synchronous DRAM (SDRAM) – Synchronous Burst SRAM (SBSRAM)
On-Chip Peripherals	– Two 20-Bit Timers – Six-Channel Direct Memory Access(DMA) Controller – Three Multichannel Buffered Serial Ports (McBSPs) – 16-Bit Parallel Enhanced Host-Port Interface (EHPI) – Programmable Digital Phase-Locked Loop (DPLL) Clock Generator – Eight General-Purpose I/O (GPIO) Pinsand Dedicated General-Purpose Output (XF)
On-Chip Scan-Based Emulation Logic	
IEEE Std 1149.1 (JTAG) Boundary ScanLogic	
3.3-V I/O Supply Voltage	
1.6-V Core Supply Voltage	

Table 4 Parameters of The RSA Decryption

RSA modulus *n*	2048bits	1024bits	512bits
RSA exponent length	2048bits	1024bits	512bits
message length	2048bits	1024bits	512bits
level-1 prime length	1024bits	512bits	256bits
level-2 prime length	512bits	256bits	128bits

Table 5 Numbers of CPU Clock Cycles for Realization RSA Decryption between Three Methods (Key Length = 2048 bits)

	Clock Cycles	Computational Cost
Original Decryption Method	269083092	100%
Decryption Method Based on CRT	55491364	20.62%
New Decryption Method	30634747	11.37%

Table 6 Numbers of CPU Clock Cycles for Realization RSA Decryption between Three Methods (Key Length = 1024 bits)

	Clock Cycles	Computational Cost
Original Decryption Method	35635338	100%
Decryption Method Based on CRT	9985827	28.02%
New Decryption Method	5506659	15.45%

Table 7 Numbers of CPU Clock Cycles for Realization RSA Decryption between Three Methods (Key Length = 512 bits)

	Clock Cycles	Computational Cost
Original Decryption Method	5145156	100%
Decryption Method Based on CRT	1633335	31.75%
New Decryption Method	897929	17.45%

computational cost needed by each kind of decryption method is almost the same with the computational cost that we assessed before. Comparing the results in Table 5, Table 6, and Table 7, we find that, with the increase of key length, the computational costs which can be saved by decryption method based on the CRT and by our new decryption method also increase. Let's take another example. In table 7, when the key length is 2048bits, the decryption method based on the CRT only needs 20% of the computational cost of original decryption method. Furthermore, our new decryption method only needs 11% of the computational cost of original decryption method.

The memory spaces needed for processing the original RSA decryption method, decryption method based on the CRT, and our proposed method are given in Table 8, Table 9, and Table 10. As demonstrated in Table 8, Table 9, and Table 10, our new decryption method and decryption method based on the CRT need more memory space respectively than original decryption method does. When key length is 1024bits, the memory space needed by decryption method based

Table 8 The Memory Spaces for Realization RSA Decryption between Three Methods (Key Length = 2048 bits)

	Memory Space	Increase Percentage
Original Decryption Method	11347bytes	100%
Decryption Method Based on CRT	12440bytes	109.63%
New Decryption Method	14704bytes	129.58%

Table 9 The Memory Spaces for Realization RSA Decryption between Three Methods (Key Length = 1024 bits)

	Memory Space	Increase Percentage
Original Decryption Method	6611bytes	100%
Decryption Method Based on CRT	7558bytes	114.23%
New Decryption Method	8985bytes	135.91%

Table 10 The Memory Spaces for Realization RSA Decryption between Three Methods (Key Length = 512 bits)

	Memory Space	Increase Percentage
Original Decryption Method	4243bytes	100%
Decryption Method Based on CRT	5048bytes	118.97%
New Decryption Method	6005bytes	141.53%

on the CRT is 14% more than the memory space needed by the original decryption method. The memory space needed by our proposed method is 36% more than the memory space needed by the original decryption method. Comparing the results in Table 8, Table 9, and Table 10, we find that, with the increase of key length, the memory space increased by decryption method based on the CRT and by our new decryption method is lessening respectively. For example, in Table 8, when key length is 2048bits, the memory space of decryption method based on the CRT increases 10% and the memory space of our proposed method only increases 30%.

As demonstrated by the results of implementation, when little amount of memory space is increased, the speed of RSA decryption operation can be greatly accelerated. Furthermore, when the key length is increased, the memory space needed to increase is reducing, but the speed of RSA decryption operation can be accelerated much significantly. Therefore, as it can be seen from the results of implementation, our new decryption method is more efficient than other two kinds of decryption method.

Chapter 6 Conclusion

In this thesis, we propose an efficient method to implement RSA decryption algorithm. This decryption method is not only based on CRT but also on the strong prime of RSA criterion. By our computational performance analysis, the 1024 bits RSA original decryption method without any tricks must require 18256896 clock cycles. The decryption method based on CRT takes 5447159 clock cycles. However, our proposed decryption method only takes 2995062 clock cycles. The complexity of our proposed method is only 16% of that of the original decryption method. Compared with decryption method based on CRT, our proposed method reduces approximately 45% computational costs. In a word, on the TMS320C55x family of signal processors, the speed of our proposed method is almost 1.8 times faster than the speed of the decryption method based on CRT. Our method can be applied not only decryption operation but also signing phase of digital signature. This efficient decryption method can enhance the performance of the RSA algorithm.

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